Problem set for Week 6

Continuum mechanics 2023

Exercise 1

Flow velocity and density are fields we use in continuum mechanics. Can you think of other properties we may need to formulate continuum mechanics?
(Think in analogues to classical mechanics.)

Exercise 2

In this exercise the goal is to get a sense for how small scales the continuum approximation makes sense. Let's say we want to write a computer model of water flow in a really tiny pipe (e.g. a capillary blood vessel). We want to subdivide the space into a grid of smaller volumes. We will here consider grid cell volumes of $1\ \mathrm{nm}^3$ and $1\ \mu\mathrm{m}^3$. The smaller the volume; the fewer the molecules; hence the more their average velocity will be affected by random fluctuations of individual molecules.

Q1: Estimate the average number of water molecules (N) in the two grid cell volumes.

At a given temperature (T=300K) the molecules in these volumes will have a range of different velocities. This is described by the Maxwellian distribution. This is gaussian with a certain standard deviation.

Q2: Calculate the standard deviation of the x-velocities. (The distribution for |v| and vx are be different. Make sure you use the right version.)

For the continuum approximation to make sense the typical average velocity of a random sample of N particle velocities must be very small compared to velocities we aim to resolve with the continuum approximation.

Q3: What is the standard deviation of the grid cell average velocity arising to temperature. Calculate for both volumes. Compare that to a typical

THIS EXERCISE IS DIFFERENT FROM PAGE 8 In the book. Pg8 looks at rms velocities derived from f(|**v**|), and this exercise looks at f(vx). Just use google to find the equations you need.

Exercise 3

Global warming is melting ice on land which is causing sea level rise. In sea level modelling we usually assume that water is incompressible. However, as we add mass to the oceans, we will also place an additional load on the entire water column which will increase pressure throughout. That will compress the water slightly. But by how much?

What happens to the height of a 4000m tall water column, if you add one more metre of water on top? Consider the column to be constrained in the x and y directions.

(The average depth of the oceans is ~4000 m.)

Selected book problems from ch2

2.2 Consider a canal with a dock gate that is 12 m wide and has water depth 9 m on one side and 6 m on the other side. Calculate

(a) The pressure in the water on both sides of the gate at a height z over the bottom.
(b) The total force on the gate.
(c) The total moment of force around the bottom of the gate.
(d) The height over the bottom at which the total force acts.

2.7 The equation of state due to van der Waals is

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT,$$

where a and b are constants. It describes gases and their condensation into liquids. **(a)** Calculate the isothermal bulk modulus. **(b)** Under which conditions can it become negative, and what does that mean?

https://en.wikipedia.org/wiki/Van_der_Waals_equation

Exercise 4

Consider a 2D iceberg in a 2D world. We assume a flat static ocean. The iceberg can be represented by a simple polygon. Outline how you would code a program would find a floating equilibrium position of the iceberg. Discuss your idea for solutions in groups of ~2-3. Code it up.

You can code it yourself from scratch, or you can get a python notebook starter template is available. But before you fetch the template then please think about how you would solve it.

Selected book problems from ch3

3.1 A stone weighs $\mathcal{F}_1 = 1{,}000$ N in vacuum and $\mathcal{F}_0 = 600$ N when submerged in water of density ρ_0. **(a)** Calculate the volume V and **(b)** average density ρ_1 of the stone.

3.3 A cylindrical wooden stick with density $\rho_1 = 0.65$ g cm^{-3} floats in water (density $\rho_0 = 1$ g cm^{-3}). The stick is loaded down with a lead weight with density $\rho_2 = 11$ g cm^{-3} at one end such that it floats in a vertical orientation with a fraction $f = 1/10$ of its length out of the water. **(a)** What is the ratio M_1/M_2 between the masses of the wooden stick and the lead weight? **(b)** As a function of the density of the wood, how large a fraction of the stick can be out of the water in hydrostatic equilibrium (disregarding questions of stability)?