CMP1 PS4

Using the hint, I look at the effective Schrödinger equation:

$$\underline{\underline{H}}\,\underline{\phi} = E\,\underline{\phi}$$

Where we define $H_{n,m}$ as follows:

$$H_{n,m} = \begin{cases} \varepsilon & \text{for } n = m \\ -t & \text{for } n = m \pm 1 \\ 0 & \text{for all else} \end{cases}$$

We now take inspiration from section 11.2 and make an educated guess for ansatz:

$$\phi_n = \frac{e^{-ikna}}{\sqrt{N}}$$

Using this on the left hand side of our Schrödinger equation:

$$\underline{\underline{H}}\,\underline{\phi} = \sum_{n,m} H_{n,m}\phi_m = -t\,\phi_{(n-1)} + \varepsilon\phi_n - t\,\phi_{(n+1)}$$

▷ To find the amount of eigenstates (which my intuition tells me is N)

I look on p. 10 and see:

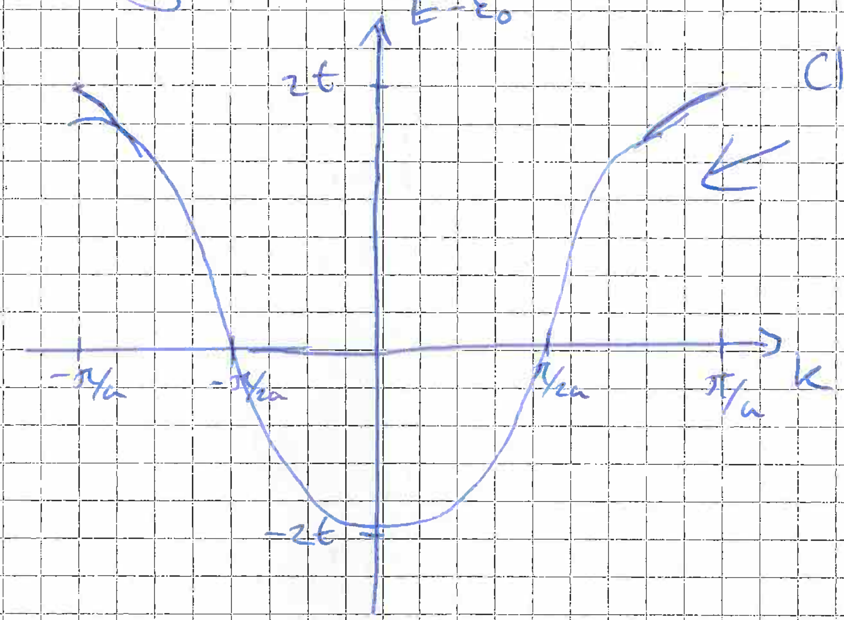
$$n_{states} = \frac{L}{2\pi}\int_{-\pi/a}^{\pi/a} dk = \frac{L}{2\pi}\frac{2\pi}{a} = \frac{L}{a}$$

I can then use that $L \equiv N \cdot a$, and I find:

$$n_{states} = N$$

Exactly as my intuition told me!

Sketching this gives:



Clearly a cosine-function!

▷ This time drawing inspiration from page 103 I expand the energy around $k=0$:

$$E(k) = \sum_n \frac{E^{(n)}(k=0)}{n!} k^n \approx (\varepsilon_0 - 2t) + \frac{2}{2!} a^2 k^2 t$$

As we are looking at the behaviour at the bottom of the band, we choose to equate the dynamic part with the energy of a free electron:

$$ta^2k^2 = \frac{\hbar^2 k^2}{2 m_{free}} \Leftrightarrow m^*_{free} = \frac{\hbar^2}{2ta^2}$$

▷ Per definition, the density of states is:

$$g(\varepsilon) = \frac{\partial N}{\partial \varepsilon}$$

This is easier to work with if I use the chain rule:

$$g(\varepsilon) = \frac{\partial N}{\partial \varepsilon} = \frac{\partial N}{\partial k}\frac{\partial k}{\partial E}$$

I will now use $\varepsilon = E$ and look at the expression I previously found:

$$E = \varepsilon - 2t\cos(ka)$$

Isolating for k gives:

$$2t\cos(ka) = \varepsilon - E \Rightarrow \cancel{k = [a\cos} \quad k = a^{-1}\cos^{-1}\left(\frac{\varepsilon - E}{2t}\right)$$

Finding $\frac{\partial k}{\partial E}$ is now trivial with the use of Schaums:

$$\frac{\partial k}{\partial E} = \frac{+a^{-1}}{\sqrt{1-\left(\frac{\varepsilon-E}{2t}\right)^2}}\frac{1}{2t} = \left[a\,2t\sqrt{1-\left[\frac{\varepsilon-E}{2t}\right]^2}\right]^{-1}$$

I have previously found that

$$N = \frac{L}{2\pi}\int dk = \frac{L}{2\pi}k$$

This gives $\frac{\partial N}{\partial k} = \frac{L}{2\pi} = \frac{Na}{2\pi}$ Granting us the full:

$$\frac{\partial N}{\partial E} = \frac{\partial N}{\partial k}\frac{\partial k}{\partial E} = \frac{Na}{2\pi}\left[2at\sqrt{1-\left(\frac{\varepsilon_0 - E}{2t}\right)^2}\right]^{-1} \cdot 2 \cdot 2$$

spin

← 2 k's for every E

▷ Finding the density of states at the Fermi-surface simply requires finding the Fermi-k. To find this, I will use fig. 11.4 as this is also a valence 1-system.

Per definition I can see k_{fermi} will be $k_{fermi} = \frac{\pi}{2a}$.

Inserting this I get:

$$\frac{\partial N}{\partial E} = \frac{N}{2\pi}\left(2at\sqrt{1-\left(\frac{\varepsilon_0-\varepsilon_0}{2t}\right)^2}\right)^{-1} \cdot 4$$

where I have used $E(k=\frac{\pi}{2a}) = \varepsilon_0 - 2t\cos\left(\frac{\pi}{2}\right) = \varepsilon_0$