Continuum Mechanics
Hand In 3

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1 Solve the problem

1.1 which regions experience compressional and extensional deviatoric (shear) stresses?

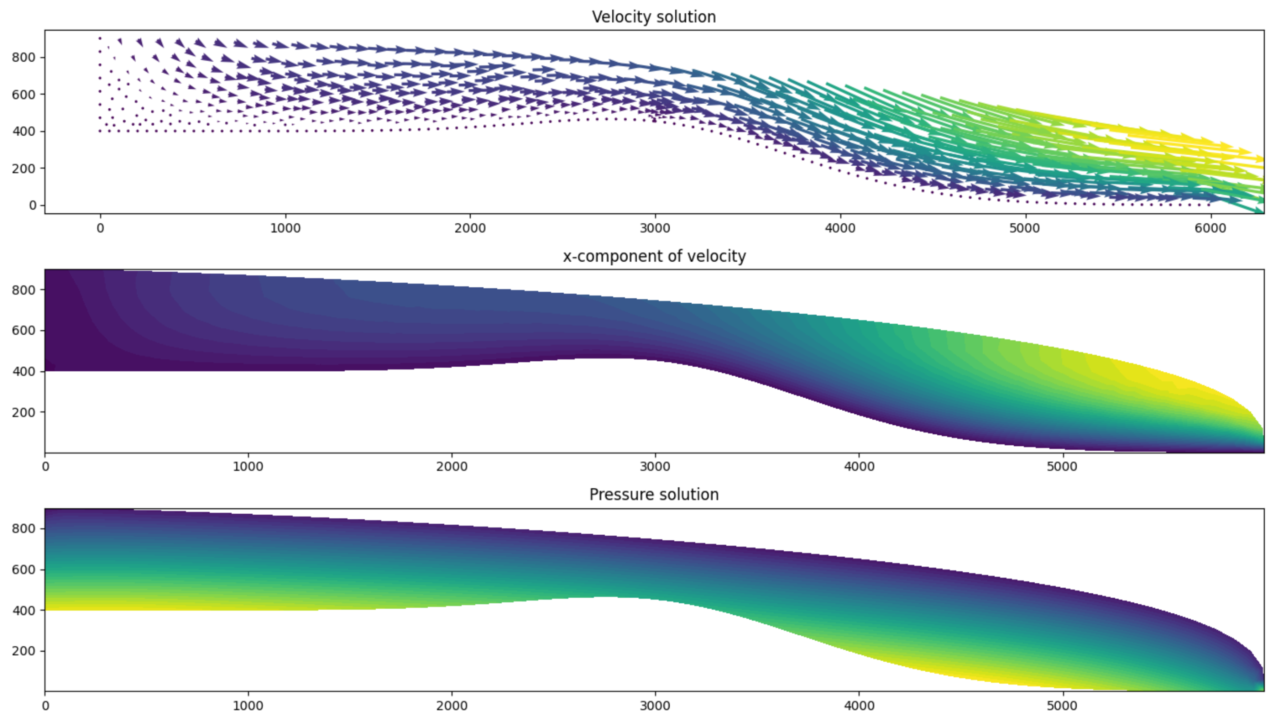

Figure 1: Plots of the velocity and pressure solution

Figure 1 displays the pressure and velocity solution to the simplified problem problem. Observing these figures we see, that we have the largest compressional stresses near the bottom on the $\Gamma_{B,1}$ and $\Gamma_{B,2}$ boundaries. The velocity gradient is greatest on the top, a bit away from the topological bumb. We therefore have the greatest deviatoric stresses here.

2 Impose that u_x must vanish on Γ_L

2.1 is the solution, before or after the bump, sensitive to changing the noslip condition on Γ_L to a slip condition?

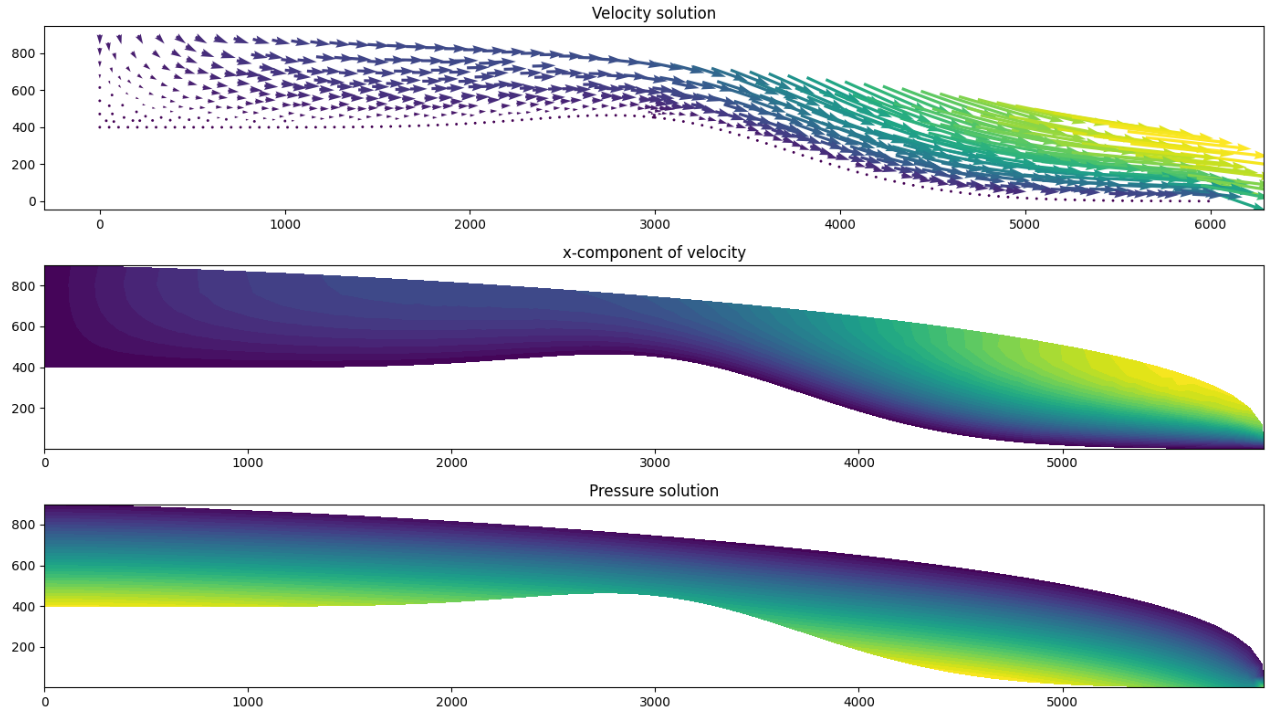

Figure 2: Velocity and solution to the problem where the slip condition is imposed on Γ_L

Figure 2 displays the velocity and pressure solution to the problem where the slip condition is imposed on Γ_L. Observing the figure we see, that the new boundary condition only changes a very small area in the topmost left corner, we therefore conclude that the solution is not sensitive to this new boundary condition.

2.2 Why does specifying $u \cdot n = 0$ on Γ_L in our model actually constitute a free-slip condition rather than a regular slip condition, as it would seem to imply?

As we have not included the integral $\int_{\Gamma_L} \mathbf{v} \cdot \boldsymbol{\sigma} \cdot \mathbf{n} dl$ thus far we assume that the integrand is zero, i.e. $\boldsymbol{\sigma} \cdot \boldsymbol{n} = 0$. We therefore have a free slip condition.

3 Let us model the effect of temperature in terms of the simple linear viscosity–temperature relation

3.1 How do the longitudinal- (τ_{xx}) and shear-stress (τ_{xy}) components change in response to including the temperature-dependant viscosity? Put differently, is there a change in which internal resistive stresses balance gravity?

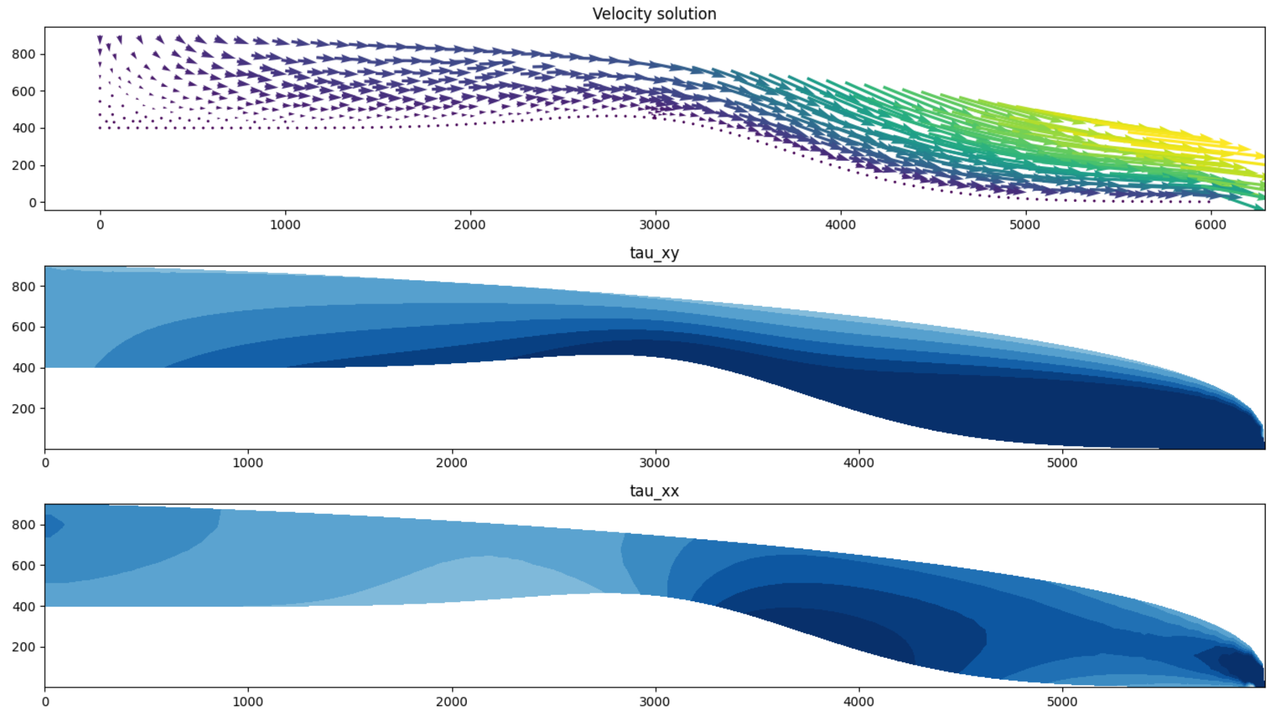

Figure 3: Velocity and τ_{xx} and τ_{xy} components of stress when not including the temperature-dependant viscosity.

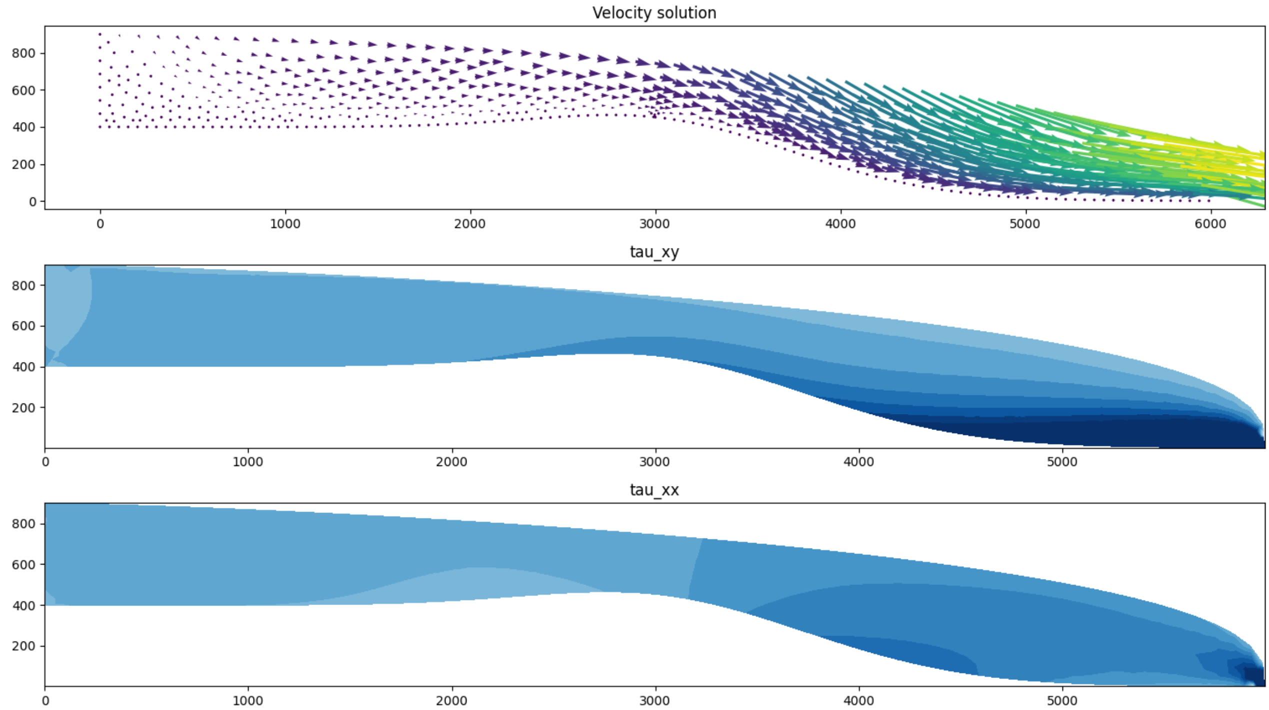

Figure 4: Velocity and τ_{xx} and τ_{xy} components of stress when including the temperature-dependant viscosity.

Comparing the τ_{xy} component of figure 3 and 4 we see, that this stress moves a lot more forward and down towards the substrate. This makes physical sense, as the viscosity changes a lot more in this valley than at the top of the glacier.

Comparing the τ_{xx} component of figure 3 and 4 we see, that the stress in general gets smaller and moves towards the tip of the glacier. The tip of the glacier has a very steep slope, and thus changes viscosity quickly, it thus makes sense that the velocity gradient in the x-direction would change quickly here.

4 Snowfall

4.1 How do the compressional/extensional regimes change in response to the new geometry?

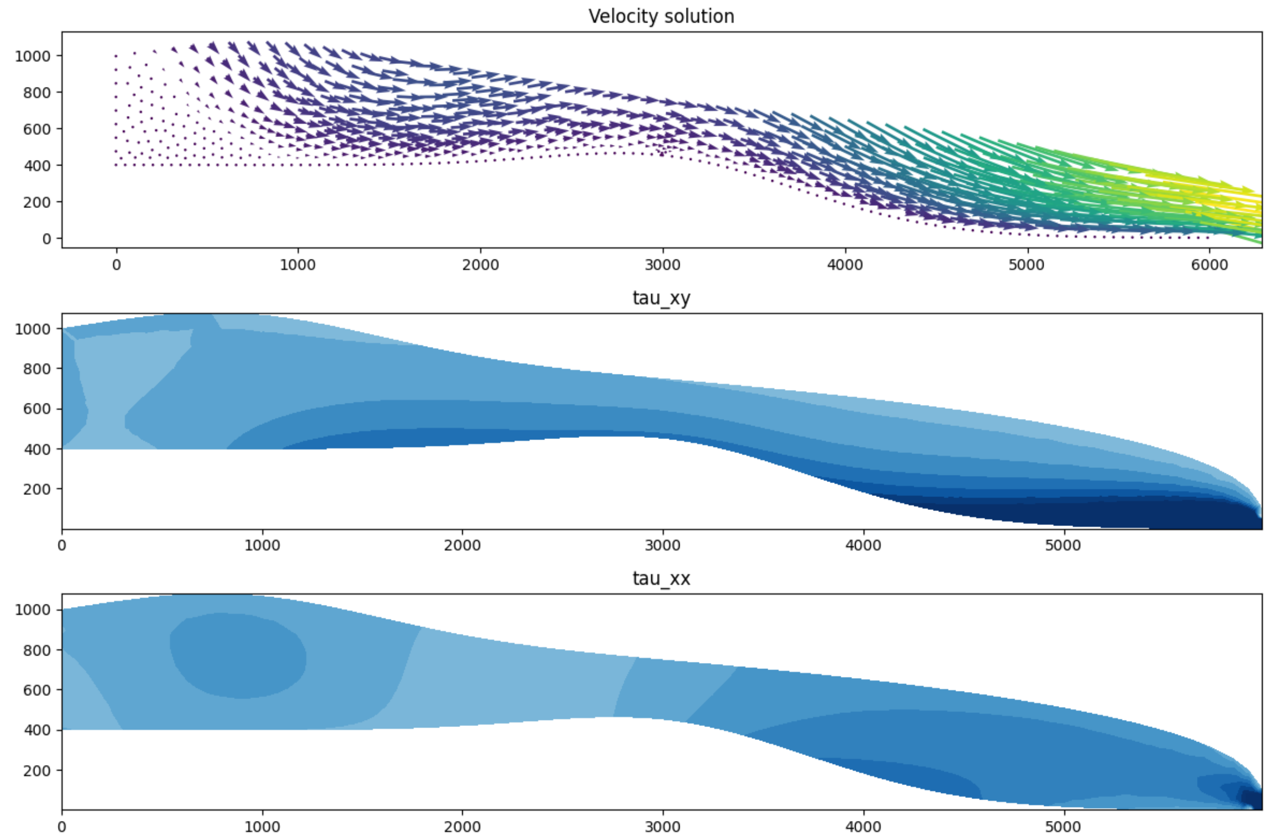

Figure 5: Velocity and τ_{xx} and τ_{xy} components of stress when including the temperature-dependant viscosity with new geometry from snowfall.

Comparing the different components of stress in figure 4 and 5 we see that the compressional and extensional stresses change quite a bit further up on the glacier, where the snow has fallen, but it does not seem to have any effect on the lower part of it. Naively, this makes sense, as the new geometry only really affects this area.

5 Crevasses can form in glaciers

5.1 Since crevasses represent a break in the continuum across which stresses can not be transmitted, how does the formation of a crevasse locally influence the stress components τ_{ij}? How far upstream can the crevasse be "felt" (exert influence) in the components τ_{ij}?

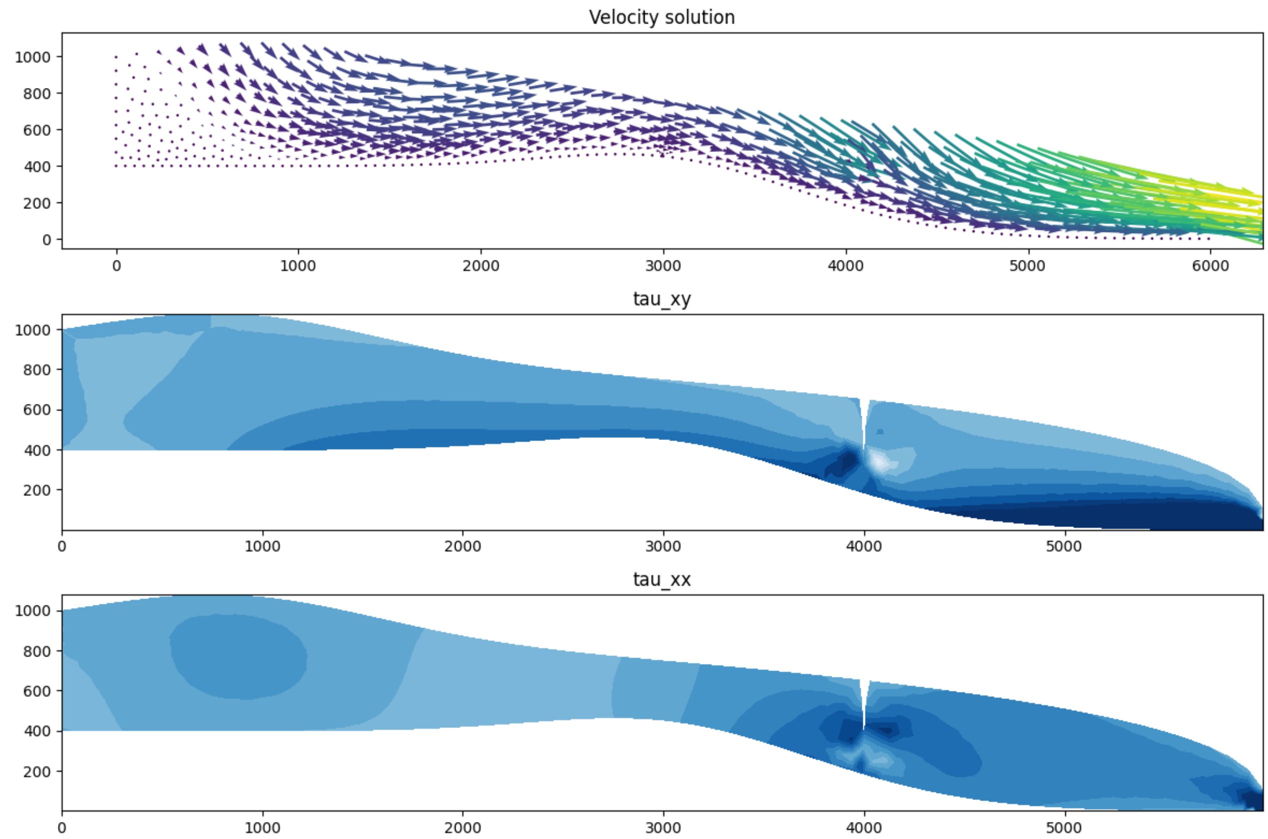

Figure 6: Caption

The crevasse both adds a lot of pressure (t_{xx}), but only to the left and right. Below the crevasse, no pressure is felt. The crevasse also add a sheering force to the left of where the glacier is broken. From what we visually/qualitatively can see, the crevasse also 'relieves' the glacier of pressure down in the valley, but has no obvious effect further up.

6 On the lower subglacial boundary, $\Gamma_{B,2}$, the ice rests on a deformable substrate

6.1 Above what uniform sliding speed ux $= u_{slide}$ on $\Gamma_{B,2}$ does sliding affect τ_{xx}?

I have used a for loop over the different velocitites:

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u_x = ky2ms*velocities_im_km_y[i]
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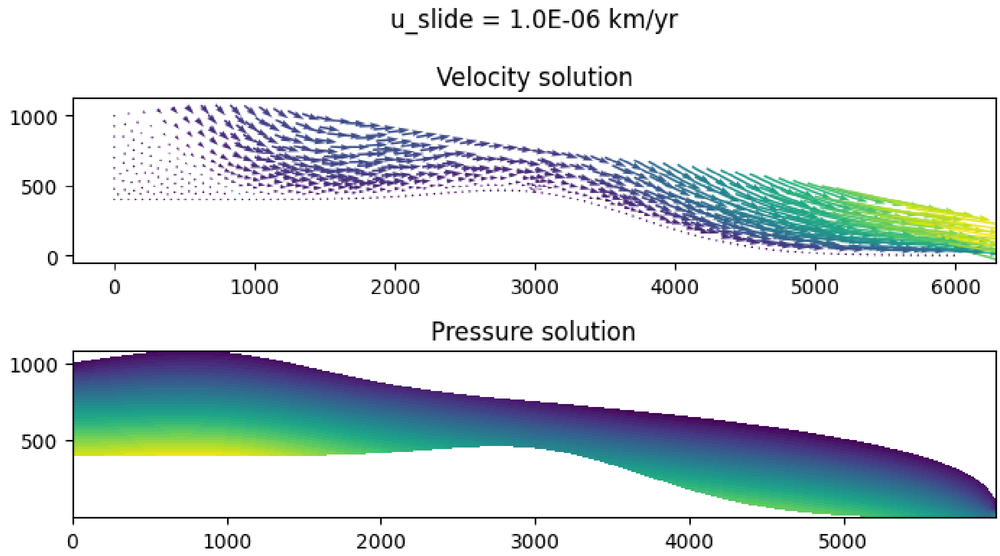
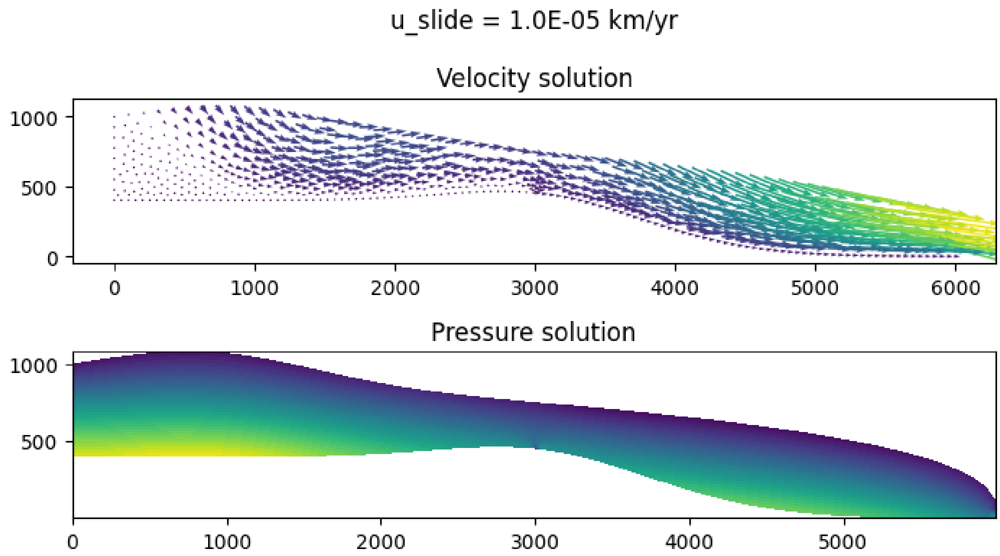
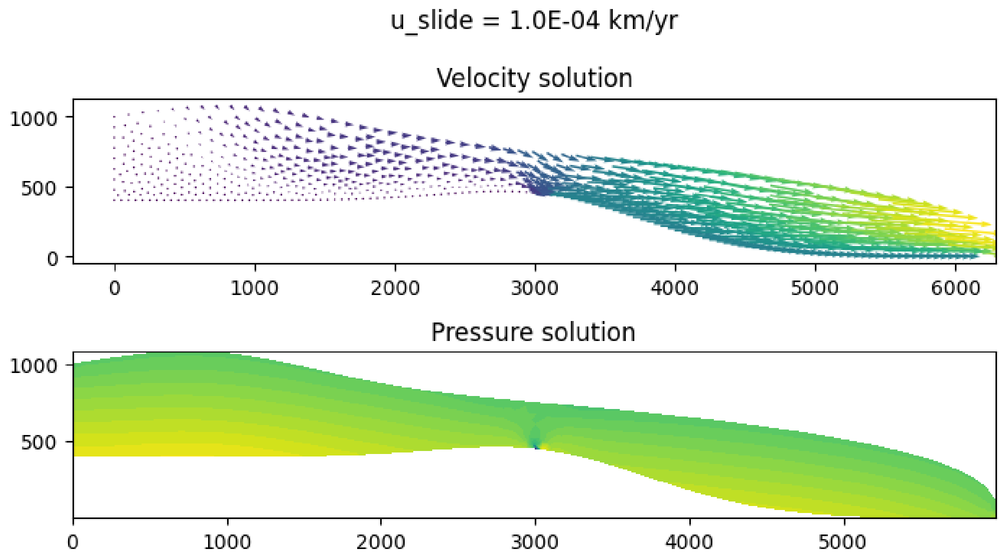
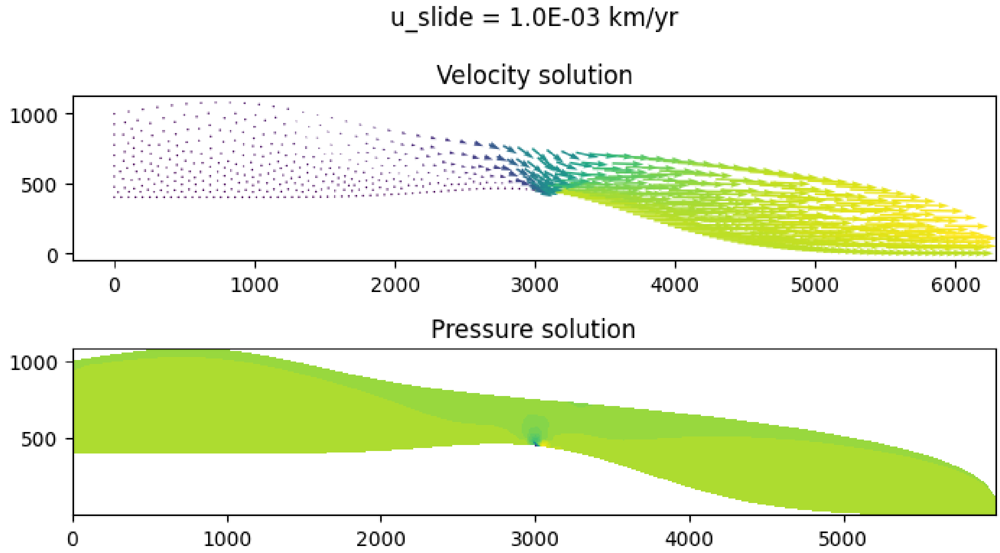
It gave me these:









It is clear that the change is somewhere on the order of 10^{-4} km/y

6.2 Where do the largest gradients in τ_{xx} and τ_{xy} occur for large sliding speeds? Can you qualitatively explain why

Using a high bottom-velocity we achieve the following plot:

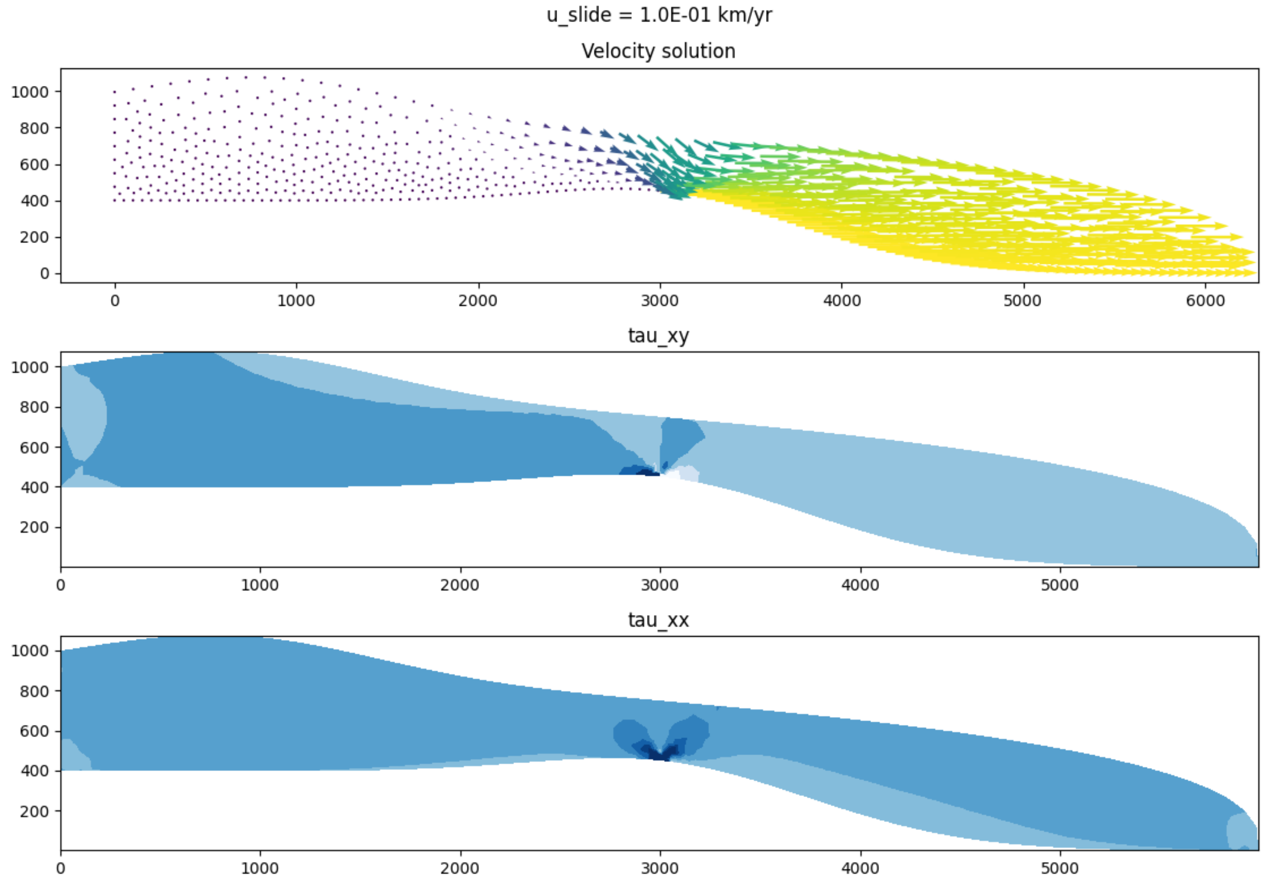


Figure 7: The x-component of the velocity found by solving the problem with a velocity above the critical 10^{-3}

Using this plot, it is easy to see, that the highest τ_{xy} is not at the bottom rightmost edge of the glacier. This makes sense, as the softer, warmer glacier now slides as fast as the artificially slipping bottom.