

PLACEHOLDER-TITLE: Functional Linear Regression in a Scalar-on-Function Setting with Applications to SOMETHING

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whenever

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1 Introduction

- Describe the idea of regressing a scalar on functional data
- Describing the difference to multiple linear regression intuitively
- Giving an intuitive example

2 Theory

2.1 Draft-Overview

- Introduce the concept of random functions
- Introduce the concept of square integrable deterministic & random functions
- Explain basis expansions (so basis of the vector space L^2 and b-spline basis as an example)
- Motivate Karhunen-Loeve-Expansion and Eigenbasis from PCA
- Explain Scalar-on-Function Regression
- Estimation through basis-expansion (incl. Eigenbasis) [and estimation with roughness penalty]
- Address approximation error due to basis-truncation

2.2 Literature

- Kokoszka and Reimherr 2017
- Hsing and Eubank 2015
- Ramsay and Silverman 2005
- Horváth and Kokoszka 2012
- Cai and Hall 2006
- Levitin et al. 2007

3 Simulation

3.1 Draft-Overview

- Motivate Simulation for some data generating process from application
- Describe Simulation Setting from technical standpoint (DGP, set-up for replication, ...)
- Compare estimation with
 1. b-spline basis without addressing approximation error
 2. ... including proper treatment of approximation error
 3. Eigenbasis constructed from observations
- Prediction not Inference (Alternative: Focused on a testing procedure motivated by the application)
- Present Results
- Explain relevance for application

3.2 Literature

- Shonkwiler and Mendivil 2009
- R-packages: fda, refund, mgcv

4 Application

4.1 Draft-Overview

- Prediction not Inference (Alternative: Focused on a testing procedure motivated by the data set)
- IID data set (no dependence between the curves, don't want to do functional time series)
- Not necessarily data from economics (like biology, sports, whatever)
- Smooth curves or random walk (both fine)
- <https://functionaldata.wordpress.ncsu.edu/resources/>

4.2 Literature

- Carey et al. 2002

5 Outlook

5.1 Literature

- James, Wang, and Zhu 2009 (shape-restrictions)

6 Appendix

7 Bibliography

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