Homework 12 Quantitative Risk Management

Group G03
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Question 1

The transition rate matrix was copied from table 23 on pages 60-61 of 2016 Annual Global Corporate Default Study and Rating Transitions. We start off by addding a default row with zeros everywhere except for the last entry, and divide all entries by 100 to obtain probabilities in the range 0 to 1. After normalization such that all rows add to 1, we obtain the complete 18×18 transition rate matrix. We proceed by raising this matrix to the 5th power to obtain the 5-year transition probabilities. Since we are assuming a zero interest rate environment and a face value of 1, the fair price of the bond is simply the probability of it being alive, i.e. 1 minus the default probability. The 5-year default probabilites can be retrieved in the last column of the 5-year transition rate matrix. This results in the bond prices seen in table 1. Interestingly, the AA+ bonds have a higher price than the AAA bonds. This is likely explained by the fact that AAA bonds have a 0.05% yearly probabilty of transitioning to the CCC/C-rating, while the AA+ rated bonds do not. The reason for this would be interesting to know.

Rating	Bond price
AAA	0.9986
AA+	0.9993
AA	0.9973
AA-	0.9974
A+	0.9965
A	0.9952
A-	0.9938
BBB+	0.9897
BBB	0.9859
BBB-	0.9753
BB+	0.9605
BB	0.9413
BB-	0.9032
B+	0.8291
B	0.7196
B-	0.5646
CCC/C	0.2894

Table 1

Question 2

This question is largely done the same way as the previous one, but we add or subtract the standard deviation matrix to the transition rate matrix. The standard deviation matrix can be seen in the code section below. When subtracting the standard deviation matrix, there is a chance of obtaining a negative entry in the resulting transition rate matrix, so we set all negative entries to 0 before normalizing such that the rows add to 1.

The bond prices for plus one standard deviation can be seen in table 2, and the bond prices for minus one standard deviation can be seen in table 3. We notice that the bond prices for the top 11 rated bonds are all 1.0000 for the minus standard deviation prices. Our guess is that they are not really 1, but rounding causes them to be reported as 1 in Matlab. Regardless, we would expect them to be higher than in question 1. This is because the overall probability of transitioning to the lower rated states, and therefore ultimately default, is much higher when adding a standard deviation, but often goes from a positive probability to 0 when subtracting one standard deviation. For small transition probabilities, the uncertainty in percentage points is often larger than the estimated probability, which causes this to happen. The only rating for which the bond price is higher for minus one standard deviation is for CCC/C-rated bonds. This is probably because normally, these bonds have a small estimated probability of going to relatively highly rated states, but when subtracting one standard deviation, this probability is 0.

Rating	Bond price
AAA	0.9912
AA+	0.9956
AA	0.9890
AA-	0.9889
A+	0.9869
A	0.9838
A-	0.9784
BBB+	0.9695
BBB	0.9631
BBB-	0.9400
BB+	0.9134
BB	0.8903
BB-	0.8382
B+	0.7522
B	0.6315
B-	0.5221
CCC/C	0.3204

Table 2: Bond prices: plus one standard deviation

Rating	Bond price
AAA	1.0000
AA+	1.0000
AA	1.0000
AA-	1.0000
A+	1.0000
A	1.0000
A-	1.0000
BBB+	1.0000
BBB	1.0000
BBB-	1.0000
BB+	1.0000
BB	0.9996
BB-	0.9974
B+	0.9813
B	0.9484
B-	0.7389
CCC/C	0.2590

Table 3: Bond prices: minus one standard deviation

Code

```
close all
clear all

% Transition rate and standard deviation matrices, copied from table 23 on page 60-61 of 2016 Annual Global
% Corporate Default Study and Rating Transitions.

transition_1y = [87.05 5.78 2.56 0.69 0.16 0.24 0.13 0.00 0.05 0.00 0.03 0.05 0.00 0.00 0.03 0.00 0.05 0.00;
                 2.42 77.53 11.54 3.78 0.76 0.40 0.20 0.05 0.10 0.05 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00;
                 0.44 1.29 80.25 8.71 2.83 1.21 0.39 0.40 0.13 0.08 0.05 0.03 0.02 0.02 0.00 0.02 0.05 0.02;
                 0.04 0.12 3.97 78.01 10.07 2.34 0.61 0.28 0.16 0.07 0.03 0.00 0.00 0.03 0.09 0.00 0.00 0.03;
                 0.00 0.06 0.48 4.58 77.51 9.10 2.29 0.66 0.35 0.09 0.06 0.10 0.01 0.07 0.03 0.00 0.00 0.05;
                 0.04 0.05 0.24 0.46 5.26 78.04 7.04 2.57 0.93 0.29 0.12 0.11 0.08 0.10 0.02 0.00 0.02 0.06;
                 0.04 0.01 0.07 0.17 0.48 6.72 76.84 7.62 2.22 0.62 0.15 0.15 0.13 0.12 0.03 0.01 0.03 0.07;
                 0.00 0.01 0.06 0.07 0.23 0.86 7.26 74.40 8.41 1.80 0.41 0.34 0.15 0.18 0.12 0.03 0.07 0.12;
                 0.01 0.01 0.05 0.03 0.11 0.34 1.12 7.68 75.01 6.41 1.41 0.66 0.30 0.25 0.13 0.04 0.06 0.17;
                 0.01 0.01 0.02 0.05 0.06 0.16 0.31 1.26 9.11 71.63 5.85 2.18 0.92 0.41 0.25 0.17 0.23 0.26;
                 0.05 0.00 0.00 0.03 0.02 0.10 0.08 0.46 1.84 11.51 63.56 7.80 2.95 1.04 0.65 0.26 0.43 0.36;
                 0.00 0.00 0.04 0.01 0.00 0.07 0.05 0.19 0.56 2.26 9.67 64.74 8.13 2.34 1.07 0.35 0.60 0.58;
                 0.00 0.00 0.00 0.01 0.01 0.01 0.05 0.11 0.25 0.39 1.87 9.34 63.09 8.64 3.19 0.83 0.75 1.05;
                 0.00 0.01 0.00 0.03 0.00 0.03 0.07 0.05 0.06 0.12 0.31 1.51 8.07 63.14 8.91 2.55 1.76 2.15;
                 0.00 0.00 0.01 0.01 0.00 0.04 0.05 0.02 0.07 0.04 0.14 0.26 1.28 7.94 61.36 8.55 4.17 3.89;
                 0.00 0.00 0.00 0.00 0.02 0.04 0.00 0.08 0.06 0.12 0.10 0.18 0.47 2.32 10.16 53.36 11.77 7.49;
                 0.00 0.00 0.00 0.00 0.03 0.00 0.10 0.06 0.06 0.06 0.03 0.16 0.44 1.08 2.73 9.11 43.97 26.78;
                 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 100];

std=             [7.14 6.00 3.30 1.04 0.45 0.56 0.35 0.00 0.25 0.00 0.17 0.19 0.00 0.00 0.17 0.00 0.35 0.00;
                 3.61 10.60 7.14 4.10 2.38 0.85 0.49 0.25 0.68 0.23 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00;
                 0.51 1.57 8.79 6.14 2.60 1.23 0.65 0.82 0.35 0.24 0.16 0.13 0.10 0.12 0.00 0.09 0.15 0.09;
                 0.13 0.31 4.32 7.31 4.85 2.61 0.83 0.50 0.45 0.25 0.20 0.00 0.00 0.15 0.38 0.00 0.00 0.10;
                 0.00 0.20 0.70 2.56 5.41 3.02 1.49 0.65 0.44 0.19 0.17 0.25
```

```
                 0.05 0.19 0.14 0.00 0.00 0.15;
                 0.13 0.14 0.51 0.48 2.02 5.14 3.04 1.69 0.94 0.39 0.21 0.28 0.31 0.34 0.10 0.00 0.06 0.12;
                 0.20 0.05 0.15 0.28 0.62 3.24 5.63 3.08 1.54 0.63 0.34 0.36 0.24 0.30 0.08 0.08 0.15 0.19;
                 0.00 0.05 0.16 0.19 0.45 1.04 3.01 5.92 3.35 1.44 0.53 0.59 0.22 0.43 0.31 0.10 0.18 0.27;
                 0.07 0.07 0.14 0.13 0.22 0.69 0.99 3.19 4.42 2.31 1.06 0.61 0.49 0.46 0.39 0.10 0.13 0.28;
                 0.08 0.05 0.06 0.21 0.17 0.40 0.58 1.15 3.25 5.16 2.59 1.52 0.78 0.83 0.48 0.46 0.57 0.41;
                 0.23 0.00 0.00 0.13 0.09 0.40 0.29 0.72 1.87 4.47 5.95 4.32 1.96 1.64 1.19 0.39 0.96 0.64;
                 0.00 0.00 0.22 0.07 0.00 0.38 0.22 0.43 0.87 2.19 4.36 5.24 2.96 1.53 1.36 0.59 1.00 0.69;
                 0.00 0.00 0.00 0.10 0.08 0.08 0.29 0.25 0.45 0.65 1.67 3.91 5.48 3.85 1.60 0.85 0.85 1.45;
                 0.00 0.06 0.00 0.14 0.00 0.09 0.21 0.13 0.17 0.21 0.35 1.09 3.48 5.67 3.66 1.31 1.65 2.04;
                 0.00 0.00 0.09 0.06 0.00 0.21 0.39 0.08 0.31 0.11 0.39 0.59 1.27 3.26 7.42 3.46 3.36 4.30;
                 0.00 0.00 0.00 0.00 0.32 0.31 0.00 0.35 0.19 0.46 0.47 0.92 0.92 2.32 5.52 7.13 4.28 6.37;
                 0.00 0.00 0.00 0.00 0.24 0.00 0.39 0.49 0.33 0.39 0.24 0.53 0.79 1.60 3.12 5.65 9.03 11.48;
                 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];

% We want probabilities in the range 0 to 1.

transition_1y=transition_1y/100;
transition_plus_1y=transition_1y+std/100;
transition_minus_1y=transition_1y-std/100;

% Set negative entries to 0, only the matrix with minus one standard deviation can have negative entries.

transition_minus_1y=transition_minus_1y.*(transition_minus_1y>0);

% Normalize such that the rows add to 1

for i=1:17
    transition_1y(i,:)=transition_1y(i,:)/sum(transition_1y(i,:));
    transition_plus_1y(i,:)=transition_plus_1y(i,:)/sum(transition_plus_1y(i,:));
    transition_minus_1y(i,:)=transition_minus_1y(i,:)/sum(transition_minus_1y(i,:));
end

% We raise the 1-year transition matrices to the 5th power to retrieve the 5-year transition rate matrices.

transition_5y=transition_1y^5;
transition_plus_5y=transition_plus_1y^5;
transition_minus_5y=transition_minus_1y^5;
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```
% The last column is the 5-year probability of default.

default_probability_5y=transition_5y(:,end);
default_probability_plus_5y=transition_plus_5y(:,end);
default_probability_minus_5y=transition_minus_5y(:,end);

% In this simplified case, the bond price is just 1 minus the
% probability of default.

bond_prices=1-default_probability_5y
bond_prices_plus=1-default_probability_plus_5y
bond_prices_minus=1-default_probability_minus_5y
```