Adversarial Machine Learning

Machine Learning Course - CS-433

Nov 15, 2023

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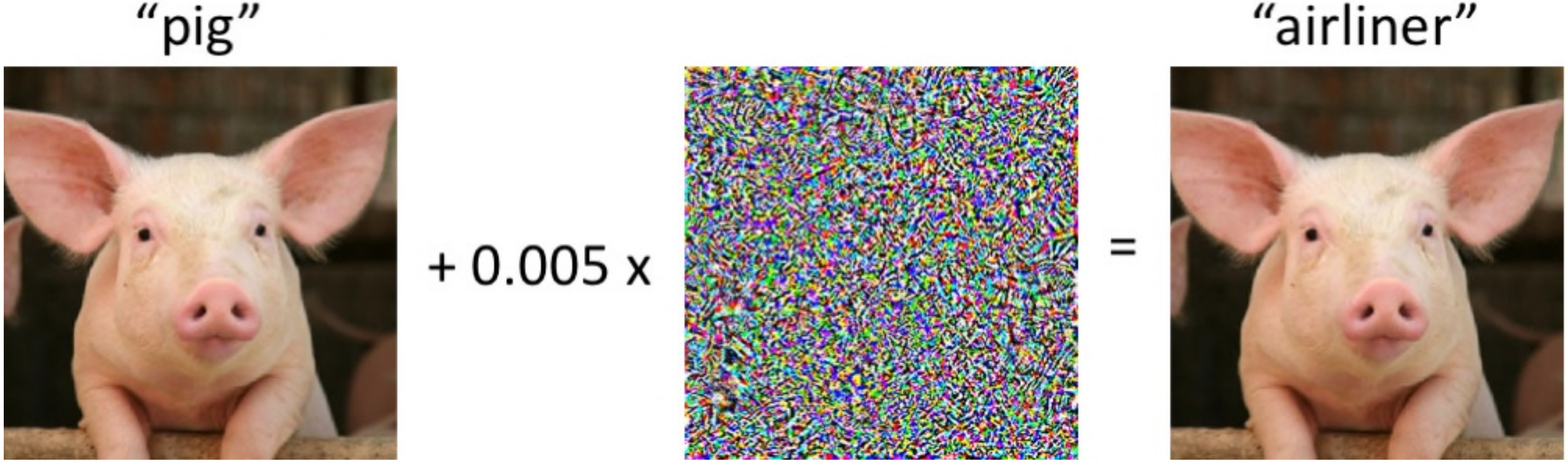
Some input examples are hard for humans

Dog or mop?

- Some examples may be challenging for humans
- NNs typically have **no problem** with them
- However, NNs are not always **robust** in their decisions

Adversarial examples are small perturbations that cause **misclassification with high confidence**

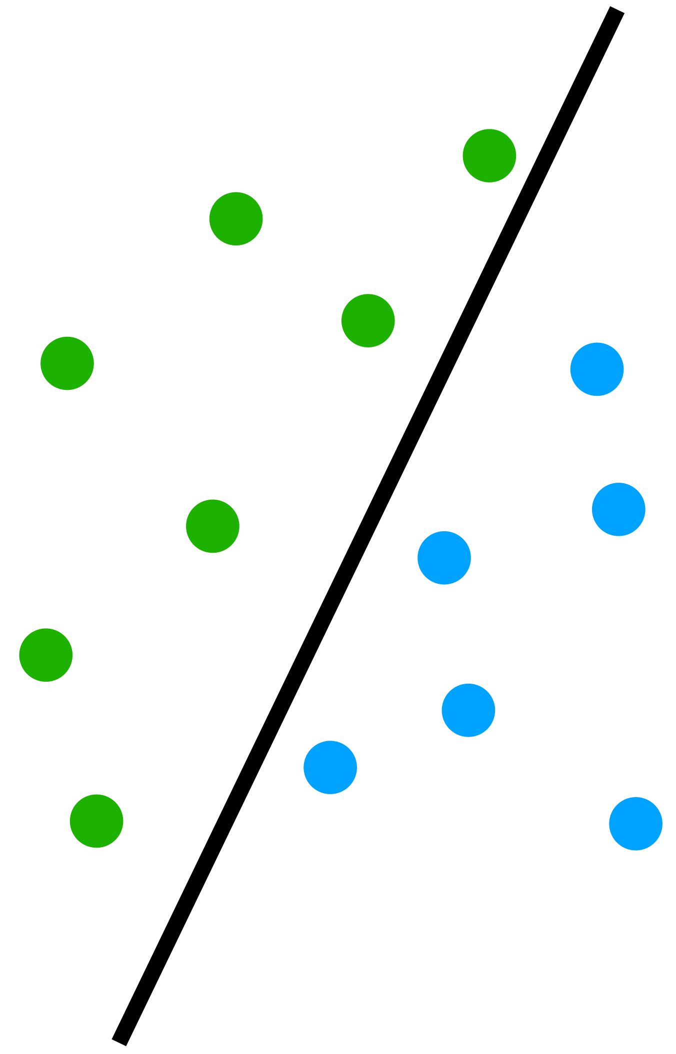


Source: Z. Kolter, A. Madry, NeurIPS'18 tutorial on adversarial robustness

NNs have struggle with imperceptible yet very specific inputs known as **adversarial examples**

- ➡ **Security issue**: consider the implications for a self-driving car and its ability to detect stop signs
- ➡ We don't understand how these models **generalize** and react to shifts in the distribution of data (i.e., **distribution shifts**)

Standard risk

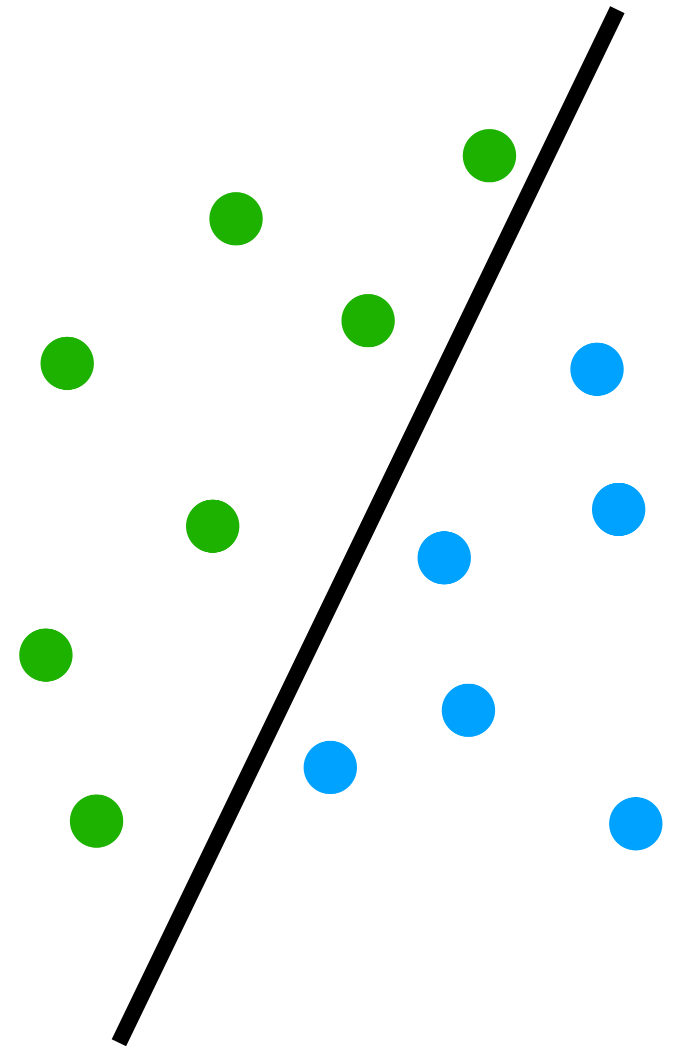


Classification problem: $(X, Y) \sim \mathcal{D}$, Y with range $\{-1,1\}$

Standard risk: average zero-one loss over X

$$R(f) = \mathbb{E}_{\mathcal{D}}\left[1_{f(X)\neq Y}\right] = \mathbb{P}_{\mathcal{D}}\left[f(X) \neq Y\right]$$

Standard risk vs. adversarial risk



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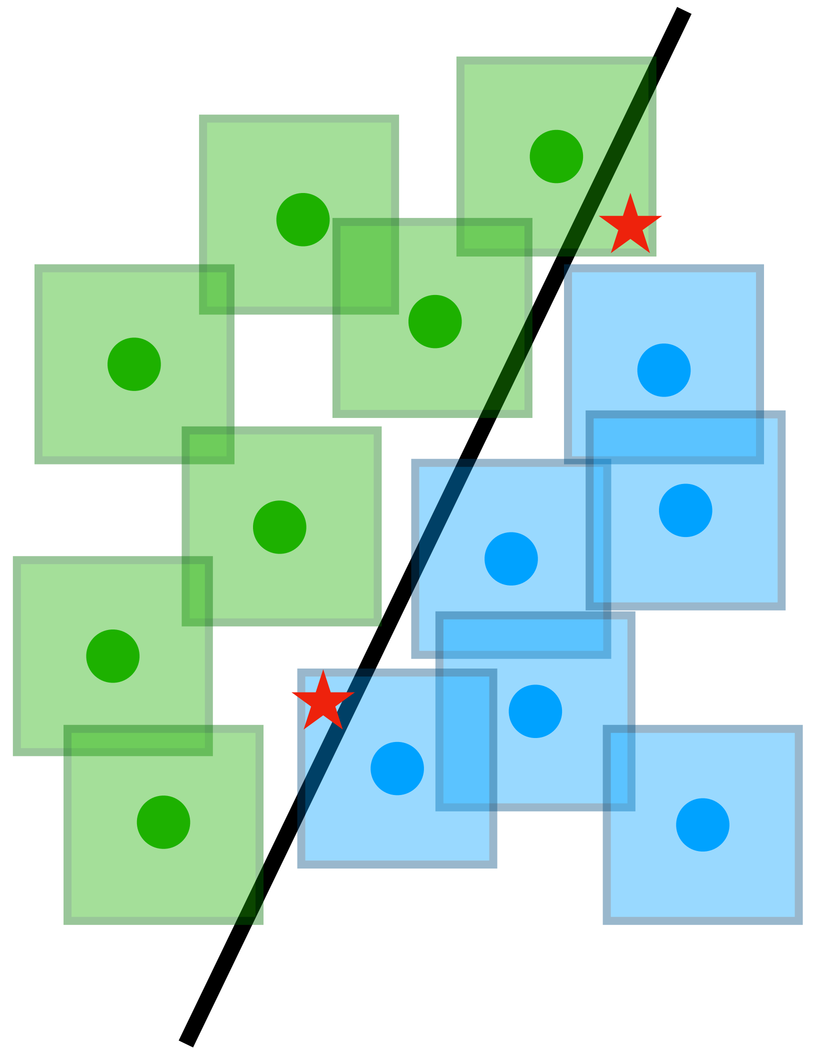
$$R(f) = \mathbb{E}_{\mathcal{D}}\left[1_{f(X)\neq Y}\right] = \mathbb{P}_{\mathcal{D}}\left[f(X) \neq Y\right]$$

Adversarial risk: average zero-one loss over **small, worst-case perturbations of** X

$$R_\varepsilon(f) = \mathbb{E}_{\mathcal{D}}\left[\max_{\hat{x},\|\hat{x}-X\|\leq\varepsilon} 1_{f(\hat{x})\neq Y}\right]$$



Adversarial vulnerability raises many questions

$$R_\varepsilon(f) = \mathbb{E}_{\mathscr{D}}\left[\max_{\hat{x}, \|\hat{x}-X\|\leq\varepsilon} 1_{f(\hat{x})\neq Y}\right]$$

- Threat model:
  - How should we define the adversary's power?
  - Which norm should we consider? $\ell_\infty, \ell_2, \ell_1, \ell_0, \ldots$
  - What set of perturbations?
- If $R(f) \leq \delta$, then how large can $R_\varepsilon(f)$ be?

Adversarial vulnerability raises many questions

- How can we compute an adversarial example?
- What level of access do we have to the model to attack it?
- How can we design a classifier f so that it is robust?
  Related: given a non-robust classifier, how can we make it robust?
- Why are neural networks non-robust?

Generating adversarial examples

<u>Task:</u> given an input (x, y) and a model $f : \mathcal{X} \to \{-1,1\}$, find an input $\hat{x}$, such that

(a) $\|\hat{x} - x\| \leq \varepsilon$

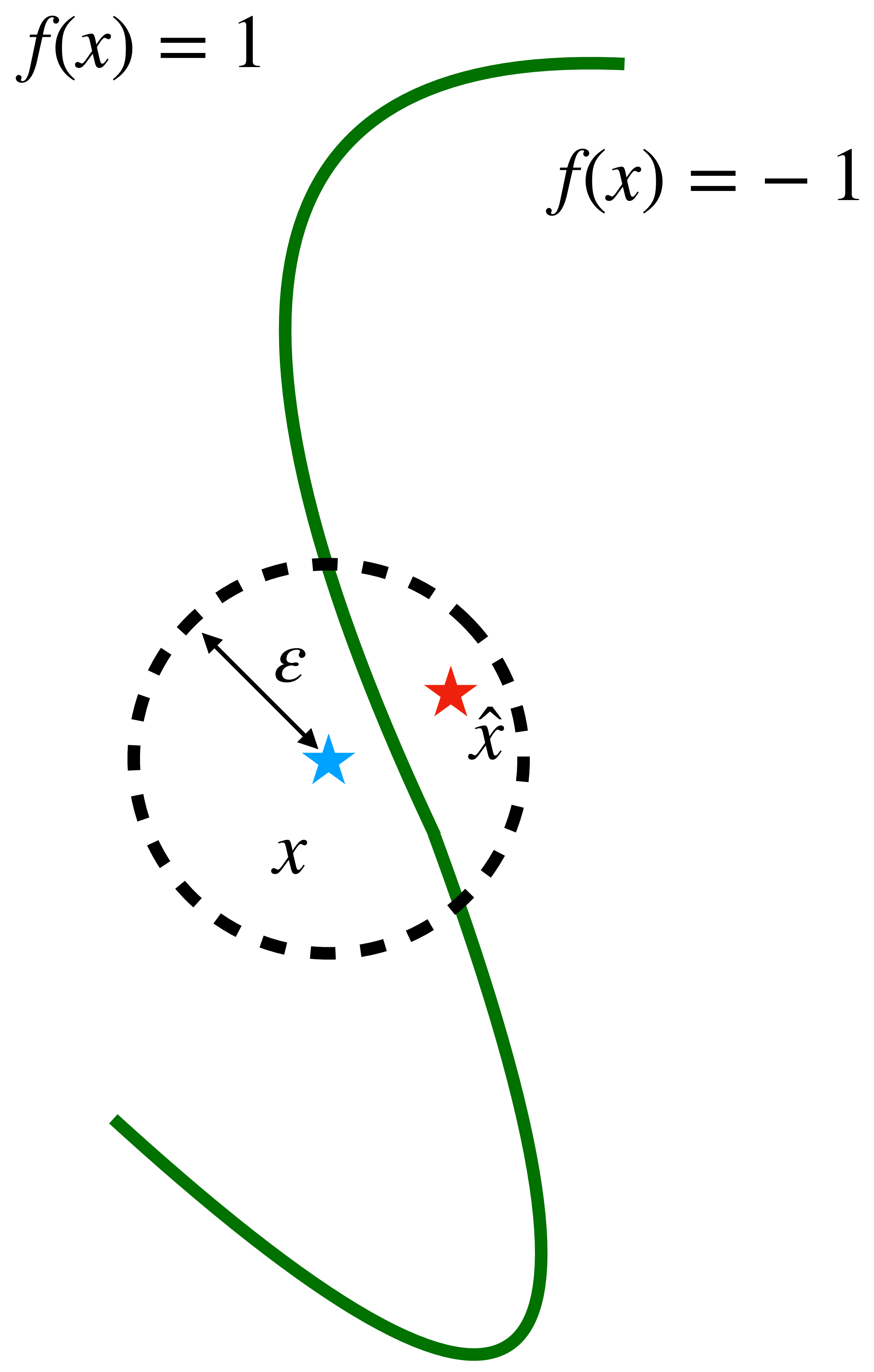
(b) the model f makes a mistake on it

Trivial case: x is already misclassified

➡ No action required

General case: x is correctly classified

➡ find $\hat{x}$ such that $f(\hat{x}) \neq y$ and $\|\hat{x} - x\| \leq \varepsilon$
i.e., $\hat{x} \in B_x(\varepsilon) \cap \{x', f(x') = -y\}$



Generating adversarial examples amounts to maximizing the classification loss w.r.t. the inputs

Find an adversarial example by solving

$$\max_{\hat{x}, \|\hat{x}-x\| \leq \varepsilon} 1_{f(\hat{x}) \neq y}$$

➡ Optimization problem with respect to the inputs

<u>Problem</u>**:** optimizing the indicator function $1_{f(\hat{x}) \neq y}$ is difficult:

1. The indicator function 1 is not continuous
2. The NN prediction f outputs discrete class values $\{-1, 1\}$

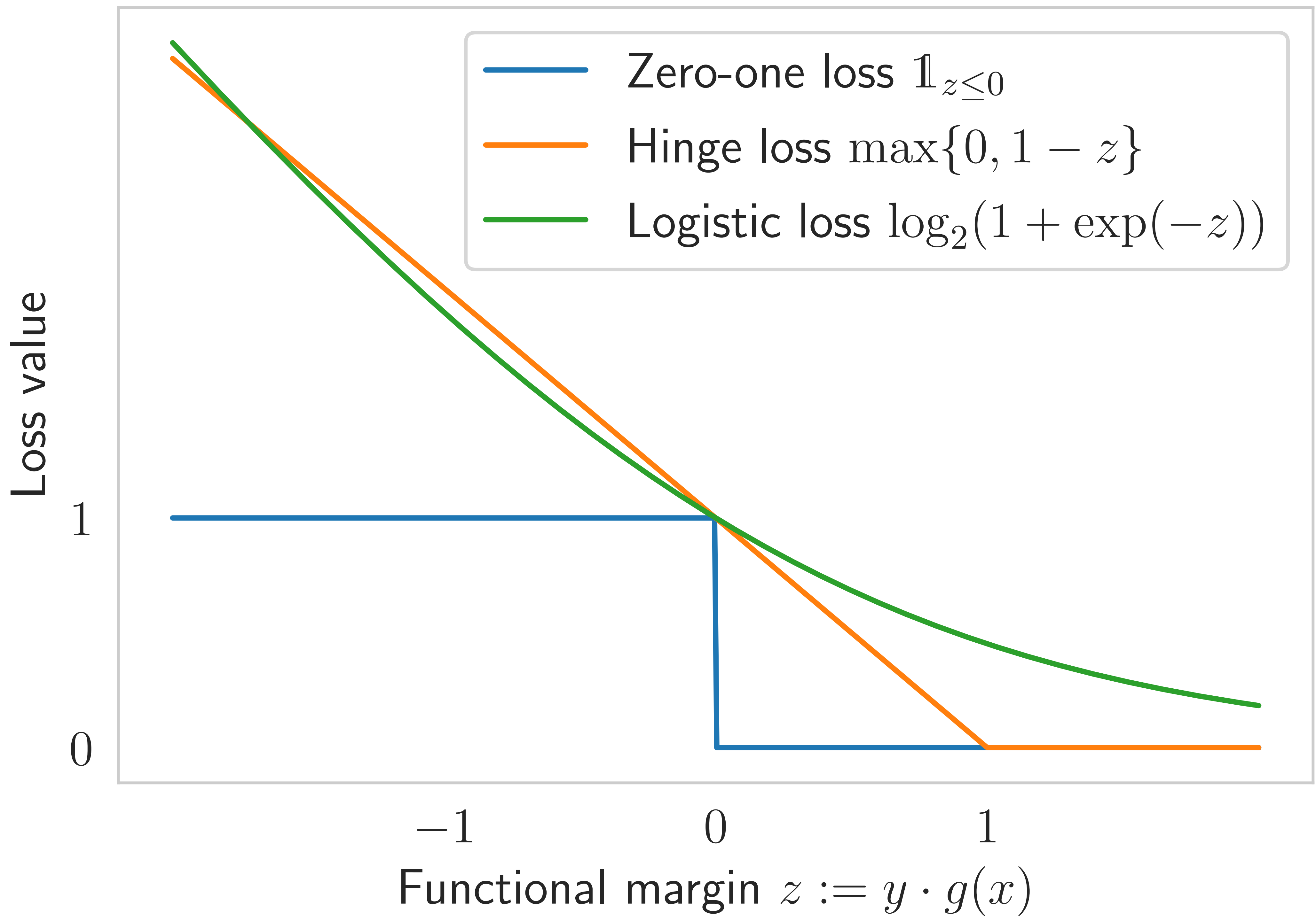
Generating adversarial examples amounts to solving a constrained optimization problem

Solution:

1. Use a smooth classification loss ℓ (e.g., logistic or hinge loss) instead
2. Consider the output g of the NN before classification (i.e., $f(x) = \text{sign}(g(x))$)

Main idea: Replace the difficult problem involving the indicator with a smooth problem

$$\max_{\hat{x}, \|\hat{x}-x\| \leq \varepsilon} 1_{f(\hat{x}) \neq y} \quad \longrightarrow \quad \max_{\hat{x}, \|\hat{x}-x\| \leq \varepsilon} \ell(yg(\hat{x}))$$



Reminder: decreasing, margin-based (i.e., dependent on $y \cdot g(x)$) classification losses

Generating adversarial examples: white-box case

How to solve $\max_{\hat{x}, \|\hat{x}-x\| \leq \varepsilon} \ell(yg(\hat{x}))$ in the **white-box** case, i.e., if we know the model g?

Compute its gradient: $\nabla_x \ell(yg(x)) = y \underbrace{\ell'(yg(x))}_{\leq 0 \text{ since classification loss are decreasing}} \nabla_x g(x)$

We should move in the direction $\propto -y \nabla_x g(x)$

<u>Interpretation:</u> $f(x) = \text{sign}(g(x))$

- If $y = 1$, we want to decrease $g(x)$ and follow $-\nabla_x g(x)$
- If $y = -1$, we want to increase $g(x)$ and follow $\nabla_x g(x)$

⚠ Why use ℓ, and not directly minimize $yg(\hat{x})$?

→ It won't extend to multi-class classification and robust training.

Generating adversarial examples: taking into account the constraints

We can linearize the loss $\tilde{\ell}(x) := \ell\left(yg(x)\right)$ to derive an iteration:

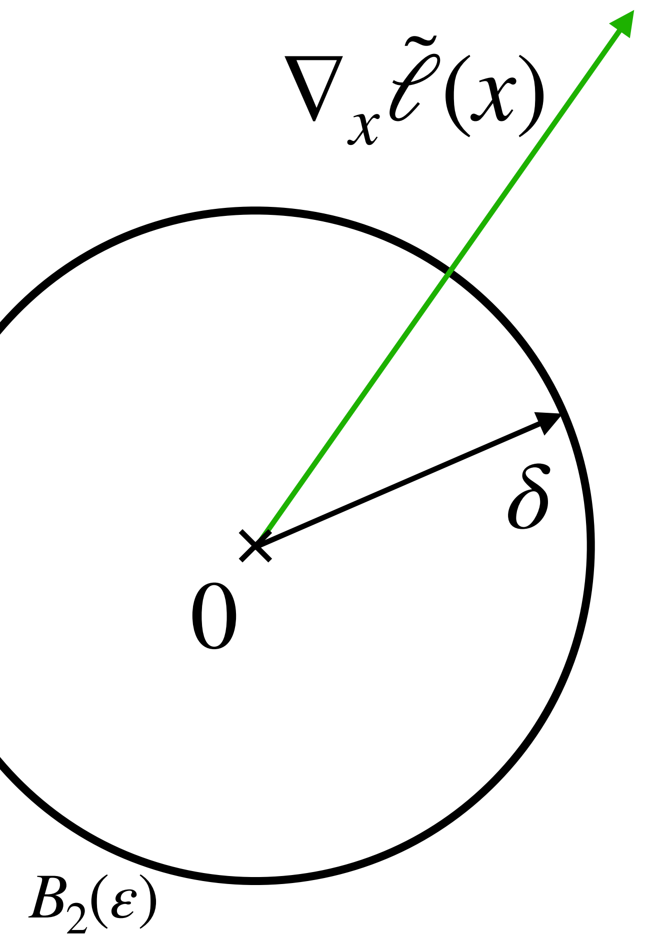
$$
\begin{aligned}
\max_{\|\hat{x}-x\|\leq\varepsilon} \tilde{\ell}(\hat{x}) &\approx \max_{\|\hat{x}-x\|\leq\varepsilon} \tilde{\ell}(x) + \nabla_x \tilde{\ell}(x)^T(\hat{x}-x) \\
&= \tilde{\ell}(x) + \max_{\|\hat{x}-x\|\leq\varepsilon} \nabla_x \tilde{\ell}(x)^T(\hat{x}-x) \\
&= \tilde{\ell}(x) + \max_{\|\delta\|\leq\varepsilon} \nabla_x \tilde{\ell}(x)^T \delta
\end{aligned}
$$

- We need to maximize the inner product under a norm constraint, i.e. find the optimal local update
- This is a simple problem for which we can get a closed-form solution depending on the norm used to measure the perturbation size $\|\delta\|$

Generating adversarial examples: one-step attack

Problem:

$$\max_{\|\delta\| \leq \varepsilon} \nabla_x \tilde{\ell}(x)^T \delta$$



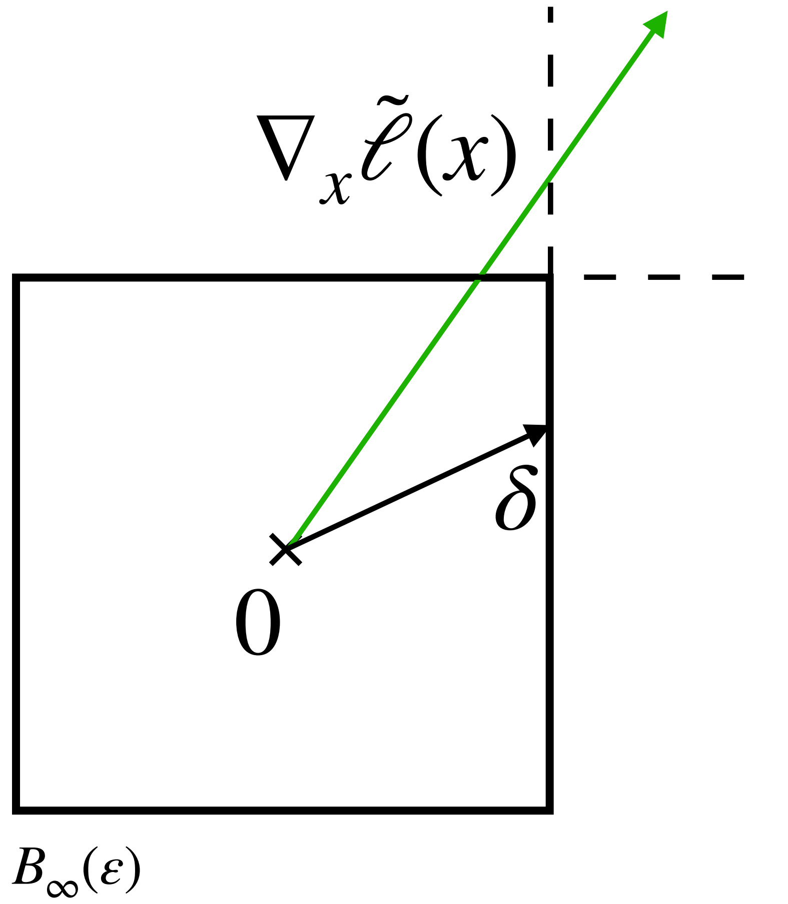
- Solution for the ℓ_2 norm: $\delta_2^\star = \varepsilon \cdot \frac{\nabla_x \tilde{\ell}(x)}{\|\nabla_x \tilde{\ell}(x)\|_2} = -\varepsilon y \cdot \frac{\nabla_x g(x)}{\|\nabla_x g(x)\|_2}$

  ➡ $\hat{x} = x - \varepsilon y \cdot \frac{\nabla_x g(x)}{\|\nabla_x g(x)\|_2}$

- Solution for the ℓ_∞ norm: $\delta_\infty^\star = \varepsilon \cdot \text{sign}(\nabla_x \tilde{\ell}(x)) = -\varepsilon y \cdot \text{sign}(\nabla_x g(x))$

  ➡ $\hat{x} = x - \varepsilon y \cdot \text{sign}(\nabla_x g(x))$

  ➡ **Fast Gradient Sign Method** [Goodfellow et al., 2014]



Generating adversarial examples: multi-step attack

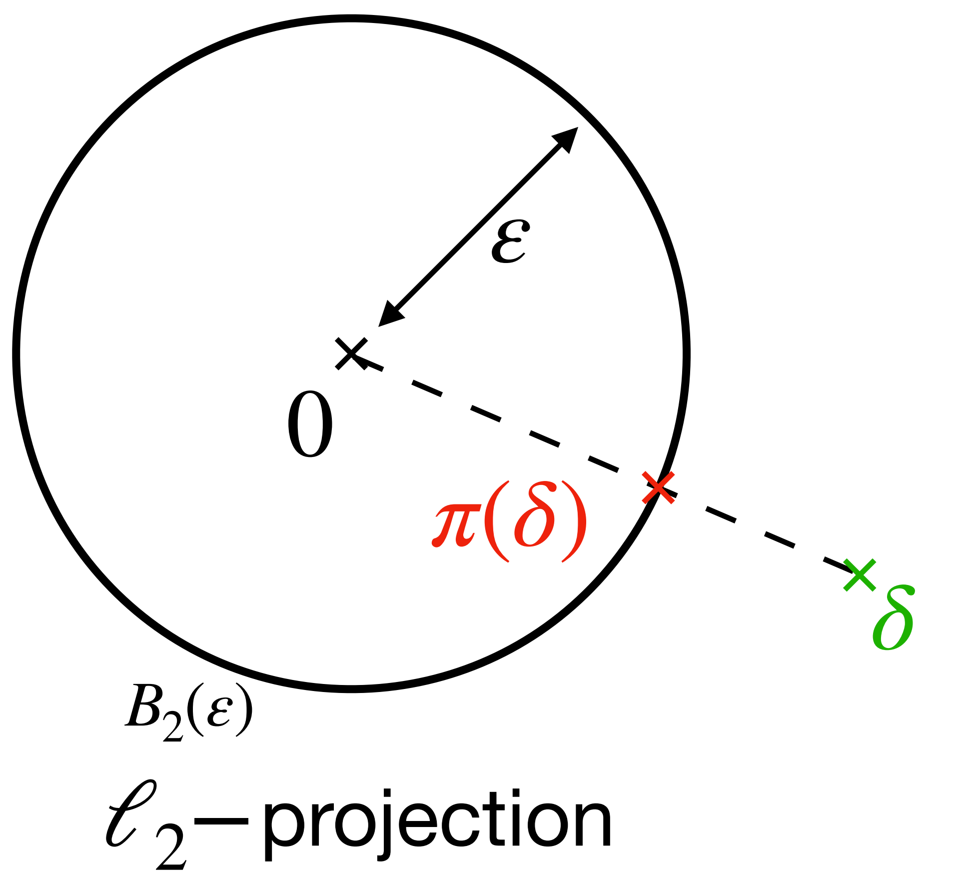
These updates can be done iteratively and combined with a projection Π on the feasible set (i.e., ℓ_2/ℓ_∞ balls here)

Projected Gradient Descent (PGD attack):

- ℓ_2 norm:

$$\delta^{t+1} = \Pi_{B_2(\varepsilon)}\left[\delta^t + \alpha \cdot \frac{\nabla \tilde{\ell}(x+\delta^t)}{\|\nabla \tilde{\ell}(x+\delta^t)\|_2}\right],$$

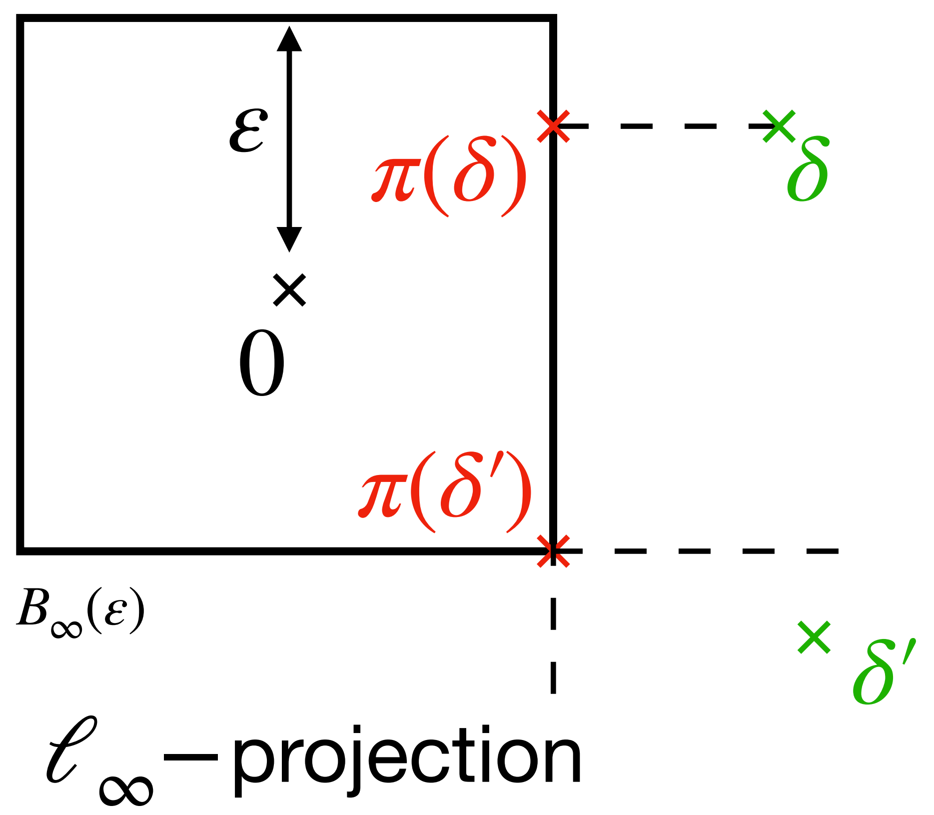
$$\text{where } \Pi_{B_2(\varepsilon)}(\delta) = \begin{cases} \varepsilon \cdot \delta/\|\delta\|_2, & \text{if } \|\delta\|_2 \geq \varepsilon \\ \delta, & \text{otherwise} \end{cases}$$



- ℓ_∞ norm:

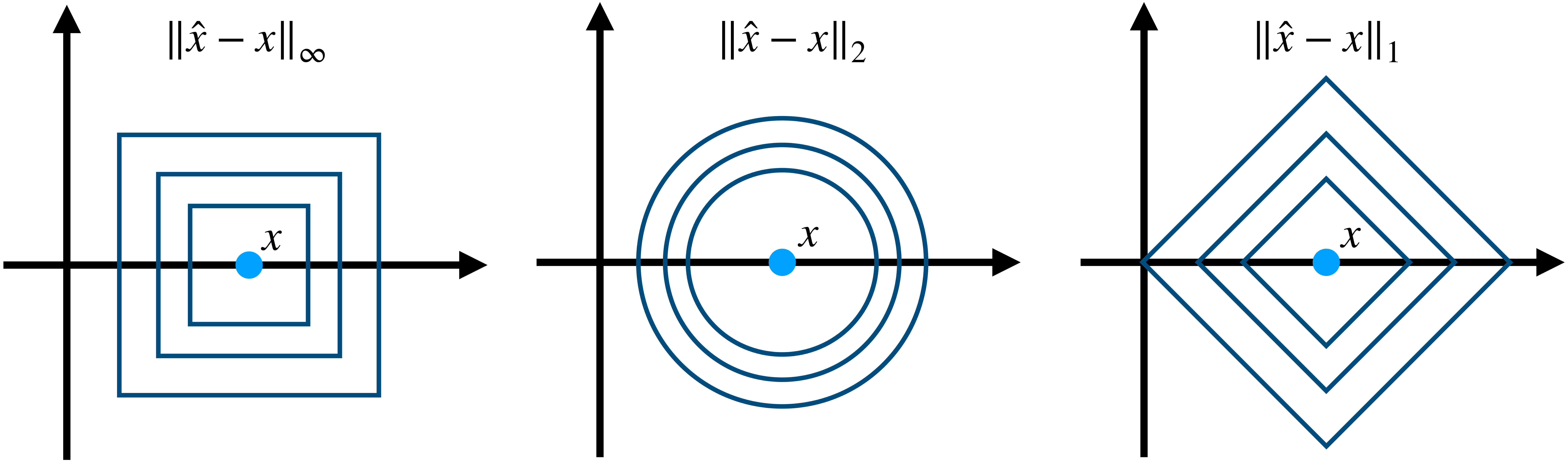
$$\delta^{t+1} = \Pi_{B_\infty(\varepsilon)}\left[\delta^t + \alpha \cdot \text{sign}(\nabla \tilde{\ell}(x+\delta^t))\right],$$

$$\text{where } \Pi_{B_\infty(\varepsilon)}(\delta)_i = \begin{cases} \varepsilon \cdot \text{sign}(\delta_i), & \text{if } |\delta_i| \geq \varepsilon \\ \delta_i, & \text{otherwise} \end{cases}$$



Reminder: ℓ_p norms

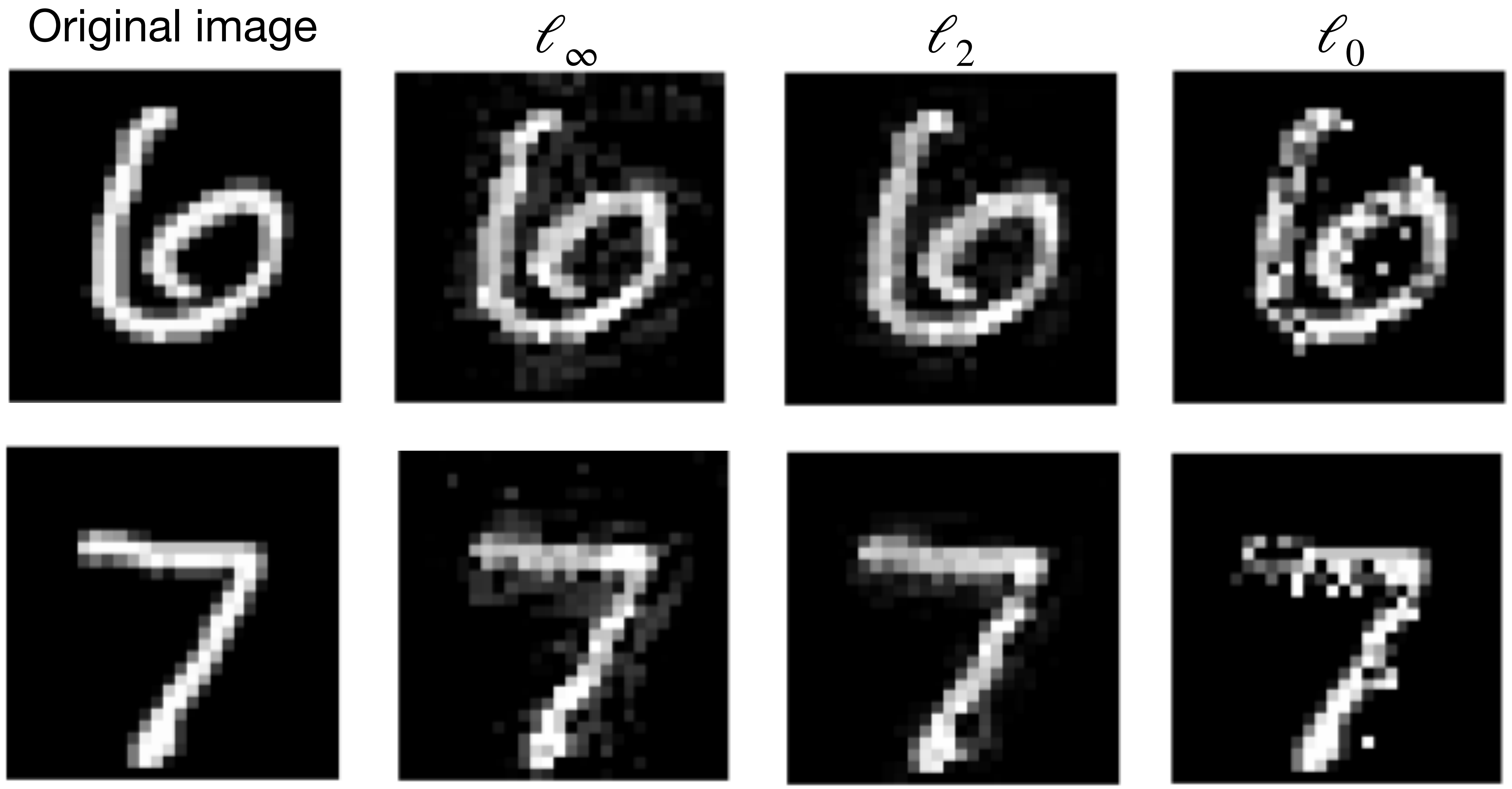
Different ℓ_p norms have different geometry



The difference is especially pronounced in high dimensions!

Visualizations of different ℓ_p adversarial examples

The choice of the norm leads to different properties of the resulting adversarial perturbations: e.g. ℓ_∞ are **dense** and ℓ_0 are **sparse**



Source: Towards Evaluating the Robustness of Neural Networks, Carlini et al., 2018

Which perturbations should we aim to be robust against?

➡ Extensive research on formulating the 'right' perturbation set!

White-box attacks: implementation

- For a neural network, the gradients $\nabla_x g(x)$ can also be computed by **backpropagation** (note: they are taken w.r.t. **inputs**, not parameters!)
- Modern deep learning frameworks readily support this
  $\rightarrow$ **lab #9** (implement Fast Gradient Sign Method on MNIST in PyTorch)
- Now: what **if we don't know** $g(x)$**?** Is it possible to run an attack without knowing how to compute the gradient $\nabla_x g(x)$?

Black-box attacks: query-based gradient estimation

There are different assumptions on the knowledge about the model f:

- **score-based**: we can query the continuous model scores $g(x) \in \mathbb{R}$
- **decision-based**: we can query only the predicted class $f(x) \in \{-1,1\}$

In the score-based case, we can approximate the gradient by using the finite difference formula:

$$\nabla_x g(x) \approx \sum_{i=1}^{d} \frac{g(x+\alpha e_i) - g(x)}{\alpha} e_i$$

Remark: similar techniques can be adapted for the decision-based case (when x is near the decision boundary)

Black-box attacks via transfer attacks

Alternative approach: **transfer attacks**

1. train a **similar** surrogate model $\hat{f} \approx f$ on **similar** data
2. transfer the resulting white-box adversarial perturbation from $\hat{f}$ to f

- Success depends on how **similar** the model architecture and data are
- If we are allowed to query f given some **unlabeled** inputs $\{x_n\}_{n=1}^{N}$, we can obtain $\{x_n, f(x_n)\}_{n=1}^{N}$ and use that information to learn $\hat{f}$ (known as **model stealing**)
  $\rightarrow$ can facilitate **transfer attacks**

Black-box attacks: summary

General takeaway: black-box attacks are of practical concern but:

- Query-based methods often require a lot of queries (10k-100k), particularly **decision-based** attacks → easy to restrict access for the attacker!
- Obtaining a surrogate model $\hat{f}$ can be costly and there is no guarantee of success
- A critical missing element is the implementation of **physically realizable attacks**

Physically realizable attacks

For practical application, adversarial examples must meet additional requirements:

- Resilience to JPEG compression (for images input directly in a digital format)
- Resilience to photographic distortions (for real-world adversarial examples captured by a camera)
- Resilience to varying camera angles (for a moving camera, e.g., on a self-driving car)

→ a surge of papers on how to take these requirements into account

Source: Robust Physical-World Attacks on Deep Learning Visual Classification (CVPR 2018)

How do we train robust models?

We have seen how to **generate** adversarial examples, but **how can we make our models robust to such attacks**?

➡ Simply train them on these adversarial examples, a.k.a. **adversarial training**

- **Standard training**: the goal is to minimize the **standard risk**:

$$\min_{\theta} R(f_\theta) = \mathbb{E}_{\mathscr{D}}\left[1_{f(X)\neq Y}\right]$$

- **Adversarial training**: the goal is to minimize the **adversarial risk**:

$$\min_{\theta} R_\varepsilon(f_\theta) = \mathbb{E}_{\mathscr{D}}\left[\max_{\hat{x}, \|\hat{x}-X\| \leq \varepsilon} 1_{f(\hat{x})\neq Y}\right]$$

Adversarial training: formulation

Goal:

$$\min_{\theta} R_{\varepsilon}(f_{\theta}) = \mathbb{E}_{\mathscr{D}}\left[\max_{\hat{x}, \|\hat{x}-X\| \le \varepsilon} 1_{f(\hat{x}) \neq Y}\right]$$

- The data distribution $\mathscr{D}$ is unknown → approximate it with a **sample average**
- The classification loss is non-continuous → use a **smooth loss**

This leads to the following **robust optimization** problem:

$$\min_{\theta} \frac{1}{N} \sum_{n=1}^{N} \max_{\hat{x}_n, \|x_n - \hat{x}_n\| \le \varepsilon} \ell(y_n g_{\theta}(\hat{x}_n))$$

Interpretation: minimize the risk on adversarial examples

Adversarial training: algorithm

$$\min_{\theta} \frac{1}{N} \sum_{n=1}^{N} \max_{\hat{x}_n, \|x_n - \hat{x}_n\| \leq \varepsilon} \ell(y_n g_\theta(\hat{x}_n))$$

Adversarial training: at each iteration t:

1. For each x_n, approximate $\hat{x}_n^\star \approx \arg \max_{\|x_n - \hat{x}_n\| \leq \varepsilon} \ell(y_n g_\theta(\hat{x}_n))$ via the **PGD attack**

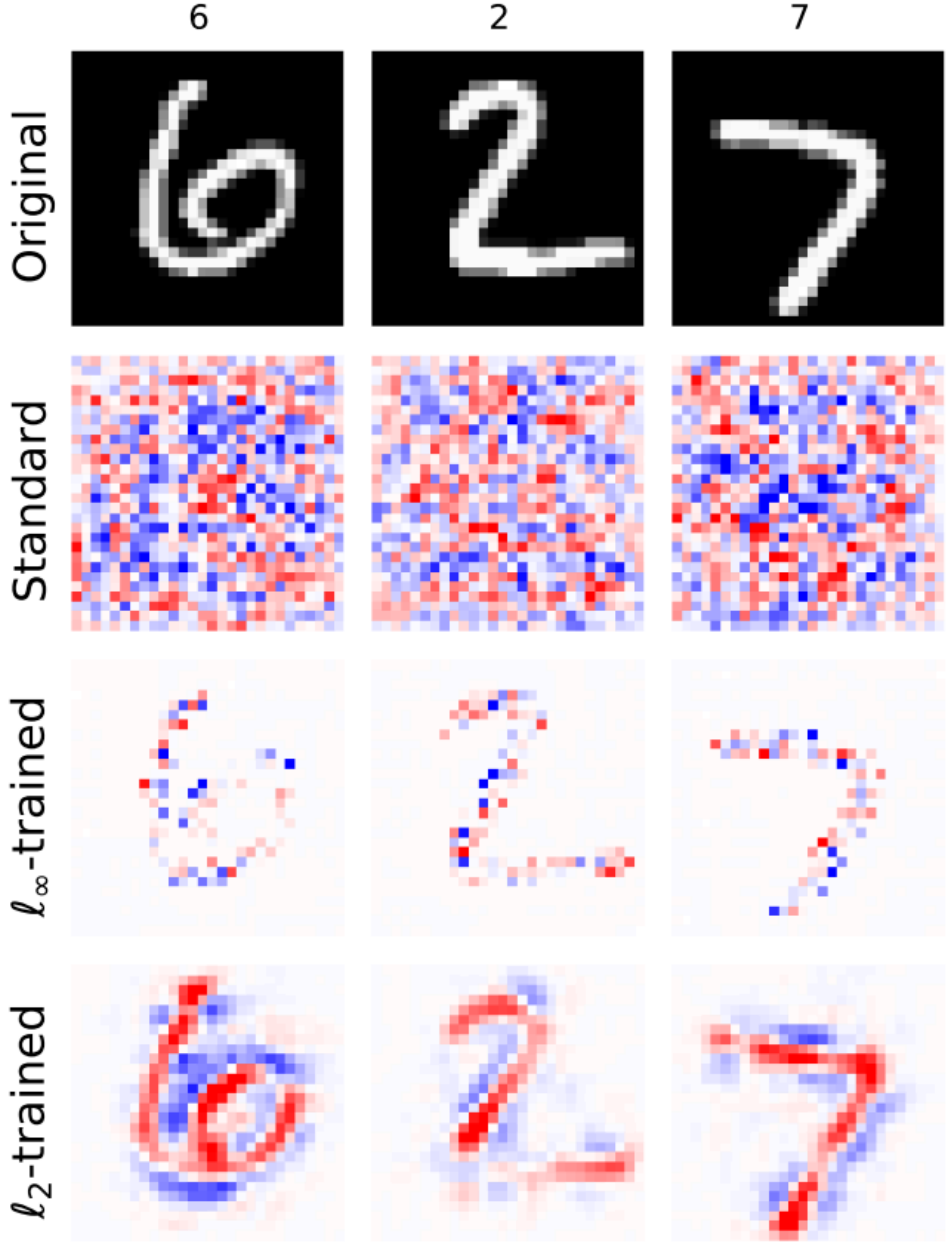
2. Perform a gradient descent step w.r.t. θ using $\frac{1}{N} \sum_{n=1}^{N} \nabla_\theta \ell(y_n g_\theta(\hat{x}_n^\star))$
   ⚠ Note you are using $\hat{x}_n^\star$ and not x_n

Adversarial training: discussion

Good news:

- Adversarial training is a state-of-the-art approach for robust classification!
- Adversarial training leads to **more interpretable gradients** $\nabla_x \ell(y g_\theta(x))$
- The algorithm is fully compatible with SGD $\rightarrow$ you will explore it in **lab #9** (adversarial training of a CNN on MNIST)

Visualization of $\nabla_x \ell(y g_\theta(x))$



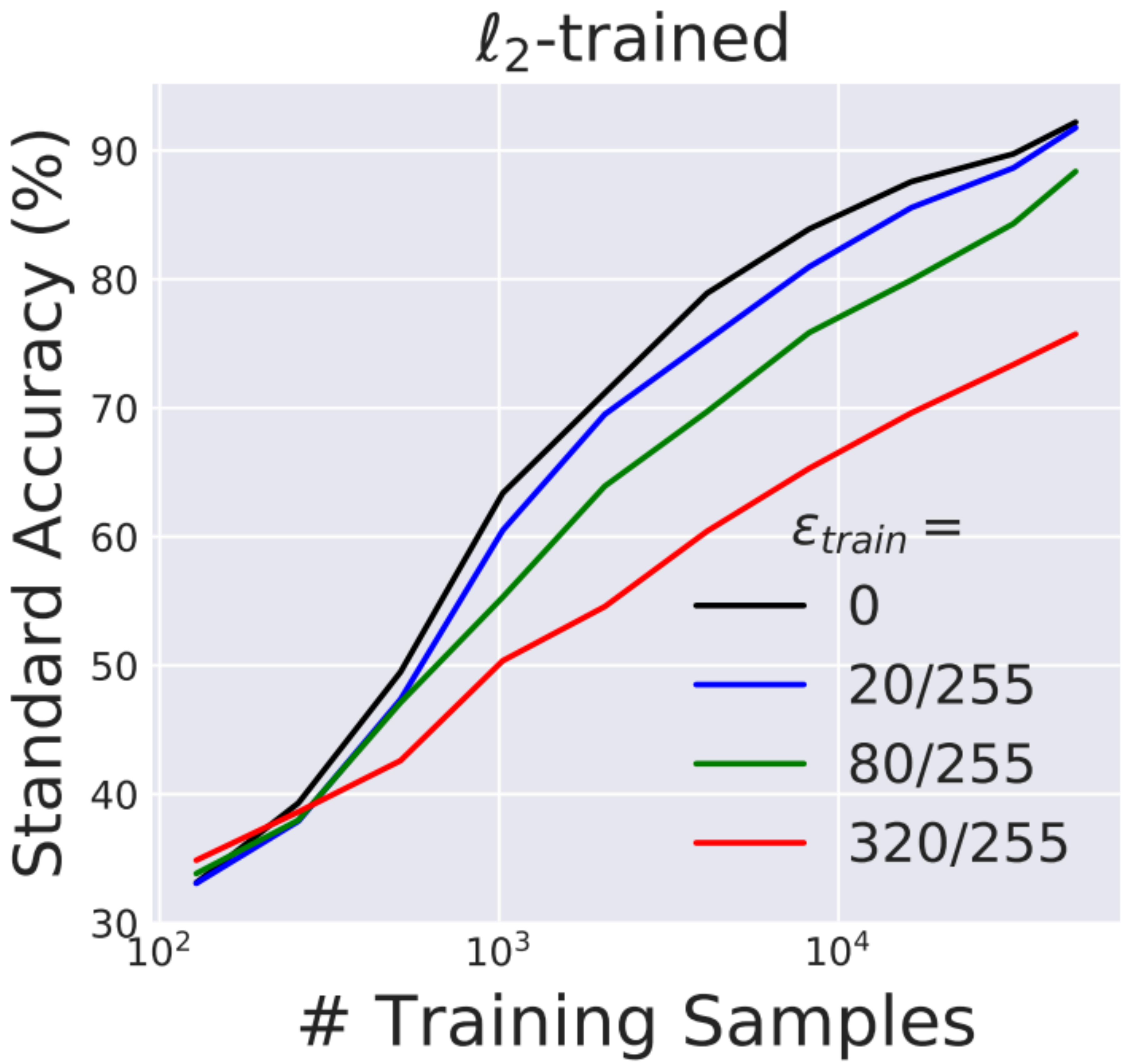
Source: Robustness May Be at Odds with Accuracy (ICLR 2019)

Adversarial training: discussion

Bad news:

- Increased computational time: proportional to the number of PGD steps
- **Robustness-accuracy tradeoff**: using too large ε lead to worse standard accuracy (right)

Deep ConvNet on CIFAR-10



Source: Robustness May Be at Odds with Accuracy (ICLR 2019)

Key question: so why do adversarial examples exist?

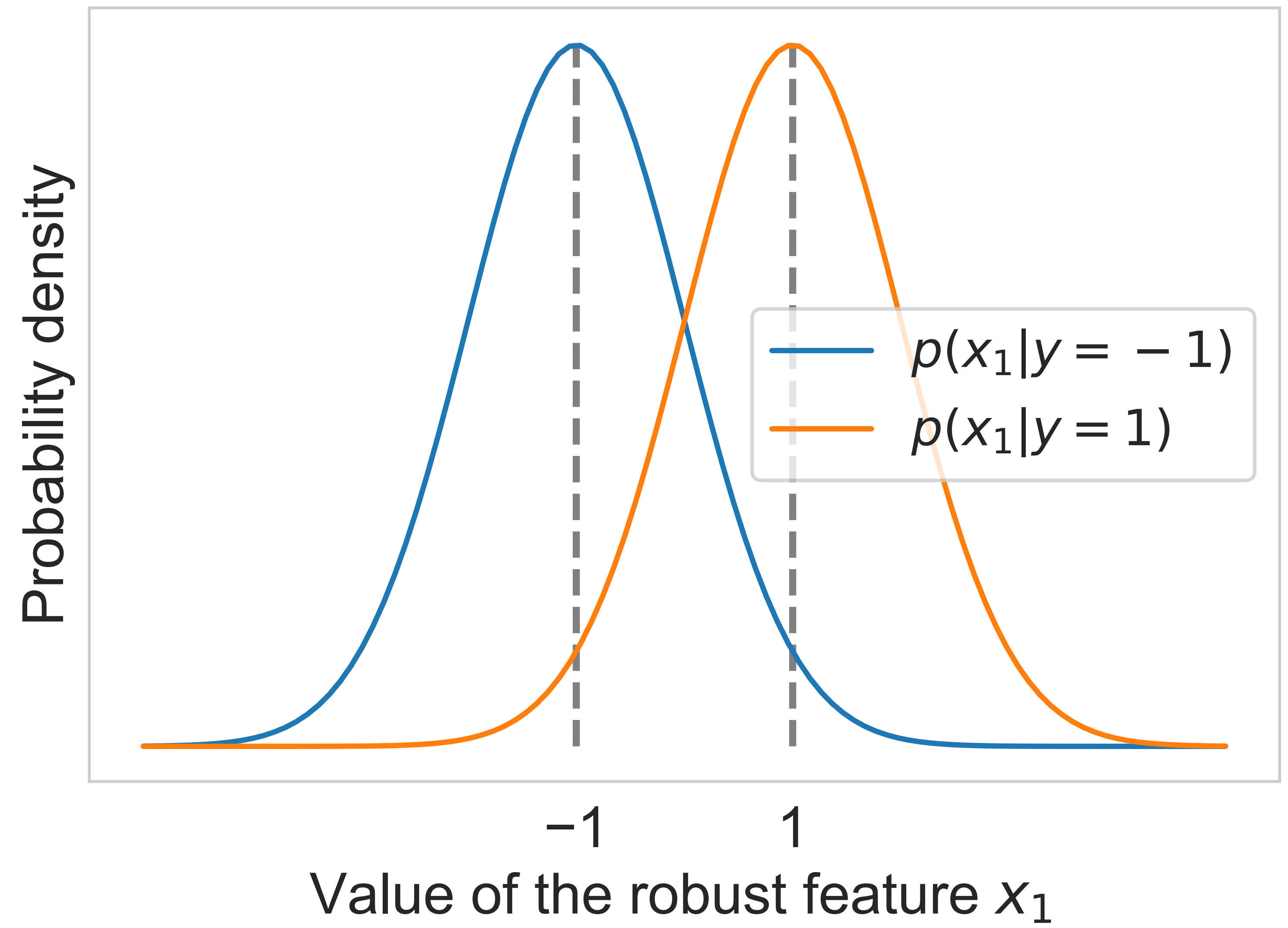
We can conceptualize it with a simple model

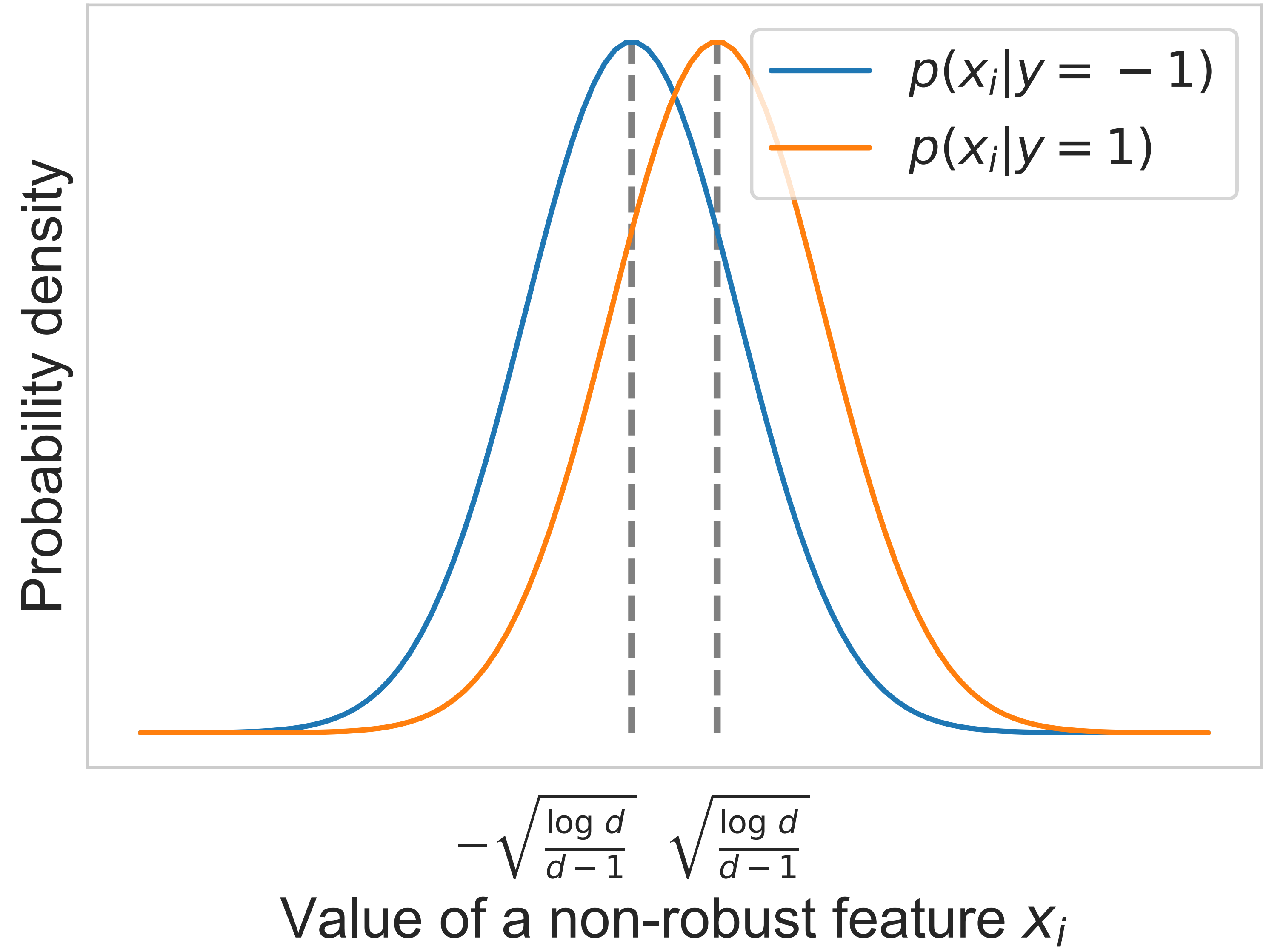
Consider $x \in \mathbb{R}^d$, $y \sim \mathcal{U}(\{-1,1\})$, $Z_i \sim \mathcal{N}(0,1)$:

- **Robust features**: $x_1 = y + Z_1$
- **Non-robust features**: $x_i = y\sqrt{\frac{\log d}{d-1}} + Z_i$ for $i \in \{2,\ldots,d\}$

We'll see that when $d \to \infty$:

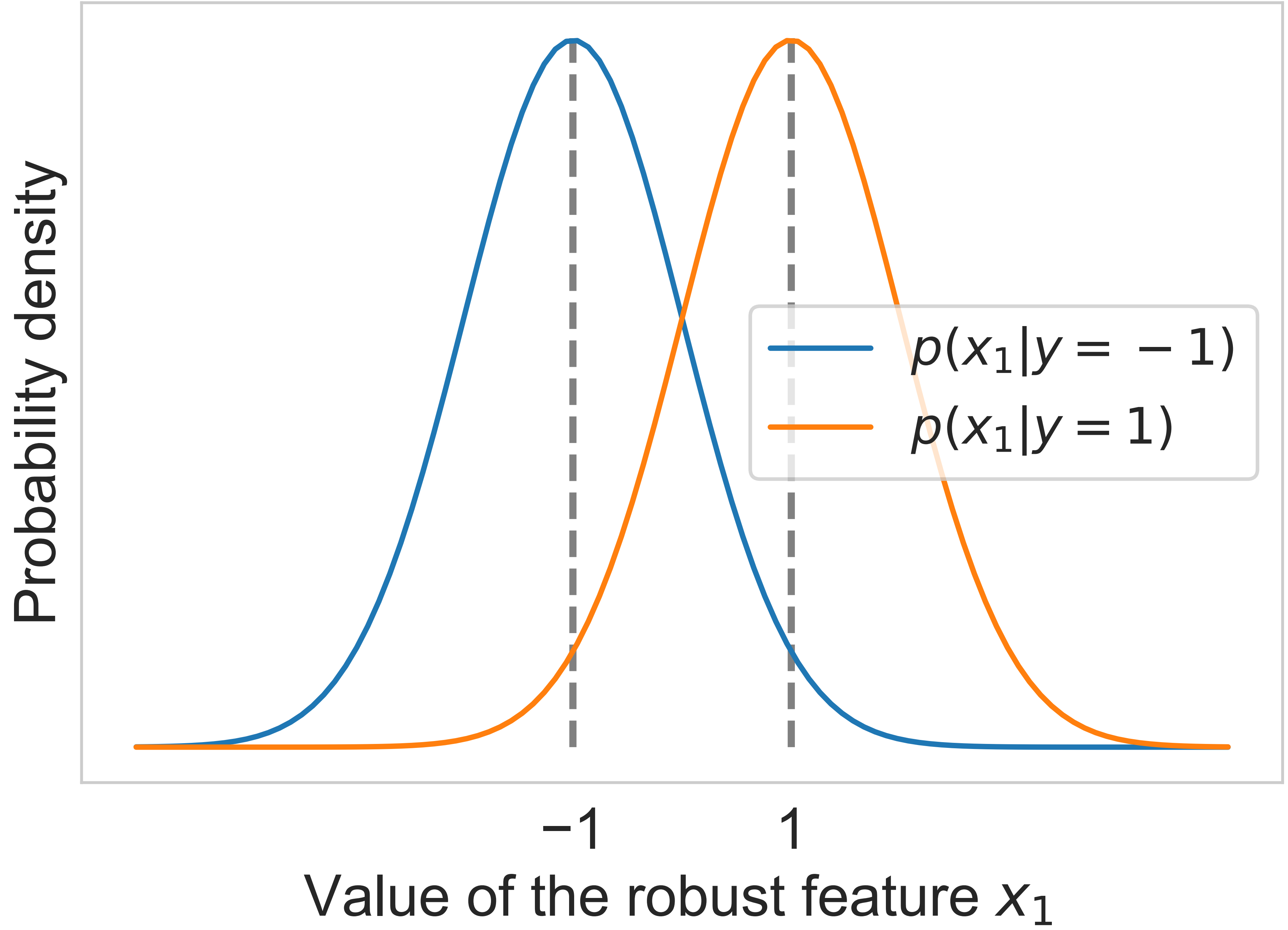
- **standard risk** can be arbitrarily **small**
- **adversarial risk** can be arbitrarily **large**





D. Tsipras, S. Santurkar, L. Engstrom, A. Turner, and A. Madry. Robustness May Be at Odds with Accuracy, ICLR 2019

Model is only using the robust feature x_1



assuming
$p(y=1) = p(y=-1)$

MLE: $\arg\max_{\hat{y}\in\{\pm 1\}} p(\hat{y} \mid x_1) = \arg\max_{\hat{y}\in\{\pm 1\}} \frac{p(x_1 \mid \hat{y})p(\hat{y})}{p(x_1)} = \arg\max_{\hat{y}\in\{\pm 1\}} p(x_1 \mid \hat{y})$

Standard risk: $\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-0.5(x+1)^2} dx \approx 0.16 \rightarrow$ good but not perfect!

Model is using both robust and non-robust features (I)

Let's derive MLE using **all** features using the shortcut notation $x_i = ya_i + Z_i$:

$$
\begin{aligned}
\arg\max_{\hat{y}\in\{\pm 1\}} p(\hat{y} \mid x) &= \arg\max_{\hat{y}\in\{\pm 1\}} \prod_{i=1}^{d} p(x_i \mid \hat{y}) \\
&= \arg\max_{\hat{y}\in\{\pm 1\}} \sum_{i=1}^{d} \log p(x_i \mid \hat{y}) \\
&= \arg\max_{\hat{y}\in\{\pm 1\}} \sum_{i=1}^{d} \log \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x_i - \hat{y}a_i)^2} \\
&= \arg\min_{\hat{y}\in\{\pm 1\}} \sum_{i=1}^{d} (x_i - \hat{y}a_i)^2 \\
&= \arg\min_{\hat{y}\in\{\pm 1\}} \sum_{i=1}^{d} (x_i^2 - 2x_i\hat{y}a_i + \hat{y}^2 a_i^2) = \arg\max_{\hat{y}\in\{\pm 1\}} \hat{y} \sum_{i=1}^{d} x_i a_i
\end{aligned}
$$

Model is using both robust and non-robust features (II)

The MLE expression we maximize over $\hat{y} \in \{-1,1\}$ becomes:

$$\hat{y}\sum_{i=1}^{d} x_i a_i = \hat{y}y(\sum_{i=1}^{d} a_i^2) + \hat{y}\sum_{i=1}^{d} a_i Z_i = \hat{y}y(1+\log(d)) + \hat{y}Z,$$

where $Z := \sum_{i=1}^{d} a_i Z_i \sim \mathcal{N}(0,1+\log d)$

Scaling by $1/(1+\log d)$, the MLE expression results in:

$$y\hat{y} + \hat{y}Z \text{ with } Z \sim \mathcal{N}(0,1/(1+\log d))$$

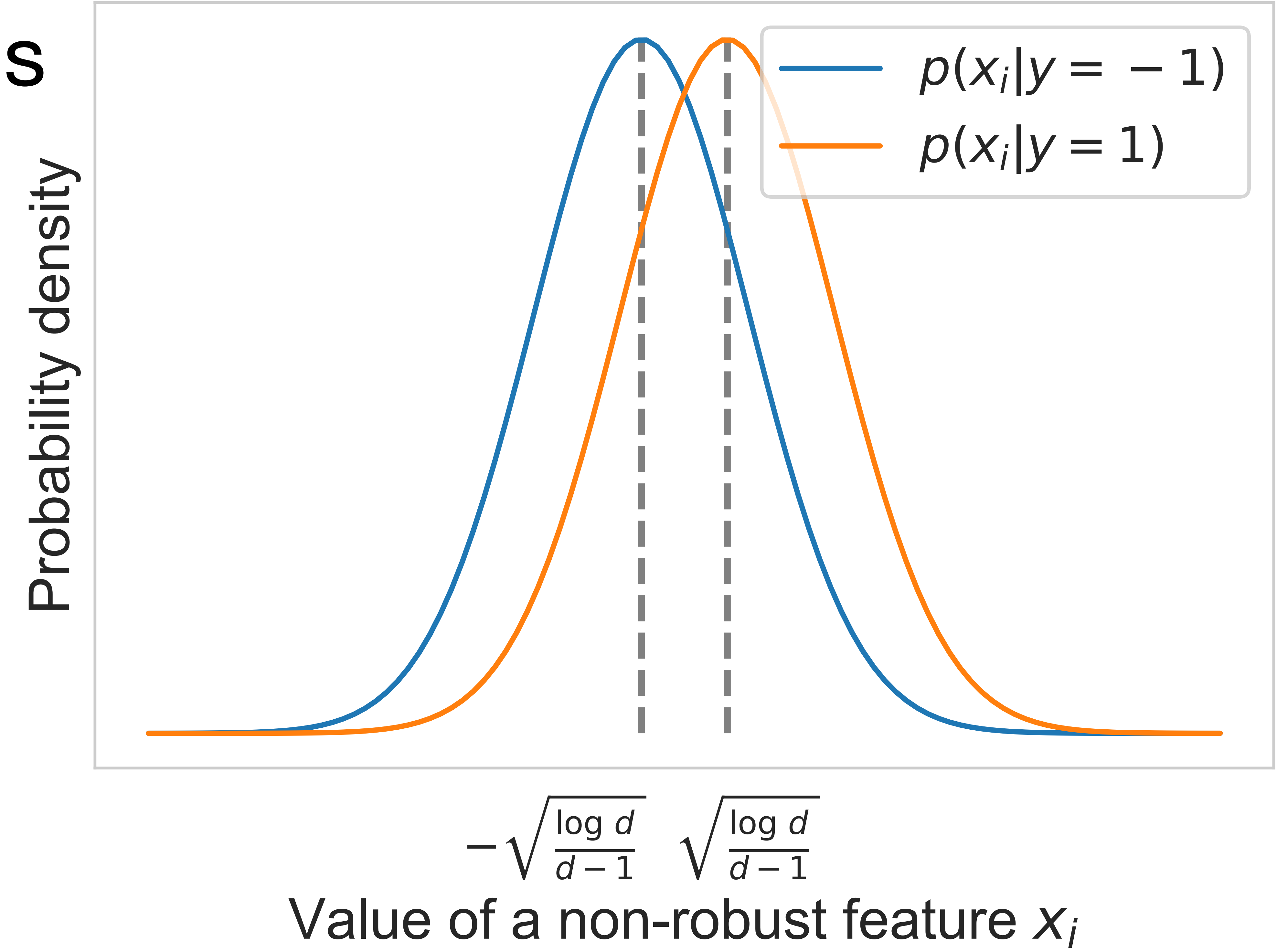
Conclusion: when the dimension $d \to \infty$, $\hat{y}Z \to 0$ and standard risk $\to 0$

Interpretation: using the non-robust features improves standard risk!

What about adversarial risk?

- The adversary can use tiny ℓ_∞- perturbations
  $\varepsilon = 2\sqrt{\frac{\log d}{d-1}}$ ($\to 0$ when $d \to \infty$)

- Optimal adversarial strategy:
  $\hat{x}_1 = \left(1 - 2\sqrt{\frac{\log d}{d-1}}\right)y + Z_1$ (almost unaffected)
  $\hat{x}_i = -\sqrt{\frac{\log d}{d-1}}y + Z_i$ (completely flipped!)



- **Adversarial risk** $R_\varepsilon(f)$ will become ≈ 1
  (due to non-robust x_i) although **standard risk** $R(f)$ is 0!

- **But**: only using the robust feature x_1 leads to $R_\varepsilon(f) \approx R(f) = 0.16$

  ➡ **tradeoff** between accuracy and robustness

Recap

- **NNs may be susceptible to adversarial examples** imperceptible to us
- Adversarial examples can be **obtained via gradient steps on the input**
- Adversarial training: Enhance model robustness by **training on adversarially perturbed input data**
- Robustness typically comes at the cost of accuracy, since **non-robust features can still be useful features**