TOTIMERSØVING NR 3 TEP 4105 FLUIDMEKANIKK

Høst 2015
Utført av: (alle i gruppa)

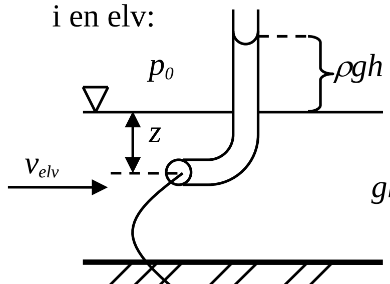
LØSNINGSFORSLAG

Problemene søkes løst på en enkel måte, (Kap. 3,) med kontrollvolumanalyse, Bernoulli. Gjør nødvendige antagelse og forenklinger, eventuelt begrunn hvorfor oppgaven ikke lar seg løse. Vær presis på hvilke likninger/prinsipper som ligger til grunn.

Oppgave 1

Finn strømningshastigheten ut fra avlest høyde H i et rør strukket inn i strømningen (Pitot-rør).

i en elv:



$$\rho g h = p_s - p(z)$$
$$\Downarrow$$
$$gh = \frac{1}{2} v_{elv}^2$$

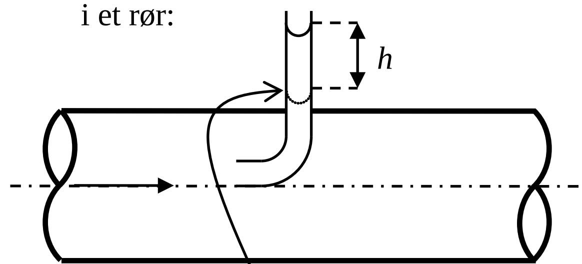
$p(z) = p_0 + \rho g h$

statisk

Bernoulli:

$$p_s = p(z) + \frac{1}{2}\rho v_{elv}^2 = \text{statisk} + \text{dynamisk}$$

i et rør:



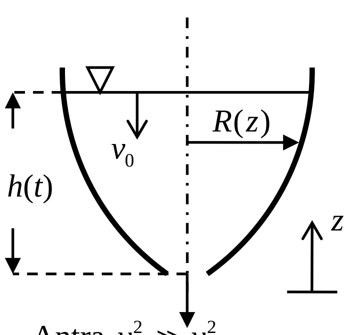
Kan ikke løses uten å vite det statiske trykket i røret.

Oppgave 2

Du skal designe et vanntimeglass (et aksesymmetrisk kar) slike de brukte i oldtiden. Finn radiusen $R(z)$ som gjør at fluidoverflaten synker med <u>konstant</u> hastighet v_0. Arealet på utløpet er A_{ut}.

Svar: Massebevarelse: $q = v_{ut} \cdot A_{ut} = v_0 \pi R(z)^2$

Kinematisk: $v_0 = -\dfrac{\mathrm{d}h}{\mathrm{d}t}$

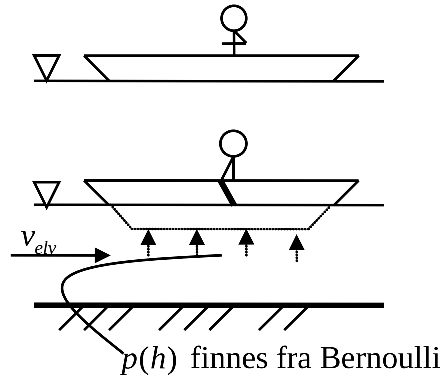


Antra $v_{ut}^2 \gg v_0^2$

Bernoulli: $v_{ut} \approx \sqrt{2gh(t)}$

$$\rightarrow R(z) = \sqrt{\frac{\sqrt{2gz}A_{out}}{v_0 \pi}}$$

Oppgave 3



En kajakk stikker en dybde h ned i vannet. Hvordan vil du beregne h dersom kajakken ligger i ro? Hvordan vil du beregne h dersom kajakken padles motstrøms opp en elv? I hvilket av de to tilfellene er h størst?

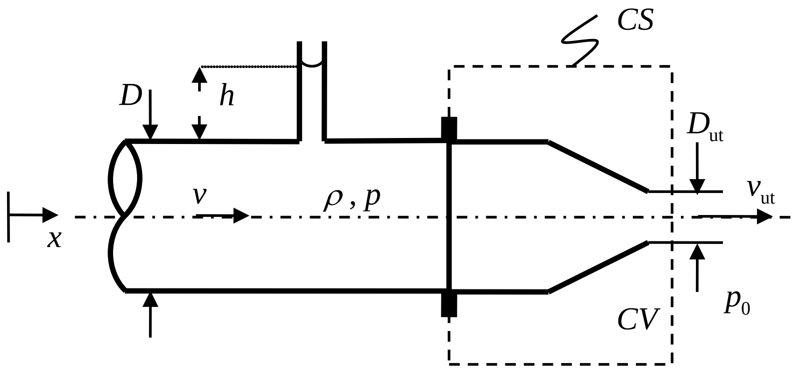
Svar:
Stillestående: Ok med Archimedes:
mg = vekt av fortrengt fluid

Padlende: Vanne akselereres langs strømlinje under båten ("dynamisk trykk.") Archmedes ikke ok – bruk Bernoulli langs en strømlinje. Siden vannhastigheten under båten er større enn foran og bak båten (sett relativt til båten) vil båtens bevegelse føre til en trykkreduksjon på undersiden slik at h øker.

Oppgave 4

Finn kraften som virker *på* dysen ut fra avlest høyde h når dysens innløpsdiameter er D og utløpsdiameter er D_{ut}. Volumstrøm/hastigheter er ukjent. Utløps- og manometertrykket er atmosfærisk.

Svar:



Massebevarelse

$$q = v\frac{\pi}{4}D^2 = v_{\text{ut}}\frac{\pi}{4}D_{\text{ut}}^2$$

Bernoulli:

$$\frac{p}{\rho} + \frac{v^2}{2} = \frac{p_0}{\rho} + \frac{v_{\text{ut}}^2}{2}$$

Hydrostatisk trykk lest fra h: $p = p_0 + \rho g\left(h + D/2\right)$

$$\rightarrow v = \sqrt{\frac{2g(\text{h} + D/2)}{(D/D_{out})^4 - 1}}$$

Kraftlov:

$$\sum F_x = F_{\text{kontakt}} + (p - p_0)\frac{\pi}{4}D^2 = -\rho v^2\frac{\pi}{4}D^2 + \rho v_{\text{ut}}^2\frac{\pi}{4}D_{\text{ut}}^2$$

$$\underline{\underline{F_{\text{dyse}} = -F_{\text{kontakt}}}}$$

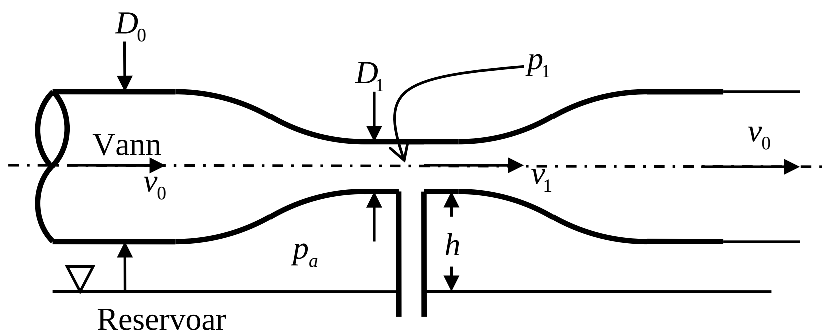
Oppgave 5

Finn vannhastigheten v_0 i røret slik at vannet i reservoaret akkurat løftes høyden h i det vertikale uten å strømme inn i det horisontale røret. Diameteren er D_0 i den brede delen og D_1 i den smale. Trykket på utløpet (høyre side) er atmosfærisk p_a.

Svar: Massebevarelse: $q = v_0 \frac{\pi}{4} D_0^2 = v_1 \frac{\pi}{4} D_1^2$

Bernoulli langs en strømlinje fra 1 til utløp: $\frac{p_1}{\rho} + \frac{v_1^2}{2} = \frac{p_a}{\rho} + \frac{v_0^2}{2}$.

Likevekt: $\rho g h = p_a - p_1$. $\rightarrow v_0 = \sqrt{\frac{2gh}{D_0^4 / D_1^4 - 1}}$



Oppgave 6

En vannturbin drives av en vannstråle med hastighet v_j. Finn turtallet Ω som gir maksimum effekt.

Svar: Vi plasserer et kontrollvolum rundt skovlen som beveger seg med rotasjonshastigheten $v_\Omega = \Omega R$

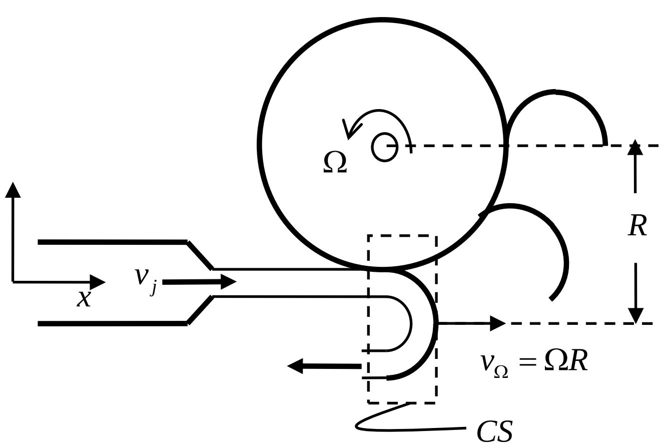
Massebevarelse: $v_{\text{inn}} = v_{\text{ut}}$.

Bernoulli: $p = p_a$ overalt.

Kraftlov:

$$\sum F_x = F_{\text{kontakt}} = \oiint_{CS} \rho v_x (\vec{v}_j \cdot \vec{n}) \mathrm{d}A$$

$$= \rho A (v_j - v_\Omega)^2 + \rho A (v_j - v_\Omega)^2$$

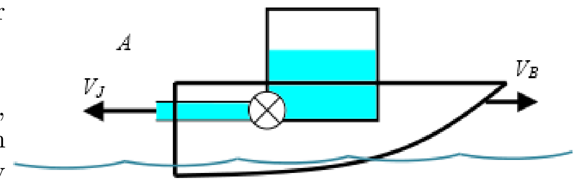


Effekt: $P = F v_\Omega = 2\rho A (v_j - v_\Omega)^2 v_\Omega$

Maks effekt for $\frac{\mathrm{d}P}{\mathrm{d}v_\Omega} = 0$, så $\Omega = \frac{1}{3} \frac{v_j}{R}$

1 Superspeedy jet powered boat

This question was originally given in the coarse TEP4100 and dealt with solving the same problem in three different frames of reference. In TEP4105 we omit some of these generalisations, only asking for a solution in the first of the three reference frames presented below. The full version is however given here for students wishing to better understand control volume analysis.



A boat is driven by a water jet. The jet is powered by a water pump, pumping water backwards with velocity V_J (relative to the boat) through a pipe of area A. The boat has reached a constant ('terminal') velocity V_B. Find the force with which the jet pushes the boat. Let the CV follow the boat and check that you obtain the same answer using both absolute and relative coordinate systems. Challenge: compute the force with a steady CV.
Solution:
There are three main choices for the coordinate system and CV for this problem:

(i) Coordinate system and CV moves relative to the boat

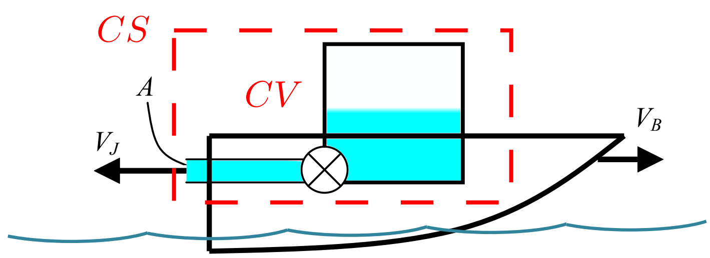
(ii) A stationary coordinate system (relative to an observer on land) and a CV moving relative to the boat.

(iii) Stationary Coordinate system and CV.

The first option yields the easiest path for computing the pressure and the third the most cumbersome.

In all approaches we apply a momentum balance (Newton's 2nd law) to the CV

$$\text{Time rate of momentum change (generalized 'acceleration} \cdot \text{mass')} = \frac{\mathrm{d}}{\mathrm{d}t}\int_{CV}\rho\vec{V}\,\mathrm{d}V + \oint_{CS}\rho\vec{V}\left(\vec{V}\cdot\vec{n}\right)\,\mathrm{d}A = \sum_{CV}\vec{F}_\text{external} \tag{1}$$



(i) **Relative coordinate system and CV**

The flow out of the CV moving with the boat is

$$\dot{\mathcal{V}} = AV_\mathrm{J},$$

where V_J was the velocity relative to the boat. Momentum is always taken *relative to the coordinate system.* The momentum per volume M of the of the water exiting the CV is $\rho\left(-V_\mathrm{J}\right)$. The volume flow and momentum are constant where the jet exits the CS and zero everywhere else. Consequently, the convective integral of (1) evaluates to

$$\oint_{CS}\rho\vec{V}\left(\vec{V}\cdot\vec{n}\right)\,\mathrm{d}A = \rho\left(-V_\mathrm{J}\right)\dot{\mathcal{V}} = -\rho AV_\mathrm{J}^2$$

Relative to the coordinate system the liquid and material inside the CV are stationary. Therefore, the momentum inside the CV does not change with time, and the transient term in (1) evaluates to zero.

Equation (1) expresses the more general form of Newton's 2nd law: the time rate change of the total momentum in the CV equals the forces exserted on the CV – in this case those forces are exserted from the boat, and originates form friction and wave generation. (1) then gives $\sum_{CV}\vec{F}_\text{external} = -\rho AV_\mathrm{J}^2$. By Newton's 3rd law, the force acting on the boat *from* the jet are equal but oppositely directed

$$\underline{\underline{F_\text{jet} = \rho AV_\mathrm{J}^2}}$$

Question: What about the forces from the pump?
Answer: The forces from the pump are internal. That is, also the *reactive* force from the pump is part of the CV and so these cancel out. Here, the force from the boat and the force from the pump are in fact the same.

Question: What makes forces internal (ignorable) and external (important)
Answer: A CV force is external if its reactive counterpart is *outside* the CV. This is the case just along the CS and for external force fields such as gravity and external magnetic fields. (When gravity pulls at a CV the CV pulls back at *the earth.*)

(ii) **Stationary coordinate system, relative** CV

The volume flow out of the CV will still be the same as in (i), namely $\dot{\mathcal{V}} = AV_{\mathrm{J}},$.
Relative to the coordinate system the momentum of the water exiting the CV is now $\rho\left(V_{\mathrm{B}} - V_{\mathrm{J}}\right)$,
yielding

$$\oint_{CS} \rho\vec{V}\left(\vec{V}\cdot\vec{n}\right)\,\mathrm{d}A = \rho\left(V_{\mathrm{B}} - V_{\mathrm{J}}\right)\dot{\mathcal{V}} = \rho A V_{\mathrm{J}}\left(V_{\mathrm{B}} - V_{\mathrm{J}}\right)$$

The water in the water jet tank now has a momentum relative to the coordinate system equalling ρV_{B} per unit volume. Its rate of depletion is the rate at which the water is removed from the CV, namely $\dot{\mathcal{V}} = AV_{\mathrm{J}}$. The time rate *increase* of momentum in the first integral of (1) is then

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{CV}\rho\vec{V}\,\mathrm{d}V = \rho V_{\mathrm{B}}\left(-\dot{\mathcal{V}}\right) = -\rho A V_{\mathrm{J}} V_{\mathrm{B}}.$$

As expected, when these integrals are summed, the reactive force is found to be

$$\underline{\underline{F_{\mathrm{jet}} = \rho A V_{\mathrm{J}}^2}}$$

(iii) **Stationary coordinate system and** CV

This one is more tricky. In evaluating the convection integral through a stationary CV with the momentum at the CS relative to a stationary coordinate system we find

$$\oint_{CS} \rho\vec{V}\left(\vec{V}\cdot\vec{n}\right)\,\mathrm{d}A = \underbrace{\left(V_{\mathrm{J}} - V_{\mathrm{B}}\right)A}_{\dot{\mathcal{V}}}\cdot\underbrace{\rho\left(-V_{\mathrm{J}} + V_{\mathrm{B}}\right)}_{\text{momentum}} = -\rho A\left(V_{\mathrm{J}} - V_{\mathrm{B}}\right)^2$$

The momentum per unit volume in the jet tank is still ρV_{B} as in (ii), and its rate of depletion form the tank is $V_{\mathrm{J}}A$, making the momentum time rate *increase* in the tank $\rho V_{\mathrm{B}}\cdot\left(-V_{\mathrm{J}}A\right)$. This does not sum to $\rho A V_{\mathrm{J}}^2$, so where has the last momentum gone to? The answer is that as the boat passes through a stationary CV it fills the CV with the ejected water. The momentum of the expelled water filling the CV is also $\rho\left(-V_{\mathrm{J}} + V_{\mathrm{B}}\right)$ and the rate at which the CV is filled with the expelled water is $V_{\mathrm{B}}A$ (imagine that the boat leaves behind it a fixed pipe shaped jet as it traverses at velocity V_{B}). The transient integral thus evaluates to

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{CV}\rho\vec{V}\,\mathrm{d}V = \underbrace{\rho V_{\mathrm{B}}\cdot\left(-V_{\mathrm{J}}A\right)}_{\text{in tank}} + \underbrace{\rho\left(-V_{\mathrm{J}} + V_{\mathrm{B}}\right)\cdot V_{\mathrm{B}}A}_{\text{in } CV\text{, outside tank}}$$

Together, the transient and convective integrals evaluate to $-\rho A V_{\mathrm{J}}^2$, yielding the appropriate jet force

$$\underline{\underline{F_{\mathrm{jet}} = \rho A V_{\mathrm{J}}^2}}$$