TEP 4105 FLUIDMEKANIKK

Løsning Øving 1

1. Kaller $\vec{C} = \nabla \times \nabla f$, og regner ut komponenten

$$C_z = \frac{\partial}{\partial x}\frac{\partial f}{\partial y} - \frac{\partial}{\partial y}\frac{\partial f}{\partial x} = \underline{0}.$$ Tilsv. for de andre komp.

2 1) $$\frac{\partial \vec{V}}{\partial t} = \frac{\partial u}{\partial t}\vec{i} + \frac{\partial v}{\partial t}\vec{j}$$, hvor

$$\frac{\partial u}{\partial t} = \frac{U}{L^3}\cdot\frac{2\pi U}{L}\cos\left(\frac{2\pi U}{L}t\right)\cdot(-x^2 y),$$

$$\frac{\partial v}{\partial t} = \frac{U}{L^3}\cdot\frac{2\pi U}{L}\cos\left(\frac{2\pi U}{L}t\right)\cdot xy^2.$$

2) $$(\vec{V}\cdot\nabla)\phi = \left(u\frac{\partial\phi}{\partial x} + v\frac{\partial\phi}{\partial y}\right) = \frac{U}{L^3}\sin\left(\frac{2\pi U}{L}t\right)\phi_0\exp\left(-\frac{x^2+y^2}{L^2}\right)\cdot\frac{2}{L^2}\cdot(x^3y - xy^3).$$

3) $$(\vec{V}\cdot\nabla)\vec{V} = (\vec{V}\cdot\nabla)u\,\vec{i} + (\vec{V}\cdot\nabla)v\,\vec{j}$$, hvor

$$(\vec{V}\cdot\nabla)u = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{U^2}{L^6}\sin^2\left(\frac{2\pi U}{L}t\right)\cdot x^3y^2,$$

$$(\vec{V}\cdot\nabla)v = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{U^2}{L^6}\sin^2\left(\frac{2\pi U}{L}t\right)x^2y^3.$$

4) $$(\nabla\times\vec{V})_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{U}{L^3}\sin\left(\frac{2\pi U}{L}t\right)(x^2+y^2).$$

5) $$\nabla\cdot\vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{U}{L^3}\sin\left(\frac{2\pi U}{L}t\right)(-2xy+2xy) = 0.$$

6) $$\nabla^2\vec{V} = \nabla^2 u\,\vec{i} + \nabla^2 v\,\vec{j}$$, hvor

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{U}{L^3}\sin\left(\frac{2\pi U}{L}t\right)\cdot(-2y),$$

$$\nabla^2 v = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{U}{L^3}\sin\left(\frac{2\pi U}{L}t\right)\cdot 2x.$$

Løsning øving 1, forts.

7) $\nabla p = \dfrac{\partial p}{\partial x}\vec{i} + \dfrac{\partial p}{\partial y}\vec{j}$, hvor

$$\frac{\partial p}{\partial x} = \frac{2p_0}{x^2+y^2}\cdot x , \qquad \frac{\partial p}{\partial y} = \frac{2p_0}{x^2+y^2}\cdot y .$$

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1) Tid t : L/U.

2) Trykk p : ρU^2

3) Dynamisk viskositet μ : $\rho U L$.

Forholdet mellom $\rho U L$ og μ er Reynolds tall:

$$\frac{\rho U L}{\mu} = Re.$$

4 Se på $\nabla\cdot\vec{V} = \nabla\cdot(f\nabla g) = f\nabla^2 g + \nabla f\cdot\nabla g$.

Innsatt i Gauss' teorem

$$\int \nabla\cdot\vec{V}\, dV = \oint (\vec{V}\cdot\vec{n})\, dA = \oint V_n\, dA \quad \text{gir dette}$$

$$\int (f\nabla^2 g + \nabla f\cdot\nabla g)\, dV = \oint f\nabla g\cdot\vec{n}\, dA = \oint f\frac{\partial g}{\partial n}\, dA$$

La $f \leftrightarrow g$:

$$\int (g\nabla^2 f + \nabla g\cdot\nabla f)\, dV = \oint g\nabla f\cdot\vec{n}\, dA = \oint g\frac{\partial f}{\partial n}\, dA$$

Subtraherer:

$$\int (f\nabla^2 g - g\nabla^2 f)\, dV = \oint \left(f\frac{\partial g}{\partial n} - g\frac{\partial f}{\partial n}\right) dA .$$

Greens teorem.