Quantitative Risk Management
Assignment 9

Question 1: Show that the Fréchet bounds hold:

$$\max\left\{1-d+\sum_{i=1}^{d}u_i,0\right\} \leq C(u_1,\ldots,u_d) \leq \min\{u_1,\ldots,u_d\}.$$

Show that, for $d=2$, the Fréchet lower bound is the distribution of $(U,1-U)$, where $U\sim Unif(0,1)$: this is called the counter-monotonicity copula.
Provide an example (e.g. for $d=2$) for which the Fréchet upper bound is attained.

Question 2: Consider the bivariate random vector (X,Y) of non-negative random variables which have joint distribution given by:

$$F_{X,Y}(x,y)=\frac{1}{1+e^{-x}+e^{-y}}.$$

1. Compute the marginal distributions of X and Y.
2. Find an expression for the copula $C(u,v)$ of the two random variables.
3. Check that it is a valid copula.

Question 3: Let $(X_i)_{i\in\mathbb{N}}$ be independent with distribution F.

1. Let $F(x)=1-\exp(-\beta x)$ for $\beta>0$ and $x\geq 0$ so that X_i is exponentially distributed. Show that $F\in MDA(H_0)$ by using the sequences $c_n=\frac{\log(n)}{\beta}$ and $d_n=\frac{1}{\beta}$.
2. Let $F(x)=1-(\frac{\kappa}{\kappa+x})^\alpha$ for $\alpha>0$, $\kappa>0$, and $x\geq 0$ so that X_i has Pareto distribution. Show that $F\in MDA(H_{1/\alpha})$ by using the sequences $c_n=\kappa n^{1/\alpha}-\kappa$ and $d_n=\frac{\kappa n^{1/\alpha}}{\alpha}$.

Question 4. GANs. Take the daily futures data. Train a GAN on the 7-dimensional vector of returns as your samples, using $p(z)\sim N(0,I),\ z\in\mathbb{R}^7$. So, your latent space is also 7-dimensional. You end up with a trained $G(z;\theta^G)$.
Now, generate 1000 random samples from $p(z)$, and use $G(z^i;\theta^G)$ to generate random potential scenarios.

- compute marginal distributions of generated scenarios and plot them against the true ones.
- compute pairwise empirical copulas and compare them with the true ones. What do you observe?