NTNU

Det skapende universitet

Institutt for Energi- og prosessteknikk

Eksamensoppgaver i TEP4105 FLUIDMEKANIKK

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Eksamensdato: Torsdag 12. desember 2013

Eksamenstid: 09.00 – 13.00

Hjelpemiddelkode/Tillatte hjelpemidler:

C: Typegodkjent kalkulator

Matematisk formelsamling

Annen informasjon: Sensuren faller innen 13. januar 2014

Målform/språk: Bokmål/nynorsk/engelsk

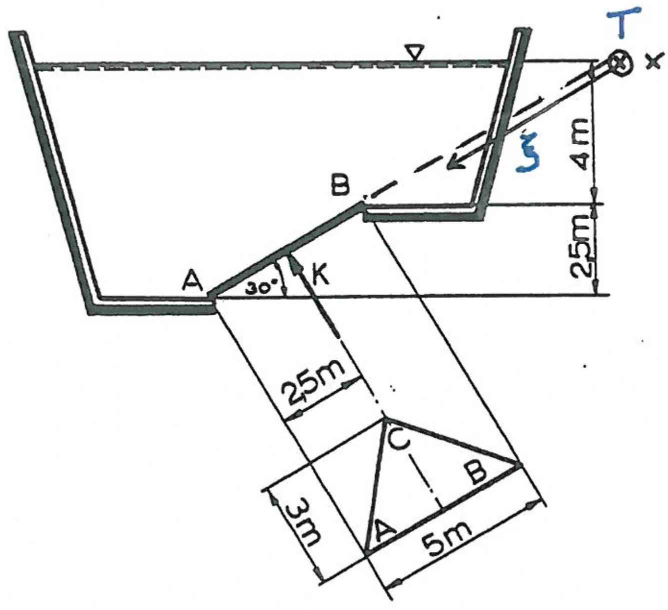
Antall sider: 5

Antall sider vedlegg: 4

Kontrollert av:

Dato Sign

Oppgave 1



I bunnen av et basseng er det en trekantet luke ABC beliggende slik at sidekanten AB er parallell med figurens plan. Luka kan åpnes med en kraft K som virker vinkelrett på lukas plan i punktet C (beliggende i retning inn i papirplanet). Bassenget er fylt med vann til en høyde 4 m over punktet B, og til 6,5 m over punktet A. Lukas plan danner vinkelen 30^0 med horisontalplanet. Se bort fra atmosfæretrykket, og sett $\gamma = \rho g = 10^4$ Pa/m.

a) Finn den hydrostatiske kraft F på luka.

b) Som vist på figuren legges ξ−aksen i lukas plan, med $\xi = 0$ i toppunktet T, og x−aksen peker vinkelrett inn i planet. Finn posisjonen ξ_{CP} til trykksentret, ved å benytte formelen

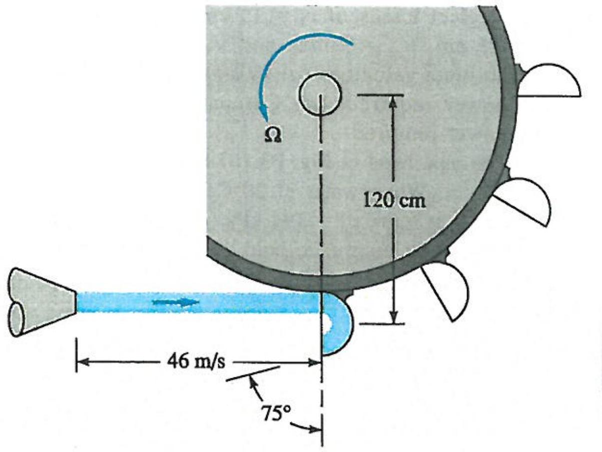
$$\xi_{CP} - \xi_{CG} = \frac{I_{xx}}{\xi_{CG} A},$$

hvor ξ_{CG} er posisjonen til centroiden (flatesentret) og A er lukas areal. Det oppgis at for en trekant med grunnlinje b og høyde h ligger centroiden $h/3$ over grunnlinjen, og at arealets treghetsmoment omkring en akse langs høyden (inn i planet) er $I_{xx} = b^3 h/48$.

c) Forklar, ved å betrakte kraftmomentet omkring grunnlinjen AB, hvorfor trykksentret CP må ligge i samme avstand fra AB som centroiden CG. Benytt dette til å beregne den kraft K som må til for å åpne luka.

d) Anta at bassenget fylles opp med mer vann slik at vannspeilets høyde over punktet B øker fra 4 m til 6 m. Hvor stor må K være nå? Gi en kort forklaring.

Oppgave 2



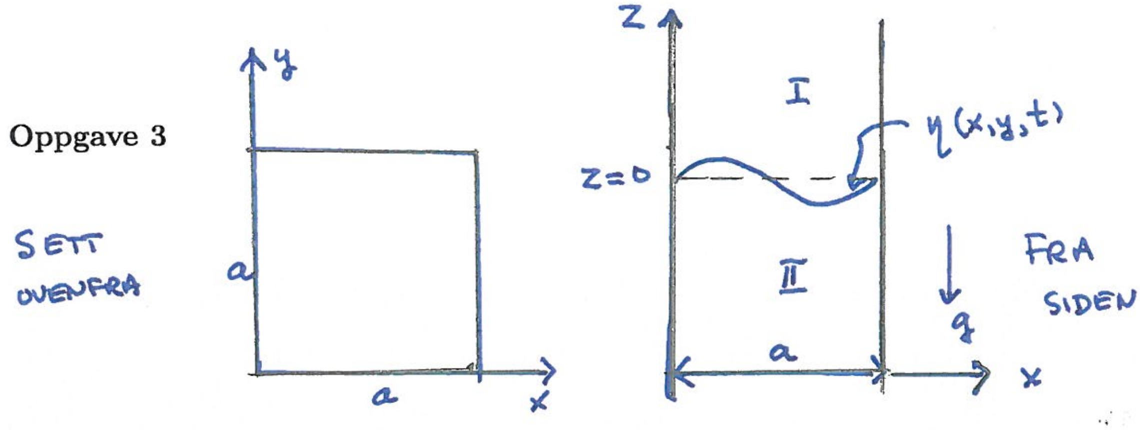
Et turbinhjul med radius $R = 120$ cm holdes i jevn rotasjon med 220 omløp/min av en horisontal vannstråle (jet) som kommer inn med hastighet $V_j = 46$ m/s relativt til laboratoriesystemet. Strålens tverrsnitt er $A = 40\ \mathrm{cm}^2$. Figuren viser strålen idet den treffer en av skovlene (engelsk 'buckets'), og forlater skovlen igjen under utgangsvinkelen $\theta = 75^0$. Anta at det er mange skovler på hjulet, slik at tilstanden kan anses for å være stasjonær. Kall hjulets vinkelhastighet Ω.

a) Finn den horisontale kraftkomponent F_{skovl} på skovlen i det medfølgende koordinatsystem, og finn den tilhørende effekt P i laboratoriesystemet. Angi svarene også numerisk. Sett $\rho = 10^3\ \mathrm{kg/m^3}$.

b) Lag en kvalitativ skisse av $P = P(\Omega)$ (de andre variablene holdes konstante). For hvilken verdi av Ω vil P ha maksimum, $P = P_{\max}$? Bare bokstavsvar kreves.

Av skissen (eller av uttrykket for P) ser du at for en bestemt verdi av Ω vil P bli lik null. Hva betyr dette tilfellet fysisk?

Oppgave 3



To ideelle væsker med konstante tettheter ρ_I og ρ_{II} er overlagret hverandre i en tank med kvadratisk grunnflate. Sidekanten er a. Begge væskelagene har uendelig dybde. Under stillevannsforhold er interfasen mellom lagene beliggende i posisjon $z = 0$. Tyngdens akselerasjon er g. De horisontale aksene er x og y.

Oppgaven i det følgende er å analysere de stasjonære svingemodene i systemet, når vinkelfrekvensen ω er gitt. Det oppgis at hastighetspotensialene i de to områdene er

$$\phi_I = Ae^{-kz} \cos px \cos qy \cos \omega t,$$

$$\phi_{II} = Be^{kz} \cos px \cos qy \cos \omega t, \qquad (1)$$

hvor A og B er konstanter.

a) Finn de horisontale hastighetskomponentene u og v i væsken, og benytt grensebetingelsene ved tankens sidevegger ($x = 0, a$ og $y = 0, a$) til å vise at de horisontale bølgetallene p og q er proporsjonale med hele tall. Kall disse tallene m og n. Benytt også inkompressibilitetsbetingelsen til å finne størrelsen k uttrykt ved a, m og n. Det er tilstrekkelig å betrakte bare område I.

b) Elevasjonen av interfasen kan skrives som $\eta = \eta(x, y, t)$. Forklar kort hvorfor den kinematiske overflatebetingelsen ved $z = \eta$ i lineær approksimasjon kan uttrykkes ved ligningene

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi_I}{\partial z}, \quad z = 0,$$

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi_{II}}{\partial z}, \quad z = 0. \qquad (2)$$

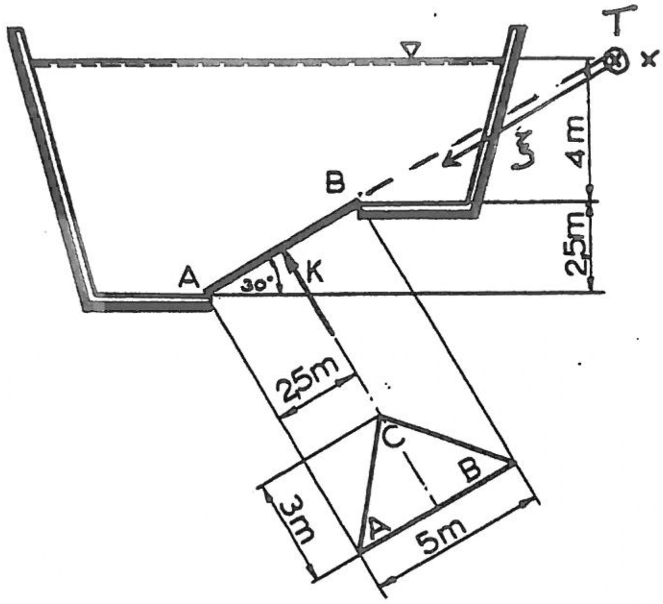
Sett opp på tilsvarende måte den dynamiske overflatebetingelsen (Bernoullis ligning) ved $z = \eta$ i lineær approksimasjon, både for fluid I og for fluid II, idet du setter Bernoulli-konstantene lik null. Vis herav at hastighetspotensialene må tilfredsstille ligningen

$$\rho_I \frac{\partial^2 \phi_I}{\partial t^2} - \rho_{II} \frac{\partial^2 \phi_{II}}{\partial t^2} + g(\rho_I - \rho_{II}) \frac{\partial \phi_{II}}{\partial z} = 0, \quad z = 0. \tag{3}$$

(Hint: Benytt at $p_I = p_{II}$ ved interfasen.)

c) Benytt ligningene ovenfor til å finne systemets dispersjonsrelasjon, $\omega = \omega(k)$, hvor også størrelsene g, ρ_I og ρ_{II} inngår. Sjekk uttrykket i spesialtilfellet $\rho_I \to 0$.

Problem 1



At the bottom of a basin is there a triangular plate ABC lying such that the baseline AB is parallel to the figure plane. The plate can be opened by a force K acting orthogonally to the plate, in the point C (lying in the direction into the figure plane). The basin is filled with water to a height 4 m above the point B, and to a height 6.5 m above the point A. The plane of inclination makes the angle 30^0 with the horizontal plane. Ignore the atmospheric pressure, and set $\gamma = \rho g = 10^4$ Pa/m.

a) Find the hydrostatic force F on the plate.

b) As shown on the figure, the $\xi-$ axis lies in the inclined plane, with $\xi = 0$ at the top point T, so that the $x-$axis points orthogonally into the figure plane. Find the position ξ_{CP} of the center of pressure, by means of the formula

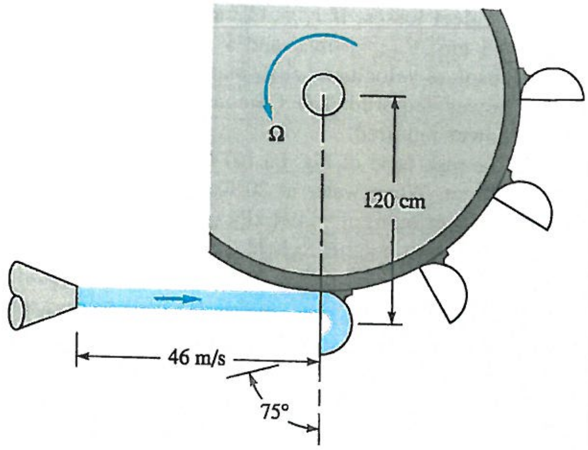
$$\xi_{CP} - \xi_{CG} = \frac{I_{xx}}{\xi_{CG} A},$$

where ξ_{CG} is the position of the centroid and A is the area of the plate. Information: For a triangle with baseline b and height h the centroid is located $h/3$ above the baseline, and the area moment of inertia around the height of the triangle (the $x-$axis) is $I_{xx} = b^3 h/48$.

c) Consider the force moment around the baseline AB, and explain why the center of pressure CP must lie in the same height from AB as the centroid CG. Make use of this to calculate the force K necessary to open the plate at the point C.

d) Suppose that the basin is filled with more water so that the height of the free surface above the point B is increased from 4 m to 6 m. How large must the force K be now? Give a short explanation.

Problem 2

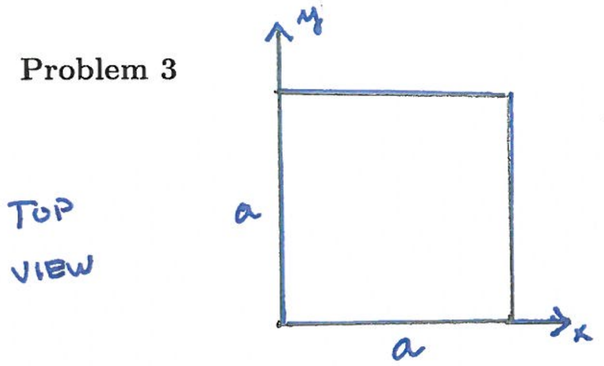


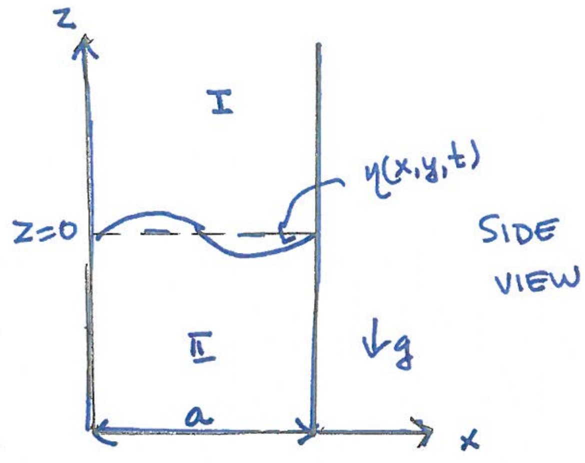
A water wheel (turbine wheel) with radius $R = 120$ cm is held in uniform rotation with 220 rotations/min by a horizontal water jet coming in with velocity $V_j = 46$ m/s relatively to the laboratory system. The cross section of the jet is $A = 40$ cm^2. The figure shows the jet when it strikes upon one of the buckets and leaves the bucket again under the angle $\theta = 75^0$. Assume that there are many buckets on the wheel, so that the state can be taken to be stationary. Let Ω be the angular velocity of the wheel.

a) Find the horizontal force component F_{bucket} on the bucket in the comoving coordinate system, and find the corresponding effect P in the laboratory system. Calculate the answers also numerically.

b) Draw a qualitative sketch of $P = P(\Omega)$ (the other variables held constant). For which value of Ω will P have a maximum, $P = P_{\text{max}}$? No numerics is required here.
From the sketch (or from the expression for P) you see that for a definite value of Ω will P be equal to zero. What does this case mean physically?

Problem 3





Two ideal fluids with constant densities ρ_I and ρ_{II} are lying above each other in a tank having a square base area. The edge of the base is a. Both fluid sheets are assumed to have infinite depths. Under still water conditions the interface between the sheets is lying at the level $z = 0$. The gravitational acceleration is g. The horizontal axes are x and y.

The task in the following is to analyze the stationary modes of oscillations in the system, when the angular frequency ω is given. The velocity potentials in the two regions are given as

$$\phi_I = Ae^{-kz} \cos px \cos qy \cos \omega t,$$

$$\phi_{II} = Be^{kz} \cos px \cos qy \cos \omega t, \tag{1}$$

where A and B are constants.

a) Find the horizontal velocity components u and v in the liquid, and use the boundary conditions at the side walls ($x = 0, a$ and $y = 0, a$) to show that the horizontal wave numbers p and q are proportional to integers. Call these integers m and n. Make also use of the incompressibility condition to find the quantity k expressed in terms of a, m and n. It is sufficient to consider the region I only.

b) The elevation of the interface can be written as $\eta = \eta(x, y, t)$. Explain briefly why the kinematic surface condition at $z = \eta$ i linear approximation can be expressed as the equations

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi_I}{\partial z}, \quad z = 0,$$

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi_{II}}{\partial z}, \quad z = 0. \tag{2}$$

Set up in a corresponding way the dynamic surface condition (the Bernoulli equation) at $z = \eta$ in linear approximation, both for fluid I and for fluid II, when you set both Bernoulli constants equal to zero. Show from this that the velocity potentials must satisfy the equation

$$\rho_I \frac{\partial^2 \phi_I}{\partial t^2} - \rho_{II} \frac{\partial^2 \phi_{II}}{\partial t^2} + g(\rho_I - \rho_{II}) \frac{\partial \phi_{II}}{\partial z} = 0, \quad z = 0. \tag{3}$$

(Tip: Make use of the property $p_I = p_{II}$ at the interface.)

c) Make use of the equations above to find the dispersion equation $\omega = \omega(k)$ for the system, where also the quantities g, ρ_I and ρ_{II} are present. Check the expression in the special case where $\rho_I \to 0$.

TEP4105 Fluid Mechanics. Exam 12.12.2013. Solution

Problem 1

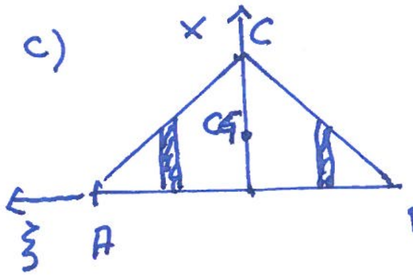
a) $F = \gamma h_{CG} A$, where the depth of the centroid is $h_{CG} = 4 + \frac{1}{2} \cdot 2{,}5 = 5{,}25$ m. The centroid lies on the symmetry line of the triangle. Area A is
$A = \frac{1}{2} \cdot 5 \cdot 3 = 7{,}5 \text{ m}^2 \Rightarrow \underline{F} = 10^4 \cdot 5{,}25 \cdot 7{,}5 = \underline{3{,}94 \cdot 10^5 \text{ N}}$

b) $\xi_{CP} - \xi_{CG} = \frac{I_{xx}}{\xi_{CG} \cdot A}$. Distance from top point T to the centroid is
$\xi_{CG} = TB + 2{,}5 = 8 + 2{,}5 = 10{,}5$ m, $I_{xx} = \frac{1}{48} b^3 h = \frac{1}{48} 5^3 \cdot 3 = 7{,}81 \text{ m}^4$.

Thus $\underline{\xi_{CP}} = 10{,}5 + \frac{7{,}81}{10{,}5 \cdot 7{,}5} = \underline{10{,}60 \text{ m}}$

$(\xi_{CP} - \xi_{CG} = 10{,}60 - 10{,}50 = 0{,}10 \text{ m})$

c)



Divide area of triangle into stripes. The pressure is the same within the same stripe (p is independent of x). Thus the CP for each stripe is coincident with the CG. As the figure is symmetric about the x-axis, the CP for the plate is located at the distance $h/3 = 1$ m from AB.

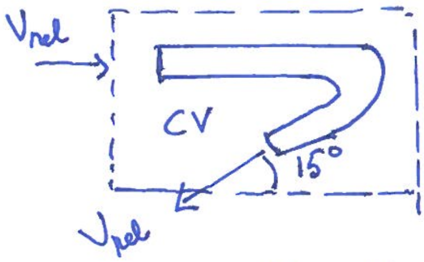
Force moment $F \cdot 1 = F$ must be equal to $K \cdot h = 3K$ when the plate opens. Equation $3K = F = 3{,}94 \cdot 10^5$ Nm $\Rightarrow \underline{K = 1{,}31 \cdot 10^5 \text{ N}}$

d) When height of surface above point B rises from 4 m to 6 m:
Depth of centroid becomes $6 + 1{,}25 = 7{,}25$ m.
Force moment around AB : $F \cdot 1 = K \cdot 3$, where now
$F = \gamma h_{CG} A = 10^4 \cdot 7{,}25 \cdot 7{,}5 = 5{,}44 \cdot 10^5$ N

$\underline{K} = \frac{1}{3} \cdot 5{,}44 \cdot 10^5 \text{ N} = \underline{1{,}81 \cdot 10^5 \text{ N}}$

Comment : When the height increases, both F and K increase, but the center of pressure CP is the same.

Problem 2



a) In comoving system the control volume CV is laid around the bucket (skovl).

Relative velocity of jet : $V_{rel} = V_j - R\Omega$,

where $\Omega = 2\pi f = 2\pi \frac{220}{60} \frac{rad}{s} = 23.0 \frac{rad}{s}$.

Thus $\underline{V_{rel}} = 46 - 1.20 \cdot 28 = \underline{18.4 \ m/s}$, $\underline{R\Omega} = \underline{27.6 \ m/s}$

Momentum flux in : $\dot{M}_{,INN} = \int \rho V_{rel}^2 dA = \rho (V_j - R\Omega)^2 \cdot A$, in x-dir.

Momentum flux out : $\dot{M}_{UT} = -\int \rho V_{rel}^2 dA \cdot \cos 15° = -\rho \cos 15° \cdot (V_j - R\Omega)^2 \cdot A$, in x-direction.

Force F on water in CV is given by momentum equation :

$F = \dot{M}_{UT} - \dot{M}_{INN} = -\rho (1 + \cos 15°)(V_j - R\Omega)^2 \cdot A$

Force on bucket $\underline{F_{skovl}} = -F = \underline{+\rho (1 + \cos 15°)(V_j - R\Omega)^2 \cdot A}$

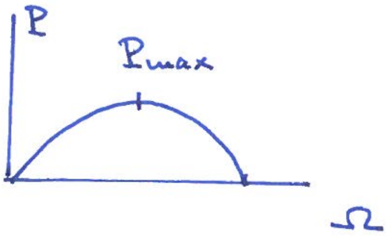
Power in lab- system: $\underline{P} = F_{skovl} \cdot R\Omega = \underline{\rho (1 + \cos 15°) R A \Omega (V_j - R\Omega)^2}$

Numerically : $\cos 15° = 0.966$, $A = 40 \, cm^2 = 40 \cdot 10^{-4} \, m^2$, $\Rightarrow$

$\underline{F_{skovl}} = 10^3 \cdot 1.966 \cdot 18.4^2 \cdot \cancel{27.6} \cdot 40 \cdot 10^{-4} \ N = \underline{2.66 \ kN}$

$\underline{P} = F_{skovl} \cdot R\Omega = 2.66 \cdot 27.6 \ kW = \underline{73.4 \ kW}$

b)



$\underline{P} \propto \Omega (V_j - R\Omega)^2$

$P = P_{max}$ when $dP/d\Omega = 0 \Rightarrow$

$(V_j - R\Omega)^2 - 2\Omega R (V_j - R\Omega) =$

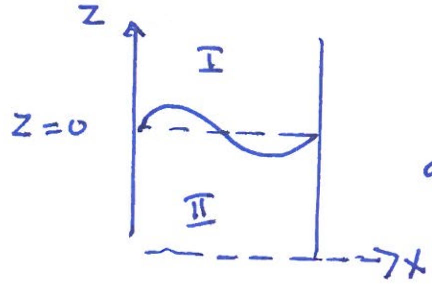
$= (V_j - R\Omega)(V_j - 3R\Omega) = 0$

Physical solution $\underline{\Omega = \frac{V_j}{3R}}$. $\underline{P_{max} = \frac{4}{27}(1 + \cos 15°) \cdot \rho A V_j^3}$

$P = 0$ also when $\Omega = V_j / R$. It corresponds to $V_{rel} = 0$.

∴ No water enters the bucket.

Problem 3



$$\phi_I = A e^{-kz} \cos px \cos qy \cos \omega t$$

$$\phi_{II} = B e^{-kz} \cos px \cos qy \cos \omega t$$

a) Consider region I:

$$u = \frac{\partial \phi_I}{\partial x} = -pA e^{-kz} \sin px \cos qy \cos \omega t$$

$$v = \frac{\partial \phi_I}{\partial y} = -qA e^{-kz} \cos px \sin qy \cos \omega t$$

Boundary condition at the walls:

$u = 0$ for $x = 0$ and $x = a$ $\Rightarrow$ $\sin pa = 0$, $pa = m \cdot \pi$,

$$\underline{p = m\pi/a}$$

$v = 0$ for $y = 0$ and $y = a$ $\Rightarrow$ $\sin qa = 0$, $qa = n\pi$,

$$\underline{q = n\pi/a}$$

Incompressibility condition

$$\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 . \qquad \text{(Considers region I)}$$

Here $w = \dfrac{\partial \phi_I}{\partial z} = -kA e^{-kz} \cos px \cos qy \cos \omega t$

Differentiating u, v and w one gets

$$-p^2 A e^{-kz} \cos px \cos qy \cos \omega t - q^2 A e^{-kz} \cos px \cos qy \cos \omega t +$$

$$+ k^2 A e^{-kz} \cos px \cos qy \cos \omega t = 0 .$$

Thus $-p^2 - q^2 + k^2 = 0$

$$\underline{k} = \sqrt{p^2 + q^2} = \underline{\frac{\pi}{a}\sqrt{m^2 + n^2}}$$

Problem 3 b)

Free surface condition $\frac{D\eta}{Dt} = w$, where $\eta = \eta(x,y,t)$.

Differentiate η, and use the chain rule,

$$\frac{D\eta}{Dt} = \frac{\partial \eta}{\partial t} + \frac{dx}{dt}\frac{\partial \eta}{\partial x} + \frac{dy}{dt}\frac{\partial \eta}{\partial y} = \frac{\partial \eta}{\partial t} + u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y}$$

Thus

$$\underbrace{\frac{\partial \eta}{\partial t}}_{O(a)} + \underbrace{u\frac{\partial \eta}{\partial x}}_{O(a^2)} + \underbrace{v\frac{\partial \eta}{\partial y}}_{O(a^2)} = \underbrace{w}_{O(a)}, \quad \text{at } z = \eta.$$

In linear approximation only the first order terms are retained:

$$\underline{\frac{\partial \eta}{\partial t} = w = \frac{\partial \phi_I}{\partial z}}, \text{ in region I, at } z = 0$$

(The difference between $z=0$ and $z=\eta$ is of higher order.)

Correspondingly in region II: $\underline{\frac{\partial \eta}{\partial t} = w = \frac{\partial \phi_{II}}{\partial z}, \quad z = 0}$

Dynamic surface condition (Bernoulli's equation) at the interface $z = \eta$, when the Bernoulli constant is zero,

$$\frac{\partial \phi_I}{\partial t} + \underbrace{\tfrac{1}{2}V^2}_{\text{NEGLECTED}} + \frac{p_I}{\rho_I} + g\eta = 0$$

Multiply with ρ_I:

$$\rho_I \frac{\partial \phi_I}{\partial t} + p_I + \rho_I g \eta = 0$$

Correspondingly

$$\rho_{II} \frac{\partial \phi_{II}}{\partial t} + p_{II} + \rho_{II} g \eta = 0$$

Use $p_I = p_{II}$ at $z = \eta$

$$\Rightarrow \quad \rho_I \frac{\partial \phi_I}{\partial t} - \rho_{II}\frac{\partial \phi_{II}}{\partial t} + g(\rho_I - \rho_{II})\eta = 0$$

Take time derivative, and use $\partial \eta / \partial t = \partial \phi_{II}/\partial z$:

$$\underline{\rho_I \frac{\partial^2 \phi_I}{\partial t^2} - \rho_{II}\frac{\partial^2 \phi_{II}}{\partial t^2} + g(\rho_I - \rho_{II})\frac{\partial \phi_{II}}{\partial z} = 0, \quad z = 0.}$$

Problem 3c)

From equation (2): $\dfrac{\partial \phi_I}{\partial z} = \dfrac{\partial \phi_{II}}{\partial z}$, $z = 0$.

Then equation (1) yields $-kA = kB \Rightarrow \underline{B = -A}$.

Calculate

$$\frac{\partial^2 \phi_I}{\partial t^2} = -\omega^2 A e^{-kz} \cos px \cos qy \cos \omega t \rightarrow -\omega^2 A \cos px \cos qy \cos \omega t, \quad z = 0$$

$$\frac{\partial^2 \phi_{II}}{\partial t^2} = -\omega^2 B e^{kz} \cos px \cos qy \cos \omega t \rightarrow -\omega^2 B \cos px \cos qy \cos \omega t, \quad z = 0$$

$$\frac{\partial \phi_{II}}{\partial z} = B k e^{kz} \cos px \cos qy \cos \omega t \rightarrow kB \cos px \cos qy \cos \omega t, \quad z = 0.$$

Insertion into eq. (3), with use of $B = -A$, yields

$$\omega^2 (\rho_I + \rho_{II}) = (\rho_{II} - \rho_I) g k$$

$$\underline{\omega^2 = \frac{\rho_{II} - \rho_I}{\rho_{II} + \rho_I} \cdot g k}$$

If $\rho_I \rightarrow 0$ (air above the interface):

$\underline{\omega^2 = gk}$, as for deep-water standing waves.