Quantitative Risk Management
Assignment 5

Question 1: Your dataset includes data at the 2-minute frequency for 10 US stocks.

- Please use data in the bid- and ask-columns.
- Build the mid prices = 0.5(bid+ask). This is c_t.
- First, we build the rolling maximum of the price over five periods (i.e., over a 10-minute interval). This is high over 10 minutes. Call it high prices. This is h_t.
- Now, please do the same for the rolling minimum. This is ℓ_t (the low over the past ten minute-interval).
- Now, sub-sample the data keeping only every 5-th observation. Now you have $[c_t, h_t, \ell_t]$ and a 10-minute frequency. Define $o_t = c_{t-1}$ (that is, the close price ten minutes ago).
- Please be careful not to include overnight returns as they can be huge in magnitude (because overnight = several hours). So, please drop all observations where t and $t-1$ are on different dates.
- Now, pick an interval T from the list $T \in \{10, 20, 100, 200\}$ and implement the Garman-Klass estimator using a rolling window of T periods (each period is 10 minutes). This will be your $\sigma_t(T)$ estimator, based on the data in $[t-T, t]$.
- Now, forecast volatility of h-period returns $\sigma_{t,t+h}$ using

$$E_t[\sigma^2_{t,t+h}] \;=\; h\sigma_t^2, \tag{1}$$

  so that

- Having constructed the σ_t time series, you can now implement the rough volatility estimators: Build

$$E_t[\log(\sigma^2_{t+\Delta})] \;=\; \frac{\cos(H\pi)}{\pi}\Delta^{H+0.5}\int_{-\infty}^{t}\frac{\log(\sigma_s^2)}{(t-s+\Delta)(t-s)^{H+0.5}}ds$$

  where you replace the integral with discrete sum.

- Estimate $Q_t \;=\; \sigma(\log(\sigma_t^2))$ as the realized standard deviation of log volatility over a rolling window T.
- Predict volatility using Gaussian moment generating function:

$$E_t[\sigma^2_{t+\Delta}] \;=\; E_t[e^{\log\sigma^2_{t+\Delta}}] \;=\; e^{E_t[\log(\sigma^2_{t+\Delta})]+0.5Q_t} \tag{2}$$

- experiment with $H = 0.01, 0.05, 0.1, 0.2, 0.5, 1$.
- Now you can compare the quality of your predictors, first (1) and then (2), using the ratio of the mean-squared error of the predictor and the approximate variance of the variance:

$$P = \sqrt{\frac{\sum_k\left(\sigma^2_{k+\Delta}-\hat\sigma^2_{k+\Delta}\right)^2}{\sum_k\left(\sigma^2_{k+\Delta}-\mathbb{E}[\sigma^2_{t+\Delta}]\right)^2}},$$

  where $\hat\sigma^2_{k+\Delta}$ is the predicted volatility, and $\sigma^2_{k+\Delta}$ is the realised volatility. Lastly, $\mathbb{E}[\sigma^2_{t+\Delta}]$ is the unconditional mean.