TEP4105 FLUIDMEKANIKK

Formelliste basert på White, Fluid Mechanics

Overflatespenning: $\Delta p = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

Strømlinjer: $dy/dx = v/u$

Atmosfæren: $p(z)/p_a = [T(z)/T_0]^{5,26}$

Kraft på plane flater: $F = \gamma h_{CG} A$

$$\xi_{CP} = \xi_{CG} + \frac{I_{xx}}{\xi_{CG} A}$$

Med $y_{CP} = \xi_{CG} - \xi_{CP}$:

$$y_{CP} = -\frac{I_{xx}}{\xi_{CG} A} = -\frac{I_{xx} \sin\theta}{h_{CG} A}$$

Tilsvarende $x_{CP} = -\frac{I_{xy}}{\xi_{CG} A} = -\frac{I_{xy} \sin\theta}{h_{CG} A}$

Kraft på krumme flater:

$$F_H = \gamma h_{CG} A_x \ , \ F_V = \gamma \mathcal{V}$$

Reynolds' transportteorem:

$$\frac{d}{dt}\int_{SYST} \phi \, d\mathcal{V} = \frac{\partial}{\partial t}\int_{CV} \phi \, d\mathcal{V} + \int_{CS} \phi \, \vec{V} \cdot \vec{n} \, dA$$

Impulsligningen: $\sum \vec{F} = \dot{\vec{M}}_{UT} - \dot{\vec{M}}_{INN}$

hvor $\dot{\vec{M}}_{UT} = \int_{UT} \rho\, \vec{V}(\vec{V} \cdot \vec{n})dA$

$$\dot{\vec{M}}_{INN} = - \int_{INN} \rho\, \vec{V}(\vec{V} \cdot \vec{n})dA$$

Energiligningen:

$$\dot{Q} - \dot{W}_s = \int_{cs} \rho\left(\hat{h} + \frac{1}{2}V^2 + gz\right)\vec{V} \cdot \vec{n}\, dA \ ,$$

hvor $\hat{h} = \hat{u} + p/\rho$ er spesifikk entalpi.

Mekanisk energiligning for inkompressibel strømning langs strømlinje:

$$\frac{p_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \left(\frac{p_2}{\rho} + \frac{1}{2}V_2^2 + gz_2\right) + w_s + gh_f \ ,$$

hvor $\hat{u}_2 - \hat{u}_1 = q + gh_f$.

Bernoulli: $\frac{p}{\rho} + \frac{1}{2}V^2 + gz = C$, langs strømlinje

Kontinuitetsligningen:

$$\partial\rho / \partial t + \nabla \cdot (\rho\vec{V}) = 0$$

Euler: $\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V} = -\frac{1}{\rho}\nabla p + \vec{g}$

Navier-Stokes: $$\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V} = -\frac{1}{\rho}\nabla p + \vec{g} + \nu \nabla^2 \vec{V} \quad , \quad \nu = \mu / \rho$$

Strømfunksjonen ψ, kartesiske koordinater:

$$u = \frac{\partial \psi}{\partial y} \quad , \quad v = -\frac{\partial \psi}{\partial x}$$

$$\nabla^2 \psi = -\varsigma_z \quad , \text{ hvor } \vec{\varsigma} = \nabla \times \vec{V} \text{ er virvlingen}$$

Planpolare koordinater:

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \quad , \qquad v_\theta = -\frac{\partial \psi}{\partial r}$$

$$\nabla^2 \psi = -\varsigma_z \quad , \qquad \text{hvor} \qquad (\nabla \times \vec{V})_z = \frac{1}{r}\frac{\partial}{\partial r}(r v_\theta) - \frac{1}{r}\frac{\partial v_r}{\partial \theta}$$

$$\nabla^2 \psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r \, \frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2}$$

Hastighetspotensial ϕ: $\vec{V} = \nabla \phi$

Singulariteter:

$$\psi_{\text{kilde}} = m\theta \quad , \quad \phi_{\text{kilde}} = m \, \ell n \, r$$

$$\psi_{\text{virvel}} = -K \, \ell n \, r \quad , \quad \phi_{\text{virvel}} = K\theta$$

Sirkulasjon: $$\Gamma = \oint \vec{V} \cdot d\vec{s}$$

Løft og drag: $$L = C_L \cdot \frac{1}{2}\rho U^2 \cdot A \quad , \quad D = C_D \cdot \frac{1}{2}\rho U^2 \cdot A$$

Reynolds tall: $\mathrm{Re} = UL/\nu$

Kutta-Joukowski: $L = -\rho U \Gamma$ (per lengdeenhet).

Vannbølger (G. Moes kompendium):

$$\phi = \frac{ga}{\omega}\frac{\cosh k(z+d)}{\cosh kd}\cos(\omega t - kx) = \frac{a\omega}{k}\frac{\cosh k(z+d)}{\sinh kd}\cos(\omega t - kx)$$

Dispersjonsrelasjon:

$$\omega^2 = gk \tanh kd$$

Bernoulli:

$$\frac{\partial \phi}{\partial t} + \frac{p}{\rho} + \frac{1}{2}V^2 + gz = \text{konst.}$$

Kinematisk overflatebetingelse:

$$\frac{\partial \eta}{\partial t} + u\frac{\partial \eta}{\partial x} = w, \quad \text{ved} \quad z = \eta$$

Dynamisk trykk: $p_d = -\rho \dfrac{\partial \phi}{\partial t}$

Komplekst potensial: $w(z) = \phi(x,y) + i\psi(x,y)$

Kompleks hastighet: $w'(z) = u - iv = Ve^{-i\theta}$

Blasius' teorem: $F_x - iF_y = \dfrac{1}{2} i\rho \oint_c \left(\dfrac{dw}{dz}\right)^2 dz$