Quantitative Risk Management
Assignment 2

Question 1: This question deals with a portfolio of three stocks. At time t, the values of the stocks are $S_{1,t} = 100$, $S_{2,t} = 50$, and $S_{3,t} = 25$. The portfolio consists of 1 share of S_1, 3 shares of S_2, and 5 shares of S_3. The risk factors are logarithmic prices and the risk factor changes have mean zero and standard deviations 10^{-3}, $2 \cdot 10^{-3}$, and $3 \cdot 10^{-3}$ respectively. The risk factors are independent.

1. Compute VaR_α, VaR_α^{mean}, and ES_α using Monte Carlo with 10,000 simulations. Do this for $\alpha = \{0.90, 0.91, \ldots, 0.99\}$. Also use the following distributions for the risk factor changes:
   (a) For each $i \in \{1, 2, 3\}$, $X_{i,t+\Delta} \sim t(3, \mu, \sigma)$ for appropriate values of μ and σ
   (b) For each $i \in \{1, 2, 3\}$, $X_{i,t+\Delta} \sim t(10, \mu, \sigma)$ for appropriate values of μ and σ
   (c) For each $i \in \{1, 2, 3\}$, $X_{i,t+\Delta} \sim t(50, \mu, \sigma)$ for appropriate values of μ and σ
   (d) For each $i \in \{1, 2, 3\}$, $X_{i,t+\Delta}$ has a normal distribution

   and plot the results.
2. Comment on the following:
   (a) The value of VaR_α compared to VaR_α^{mean}.
   (b) The value of VaR_α and ES_α as compared between the four distributions. Are the results what you expected?

Question 2: This question deals with a delta hedged call option. The following are the Black-Scholes parameters for a European call option at time $t = 0$:

$$
\begin{aligned}
T &= 0.5 \\
r_t &= 0.05 \\
\sigma_t &= 0.2 \\
S_t &= 100 \\
K &= 100
\end{aligned}
$$

The portfolio consists of a long position in the call option, and the corresponding position in the stock which makes the portfolio delta neutral. Let $\Delta = 1$day, $Z_1 = \log(S)$, and $Z_2 = \sigma$ (r will be considered unchanging in this problem). The risk factor changes are normally distributed with mean zero. Their standard deviations over one day are 10^{-3} and 10^{-4}, and their correlation is -0.5.

1. Compute VaR_α, VaR_α^{mean} and ES_α for $\alpha = 0.95$ and $\alpha = 0.99$ using the following methods:
   (a) Monte Carlo full revaluation with 10,000 simulations
   (b) Monte Carlo on the linearlized loss with 10,000 simulations
   (c) Variance-covariance method

Do not neglect the time derivative in any linearization in this question.

Question 3: Let L have the Student t distribution with ν degrees of freedom. Derive the formula:

$$ES_\alpha(L) = \left(\frac{g_\nu(t_\nu^{-1}(\alpha))}{1-\alpha}\right)\left(\frac{\nu + (t_\nu^{-1}(\alpha))^2}{\nu - 1}\right)$$

You will need to look up the probability density function of the distribution at hand.

Question 4. In the futures data, you will find back-adjusted futures prices, $P_{i,t}$ for N futures contracts, $i = 1, \cdots, N$.
`https://portaracqg.com/continuous-futures-data/`

- Pick a rolling window T. Say, $T = 256$. Compute the rolling STD and use it to compute the Gaussian VAR based on the formula in class. You will get a time series of GaussianVAR. Plot it together with the next period realized return
- Now, use the non-parametric VAR based on the empirical realized VAR over the same rolling window. Repeat the same plotting exercise. How are they related? Which one is better?