DMFS - Problem Set 3

pwn274

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1

1.1

What is the probability that you get a flush, i.e., 5 cards of the same suit but not all in sequence with respect to rank

There are $\binom{13}{5}$ was to draw 5 cards of any given suit. I count 10 ways to make a straight. Given 4 suits, this gives a total of $4(\binom{13}{5} - 10)$ ways to draw 5 cards that are not in sequence. To find the propability of an event simply divide the number of 'positive' outcomes by the totalt number of outcomes: $4(\binom{13}{5} - 10)/\binom{52}{5} \approx 0.197\%$

1.2

What is the probability that you get a straight, i.e., 5 cards of sequential rank but not all of the same suit?

Again, I count 10 distinct straights (I had to Google, and apparently Ace can be the low starting point of a straight). Now each can be one of four suits, giving $10 \cdot 4^5$ combinations. As we will not allow for us to get a straight flush, we subtract the four combinations where all cards are of the same suit, giving a total of $10(4^5 - 4)$ ways to draw a straight. Again, finding the probability amounts to: $(10(4^5 - 4))/\binom{52}{5} \approx 0.39\%$

2

Okay. Once I was able to figure this out, it is actually quite elegant:

Using the notation from the question, take any two distinct numbers 1^m and 1^n that have the same remainder. Given that we can make an infinite number of these, the pigeon hole principle tells us that these must exist. Now, subtract the lower from the higher. The resulting number must be of the form 1^m0^n (or 1^n0^m) and the remainder must be 0. Therefore, there does not only exist a number that solves the problem - there will be infinitely many of them!

3

(40p) Let $a \in \mathbb{R}^+$be any positive real number. Show that for any integer $n \geq 2$ there is a rational number $\frac{c}{d}, c, d \in \mathbb{Z}, d \leq n$, that approximates a to within error $\frac{1}{dn}$, i.e., $\left|a - \frac{c}{d}\right| \leq \frac{1}{dn}$. Hint: Consider the numbers $a, 2a, \ldots, n \cdot a$ and show that one of these numbers is at distance at most $1/n$ from some integer.

For the numbers $a, 2a, \ldots, n \cdot a$ hmmm

for some $z \in \mathbb{Z}$

$$|na - z| \leq 1/n$$

4

We have the relation:

$$M_R = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{1}$$

4.1

The matrix representation of the symmetric closure of this relation is:

$$S = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \tag{2}$$

If R had a connection from a to b then S also has a connection from b to a.

4.2

$$T = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \tag{3}$$

Transitiveness: If a has a connection to b and b has a connection to c, then a also has a connection to c.

4.3

Now I start out by finding the transitive closure:

$$M_R = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \tag{4}$$

This is already symmetric. I can here see that there is no reflexive closure, i.e. the diagonal is purely zeros. This is an obvious difference between doing it the opposite order.

5

I have taken the liberty to try to rewrite all the logical statements into natural language. For all, we expect a S to be of size k:

(a) setsize $(S, k) \wedge \forall v \forall w(E(v, w) \rightarrow S(v) \vee S(w))$
For all v and w there exist an edge between them if and only if some property holds for at least one of them
(b) setsize $(S, k) \wedge \forall v(S(v) \vee \exists w(S(w) \wedge E(v, w)))$
For all v; either some property holds or there exist a w for which a property holds and it has an edge to v.
(c) setsize $(S, k) \wedge \forall u \forall w((u \neq w \wedge S(u) \wedge S(w)) \rightarrow \exists v(E(u, v) \wedge E(v, w))$
For all distinct u and w, if some property holds for both there exist some v which connects to both via an an edge.
(d) setsize $(S, k) \wedge \forall v \forall w(E(v, w) \rightarrow ((S(v) \wedge \neg S(w)) \vee (\neg S(v) \wedge S(w))))$
For all v and w there is an edge between them if some property holds for only one of them.
(e) setsize $(S, k) \wedge \forall v(S(v) \rightarrow \exists w(\neg S(w) \wedge E(v, w)))$
For all v, if some property holds there exist a w where the property does not hold and they are connected by an edge
(f) setsize$(S, k) \wedge \forall v \forall w((v \neq w \wedge S(v) \wedge S(w)) \rightarrow E(v, w))$
For all distinct v and w, if a property holds for both there is an edge between them.

My answers are:

- Looking at (a), I am surmising that if the property $S(v)$ encodes having endpoints in S, (a) and (4) are equivalent.
- In (b), all v is either in S or is a neighbor to S. This fits with the text in (1).
- (c) is clearly the description for (6), as it connects all distinct pairs in S by a node.
- I cannot find a clear pairing for (d).
- (e). Here, if v is in S and w is not then they are connected, just as (5) describes.
- For (f), we can see, that if both v and w are in S they are connected, perfectly describing (2).