Quantitative Risk Management
Assignment 7

Question 1: Let X_1 and X_2 have the following marginal lognormal distributions: $\log(X_1) \sim \mathcal{N}(0, 1)$ and $\log(X_2) \sim \mathcal{N}(0, \sigma^2)$. Find the minimum and maximum attainable correlations between X_1 and X_2. Plot these values as a function of σ for $\sigma \in (0, 5)$.

Question 2: Recall that $\lambda_l(X_1, X_2) = \lim_{q \to 0^+} \frac{C(q,q)}{q}$. Find the corresponding expression for $\lambda_u(X_1, X_2)$.

Question 3: Compute $\lambda_u(X_1, X_2)$ and $\lambda_l(X_1, X_2)$ in terms of θ for the Gumbel and Clayton copulas.

Question 4. Take the futures data from Moodle. It contains the time series for multiplicatively back-adjusted futures prices on Wheat, Corn, Dow Jones, British Pound, Swiss Frank, SP500, and Nikkei.

- For each of these, build the returns as $X_i = p_{i,t}/p_{i,t-1} - 1$.
- For each time series, replace the returns with their historical ranks. Now each return is a number between 0 and T_i, where T_i is the number of observations for the i-th futures contract. Divide them by T_i. Now, each return has become a number on $[0, 1]$. This corresponds to the map $X \to F_X(X)$, and the transformed returns are now uniformly distributed on $[0, 1]$.
- You can now study their copulas $C_{i,j}(x_1, x_2)$ for each i, j: for each pair of transformed returns, plot the scatter plot of their returns. What do these scatter plots look like? Does it look elliptical?
- Now, estimate the pairwise correlations between the returns X_i. Based on the estimated correlations, obtain samples from a bi-variate Gaussian copula with the same correlation, show the scatter plots and compare them to the scatter plots you obtained in the previous point. What do you observe?
- Plot $C(q, q)/q$ for a grid of $q \in [0, 1]$. What does λ_l look like for each pair?
- Based on the formula you derived for λ_u in Question 2, repeat the analysis as in the previous point: What does λ_u look like for each pair?