Quantitative Risk Management
Assignment 3

October 11, 2022

Question 1: In this question you will perform an exercise similar to the backtesting example from the lecture slides.

1. Use the data on daily prices of 580 large US stocks available on Moodle (column names are tickers; e.g., AAPL = Apple)
2. Generate a fixed random portfolio $q \in \mathbb{R}^{580}$
3. Pick a rolling window of 250 days. Pick a shrinkage $\theta \in [0, 0.1, 0.3, 0.5]$ (industry standard is 0.5). Estimate the rolling covariance over this period using the chosen shrinkage; call it Σ_t. Then use formulas in the lectures based on $q'\Sigma_t q$ to compute VaR one day ahead. Do this every day.
   (a) Assume risk factor changes (returns) have a multivariate normal distribution.
   (b) On each day that you estimate VaR_α calibrate the mean and covariance of the risk factor changes to the previous 250 days.
4. In the time period, how many times did the loss of the portfolio exceed VaR_α? Does it appear that this method of VaR_α estimation is valid?

Question 2:

1. Suppose L has geometric distribution with parameter p. Here, we consider a geometric distribution that includes 0 in the support.
   (a) If $p = 0.5$, what is $VaR_{0.95}$?
   (b) Plot VaR_α for values of α ranging from 0.9 to 0.99 ($p = 0.5$).
2. Suppose X and Y are independent with Poisson distributions with parameters $\lambda_X = 1$ and $\lambda_Y = 2$. Let $L = X + Y$.
   (a) Plot $VaR_\alpha(X)$ for values of α ranging from 0.9 to 0.99.
   (b) Plot $VaR_\alpha(Y)$ for values of α ranging from 0.9 to 0.99.
   (c) Plot $VaR_\alpha(L)$ for values of α ranging from 0.9 to 0.99.

Question 3: Give an example that shows that VaR_α is not subadditive. That is, find two random variables L_1 and L_2 with a joint distribution such that the following does not hold:

$$VaR_\alpha(L_1 + L_2) \leq VaR_\alpha(L_1) + VaR_\alpha(L_2)$$

Question 4: Consider $d = 100$ defaultable corporate bonds having each a face value of CHF1000, an annual coupon of 5% and a time to maturity of one year. Suppose that the current price of each bond is CHF1000; they all trade at par. Assume that defaults on the different bonds are independent; the default probability is identical for all bonds and is equal to 0.02. Denote by L_i the loss on a bond of company i over the next year and I_i the default indicator of firm i ($I_i = 1$ if firm i defaults).

1. Write L_i in terms of the risk factor I_i.
2. What is the probability distribution of L_i?
3. Compare the following two portfolios, each worth CHF100000: V_a consists of 100 units of a single bond, and V_b consists of 1 unit of each bond. Write L_a and L_b in terms of the risk factors. What are the distributions of L_a and L_b? Compute VaR_α for both portfolios with $\alpha = 0.95$ and $\alpha = 0.99$. Comment on the results.