# TOTIMERSØVING NR 2 TEP 4105 FLUIDMEKANIKK

**Høst 2015**
**Utført av:** (alle i gruppa)

***LØSNINGSFORSLAG***

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## *Oppgave 1*

Hvordan kan man finne funksjonen $f(x, y)$ ut fra uttrykkene

$$\frac{\partial f}{\partial x} = a \ , \ \frac{\partial f}{\partial y} = b \ ?$$

**Svar:** Alt 1: Integrere begge og sammenligne. Integrasjon gir $f = ax + F(y)$ og $f = by + G(x)$. Begge uttrykkene må gjelde samtidig. Dermed er $f = ax + by + C$ hvor $C$ er en konstant. Alt 2: Integrere den ene og sett inn i den andre. $f = ax + F(y)$ som gir

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(ax + F(y)\right) = \frac{\mathrm{d}F}{\mathrm{d}y} = b \Rightarrow F = by + C \, .$$

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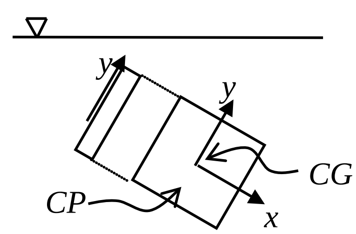
## *Oppgave 2*

$$y_{CP} = -\gamma \sin\theta \frac{I_{xx}}{p_{CG} A} \quad (1)$$

$$y_{CP} = -\sin\theta \frac{I_{xx}}{h_{CG} A} \quad (2)$$

Hva menes med punktene *CG* og *CP*?
**Svar:** *CG*: Arealsenter (Center of Gravity)
*CP*: Angrepspunkt (Center of Pressure)




Hvordan er *y*-aksen orientert?
**Svar:** Positiv opp mot overflaten langsmed flaten.
(Obs: Negativ i formelsamlingen til Irgens)

Når kan vi bruke den enkleste varianten (2) i stedet for (1)?
**Svar:** Hvis atmosfæretrykket kanselleres i problemet. Da er $p_{CG} = \gamma h_{CG}$ og $\gamma$ kan forkortes vekk.

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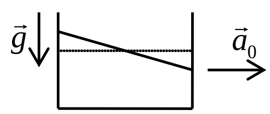
## *Oppgave 3*

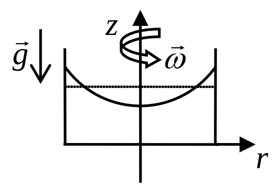
$$0 = -\nabla p + \rho \vec{g}_{eff}$$

I kvasi-statiske problemer (stivt legeme bevegelse) kan vi justere tyngdens akselerasjon $\vec{g}$ til en $\vec{g}_{eff}$ på to måter. Hvilke?

**Svar:** Konstant akselerasjon $\vec{a}_0$ eller konstant rotasjon $\vec{\omega} \parallel \vec{g}$ vil kunne gi statiske forhold i et relativt system.

Og hvordan?




$0 = -\nabla p + \rho(\vec{g}_{eff} - \vec{a}_0)$ $\qquad$ $0 = -\nabla p + \rho(\vec{g}_{eff} + \omega^2 r \cdot \vec{e}_r)$

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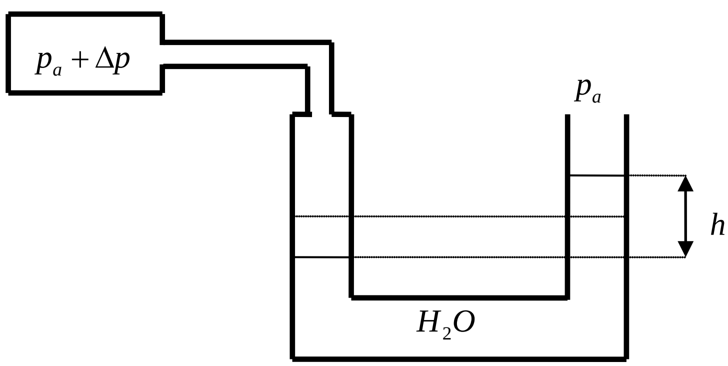
## *Oppgave 4*

Vi skal måle lave overtrykk $\Delta p$ ned til ca. 1 Pa ved hjelp av væskemanometri på tre måter.

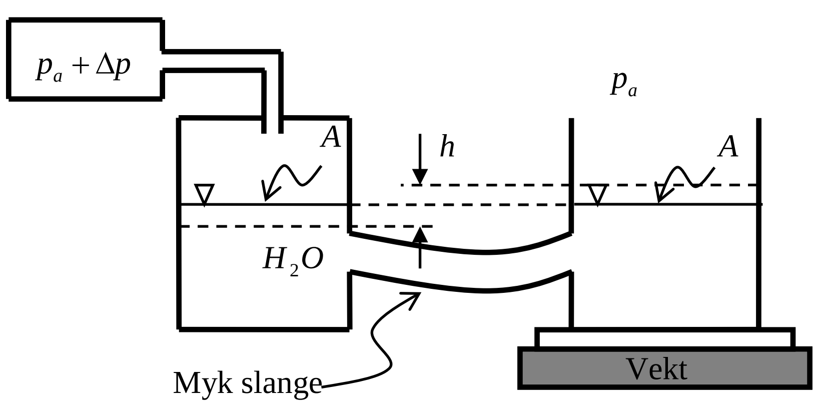
1. Enkelt U-rør




Estimer høyden $h$ ved $\Delta p \approx 1\text{Pa}$.

**Svar:** $\rho g h = \Delta p \Leftrightarrow h = \dfrac{\Delta p}{\rho g} \approx \underline{\underline{\dfrac{1}{1000 \cdot 10}\text{m}}}$

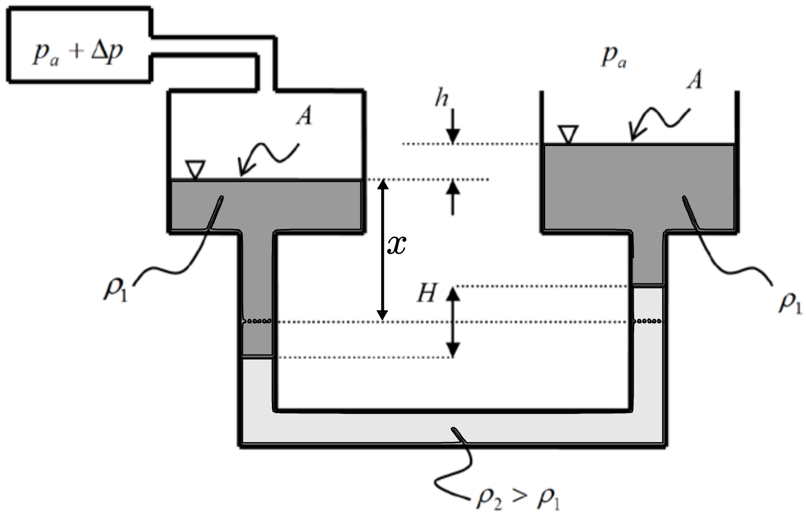
2. Med brev-vekt



Her er $A$ tverrsnittsarealet av vannbeholderene. Med hvor mange gram øker vekten med ved $\Delta p \approx 1\text{Pa}$?

**Svar:** $\Delta p = \rho g h \Rightarrow h = \dfrac{\Delta p}{\rho g}$. Masseøkning: $m = \dfrac{1}{2} h A \rho = \underline{\underline{\dfrac{\Delta p A}{2g}}}$. Anta $A = 10\text{cm} \times 10\text{cm}$, da blir masseøkningen $m = \dfrac{1\text{Pa} \cdot 0.01\text{m}^2}{2g} \approx \dfrac{1}{2}\text{gram}$.

3. Med to væsker




Hvis arealet $A$ er mye større enn tverrsnittsarealet på U-røret så kan høyden $h$ neglisjeres. Finn $H$ uttrykt ved tetthetsforskjellen og $\Delta p$.

**Svar:** Trykket nede ved der fluidene møtes i venstre kolonne er $p_B = p_a + \Delta p + \rho_1 g(x + H/2)$. Trykket ved samme høyde i høyre kolonne er $p_C = p_a + \rho_1 g(h + x - H/2) + \rho_2 g H$. Vi har antatt $h \approx 0$, så likevekt $p_B = p_C$ gir

$$\underline{\underline{H = \frac{\Delta p}{(\rho_2 - \rho_1) g}}}.$$

## Exercise 5: Pressure Force

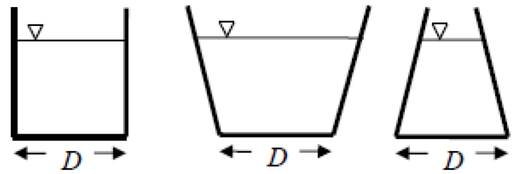


Figure 1: Pressure force in water containers.

a) The pressure forces against the bottom of the three containers in Figure 1 are all the same. How can the amount of water then be different in the three containers?

Solution:
The pressure force is the hydrostatic force acting normal to the surface and depends on its area and on the local water depth. At the horizontal bottom of all three containers the areas $A = \pi\frac{D^2}{4}$ and pressures $P_{atm} + \rho g h$ are the same ($h$ is here the total depth same for the three containers.)

When concerning the cylindrical section of the containers we are only interested in the vertical components of the pressure forces. The first container has a completely vertical sides – the normal vector at any point on this surface has no horizontal component and therefore does not contribute with any pressure force in the horizontal direction (in fact, the net side wall pressure force is zero.
The second container has a larger opening. If we consider the resultant pressure force on the cylindrical section, the vertical pressure force component will point downward – the sidewall carries the extra weight due to the large opening.
The third container has a narrow opening. The vertical component of the resultant pressure force on the sidewall is thus upward – the resulting force on the sidewall is the buoyant force.

b) The middle container of Figure 1 is placed on a weight which register its mass to 1 kg exactly. For fun, we stick an index finger 5 cm down into the water of the container. What will the weight register now?
You may assume your finger to have a cylindrical shape with a $2\,\text{cm}^2$ cross-sectional area. Use $\rho = 1000\,\text{kg/m}^3$, $g = 10\,\text{m/s}^2$.

Solution:
A weight (as the name suggests) registers the *force* $F$ pushing down on it. When it returns a mass in kg it does so by assuming that the force is gravitational, caused by an object exposed to a fixed gravitational acceleration: $m = F/g$. We need therefore find the downward force on the weight.
When we push a finger into the water we experience a buoyancy force from the water onto the finger:

$$F_{\text{buoyancy}} = g\rho_{\text{water}}\mathcal{V}_{\text{finger}} = g = 10\,\text{m/s}^2 \cdot 1000\,\text{kg/m}^3 \cdot 5\,\text{cm} \cdot 2\,\text{cm}^2 = 0.1\,\text{N}.$$

By Newton's 3rd law, an equal but oppositely directed force must then be acting on the water (and weight) from the finger. The new total force acting on the weight $F_{\text{new}}$ is then the weight of water and container $W_{\text{water+container}}$ plus $F_{\text{buoyancy}}$. We already know the combined weight of the container and water. This was the force registered at the first weighing:

$$W_{\text{water+container}} = F_{\text{old}} = m_{\text{old}} \cdot g = 1\,\text{kg} \cdot 10\,\text{m/s}^2 = 10\,\text{N}.$$

The newly registered force is then

$$F_{\text{new}} = W_{\text{water+container}} + F_{\text{buoyancy}} = 10.1\,\text{N},$$

and the weight will register a mass

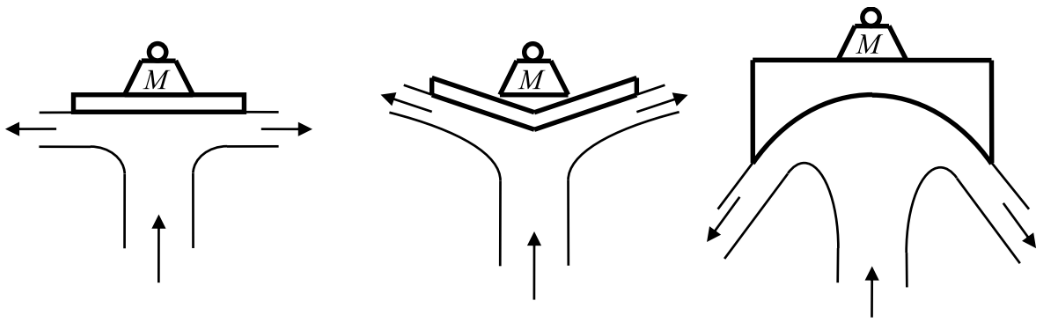
$$\underline{\underline{m_{\text{new}}}} = \frac{F_{\text{new}}}{g} = \frac{10.1\,\text{N}}{10\,\text{m/s}^2} = \underline{\underline{1.01\,\text{kg}}}.$$

(Do not be fooled by the weight/density of the finger itself; the weight of the finger merely reduce the effort with which *we* must push it down.)

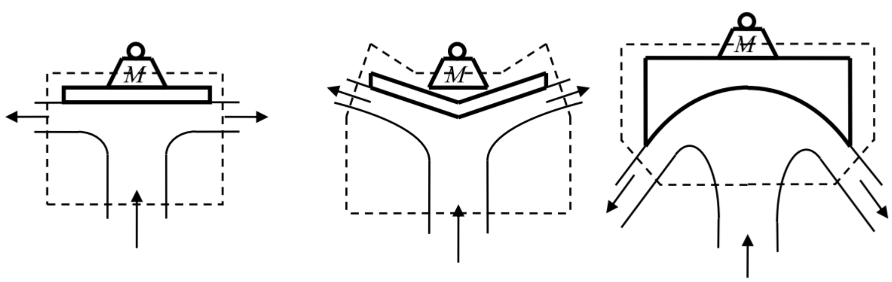
## Exercise 5: Choosing control volumes

Below is a set of problems. Draw the control volumes ($CV$) you *would* use were you to solve them. Discuss amongst yourselves how you would go about solving the problems and explain your choices of $CV$. *You are not required to actually solve the problems presented. Hint:* Simen's '5-trinns plan for løsning av kraftlovoppgaver' serves as a useful guide. It can be found under 'Auditorieøvinger' on It's Learning.

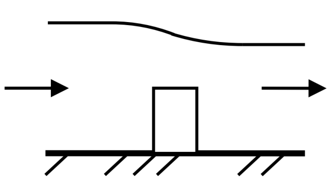
a) Find the mass M that a water jet can hold up. Which of these cases can lift the heaviest mass M and why?




Solution:
Set the control volume 90° on the inlets and the outlets. Don't set any control surfaces up to the walls since the friction and the pressure are not known at these specific locations.

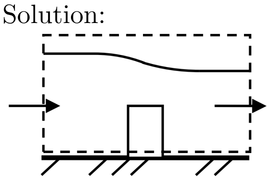



The last example (on the right) is the one that is able to lift the heaviest mass because both inlet and outlet momentum flow rates contribute to an upward force. The water jet for that last example generates the largest y-component of $\dot{m}\Delta\vec{v}$.

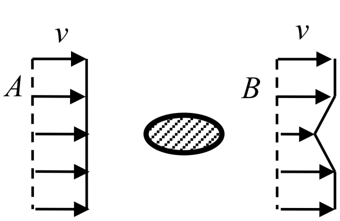
b) 

River over a cube
A river flows without friction over a cube fixed on the river bottom. The pressure of the water upstream and downstream the cube is assumed to be hydrostatic. Find the force acting on the cube.

Solution:

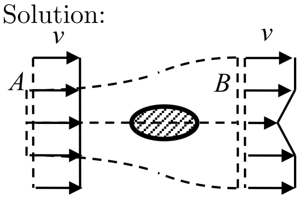


A control surface can be set on the bottom of the river because there is no friction force, but only a contact force acting from the cube on the river. Thus, it would also be possible to draw the control volume under the bottom of the river.

c) 

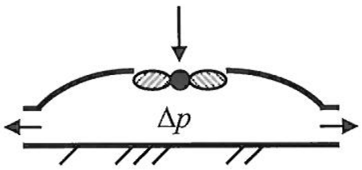

Uniform stream passing a bridge pier
An oval bridge pier is installed in a uniform stream. A wake is created behind the bridge pier with a velocity profile which is approximately drawn in the cross section B. The pressures at cross section A and cross section B are the same. Find the force on the bridge pier.

Solution:




Draw the control surfaces along streamlines to avoid the fluxes which are unknown vertically. It is also possible to use the symmetrical properties of this situation and thus use the plane of symmetry as a control surface. In either case, the control surface should cut the bridge pier in the pier's interior. The opposite of the contact force from the bridge pier on the water yields the force from the water on the bridge pier.

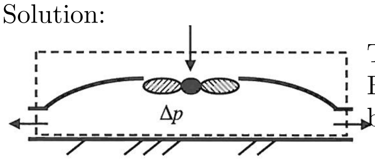
d)




Air-cushion vehicle

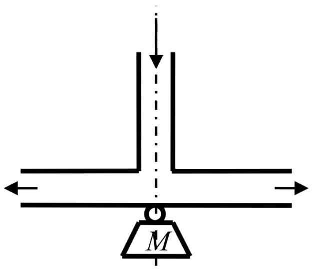
An air-cushion vehicle is hold up by keeping a constant overpressure $\Delta p$ under its flexible cover. Find the power that the motor has to deliver to lift the mass $M$ of the vehicle.

Solution:




The control surfaces should be in contact with the cover of the vehicle. But $CS$ should not be in contact with the ground, otherwise you would basically find that the vehicle is carried by the ground.

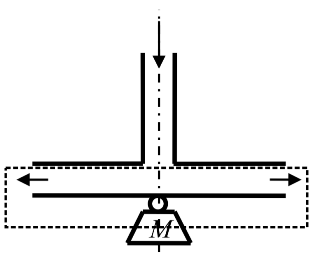
e)




Air between two plates

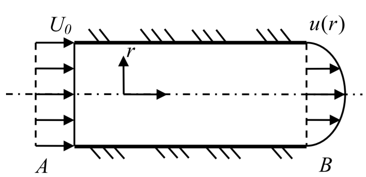
Air is injected between two circular plates. The pressure between the plates gets lower than the ambient pressure so that the lower plate can lift a mass $M$. Find $M$.

Solution:




As in the first situation, the control surfaces are 90° on the inlet and outlets and should also pass through the mass to take it into account. We make sure that the CV lies just below the upper plate so that it feels the pressure suction force inside the horizontal channel and does not feel the stretching force exserted on the vertical pipe. We would here typically assume frictionless, incompressible flow and use the Bernoulli and mass conservation equations to gain an expression for the underpressure as a function of the volume flow rate. This suction should then balance the mass weight and convection force from above.

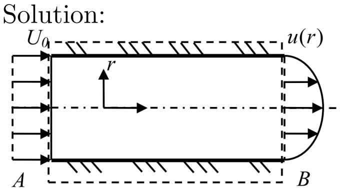
f)




Friction force in a pipe

A fluid with uniform speed enters a pipe. Because of friction, the velocity profile changes to become as $u(r)$. Given a pressure $P_A$ at point $A$ and $P_B$ at point $B$, find the friction force on the inner surface of the pipe.

Solution:




The contact force is equal to the friction force, which we do not know in detail. Therefore, we cannot integrate along the inner surface of the pipe. However, the contact force can be determined by the net momentum flow rate.