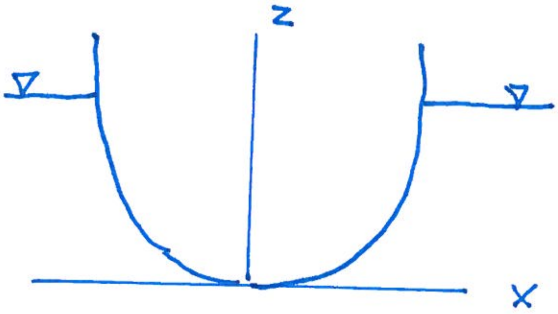
Løsning Oppgave 1



z er høyde over skipets bunn.

a) Oppdriftskraft $\rho g V$ må ved likevekt balansere tyngden mg:

$$\rho g V = mg \qquad \rho = 10^3\ \mathrm{kg/m^3}\ \text{(ferskvann)}$$

$$\underline{V} = \frac{m}{\rho} = \frac{116 \cdot 10^6}{10^3} = \underline{116 \cdot 10^3\ \mathrm{m^3}}$$

b) Antar $V = K_1 \cdot z^{11/10}$.

Skal finne $\rho_{SALTVANN}$.

$$\rho_{FERSKVANN} \cdot g \cdot V_{FERSKVANN} = \rho_{SALTVANN}\, g\, V_{SALTVANN}$$

$$\rho_{FERSKVANN} \cdot K_1 \cdot z_{FERSKVANN}^{11/10} = \rho_{SALTVANN} \cdot K_1 \cdot z_{SALTVANN}^{11/10}$$

$$\underline{\rho_{SALTVANN}} = \rho_{FERSKVANN} \left(\frac{z_{FERSKVANN}}{z_{SALTVANN}}\right)^{11/10} = 1000 \cdot \left(\frac{11}{10{,}25}\right)^{11/10} = \underline{1025{,}6\ \frac{\mathrm{kg}}{\mathrm{m^3}}}$$

c) Av formelark: $F_H = \gamma h_{CG} \cdot A_x$ (horisontal projeksjon)

Det gir her

$$\underline{F_H} = \rho_{SALTVANN} \cdot g \cdot \left(\tfrac{1}{2} z_{SALTVANN}\right) \cdot \left(W \cdot z_{SALTVANN}\right) =$$

$$= 1025{,}6 \cdot 9{,}81 \cdot \left(\tfrac{1}{2} \cdot 10{,}75\right) \cdot (10 \cdot 10{,}75)\ \mathrm{N} = \underline{5813{,}5\ \mathrm{kN}}$$

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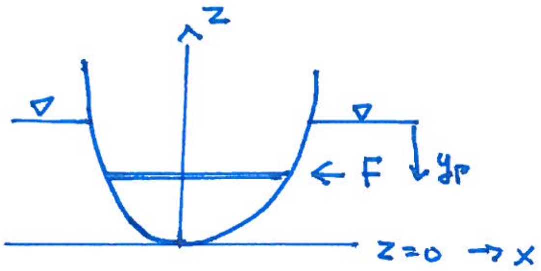
Løsning Oppgave 1 d)



Differansen mellom trykksenter y_p og centroide y_c er

$$l \equiv y_p - y_c = \frac{I_{yy}}{y_c \cdot A}.$$

Her går y-aksen inn i planet.

$$y_c = \frac{1}{2} z_p, \quad A = W \cdot z_p, \quad I_{yy} = \frac{1}{12} W z_p^3.$$

Det gir $l = \frac{1}{6} z_p$.

Trykksenteret ligger generelt lavere enn centroiden.

Trykksenterets høyde over bunnen blir dermed

$$\frac{1}{2} z_p - \frac{1}{6} z_p = \frac{1}{3} z_p = \frac{1}{3} \cdot 10{,}75 = \underline{3{,}58 \text{ m}}$$

Staget må være plassert i denne høyde over bunnen.

Exercise a): OPPGAVE 2

Incompressibility + continuity: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$

Differentiating u:

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \delta} \cdot \frac{\partial \delta}{\partial x}$$

$$= \underbrace{\frac{Uy}{2}\left[\frac{-1}{\delta^2}\left(3 - \frac{y^2}{\delta^2}\right) + \frac{2y^2}{\delta^4}\right]}_{= \frac{\partial u}{\partial \delta}} \cdot \underbrace{\frac{\delta_0}{2\sqrt{xL}}}_{= \frac{\partial \delta}{\partial x} = \frac{\delta}{2x}}$$

$$= \underline{\frac{3U}{4x}\frac{y}{\delta}\left(\frac{y^2}{\delta^2} - 1\right)}$$

Next, differentiate v:

$$\frac{\partial v}{\partial y} = \frac{U\delta}{x}\left[\frac{2Ay}{\delta^2} + \frac{4By^3}{\delta^4}\right]$$

$$= \frac{U}{x}\frac{y}{\delta}\left(4B\frac{y^2}{\delta^2} + 2A\right) \overset{\text{continuity}}{=} \frac{U}{x}\frac{y}{\delta}\left(-\frac{3}{4}\frac{y^2}{\delta^2} + \frac{3}{4}\right)$$

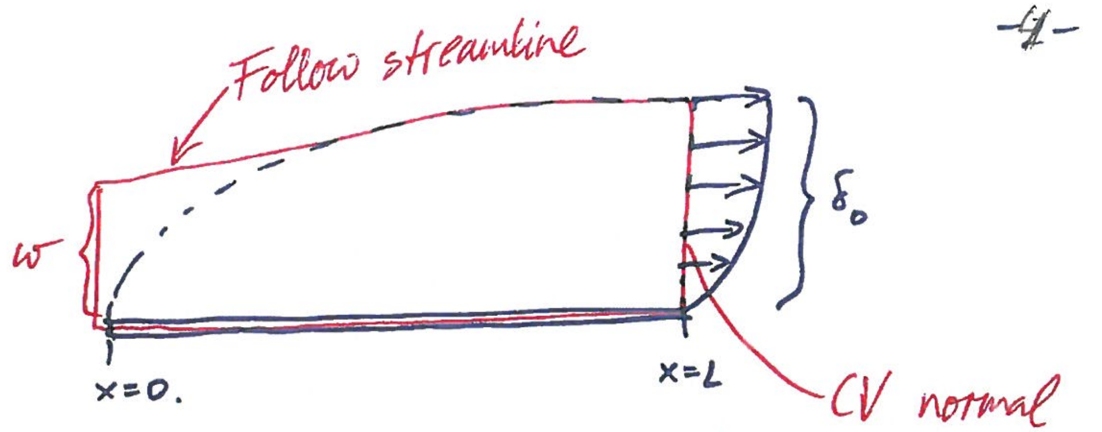
So: $4B = \frac{-3}{4} \Rightarrow \underline{\underline{B = \frac{-3}{16}}}$

and $2A = \frac{+3}{4} \Rightarrow \underline{\underline{A = \frac{+3}{8}}}$

That is,:

$$v = \frac{3U\delta}{8x}\left(\frac{y^2}{\delta^2} - \frac{y^4}{2\delta^4}\right).$$

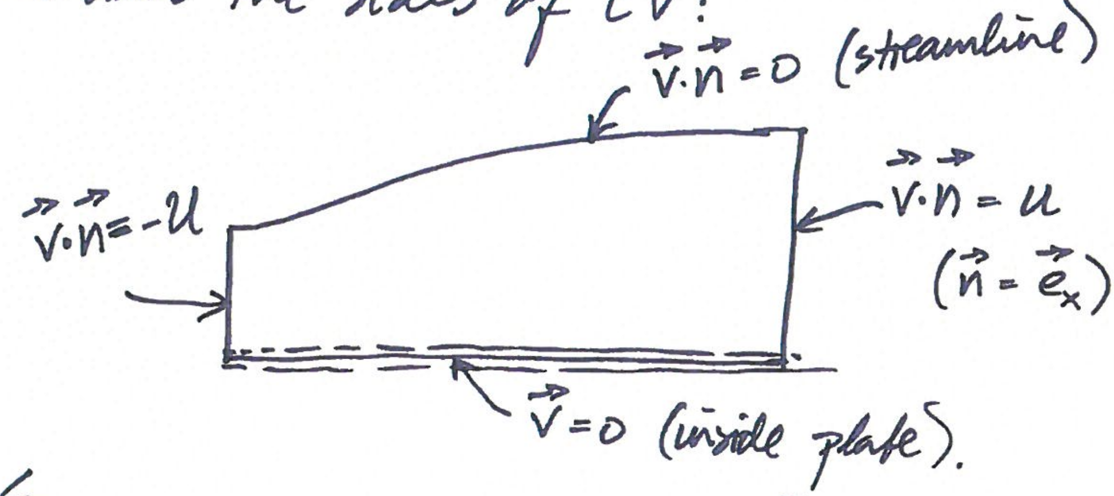
b)



First we must use mass conservation to find the inflow width w. Must have zero net flow over CS in a stationary system:

$$\oint_{CS} \vec{v}\cdot\vec{n}\,dA = 0.$$

Consider the sides of CV:



So:

$$U\cdot bw = b\int_0^{\delta_0} \underbrace{u}_{u(x=L)}\,dy = \frac{Ub}{2}\int_0^{\delta_0}\left(3\frac{y}{\delta_0}-\frac{y^3}{\delta_0^3}\right)dy$$

$$= \frac{Ub\delta_0}{2}\left(\frac{3}{2}-\frac{1}{4}\right) = \frac{5}{8}Ub\delta_0 \quad\Rightarrow\quad \underline{w = \frac{5}{8}\delta_0}$$

We can now use the momentum equation to find the force. x-component for CV above the plate

$$F_c = \oint_{CS} \rho \vec{v}(\vec{v}\cdot\vec{n})\,dA = -\rho U^2 w b + \rho b \int_0^{\delta_0} u^2\,dy$$

Contact force

$$= -\rho U^2 w b + \rho b \frac{U^2}{4} \int_0^{\delta_0} \left(3\frac{y}{\delta_0} - \frac{y^3}{\delta_0}\right)^2 dy$$

$$= -\rho U^2 \cdot \frac{5}{8}\delta_0 b + \rho U^2 \delta_0 b \cdot \frac{1}{4} \cdot \underbrace{\int_0^1 (9s^2 - 6s^4 + s^6)\,ds}_{=\frac{68}{35}}$$

$$= \rho U^2 \delta_0 b \cdot \left(\frac{17}{35} - \frac{5}{8}\right) = \underline{\frac{-39}{280} \rho U^2 \delta_0 b}$$

This is half the force required to keep the plate still. The force from the water on the plate is (x-component).

$$F_{drag} = -2F_c = \underline{\underline{\frac{39}{140} \rho U^2 \delta_0 b}}$$

Løsning Oppgave 3

a) Av kontinuitetsligningen blir det igjen bare $\frac{\partial(r v_r)}{\partial r} = 0$, altså $r v_r = \text{konst.}$

Heftbetingelser $v_r\big|_{r=a} = 0$ og $v_r\big|_{r=b} = 0$ gir konst $= 0$

Altså $\underline{v_r = 0}$

b) r-komponent av Navier-Stokes $\Rightarrow \frac{\partial p}{\partial r} = 0$, altså $p = p(\theta, z)$.

θ-komponent $\Rightarrow \frac{\partial p}{\partial \theta} = 0$, altså $\underline{p = p(z)}$

c) z-komponent $\Rightarrow 0 = -\underbrace{\frac{\partial p}{\partial z}}_{\rho g} + \rho g + \frac{\mu}{r}\frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r}\right)$

ρg (z-akse positiv nedover)

Altså $0 = \frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r}\right)$, som gir $r\frac{\partial v_z}{\partial r} = C_1$

$\frac{\partial v_z}{\partial r} = \frac{C_1}{r}$ gir $v_z = C_1 \ln r + C_2$

Heftbetingelser $v_z(a) = W$, $v_z(b) = 0$,

$$\left.\begin{aligned} C_1 \ln a + C_2 &= W \\ C_1 \ln b + C_2 &= 0 \end{aligned}\right\} \quad C_1 = -\frac{W}{\ln(b/a)}$$

$$C_2 = \frac{W}{\ln(b/a)} \cdot \ln b.$$

$\Rightarrow$

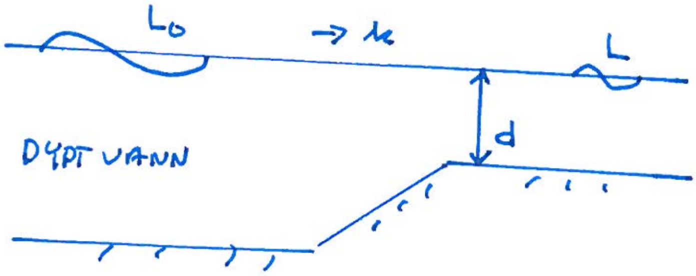
$$\underline{v_z = \frac{W}{\ln(b/a)} \cdot \ln\frac{b}{r}}$$

d) $Q = \int_a^b v_z \cdot 2\pi r\, dr = \frac{2\pi W}{\ln\frac{b}{a}}\left[\ln b \int_a^b r\,dr - \int_a^b r \ln r\, dr\right]$

$= \frac{2\pi W}{\ln\frac{b}{a}}\left[\frac{1}{2}\ln b\,(b^2 - a^2) - \frac{1}{2}b^2\left(\ln b - \frac{1}{2}\right) + \frac{1}{2}a^2\left(\ln a - \frac{1}{2}\right)\right]$

$$\underline{Q = 2\pi W\left[\frac{b^2 - a^2}{4\ln(b/a)} - \frac{a^2}{2}\right]}$$

Løsning Oppgave 4



a) For dypt vann er $\omega^2 = g k_0$. Da $\omega = 2\pi/T$ og $k_0 = 2\pi/L_0$ blir

$$\left(\frac{2\pi}{T}\right)^2 = g \cdot \frac{2\pi}{L_0} \Rightarrow \underline{L_0} = \frac{g}{2\pi} T^2 = \frac{9{,}81}{2\pi} T^2 = \underline{1{,}56\,T^2}$$

Altså $\underline{T} = \sqrt{\dfrac{L_0}{1{,}56}} = \sqrt{\dfrac{170}{1{,}56}} = \underline{10{,}4\,s}$, $\underline{\omega} = \dfrac{2\pi}{10{,}4} = \underline{0{,}604\,s^{-1}}$

Stasjonære forhold gjør at antall bølger som passerer en fiksert posisjon er konstant, dvs. $\omega =$ konst. eller $\underline{T = \text{konst.}}$

b) Gitt $L = 85$ m fra observasjon.

Dermed $\underline{k} = 2\pi/L = 2\pi/85 = \underline{\cancel{0{,}080}\ 0{,}074\ m^{-1}}$

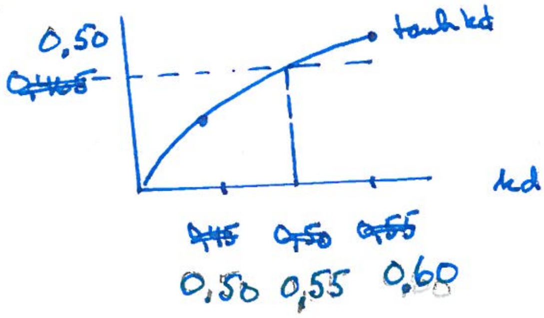
For vilkårlig dyp er $\omega^2 = gk \tanh kd$.

Regner ut $\dfrac{\omega^2}{gk} = \dfrac{0{,}604^2}{9{,}81 \cdot 0{,}074} = \cancel{0{,}465}\ 0{,}503$

Ligningen $\underline{\tanh kd = 0{,}503}$ bestemmer kd.

En finner $kd = \cancel{0{,}503}\ 0{,}55$, $\underline{d} = \dfrac{\cancel{0{,}503}\ 0{,}55}{\cancel{0{,}080}\ 0{,}074} = \underline{\cancel{6{,}3}\ 7{,}9\ m}$

Kan evt. finne kd grafisk:



kd	tanh kd
0,45	0,422
0,50	0,462
0,55	0,501
0,60	0,537