NTNU

TEP4105: Fluidmekanikk

Øving 12:
Høst 2014

Oppgave 1:

The complex potential

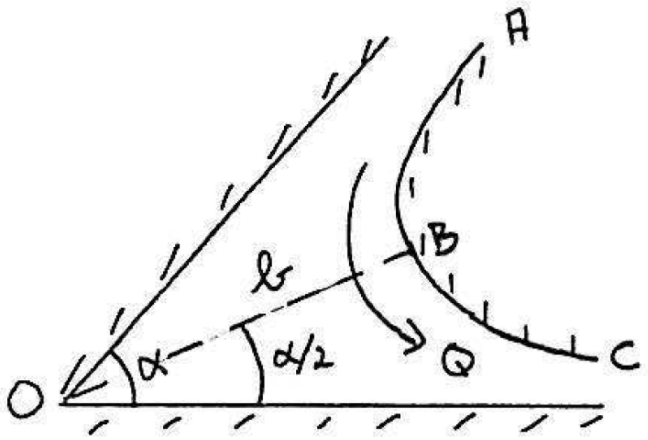
$$w(z) = Uz^{\frac{\pi}{\alpha}} \tag{1}$$

is given, where $U(>0)$ is a constant. α is a given angle on the interval $0 < \alpha < \pi/2$. The potential describes an ideal two-dimensional model for the flow in a sharp corner with angle α.

a

Set $z = r\exp(i\theta)$ and find the velocity potential Φ and the stream function Ψ, as well as the velocity components v_r and v_θ; all as a function of r and θ. Sketch the flow.

b



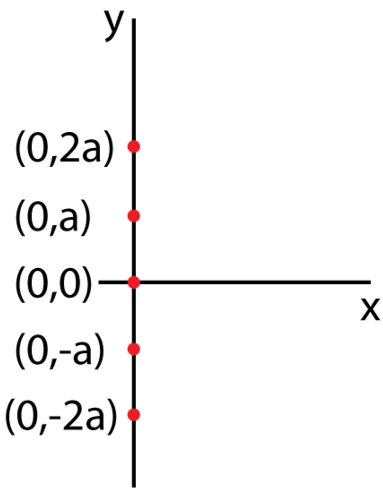
Assume that one of the stream lines (ABC in the above figure) is replaced by a solid surface. The distance between the corner O and the closest point B on the surface (corresponding to $\theta = \alpha/2$) is given as b. Find the volumetric flow Q in the channel, expressed by U, b and α.

c

Find the pressure in the fluid, when the pressure in O is known to be p_0. The density of the fluid is assumed to be constant and gravity can be neglected. Is the answer realistic for large values of r?

Oppgave 2:

An infinite number of sources are placed in positions $(0, 0), (0, \pm a), (0, \pm 2a), \ldots$, as shown below.



Find the complex potential $w(z)$ in the form of an infinite series and show, with help of the formula,

$$\sinh\left(\frac{\pi z}{a}\right) = \frac{\pi z}{a} \prod_{n=1}^{\infty} \left(\frac{z^2 + n^2 a^2}{n^2 a^2}\right), \tag{2}$$

that $w(z)$ can be written as

$$w(z) = \mathcal{C} \ln \sinh \frac{\pi z}{a}, \tag{3}$$

where $\mathcal{C}$ is a real constant. What are Φ and Ψ in this case?

Given:

$$\sinh z = \sqrt{\left(\frac{1}{2}\left(\cosh 2x - \cos 2y\right)\right)} \exp\left[i \arctan \frac{\tan y}{\tanh x}\right], \tag{4}$$

where $z = x + iy$.