

PLACEHOLDER-TITLE: Functional Linear Regression in a Scalar-on-Function Setting with Applications to SOMETHING

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1 Introduction

- Describe the idea of regressing a scalar on functional data
- Describing the difference to multiple linear regression intuitively
- Giving an intuitive example

Function Data Analysis (FDA) is a relatively new field which is getting more attention as researchers from different fields collect more data from an continuous underlying process. This data still can be processed by classical statistical methods, but only FDA allows to answer questions that are tied to the information given by the smoothness of the underlying continuous process. (see Levitin et al. 2007) As Kokoszka and Reimherr (2017) describe, FDA should be considered when one can view one or more of the variables or units of a given data set as a smooth curve or function and the interest is in analyzing samples of curves (see Kokoszka and Reimherr 2017, S. 17. To motivate the use of scalar-on-function regression, consider the case of a data set containing a scalar response and observations of an continuous underlying process. In economics, one application could be the regression of stock market correlations on the Global Crisis Index (GCI), where the regression allows to assess the relationship between the correlation and the GCI at every point within a window (see Das et al. 2019).

2 Theory

2.1 Draft-Overview

- Introduce the concept of random functions
- Introduce the concept of square integrable deterministic & random functions
- Explain basis expansions (so basis of the vector space L^2 and b-spline basis as an example)
- Motivate Karhunen-Loeve-Expansion and Eigenbasis from PCA
- Explain Scalar-on-Function Regression
- Estimation through basis-expansion (incl. Eigenbasis) [and estimation with roughness penalty]
- Address approximation error due to basis-truncation

2.2 Literature

- Kokoszka and Reimherr 2017
- Hsing and Eubank 2015
- Ramsay and Silverman 2005
- Horváth and Kokoszka 2012
- Cai and Hall 2006
- Levitin et al. 2007

3 Simulation

3.1 Draft-Overview

- Motivate Simulation for some data generating process from application
- Describe Simulation Setting from technical standpoint (DGP, set-up for replication, ...)
- Compare estimation with
 1. b-spline basis without addressing approximation error
 2. ... including proper treatment of approximation error
 3. Eigenbasis constructed from observations
- Prediction not Inference (Alternative: Focused on a testing procedure motivated by the application)
- Present Results
- Explain relevance for application

3.2 Literature

- Shonkwiler and Mendivil 2009
- R-packages: fda, refund, mgcv

4 Application

4.1 Draft-Overview

- Prediction not Inference (Alternative: Focused on a testing procedure motivated by the data set)
- IID data set (no dependence between the curves, don't want to do functional time series)
- Not necessarily data from economics (like biology, sports, whatever)
- Smooth curves or random walk (both fine)
- <https://functionaldata.wordpress.ncsu.edu/resources/>

4.2 Literature

- Carey et al. 2002

5 Outlook

5.1 Literature

- James, Wang, and Zhu 2009 (shape-restrictions)

6 Appendix

7 Bibliography

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