SPIKING NEURONS

Week 10

Dr. Luca Manneschi

So far…

Differential Equations and initial condition. How to find a unique solution?

Solutions to

Look at the exercises

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \alpha x(t)$$

$\alpha > 0$ Exponential growth

$\alpha < 0$ Exponential decay

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \alpha x(t)\big(1 - x(t)\big)$$

Saturating

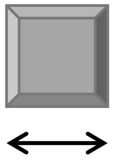
Euler's method and its error $\mathcal{O}(\delta t^2)$ $\mathcal{O}(\delta t)$

Definition of equilibrium points in one and two dimensions. Phase portraits…

Autonomous and non-autonomous: Definition

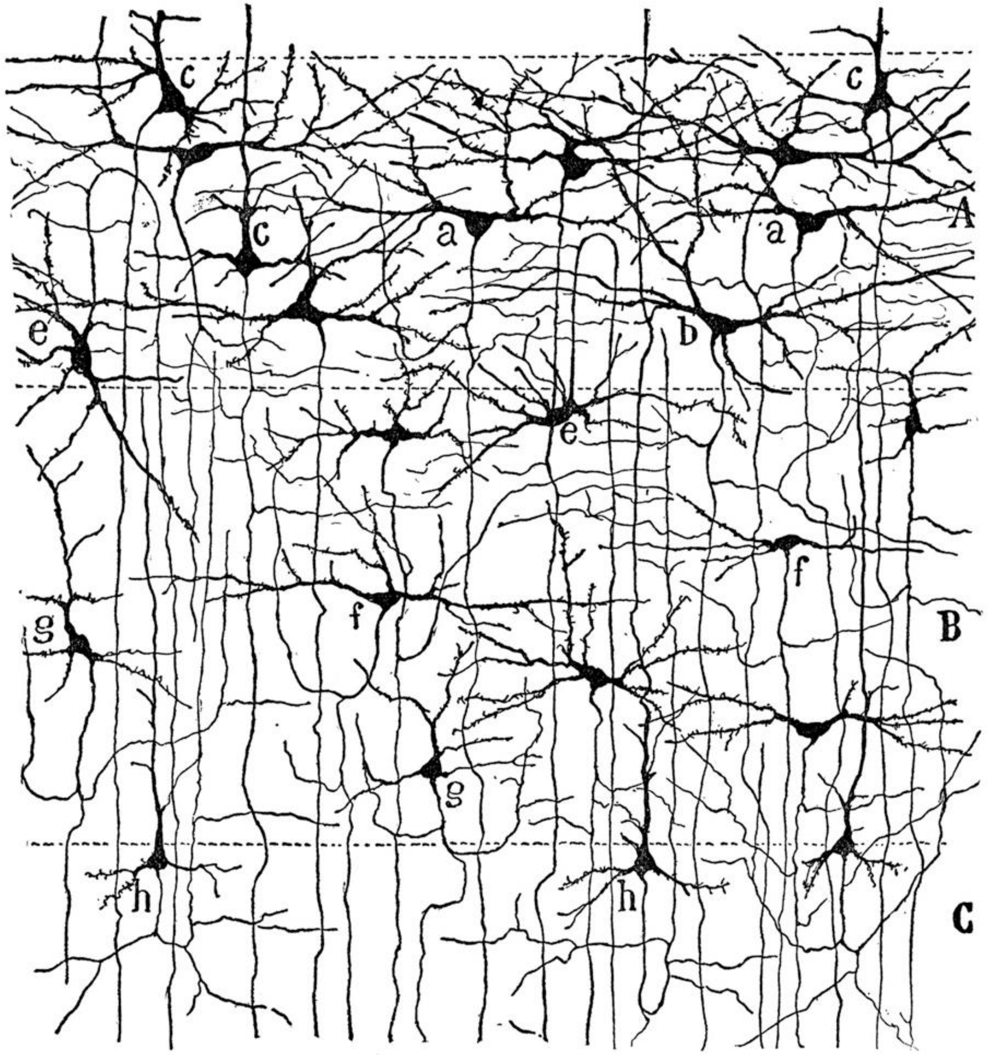
MidPoint Method, Runge-Kutta 2 and 4

Today…

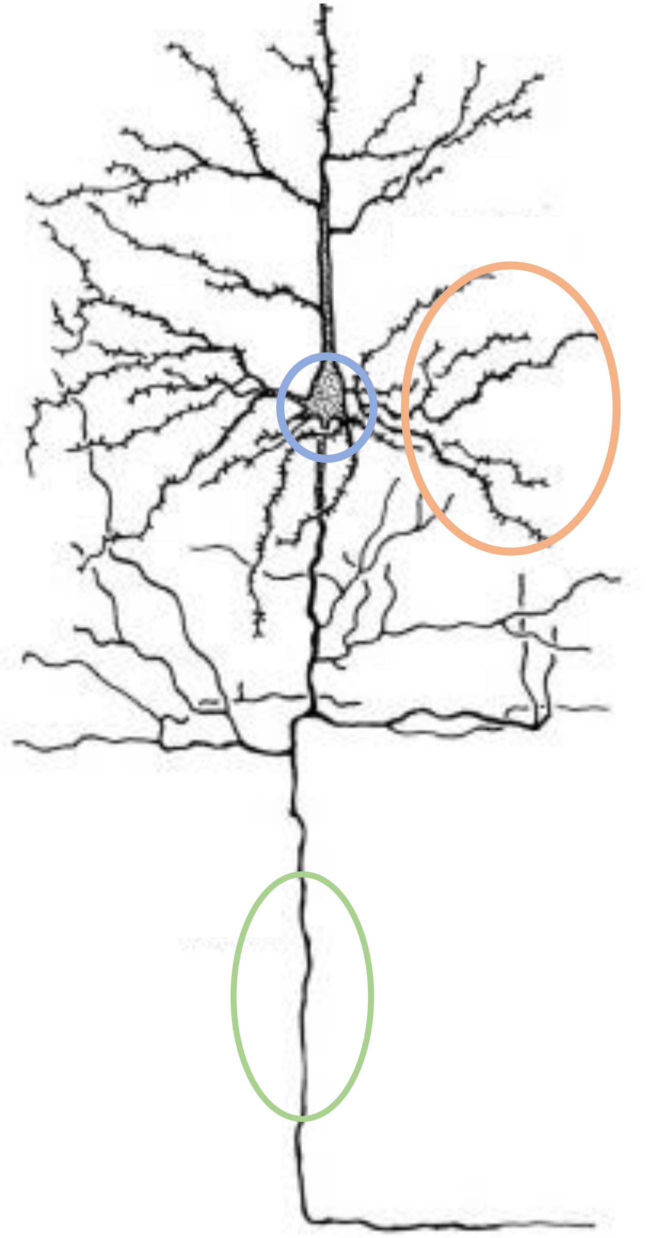


1 mm

`10 000 Neurons
3 Km of wires



Neuron: the main parts

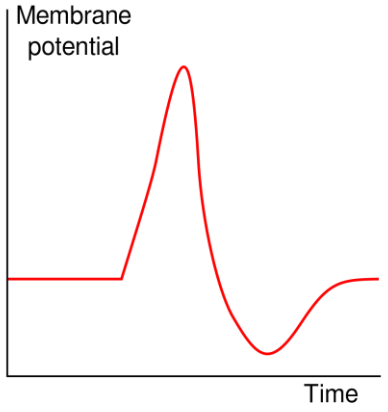


Dendrites: Input device

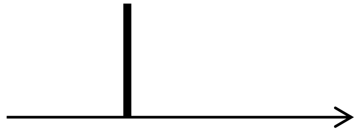
Soma: Central processing unit

Axon: Output

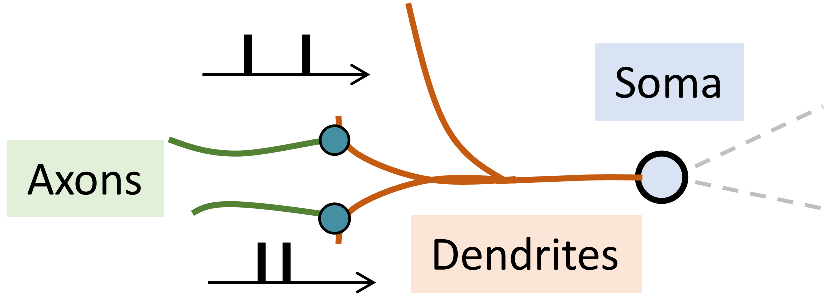
Communication? In a 'stereotype' manner…
Action potentials



Often we will treat this event as instantaneous

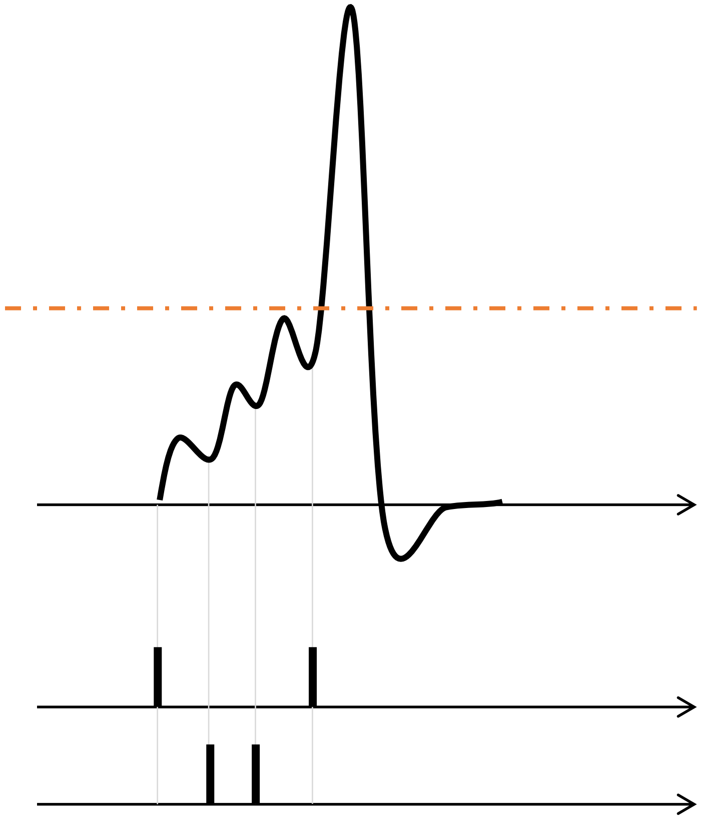


Neuron: the main parts

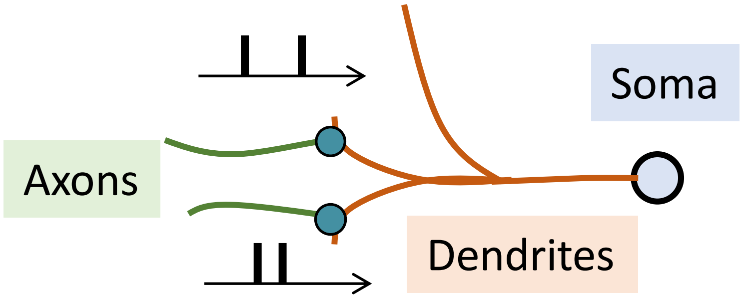


We can measure the potential difference between the inner part and outer part of the soma

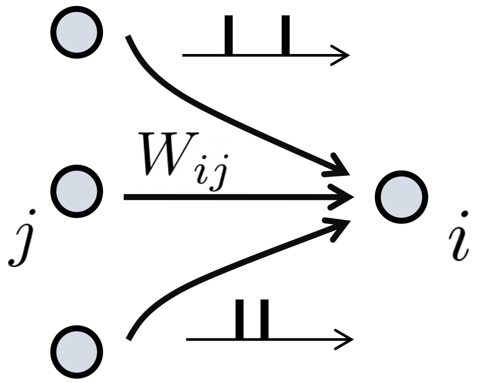
The junction between neurons is called the synapse

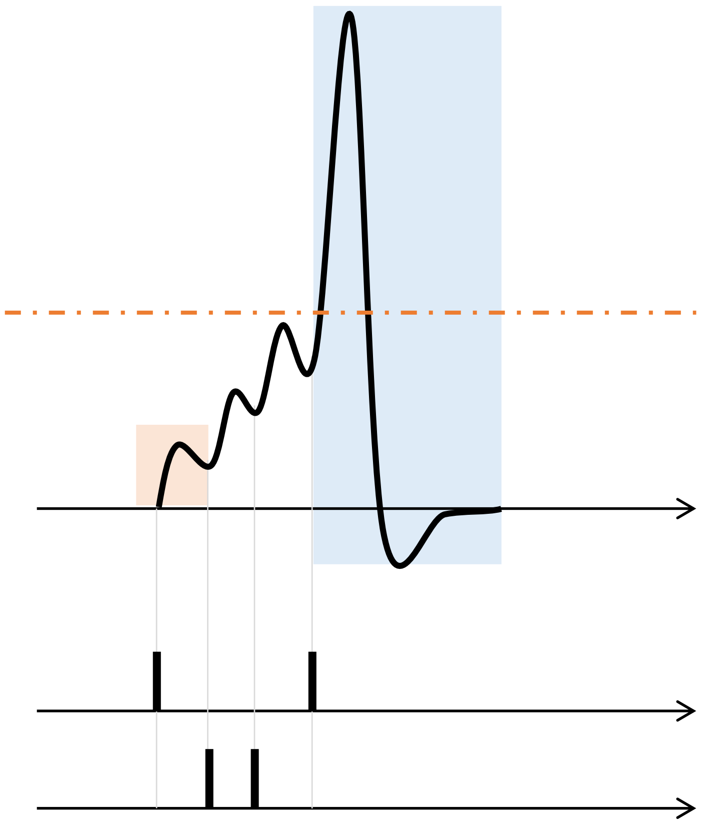


Response neuron model



We simplify the left diagram considering that there are presynaptic neurons and postsynaptic neurons connected (right diagram)





$\epsilon(t - t_j^f)$ Postsynaptic potential: the effect that an input spike has on the potential

$\eta(t - t_i^l)$ Spike of the neuron i considered

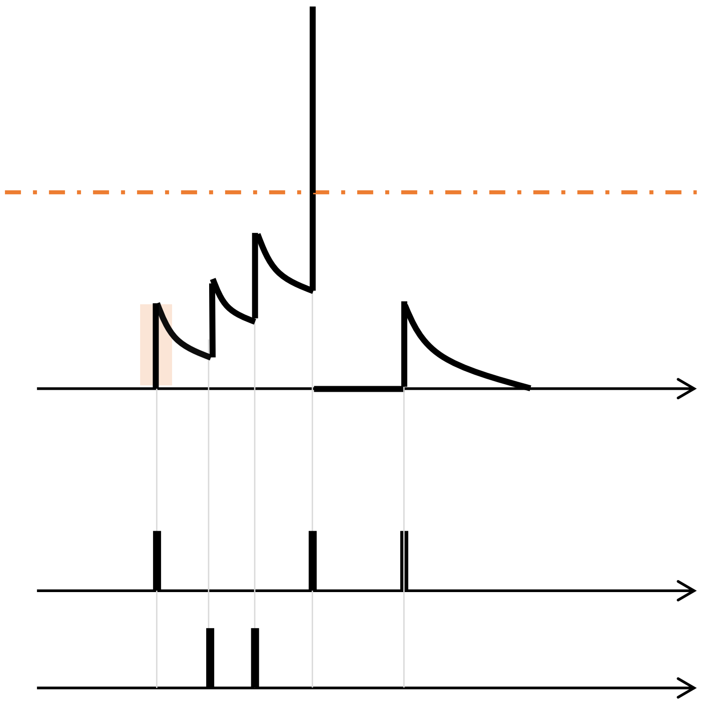
Sum across all presynaptic neurons (j) and all spikes (f)

$$u_i(t) = \eta(t - t_i^l) + \sum_j \sum_f W_{ij}\epsilon(t - t_j^f)$$

$$u_i(t) = \theta, \ \rightarrow t_i^l = t$$

Leaky integrate and fire

If we consider that the postsynaptic potential is linear we can define the leaky-integrate and fire model



Input

$$\tau \frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -u_i(t) + V(t)$$

$u_i(t) = \theta \rightarrow$ Fire + Reset

Does it sound familiar?

Autonomous or not?

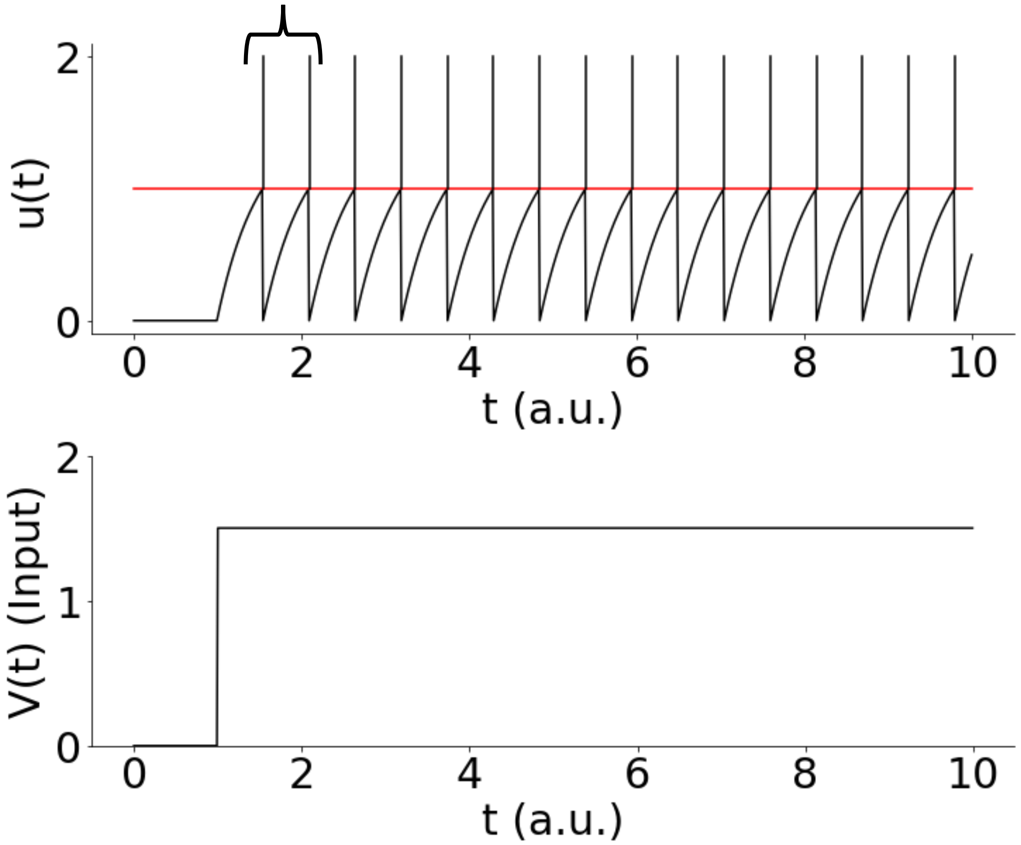
Response Examples

$$\tau \frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -u_i(t) + V(t)$$

$u_i(t) = \theta \rightarrow$ Fire + Reset

'Constant' Input

Interval between successive spikes

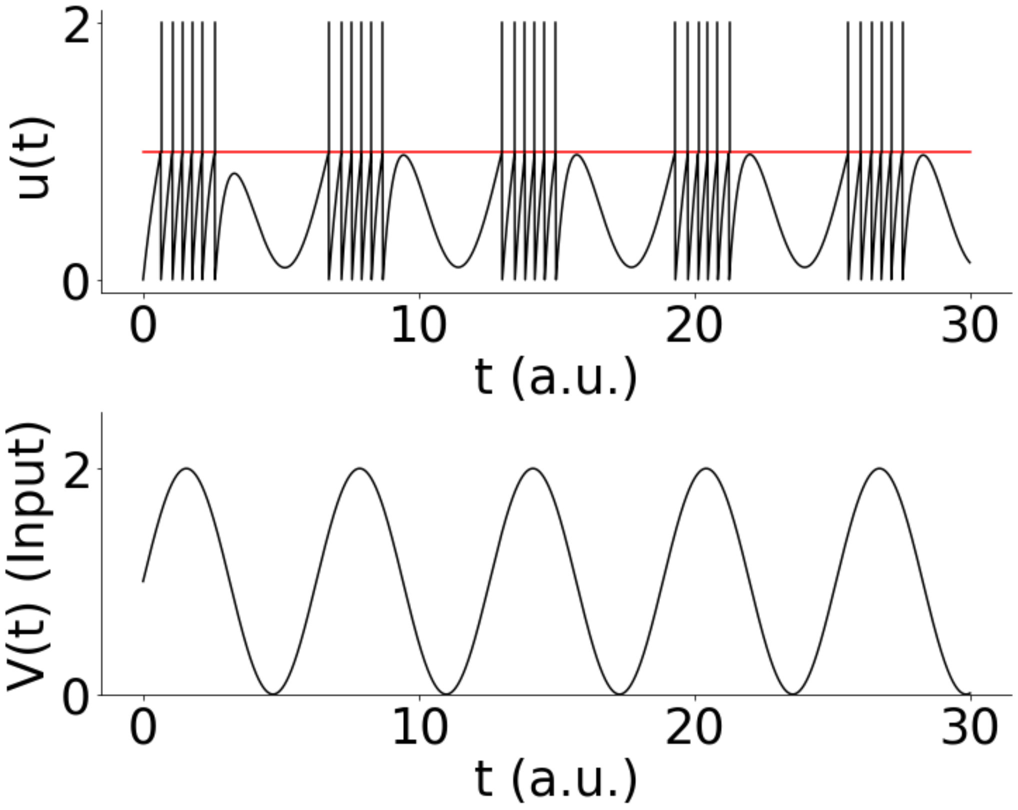


Response Examples

$$\tau \frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -u_i(t) + V(t)$$

$u_i(t) = \theta \rightarrow$ Fire + Reset

Sinusoidal Input



Response Examples

$$\tau \frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -u_i(t) + V(t)$$

$u_i(t) = \theta \rightarrow$ Fire + Reset

Poisson spike model

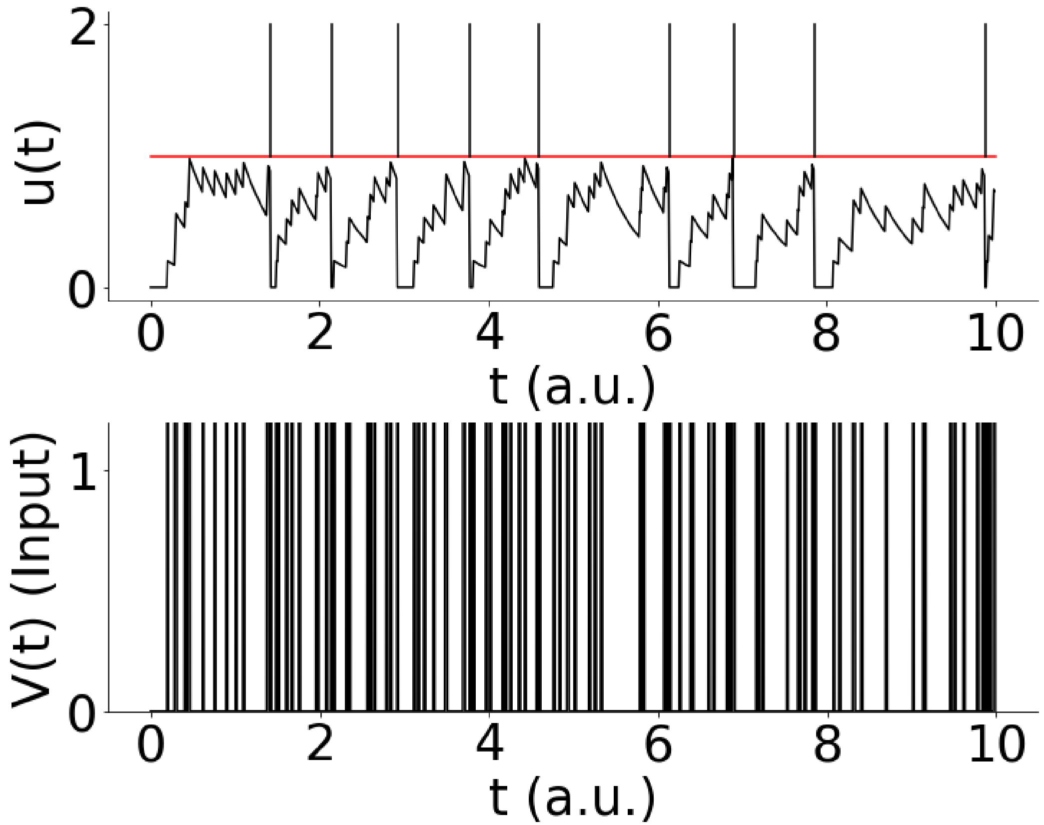
Rate

$$p_i(t) \approx r_i(t)\delta t$$

Probability of firing

Practically, you compute the probability from the rate. Then, generate a random number between [0, 1]. If the random number is less than the probability the neuron fires, otherwise it does not.

Poisson spike model as Input



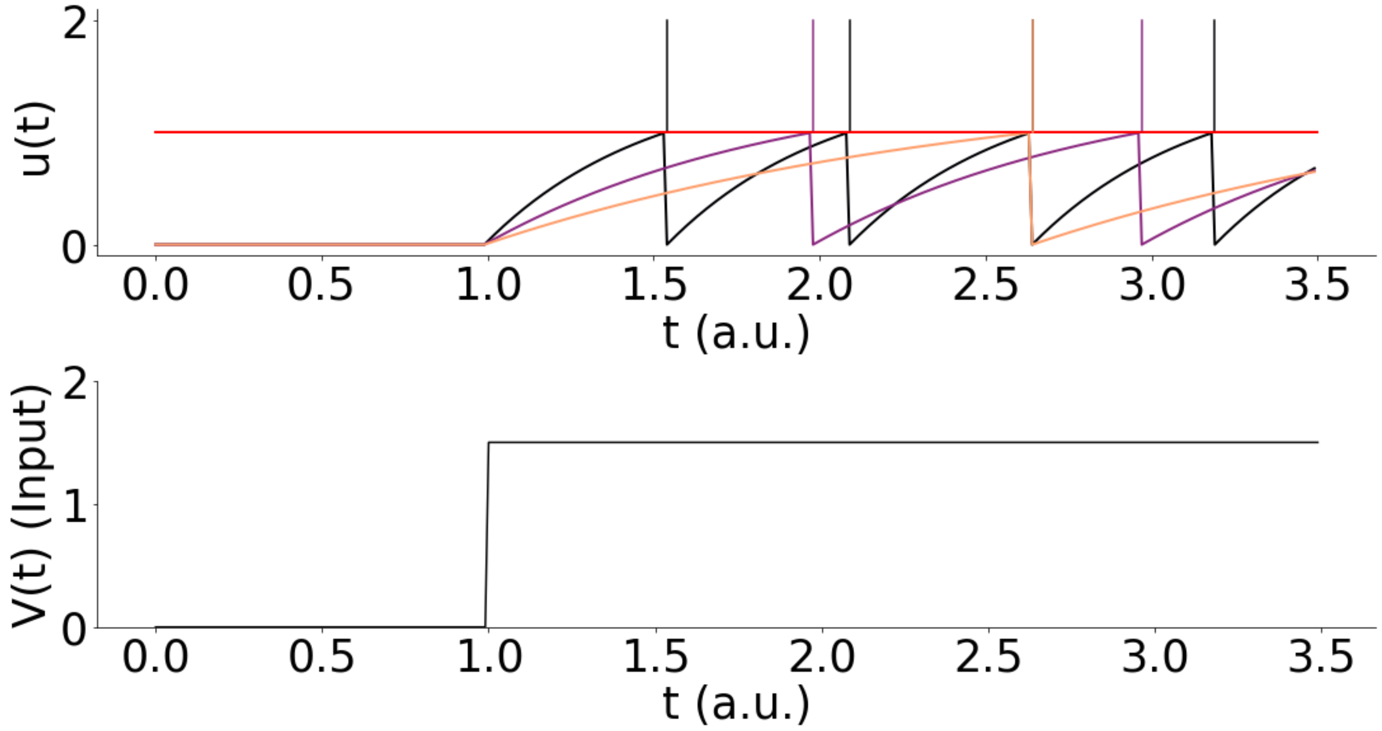
The meaning of tau

$$\tau \frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -u_i(t) + V(t)$$

It describes the rate at which information is integrated

$u_i(t) = \theta \rightarrow$ Fire + Reset

It increases from darker to brighter colours



Constant Input, Solution

$$\tau \frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -u_i(t) + C$$

$$\int \frac{\mathrm{d}u_i(t)}{-u_i(t) + C} = \int \frac{\mathrm{d}t}{\tau}$$

$$-\log(-u_i(t) + C) = \frac{t}{\tau} + k$$

$$-u_i(t) + C = e^{-t/\tau + k}$$

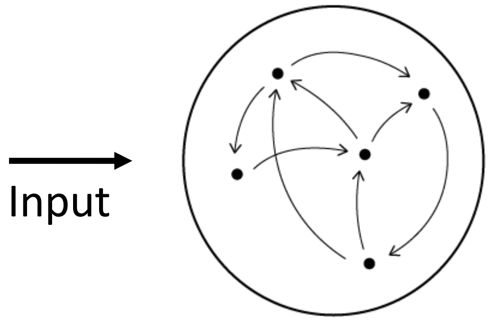
$$-u_i(t) = e^{-t/\tau} e^{k} - C$$

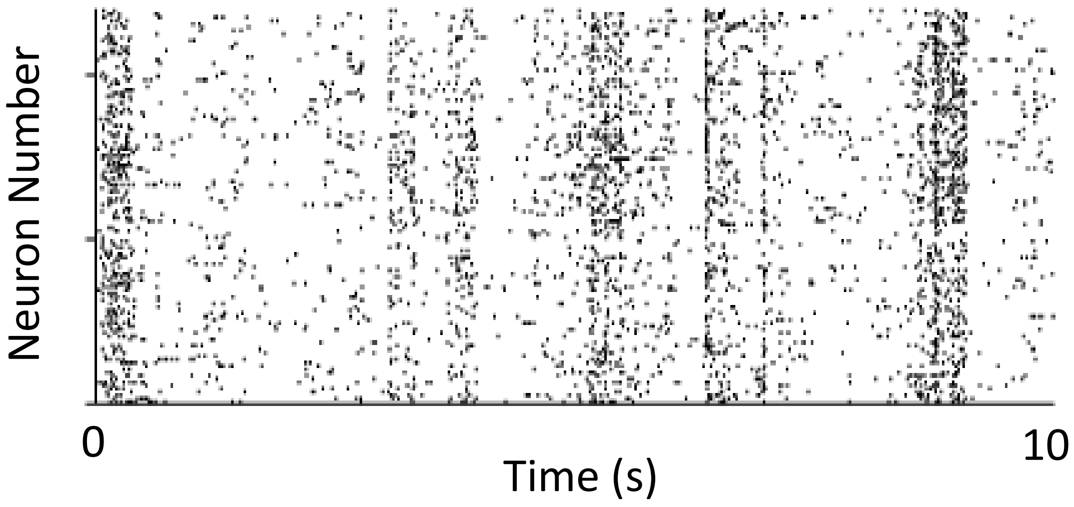
$$u_i(t) = C - e^{-t/\tau} k_1$$

Can you compute the time at which it reaches the threshold?

The problem of neuronal coding

We consider a population of neurons





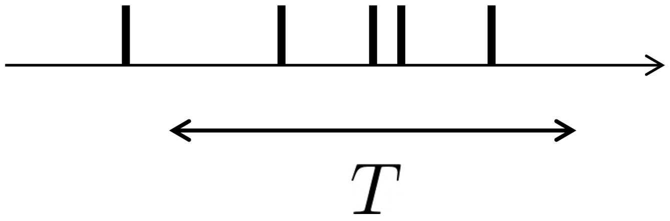
Where is the information?

The problem of neuronal coding

Rate coding. The information is in the rate at which neurons spike

How can we find the rate? We have spikes (binary information), we need to average…

For a single neuron



Temporal average

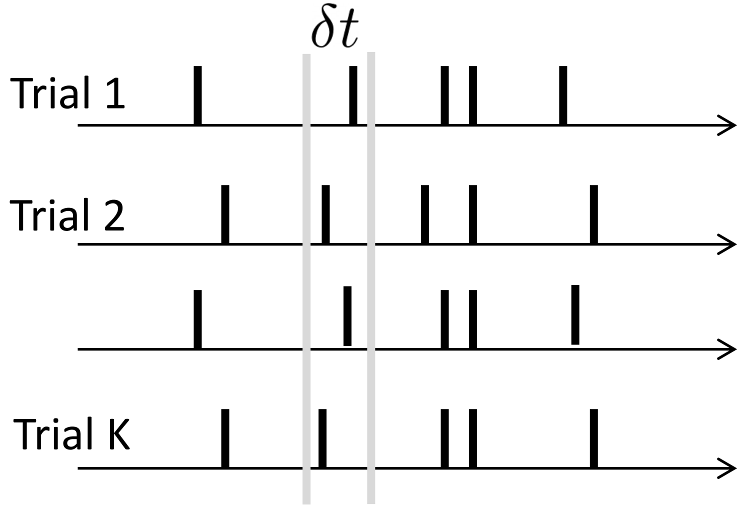
$$\nu = \frac{n(t, t+T)}{T}$$

The problem of neuronal coding

Rate coding. The information is in the rate at which neurons spike

How can we find the rate? We have spikes (binary information), we need to average…

For a single neuron



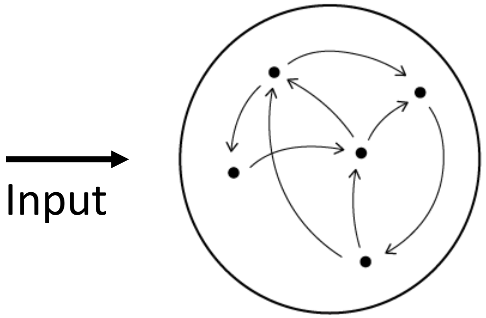
Average across repetitions

$$PSTH(t) = \frac{n(t, t+\delta t)}{K\delta t}$$

K Number of repetitions

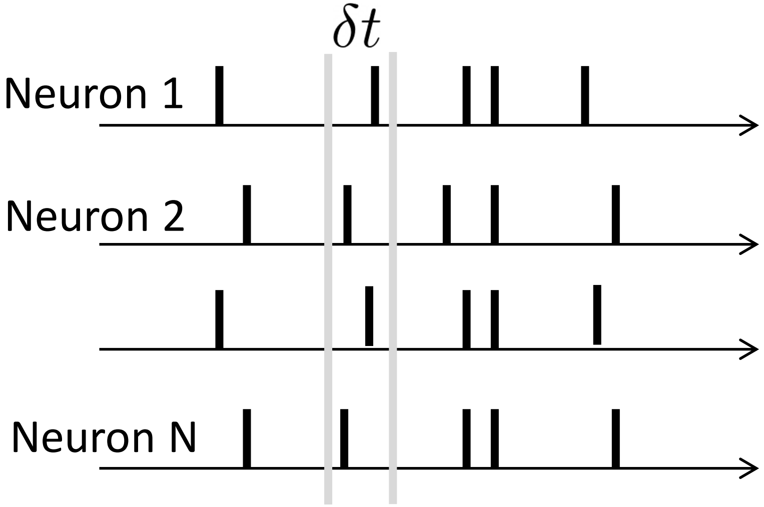
The problem of neuronal coding

Rate coding. The information is in the rate at which neurons spike



How can we find the rate? We have spikes (binary information), we need to average…

For a population



Average across neurons

$$A(t) = \frac{n(t, t + \delta t)}{N \delta t}$$

Number of neurons

The problem of neuronal coding

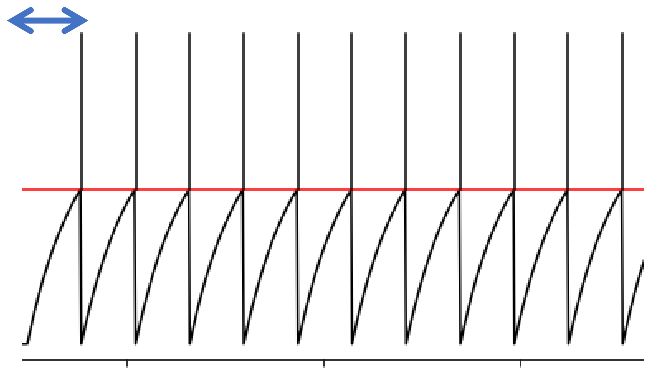
The problem with rate coding on a single neuronal level.

Imagine a frog that catches a fly. The frog reacts in approximately 70 ms. In rate coding we need to average, and averaging takes time. In this activity, there is no time for the considered neuron of the frog to encode the rate across time.

The problem of neuronal coding

Alternative to rate coding: Temporal coding. Examples:

Time-to-first-spike after input

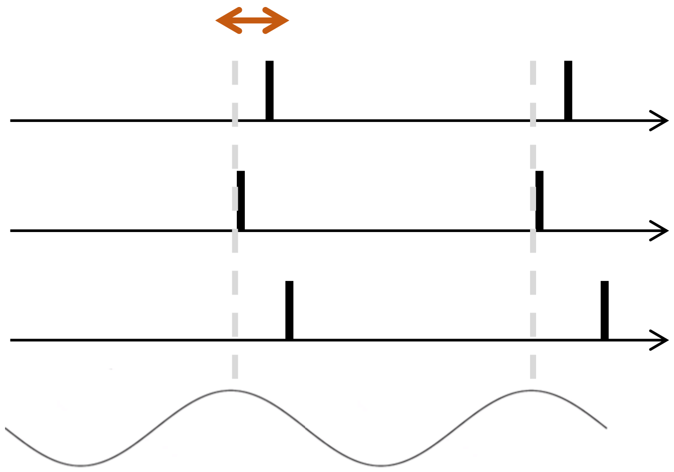


It would need only a single spike to encode this information

The problem of neuronal coding

Alternative to rate coding: Temporal coding. Examples:

Phase with respect to oscillation



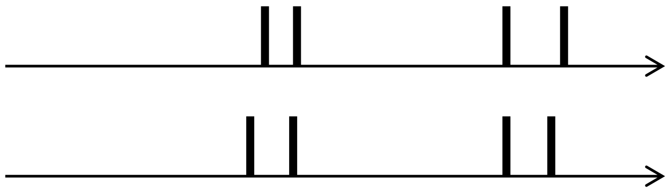
We know that parts of the brain as the hippocampus are characterized by oscillations. We can then measure the 'first spike' relative to a background oscillation

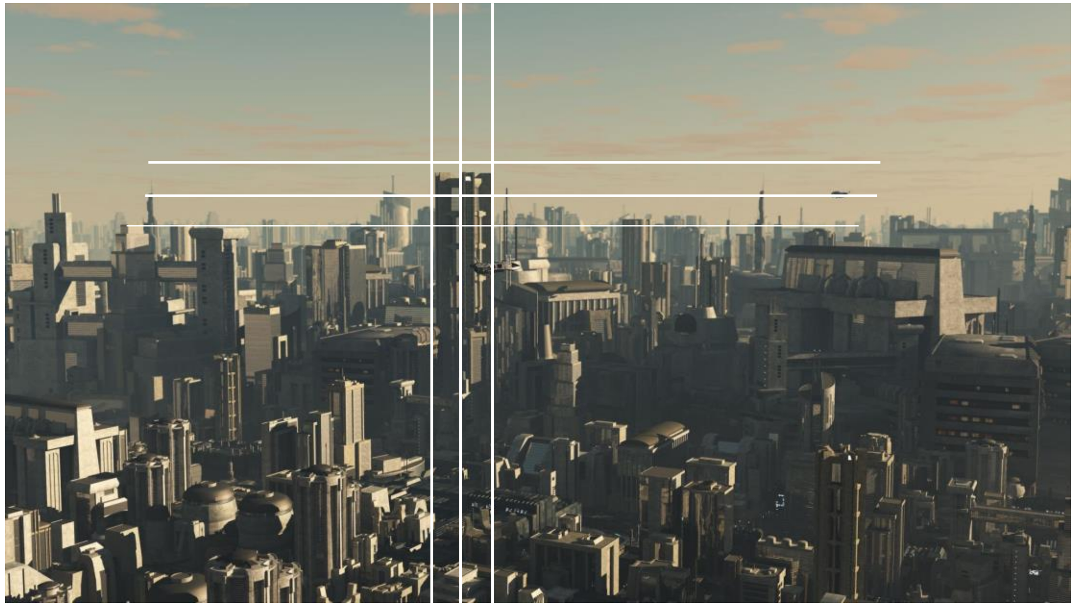
The problem of neuronal coding

Alternative to rate coding: Temporal coding. Examples:

Correlations and synchrony

The fact that a pair or an ensemble of neurons have synchronous activity could mean something. For instance, it could mean that they ‘belong together’…Imagine that neurons encode different locations in an image. Neurons that have synchronous activity could mean that they are encoding the same object





Thank you