

TOTIMERSØVING NR 3 TEP 4105 FLUIDMEKANIKK

Høst 2014

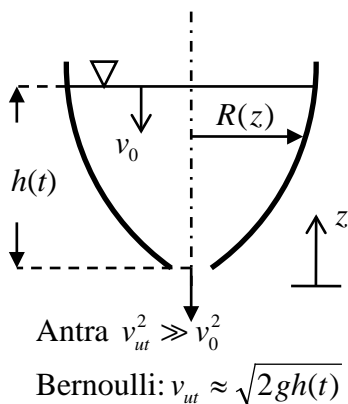
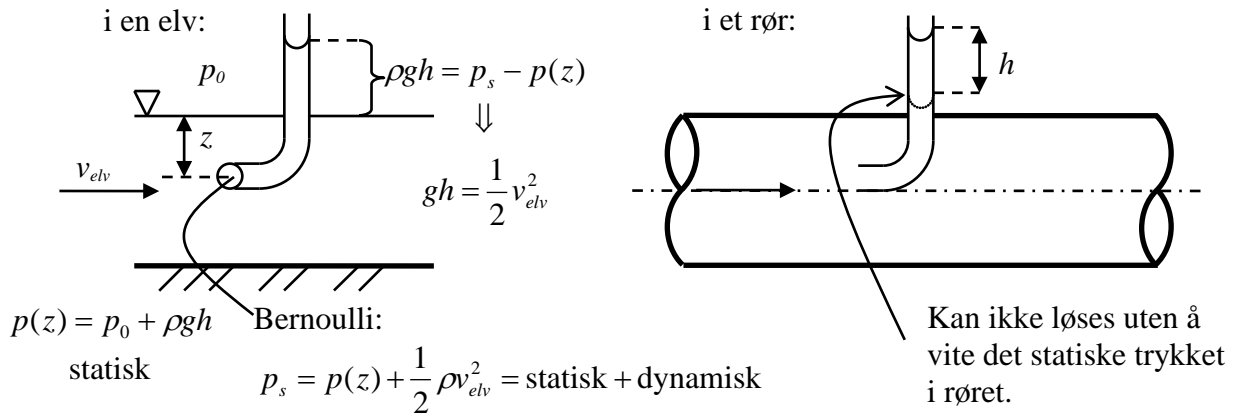
Utført av: (alle i gruppa)

LØSNINGSFORSLAG

Problemene søkes løst på en enkel måte, (Kap. 3), med kontrollvolumanalyse, Bernoulli. Vurder om dette lar seg gjøre, eventuelt hvilke forenklinger, forutsetninger, antagelser eller justeringer som i så fall må gjøres. Sett også opp hvilke likninger/prinsipper som må brukes.

Oppgave 1

Finn strømningshastigheten ut fra avlest høyde H i et rør strukket inn i strømmingen (Pitot-rør).



Oppgave 2

Lag et aksesymmetrisk kar med radius $R(z)$ slik at vannoverflaten synker med konstant hastighet v_0 . (Slike vann-ur ble brukt i oldtiden.)

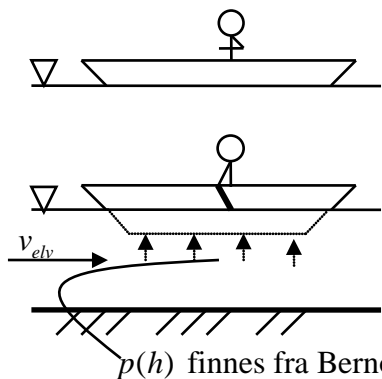
Svar: Massebevarelse: $q = v_{ut} \cdot A_{ut} = v_0 \pi R(z)^2$

Kinematisk: $v_0 = -\frac{dh}{dt}$

$$\rightarrow R(z) = \sqrt{\frac{\sqrt{2gz} A_{out}}{v_0 \pi}}$$

Oppgave 3

En kajakk stikker en dybde h ned i vannet når den ligger stille på vannet. Hvor dypt stikker den når det padles av alle krefter opp en elv?



Svar:

Stillestående: Ok med Archimedes:

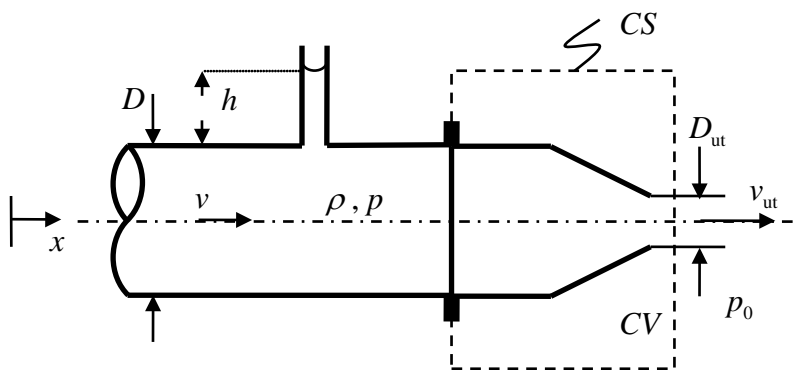
$mg =$ vekt av fortrengt fluid

Padlende: Archmedes ikke ok.

$mg =$ oppdriftskraft \equiv netto trykkraft

Oppgave 4

Finn kraften som virker på dysen ut fra avlest høyde h .



Svar:

Massebevarelse

$$q = v \frac{\pi}{4} D^2 = v_{ut} \frac{\pi}{4} D_{ut}^2$$

Bernoulli:

$$\frac{p}{\rho} + \frac{v^2}{2} = \frac{p_0}{\rho} + \frac{v_{ut}^2}{2}$$

Hydrostatisk trykk lest fra h : $p = p_0 + \rho g (h + D/2)$

$$\rightarrow v = \sqrt{\frac{2g(h + D/2)}{(D/D_{out})^4 - 1}}$$

Kraftlov:

$$\sum F_x = F_{\text{kontakt}} + (p - p_0) \frac{\pi}{4} D^2 = -\rho v^2 \frac{\pi}{4} D^2 + \rho v_{ut}^2 \frac{\pi}{4} D_{ut}^2$$

$$\underline{\underline{F_{\text{dyse}} = -F_{\text{kontakt}}}}$$

Oppgave 5

Finn vannhastigheten v_0 i røret slik at vannet i reservoaret akkurat løftes høyden h .

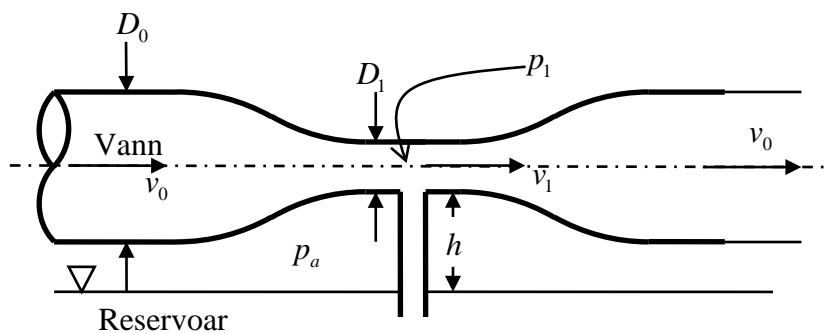
Svar: Massebevarelse: $q = v_0 \frac{\pi}{4} D_0^2 = v_1 \frac{\pi}{4} D_1^2$

$$\text{Bernoulli: } \frac{p_1}{\rho} + \frac{v_1^2}{2} = \frac{p_a}{\rho} + \frac{v_0^2}{2}$$

Likevekt: $\rho gh = p_a - p_1$

For større hastigheter i røret, finn vannmengden som transporteres opp fra reservoaret.

Svar: Går ikke! Prøv med Bernoulli fra vannoverflaten og ut! Venturi-røret fungerer nå som en pumpe (med ukjent effekt).



Oppgave 6

En vannturbin drives av en vannstråle med hastighet v_j . Finn turtallet Ω som gir maksimum effekt.

Svar: Vi plasserer et kontrollvolum rundt skovlen som beveger seg med rotasjonshastigheten $v_\Omega = \Omega R$

Massebevarelse: $v_{\text{inn}} = v_{\text{ut}}$.

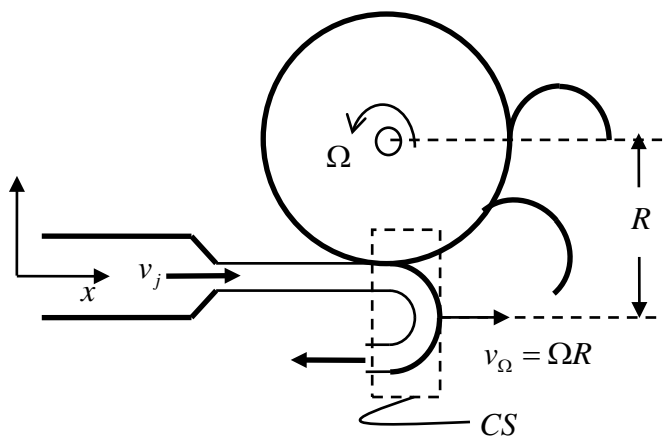
Bernoulli: $p = p_a$ overalt.

Kraftlov:

$$\begin{aligned} \sum F_x &= F_{\text{kontakt}} = \oint_{CS} \rho v_x (\vec{v}_j \cdot \vec{n}) dA \\ &= \rho A (v_j - v_\Omega)^2 + \rho A (v_j - v_\Omega)^2 \end{aligned}$$

Effekt: $P = F v_\Omega = 2 \rho A (v_j - v_\Omega)^2 v_\Omega$

Maks effekt for $\frac{dP}{dv_\Omega} = 0$, så $\Omega = \frac{1}{3} \frac{v_j}{R}$



1 Superspeedy jet powered boat

This question was originally given in the coarse TEP4100 and dealt with solving the same problem in three different frames of reference. In TEP4105 we omit some of these generalisations, only asking for a solution in the first of the three reference frames presented below. The full version is however given here for students wishing to better understand control volume analysis.

A boat is driven by a water jet. The jet is powered by a water pump, pumping water backwards with velocity V_J (relative to the boat) through a pipe of area A . The boat has reached a constant ('terminal') velocity V_B . Find the force with which the jet pushes the boat. Let the CV follow the boat and check that you obtain the same answer using both absolute and relative coordinate systems. Challenge: compute the force with a steady CV .

Solution:

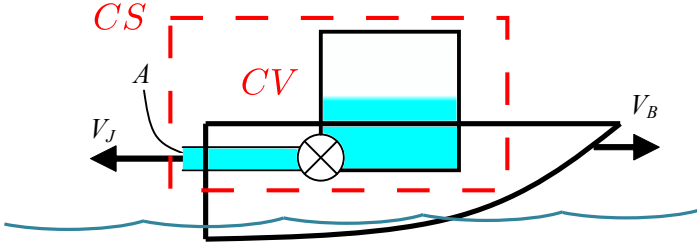
There are three main choices for the coordinate system and CV for this problem:

- (i) Coordinate system and CV moves relative to the boat
- (ii) A stationary coordinate system (relative to an observer on land) and a CV moving relative to the boat.
- (iii) Stationary Coordinate system and CV .

The first option yields the easiest path for computing the pressure and the third the most cumbersome.

In all approaches we apply a momentum balance (Newton's 2nd law) to the CV

$$\text{Time rate of momentum change (generalized 'acceleration' \cdot \text{mass})} = \frac{d}{dt} \int_{CV} \rho \vec{V} dV + \oint_{CS} \rho \vec{V} (\vec{V} \cdot \vec{n}) dA = \sum_{CV} \vec{F}_{\text{external}} \quad (1)$$



(i) Relative coordinate system and CV

The flow out of the CV moving with the boat is

$$\dot{V} = AV_J,$$

where V_J was the velocity relative to the boat. Momentum is always taken *relative to the coordinate system*. The momentum per volume M of the of the water exiting the CV is $\rho(-V_J)$. The volume flow and momentum are constant where the jet exits the CS and zero everywhere else. Consequently, the convective integral of (1) evaluates to

$$\oint_{CS} \rho \vec{V} (\vec{V} \cdot \vec{n}) dA = \rho(-V_J) \dot{V} = -\rho AV_J^2$$

Relative to the coordinate system the liquid and material inside the CV are stationary. Therefore, the momentum inside the CV does not change with time, and the transient term in (1) evaluates to zero.

Equation (1) expresses the more general form of Newton's 2nd law: the time rate change of the total momentum in the CV equals the forces exerted on the CV – in this case those forces are exerted from the boat, and originates from friction and wave generation. (1) then gives $\sum_{CV} \vec{F}_{\text{external}} = -\rho AV_J^2$. By Newton's 3rd law, the force acting on the boat *from* the jet are equal but oppositely directed

$$\underline{\underline{F_{\text{jet}} = \rho AV_J^2}}$$

Question: What about the forces from the pump?

Answer: The forces from the pump are internal. That is, also the *reactive* force from the pump is part of the *CV* and so these cancel out. Here, the force from the boat and the force from the pump are in fact the same.

Question: What makes forces internal (ignorable) and external (important)

Answer: A *CV* force is external if its reactive counterpart is *outside* the *CV*. This is the case just along the *CS* and for external force fields such as gravity and external magnetic fields. (When gravity pulls at a *CV* the *CV* pulls back at *the earth*.)

(ii) Stationary coordinate system, relative *CV*

The volume flow out of the *CV* will still be the same as in (i), namely $\dot{V} = AV_J$.

Relative to the coordinate system the momentum of the water exiting the *CV* is now $\rho(V_B - V_J)$, yielding

$$\oint_{CS} \rho \vec{V} (\vec{V} \cdot \vec{n}) dA = \rho(V_B - V_J) \dot{V} = \rho AV_J (V_B - V_J)$$

The water in the water jet tank now has a momentum relative to the coordinate system equalling ρV_B per unit volume. Its rate of depletion is the rate at which the water is removed from the *CV*, namely $\dot{V} = AV_J$. The time rate *increase* of momentum in the first integral of (1) is then

$$\frac{d}{dt} \int_{CV} \rho \vec{V} dV = \rho V_B (-\dot{V}) = -\rho AV_J V_B.$$

As expected, when these integrals are summed, the reactive force is found to be

$$\underline{\underline{F_{jet} = \rho AV_J^2}}$$

(iii) Stationary coordinate system and *CV*

This one is more tricky. In evaluating the convection integral through a stationary *CV* with the momentum at the *CS* relative to a stationary coordinate system we find

$$\oint_{CS} \rho \vec{V} (\vec{V} \cdot \vec{n}) dA = \underbrace{(V_J - V_B) A}_{\dot{V}} \cdot \underbrace{\rho(-V_J + V_B)}_{\text{momentum}} = -\rho A (V_J - V_B)^2$$

The momentum per unit volume in the jet tank is still ρV_B as in (ii), and its rate of depletion from the tank is $V_J A$, making the momentum time rate *increase* in the tank $\rho V_B \cdot (-V_J A)$. This does not sum to ρAV_J^2 , so where has the last momentum gone to? The answer is that as the boat passes through a stationary *CV* it fills the *CV* with the ejected water. The momentum of the expelled water filling the *CV* is also $\rho(-V_J + V_B)$ and the rate at which the *CV* is filled with the expelled water is $V_B A$ (imagine that the boat leaves behind it a fixed pipe shaped jet as it traverses at velocity V_B). The transient integral thus evaluates to

$$\frac{d}{dt} \int_{CV} \rho \vec{V} dV = \underbrace{\rho V_B \cdot (-V_J A)}_{\text{in tank}} + \underbrace{\rho(-V_J + V_B) \cdot V_B A}_{\text{in CV, outside tank}}$$

Together, the transient and convective integrals evaluate to $-\rho AV_J^2$, yielding the appropriate jet force

$$\underline{\underline{F_{jet} = \rho AV_J^2}}$$