UNIVERSITY OF EXETER

Modelling of Dynamical Systems

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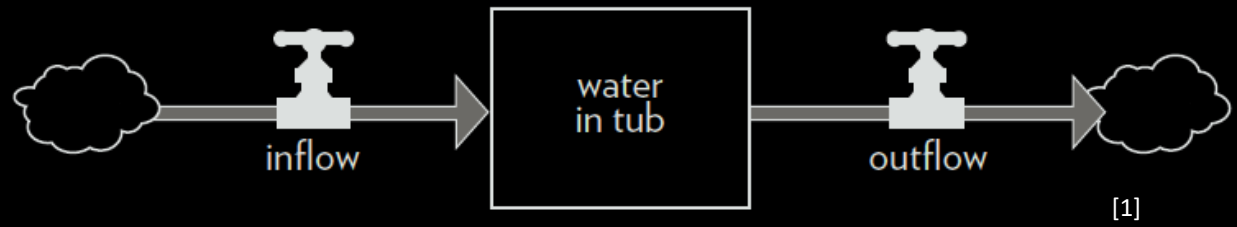
Motivation

Stock

Flow

influence

The Bathtub Example



[1]

Differential Equation

$$\frac{dW}{dt} = i - o$$

Initial Conditions:

$$W(0) = 0$$

Parameters:

$$i = 1\frac{l}{s}, \qquad o = 0.9\frac{l}{s}$$

Analytical Solution

$$W(t) = (i - o) * t$$

How to model this?

Differential Equation

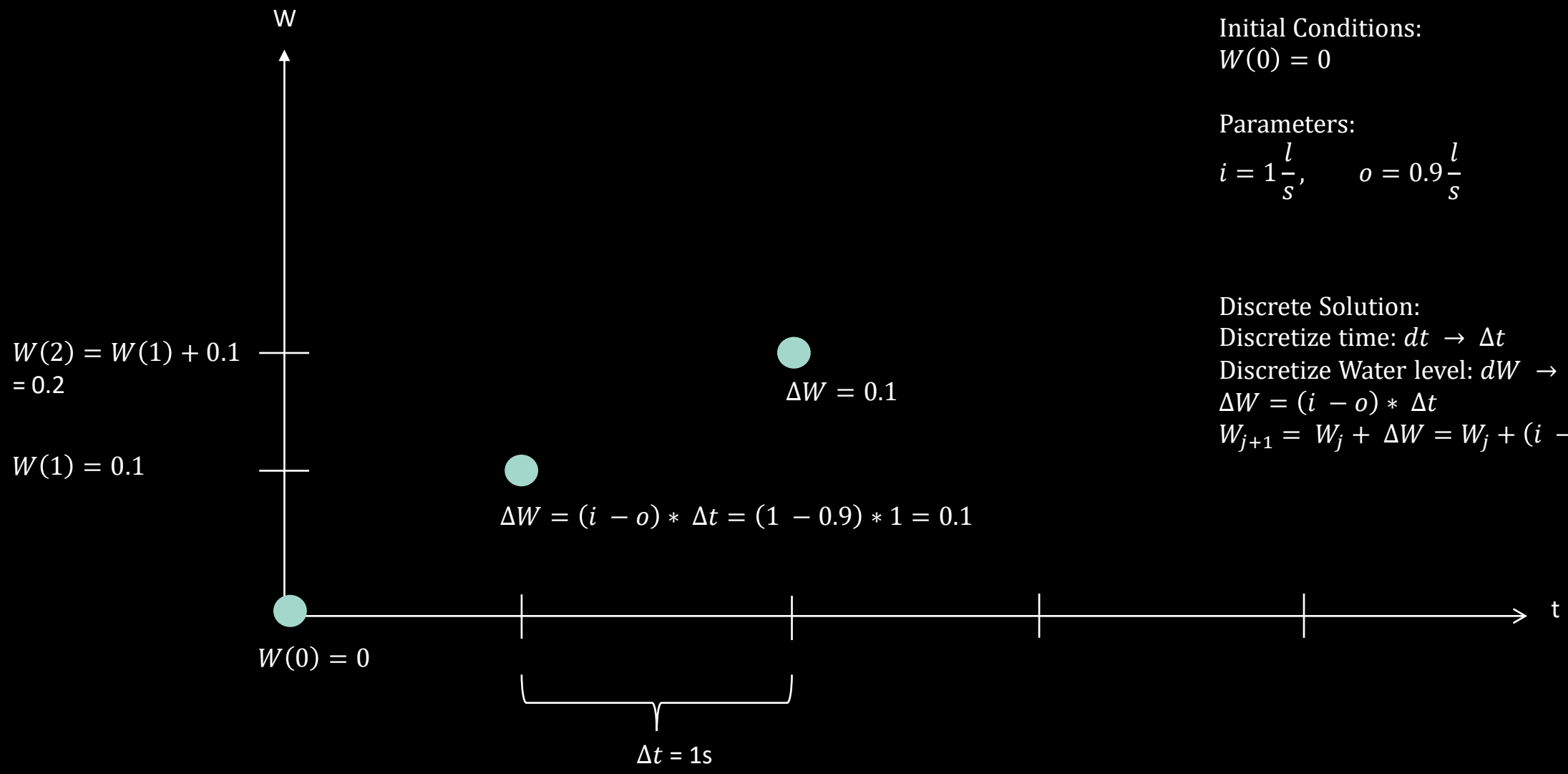
$$\frac{dW}{dt} = i - o$$

Initial Conditions:

$W(0) = 0$

Parameters:

$i = 1\frac{l}{s}, \qquad o = 0.9\frac{l}{s}$



Discrete Solution:

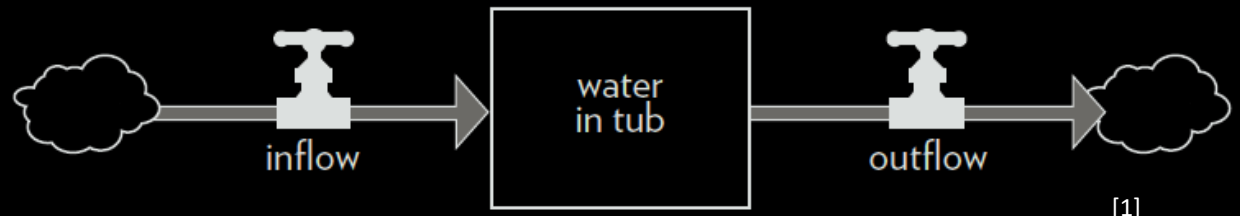
Discretize time: $dt \rightarrow \Delta t$

Discretize Water level: $dW \rightarrow \Delta W$

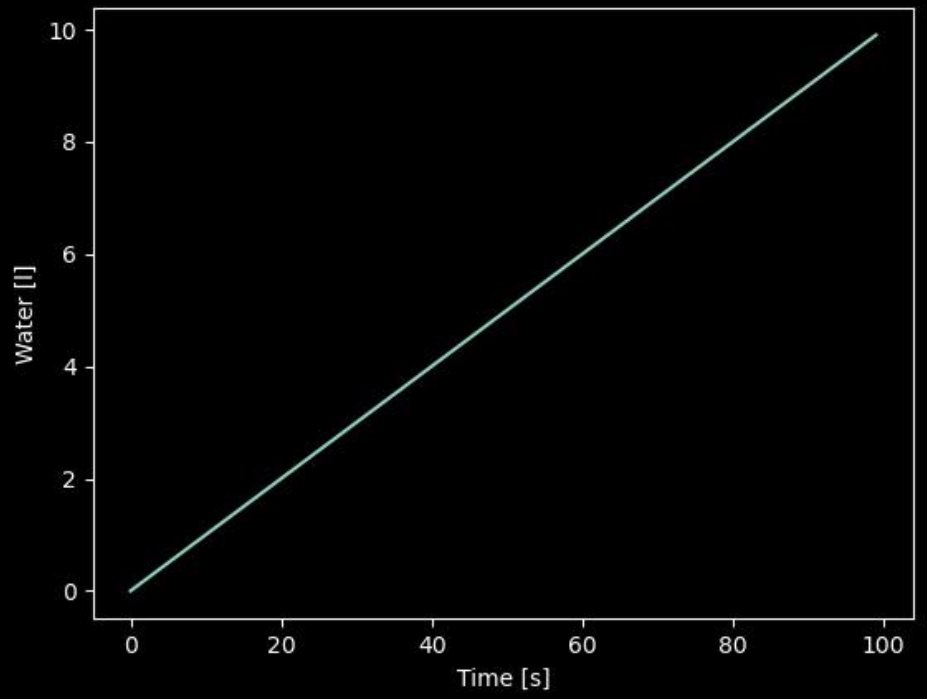
$\Delta W = (i - o) * \Delta t$

$W_{j+1} = W_j + \Delta W = W_j + (i - o) * \Delta t$

Model Output for Bathtub Example



[1]

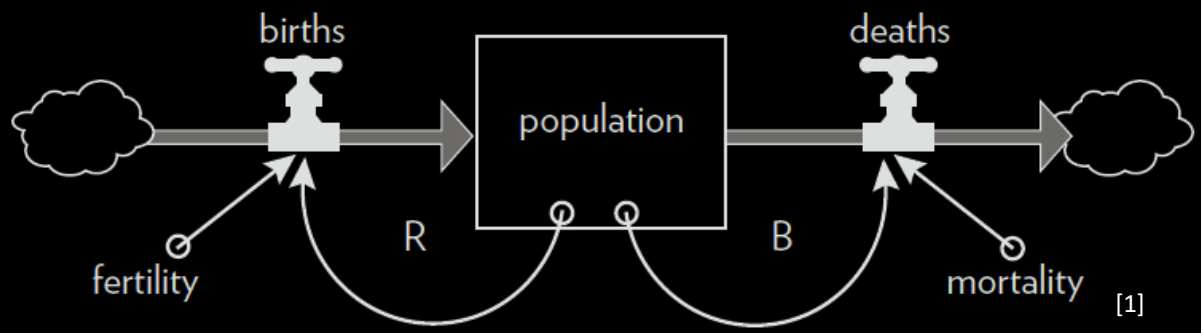


Parameters:

$$i = 1\frac{l}{s}, \qquad o = 0.9\frac{l}{s}$$

Code provided under:
GitHub

The Population Example



[1]

Differential Equation:

$$\frac{dP}{dt} = b(P) - d(P)$$

$$b(P) = f * P, \qquad d(P) = m * P$$

Initial Conditions:

$\mathrm{P}(0) = 1000\ \mathrm{P}$

Parameters:

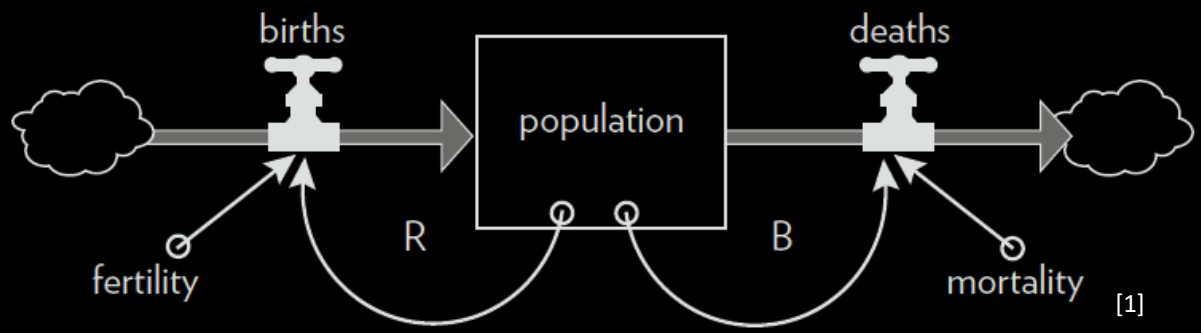
$$f = \frac{1.5}{\text{lifetime}}, \qquad m = \frac{1}{\text{lifetime}}$$

Discrete Solution:

$$\Delta P = b(P_j) - d(P_j)$$

$$\begin{aligned} P_{j+1} &= P_j + \Delta P \\ &= P_j + b(P_j) - d(P_j) \\ &= P_j + f * P_j - m * P_j \\ &= P_j * (1 + f - m) \end{aligned}$$

Model Output for Population Example

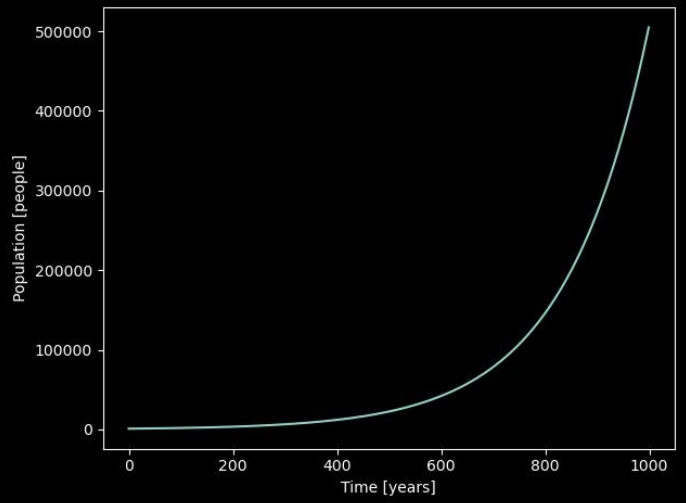


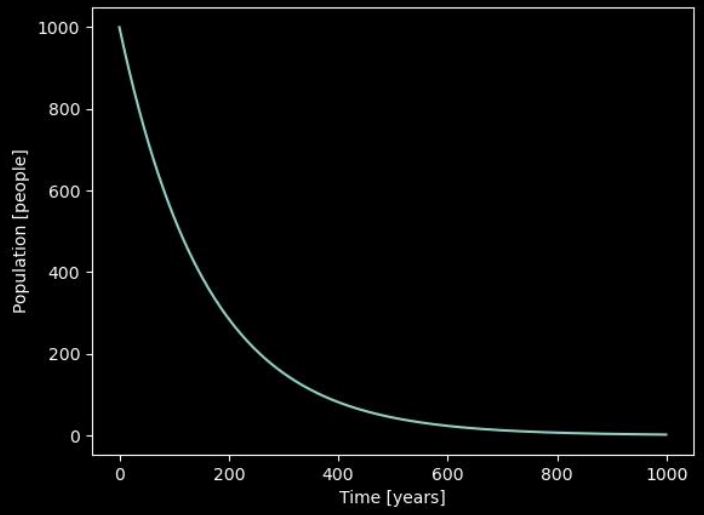
[1]

Parameters:

$$f = \frac{1.5}{\text{lifetime}}$$

$$m = \frac{1}{\text{lifetime}}$$





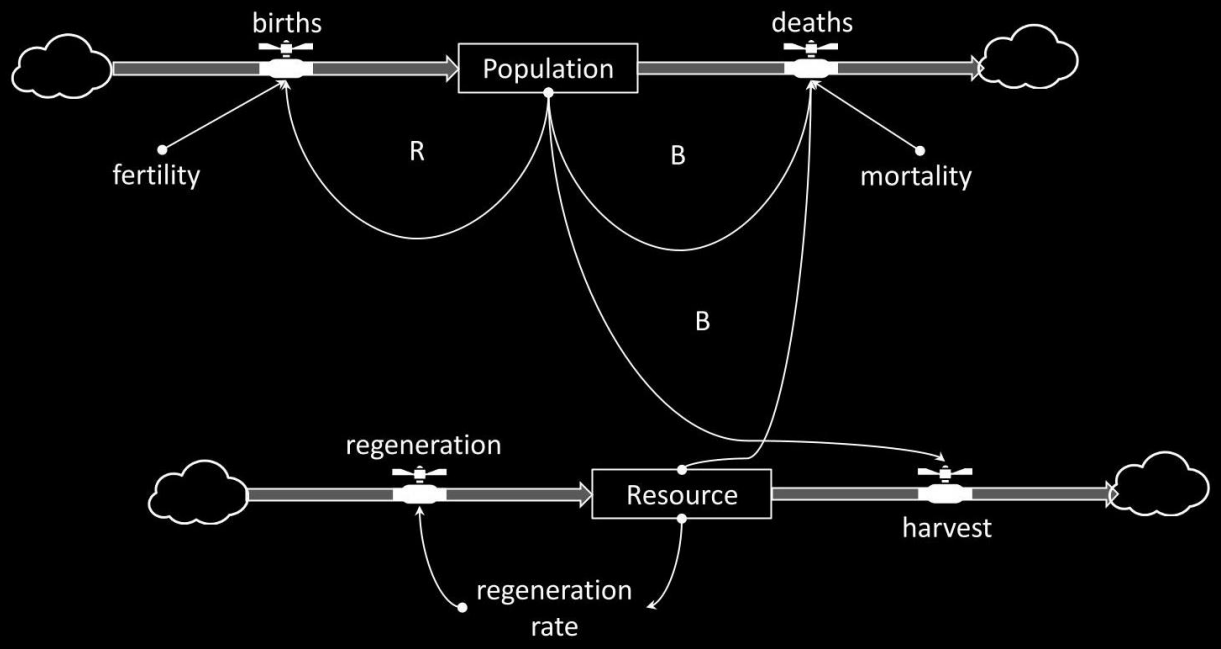
Parameters:

$$f = \frac{0.5}{\text{lifetime}}$$

$$m = \frac{1}{\text{lifetime}}$$

GitHub

The Population – Resource Example



Differential Equation:

$$\frac{dP}{dt} = b(P) - d(P,R)$$

$$b(P) = f * P$$

$$d(P) = \hat{m}(R) * P$$

$$\hat{m}(R) = m * (1 + \frac{R_0 - R}{R_0})$$

$$\frac{dR}{dt} = \hat{r}(R) - h * P$$

$$\hat{r}(R) = r * (R_0 - R)$$

Initial Conditions:

$P(0) = 10$ P

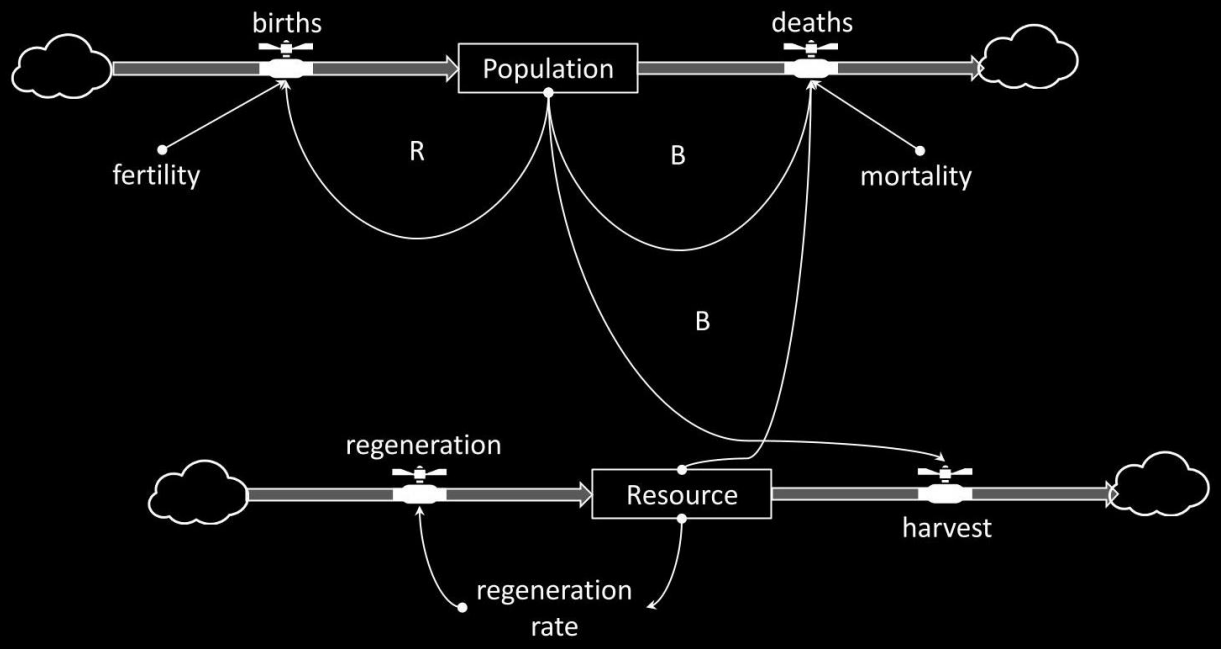
$R(0) = 1000$

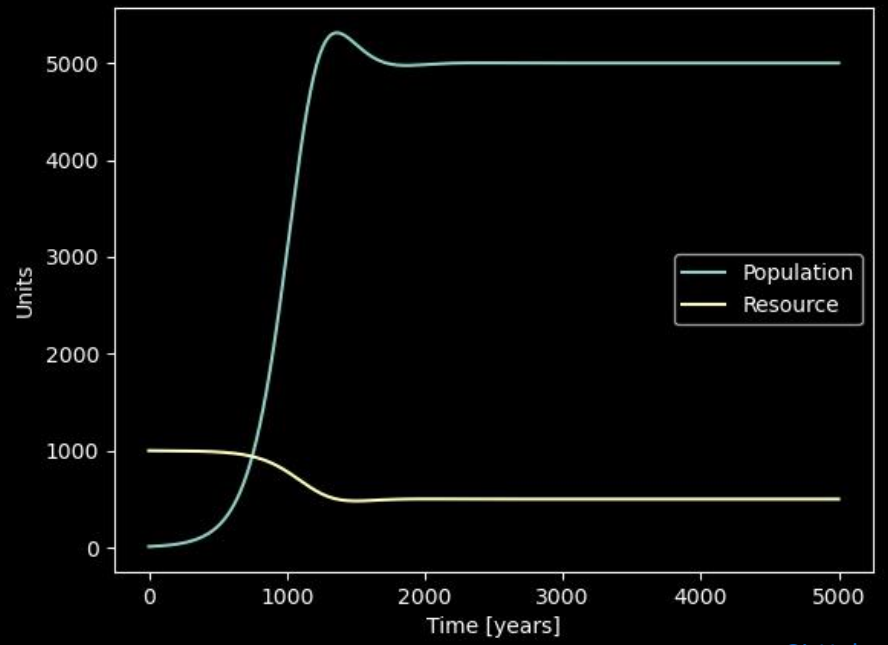
Parameters:

$$f = \frac{1.5}{\text{lifetime}}, \qquad m = \frac{1}{\text{lifetime}}$$

$r = 0.01, \qquad h = 0.001$

Model Output for Population – Resource Example





GitHub

Parameters:

$f = \frac{1.5}{\text{lifetime}}$, $m = \frac{1}{\text{lifetime}}$

$r = 0.01$, $h = 0.001$

References

Books:

[1] Meadows, Donella H (2009): Thinking in Systems – A Primer (Earthscan, UK)

Video Snippets:

https://www.youtube.com/watch?v=dvvd5RHmp7c
https://www.youtube.com/watch?v=qu0HN9rYtIw
https://www.youtube.com/watch?v=Od6EeCWytZo