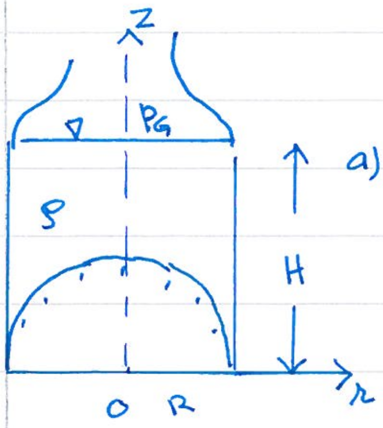
TEP4105 FLUIDMEKANIKK. EKSAMEN 6. DESEMBER 2012

Oppgave 1

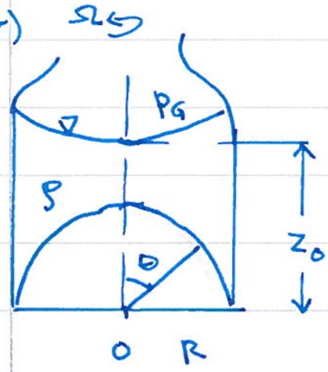


a) Volum av væsken er $\mathcal{V} = \pi R^2 \cdot H - \frac{2}{3}\pi R^3$

Vertikal kraft $F = \pi R^2 p_G + \gamma \cdot \mathcal{V}$, $\Rightarrow$

$$\underline{F = \pi R^2 \left[p_G + \gamma \left(H - \tfrac{2}{3} R \right) \right]}$$

b)



b) Med rotasjon: $p = C - \gamma z + \frac{1}{2}\rho \Omega^2 r^2$

Ved $z = z_0$, $r = 0$ er $p = p_G$, altså $p_G = C - \gamma z_0$

$$\underline{C = p_G + \gamma z_0} \quad \Rightarrow$$

$$p = p_G + \gamma (z_0 - z) + \frac{1}{2}\rho \Omega^2 r^2 \qquad (1)$$

Fri overflate $p = p_G$:

$$\cancel{p_G} = \cancel{p_G} + \gamma (z_0 - z) + \frac{1}{2}\rho \Omega^2 r^2$$

Ligning $\underline{z = z_0 + \frac{\Omega^2}{2g} r^2}$

På overflaten av halvkulen, $r = R\sin\theta$, $z = R\cos\theta$; da blir p fra (1)

$$\underline{p_s = p_G + \gamma (z_0 - R\cos\theta) + \frac{1}{2}\rho \Omega^2 R^2 \sin^2\theta}$$

c) Volum av vannet $\mathcal{V} = \int_0^R \left(z_0 + \frac{\Omega^2}{2g} r^2 \right) \cdot 2\pi r\,dr - \frac{2}{3}\pi R^3 =$

$$= 2\pi \Big[z_0 \underbrace{\int_0^R r\,dr}_{\frac{1}{2}R^2} + \frac{\Omega^2}{2g} \underbrace{\int_0^R r^3\,dr}_{\frac{1}{4}R^4} \Big] - \frac{2}{3}\pi R^3 = \pi R^2 \left(z_0 + \frac{\Omega^2}{4g} R^2 \right) - \frac{2}{3}\pi R^3$$

Fra punkt a) er $\mathcal{V} = \pi R^2 H - \frac{2}{3}\pi R^3$, altså

$$\pi R^2 \left(z_0 + \frac{\Omega^2}{4g} R^2 \right) - \frac{2}{3}\pi R^3 = \pi R^2 H - \frac{2}{3}\pi R^3$$

$$\underline{z_0 = H - \frac{\Omega^2}{4g} \cdot R^2}$$

Stillevannsdybden ligger midt imellom toppunkt og bunnpunkt for paraboloiden.

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Oppgave 1c Integrerer vertikalkraften $p_s \cdot \cos\theta$ over kuleflaten. Da flateelementet er $R^2 \sin\theta d\theta \cdot 2\pi$, blir

$$F = \int_0^{\pi/2} p_s \cos\theta \cdot R^2 \sin\theta \, d\theta \cdot 2\pi =$$

$$= 2\pi R^2 \int_0^{\pi/2} \left[p_G + \gamma(z_0 - R\cos\theta) + \tfrac{1}{2}\rho R^2 \Omega^2 \sin^2\theta \right] \cdot \cos\theta \sin\theta \, d\theta$$

$$= 2\pi R^2 \Big\{ (p_G + \gamma z_0) \underbrace{\int_0^{\pi/2} \cos\theta \sin\theta \, d\theta}_{1/2} - \gamma R \underbrace{\int_0^{\pi/2} \cos^2\theta \sin\theta \, d\theta}_{1/3} +$$

$$+ \tfrac{1}{2}\rho R^2 \Omega^2 \underbrace{\int_0^{\pi/2} \sin^3\theta \cos\theta \, d\theta}_{1/4} \Big\}$$

$$\underline{F = \pi R^2 \left[p_G + \gamma z_0 - \tfrac{2}{3}\gamma R + \tfrac{1}{4}\rho R^2 \Omega^2 \right]}$$

Setter inn $z_0 = H - \frac{\Omega^2}{4g} R^2$:

$$F = \pi R^2 \left[p_G + (\gamma H - \tfrac{1}{4}\rho \cancel{\Omega^2 R^2}) - \tfrac{2}{3}\gamma R + \tfrac{1}{4}\rho \cancel{R^2 \Omega^2} \right]$$

$$\underline{F = \pi R^2 \left[p_G + \gamma (H - \tfrac{2}{3} R) \right]}$$

Som i punkt a).

Rotasjonen har ingen innvirkning på den vertikale kraft.

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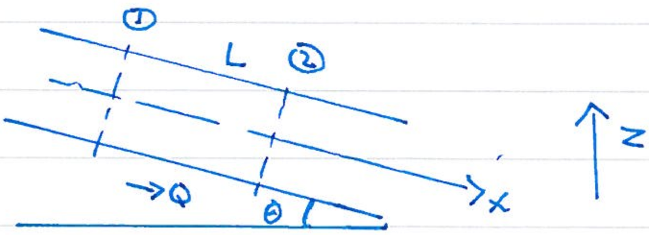
Oppgave 2



a) Energiligning

$$\cancel{\frac{p_1}{\rho}} + \frac{1}{2}\cancel{V_1^2} + gz_1 = \left(\cancel{\frac{p_2}{\rho}} + \frac{1}{2}\cancel{V_2^2} + gz_2\right) + \underbrace{w_s}_{0} + gh_f.$$

$$\Rightarrow \quad \underline{h_f} = z_1 - z_2 = \underline{\Delta z}$$

Impulsligningen $\sum F = \dot{M}_{UT} - \dot{M}_{INN} = 0.$

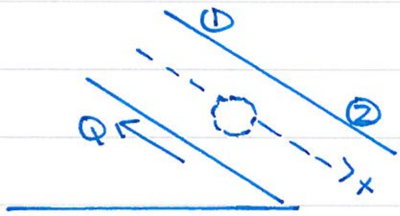
Tar hensyn til alle kreftene på vannet i CV, mellom 1 og 2:

$$\gamma(\pi R^2 L)\cdot\sin\theta - \tau_w\cdot(2\pi R)\cdot L = 0$$

Her er $L\sin\theta = \Delta z$, og da $\Delta z = h_f$, får

$$\gamma\pi R^2\cdot h_f = 2\pi R L\cdot\tau_w \quad , \quad \underline{h_f = \frac{2\tau_w}{\gamma}\cdot\frac{L}{R}}$$

b)



Når vannet pumpes oppover, med samme fart V, vil friksjonen være den samme. Dermed er friksjonshøyden $\underline{h_f = \Delta z}$ som før.

Pumpen utfører arbeid på systemet, slik at $w_s < 0$.

Energiligning, fra 2 til 1:

$$\frac{p_2}{\rho} + \frac{1}{2}\cancel{V_2^2} + gz_2 = \left(\frac{p_1}{\rho} + \frac{1}{2}\cancel{V_1^2} + gz_1\right) + w_s + \overbrace{gh_f}^{g\cdot\Delta z}$$

$\Delta z = z_1 - z_2$, $\quad \Delta p = p_2 - p_1$ er kjent.

$$\frac{\Delta p}{\rho} = g\Delta z + w_s + g\Delta z = w_s + 2g\cdot\Delta z$$

$$\underline{w_s = \frac{\Delta p}{\rho} - 2g\cdot\Delta z}$$

Her er $(-w_s)$ arbeidet per masseenhet. Effekt av pumpen

$$\underline{P} = (-w_s)\cdot\rho\cdot Q = \left(-\frac{\Delta p}{\rho} + 2g\Delta z\right)\cdot\rho Q = \underline{(-\Delta p + 2\gamma\cdot\Delta z)\cdot Q}$$

Oppgave 2c

Tyngdedrevet strømning, som i punkt a):

Navier-Stokes i x-retning

$$0 = -\frac{1}{\rho}\underbrace{\frac{\partial p}{\partial x}}_{0} + g\sin\theta + \nu\nabla^2 u \quad \Rightarrow$$

$$\nu\cdot\nabla^2 u = -g\sin\theta \quad , \quad \frac{\nu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = -g\cdot\sin\theta$$

$$\frac{d}{dr}\left(r\frac{du}{dr}\right) = -\frac{g\sin\theta}{\nu}\cdot r$$

Integrerer : $r\frac{du}{dr} = -\frac{g\sin\theta}{2\nu}r^2 + C$

Grensebetingelse ved $r=0$: $du/dr = 0 \Rightarrow C = 0$

Ny integrasjon : $u = -\frac{g\sin\theta}{4\nu}r^2 + C_1$

Heft ved $r = R$: $0 = -\frac{g\sin\theta}{4\nu}R^2 + C_1$, $C_1 = \frac{g\sin\theta}{4\nu}\cdot R^2$

$\Rightarrow$

$$u = \frac{g\sin\theta}{4\nu}R^2\left(1-\frac{r^2}{R^2}\right) = u_{max}\left(1-\frac{r^2}{R^2}\right),$$

$$u_{max} = \frac{g\sin\theta}{4\nu}R^2$$

$$Q = \int u\,dA = u_{max}\int_0^R\left(1-\frac{r^2}{R^2}\right)\cdot 2\pi r\,dr$$

$$= 2\pi\cdot u_{max}\int_0^R\left(r-\frac{r^3}{R^2}\right)dr = 2\pi u_{max}\left(\frac{1}{2}R^2-\frac{1}{4}R^2\right) = \frac{\pi}{2}u_{max}\cdot R^2$$

$u_{max} = \frac{2Q}{\pi R^2}$ gir $u = \frac{2Q}{\pi R^2}\left(1-\frac{r^2}{R^2}\right)$

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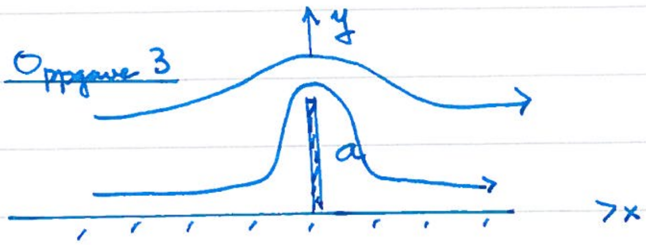
Oppgave 3



$w(z) = U\sqrt{z^2+a^2}$, $z = x+iy$
$y>0$.

a) $w^2 = (\phi + i\psi)^2 = U^2\left[(x+iy)^2 + a^2\right] \Rightarrow$

$$\phi^2 - \psi^2 + 2i\phi\psi = U^2\left(x^2 - y^2 + a^2 + 2ixy\right)$$

Sammenligner reell og imaginær del:

$$\underline{\phi^2 - \psi^2 = U^2(x^2-y^2+a^2), \quad \phi\psi = U^2 \cdot xy}$$

Løser ut ϕ: $\phi = \dfrac{U^2xy}{\psi}$, som innsatt i første ligning gir

$$\frac{U^4x^2y^2}{\psi^2} - \psi^2 = U^2(x^2-y^2+a^2), \text{ dvs.}$$

$$\underline{\psi^4 + U^2(x^2-y^2+a^2)\psi^2 - U^4x^2y^2 = 0}$$

Dette er en bikvadratisk ligning:

$$\psi^2 = -\tfrac{1}{2}U^2(x^2-y^2+a^2) + \sqrt{\tfrac{1}{4}U^4(x^2-y^2+a^2)^2 + U^4x^2y^2}$$

(Bare positivt fortegn kan brukes)

Tar kvadratroten

$$\underline{\psi = U\left[\sqrt{\tfrac{1}{4}(x^2-y^2+a^2)^2 + x^2y^2} - \tfrac{1}{2}(x^2-y^2+a^2)\right]^{1/2}}$$

b) Kompleks hastighet $\underline{w'(z) = \dfrac{U}{\sqrt{z^2+a^2}} \cdot z = u - iv}$

For $x=0$ er $z = iy$, altså $\underline{\dfrac{U \cdot iy}{\sqrt{a^2-y^2}} = u - iv}$

Hvis $y>a$: $\sqrt{a^2-y^2} \rightarrow i\sqrt{y^2-a^2}$ slik at $\dfrac{U \cdot y}{\sqrt{y^2-a^2}} = u - iv$

$$\Rightarrow \underline{u = \frac{U \cdot y}{\sqrt{y^2-a^2}}, \quad v = 0}$$

Hvis $y<a$: $\dfrac{U \cdot iy}{\sqrt{a^2-y^2}} = u - iv$ gir $\underline{u = 0, \quad v = -\dfrac{U \cdot y}{\sqrt{a^2-y^2}}}$

Ved kanten $y=a$ divergerer hastighetskomponentene.
Ikke realistisk.