Continuum Mechanics - Hand In 1

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1 Theoretical Part

1.1

Calculate the area moment (I) of the shelf (assume the size in the x-direction is 1m)

$$I = \int dA\ y^2 = \int_{-1/2}^{1/2} dx \int_{-H/2}^{H/2} dy\ y^2 = 1 \cdot [H^3/8 - (H^3/8)] = H^3/12 \tag{1}$$

1.2

The ice shelf is influenced by gravity and water pressure. Write an expression for the transverse resultant (Ky) of the distributed external forces acting on a small piece 'dz'. (Assume that the shelf is always partially submerged – i.e y is small.)

I am unsure what a transverse resultant is, so but I guess it is the "K_y" I will be using the next subpart, ie. the total resulting force per "dz-slice".

I know that for every small piece of rod we have:

$$dzF = F_{water} - F_g \tag{2}$$

Where

$$F_{water} = [H_w + y(z)] \cdot \rho_{water} \cdot dz \tag{3}$$

$$F_g = 1 \cdot H\rho_{ice} \cdot g \cdot dz \tag{4}$$

1.3

Write an ordinary differential equation for the deflection (y(z)).

I now use the following set of equations from the book and the K_y in the last sub-question

$$\mathcal{M}_x = -EI\frac{d^2y}{dz^2} = -E\frac{H^3}{12}\frac{d^2y}{dz^2}, \quad \mathcal{F}_y = \frac{d\mathcal{M}_x}{dz}, \quad K_y = -\frac{d\mathcal{F}_y}{dz}. \tag{5}$$

we get:

$$k - ay(z) = b \cdot \frac{\partial^4 y(z)}{\partial z^4} \tag{6}$$

where we have defined $k = 1 \cdot H\rho g - H_w\rho_w g$, $a = \rho_w g$ and $b = EH^3/12$ for ease of reading.

1.4

This is solved by

$$y(z) = c_1 e^{\sqrt[4]{-1}\sqrt[4]{a/b}z} + c_2 e^{\sqrt[4]{-1}\sqrt[4]{a/b}z} + c_3 e^{\sqrt[4]{-1}\sqrt[4]{a/b}z} + c_4 e^{\sqrt[4]{-1}\sqrt[4]{a/b}z} + k/a \tag{7}$$

Clearly $1/\sqrt[4]{a/b} = 1/\sqrt[4]{\frac{12\rho_w g}{EH^3}} \approx 637m$ must be the significant lengthscale (this is withing the ice etc.)

1.5

Finding the moment means integrating the 'slice' of the rod, multiplying the pressure-forces by their respective lever (the same way that gives rise to the $I = \int dyy^2$):

$$\mathcal{M}_x(L) = \int_{-H/2}^{H/2} dy\ y \cdot \rho_{ice} g y - \int_{-H/2}^{H_w + y(L) - H/2} dy\ y \cdot \rho_w y \cdot g(H_w + y(L) - H/2) \tag{8}$$

Where I am assuming no air pressure.

1.6

The boundary conditions must be no movement at the left boundary:

$$y(0) = 0 \tag{9}$$

no bending at the left boundary:

$$\frac{dy}{dz}(0) = 0 \tag{10}$$

from 1.3 and (8):

$$y''(L) = -\mathcal{M}_x/EI \tag{11}$$

Since the book kindly reminds us that $y'''(z)$ corresponds to the resulting force and we want the rod to be in equilibrium, I get the final boundary condition:

$$y'''(L) = 0 \tag{12}$$

2 Practical Part

2.1

With the boundary conditions specified, solve the elastic problem by manually plugging in different sea-level heights, Hw, until Hw is found that gives an (approximately) minimal vertical shelf displacement (i.e. the shelf does not change shape). Notice that there will always be some displacement since the rectangle is not a proper equilibrium shape.

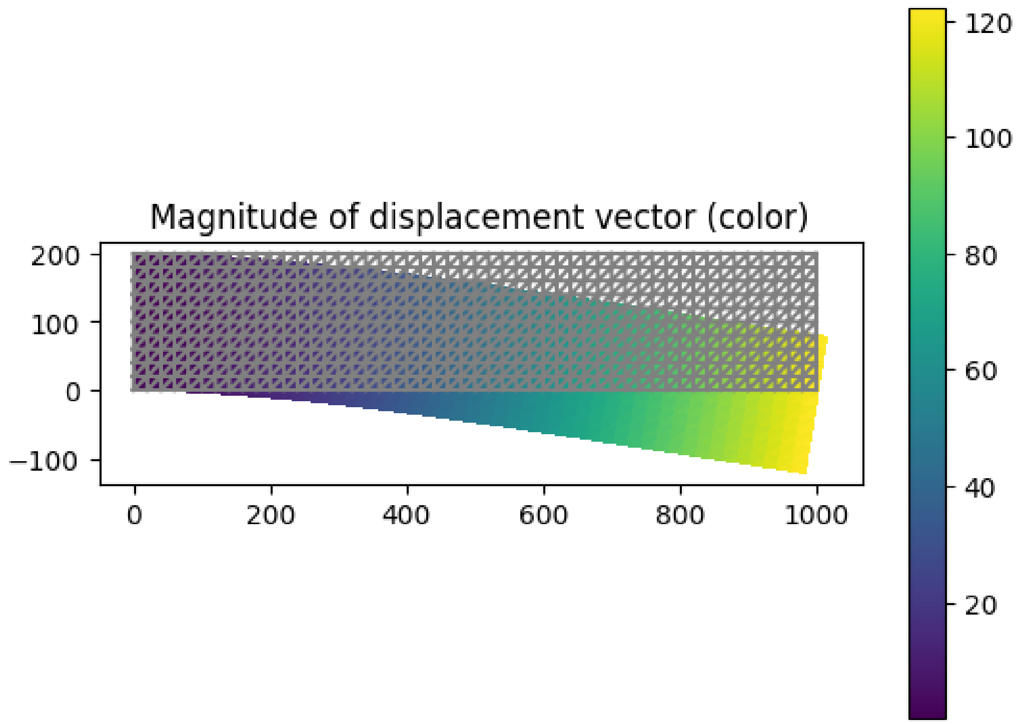

Figure 1: normal bend

Because overkill-solutions are more fun that not having Acute Stress, I wrote a simple gradient descent algorithm to find the water height that gives the minimal deflection.

I git 178.6

(this deflection can be noted to be just about $height \cdot \rho_w / \rho_{ice}$ which makes sense, as the leftmost boundary in this case is negligible (Saint-Venant's principle maybe?)

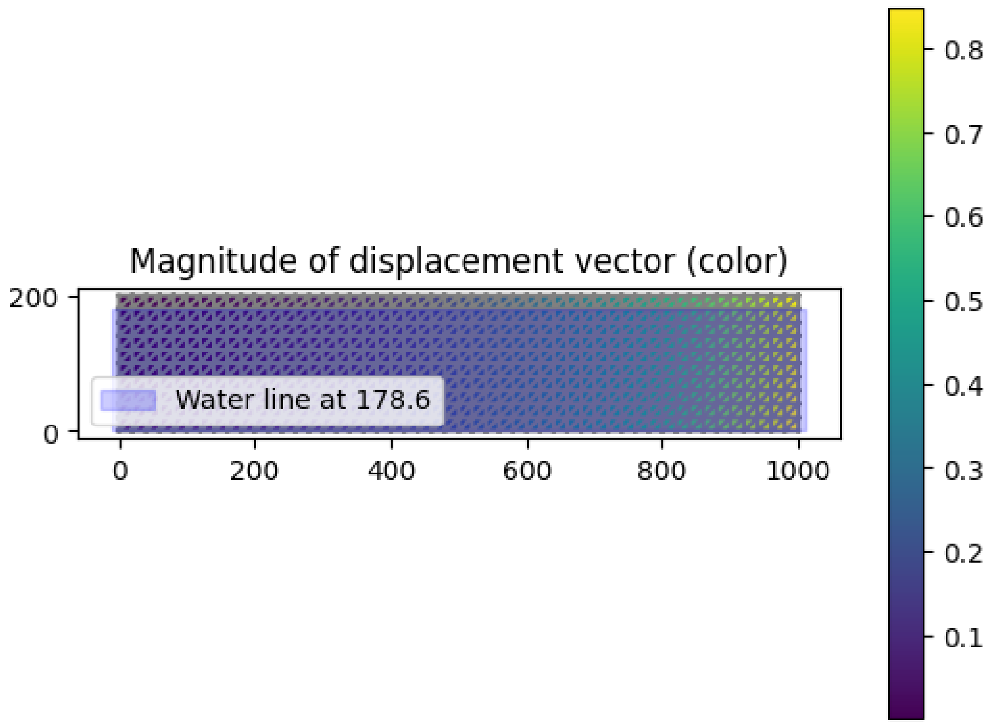

Figure 2: The minimal displacement

2.2

With the new shelf geometry, re-determine Hw such that the vertical displacement is minimal.

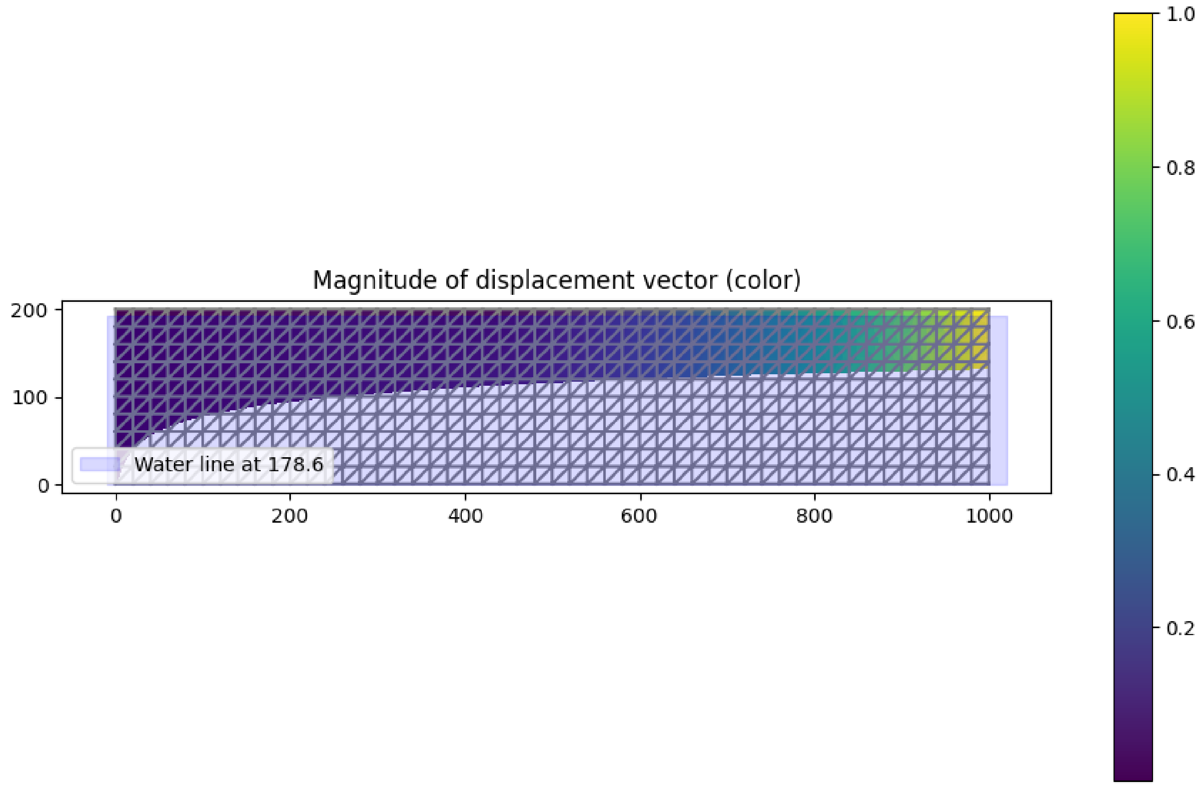

Figure 3: The minimal displacement for the full ice-shelve

2.3

How does the shelf displacement respond to a +1 m tide (i.e. Hw → Hw + 1 m) compared to the rectangular shelf?

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max displacement for normal goes from 0.0232 to 0.102
max displacement for full goes from 0.0272 to 3.48
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So the ’full’ responds much worse!

2.4

Where does the shelf experience the greatest internal stresses? Plot the scalar stress measure ”the von Mises stress”

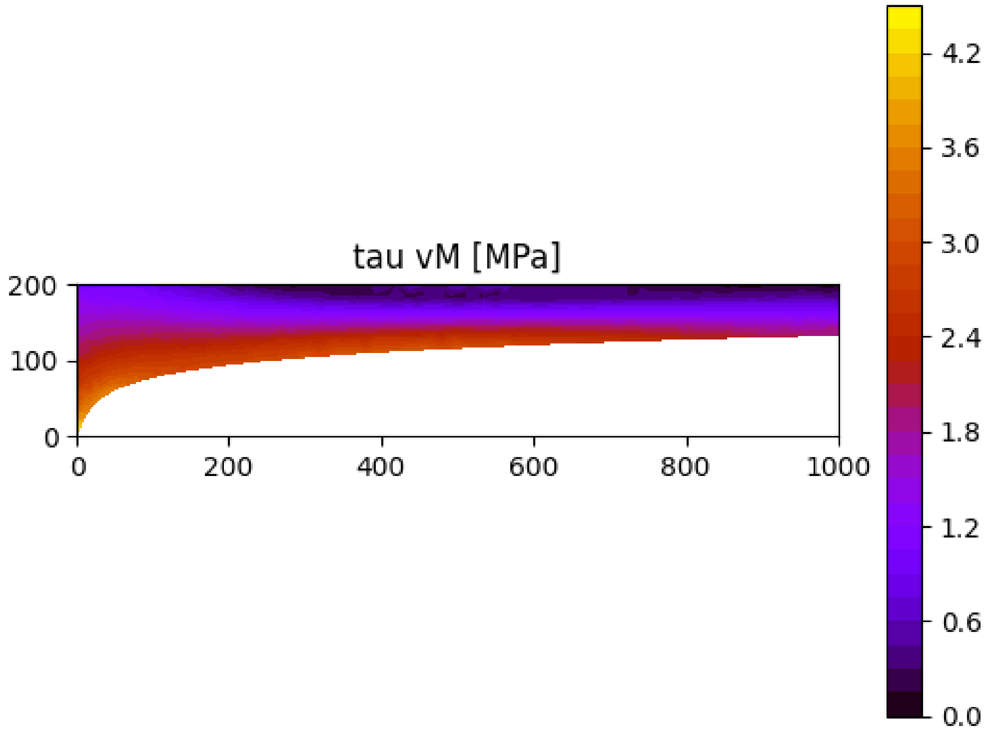

Figure 4: The von mises stress plottet on the original figure

2.5

For very large tides, is the displacement solution the new stable position? Hint: is the pressure at the ice–sea interface constant as the shelf lifts?

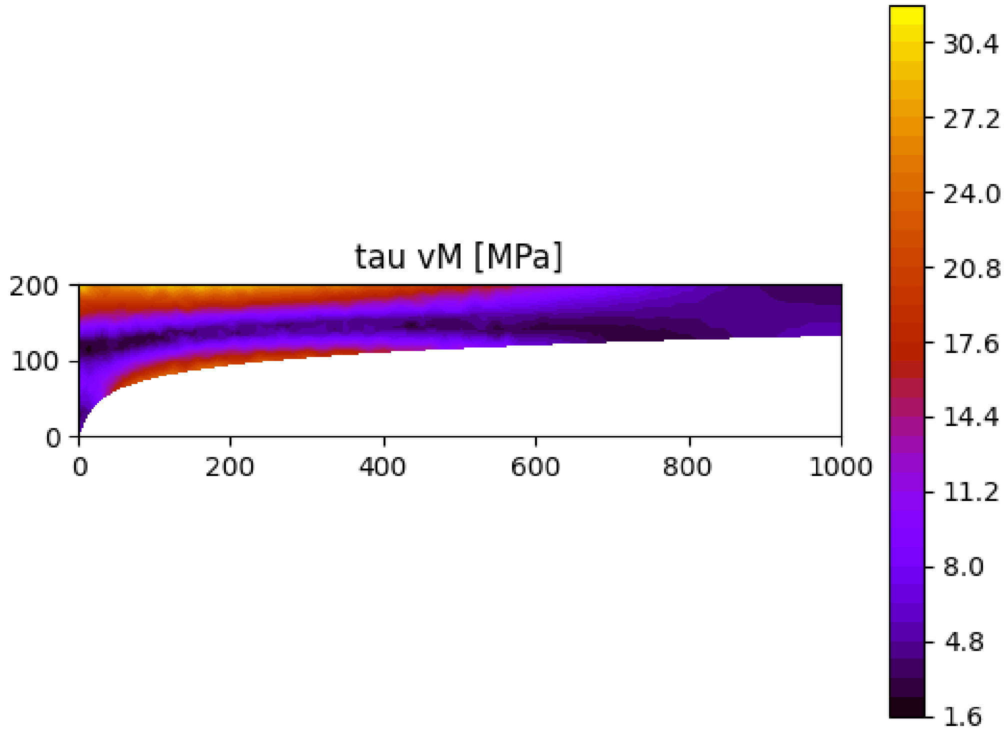

Figure 5: hw + 100 m

2.6

Re-determine Hw such that the displacement is minimal. For a 1 m tide, where is the shelf most likely to eventually break of (calve)?

hw = 191.9

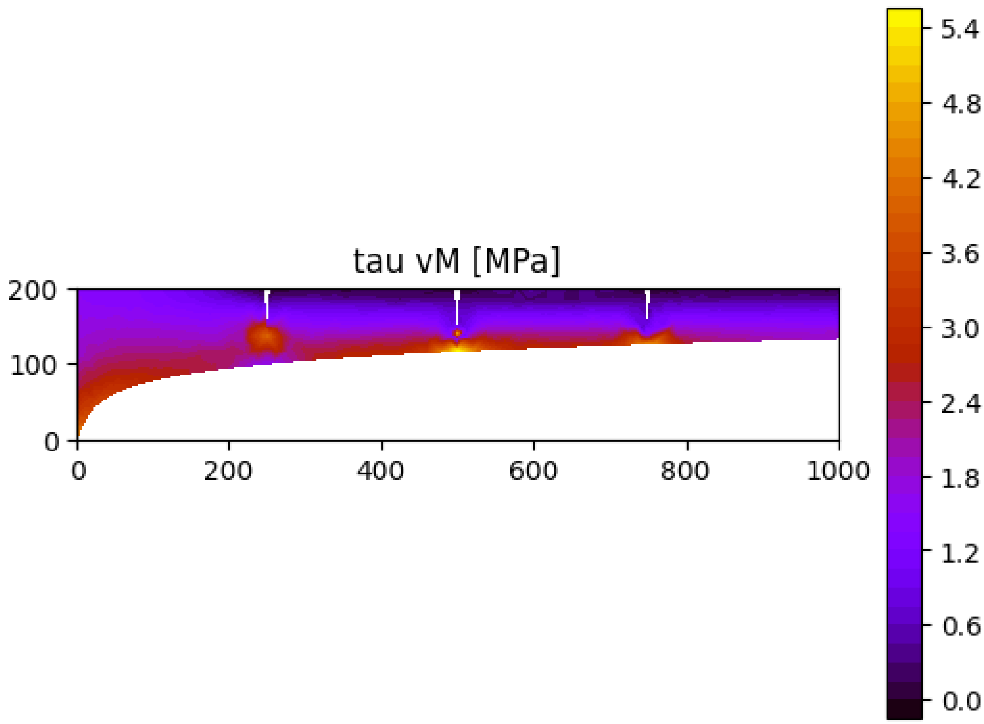

Figure 6: At water = hw = 192

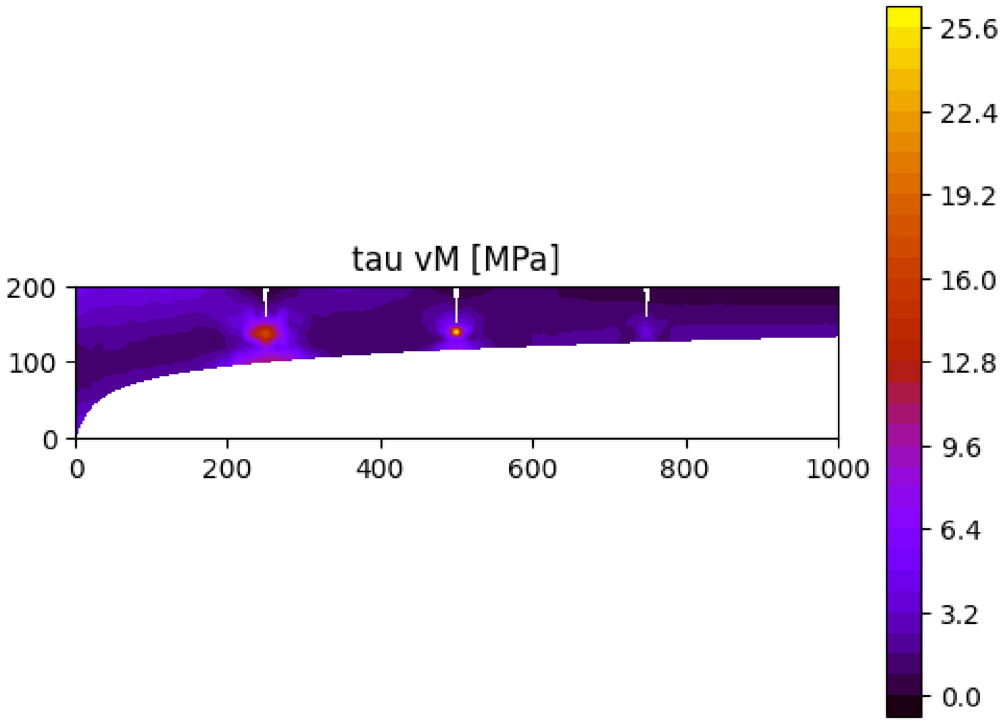

Figure 7: After 1 tide

Most likely will break at middle breaking point since it experiences highest stress there.

2.7

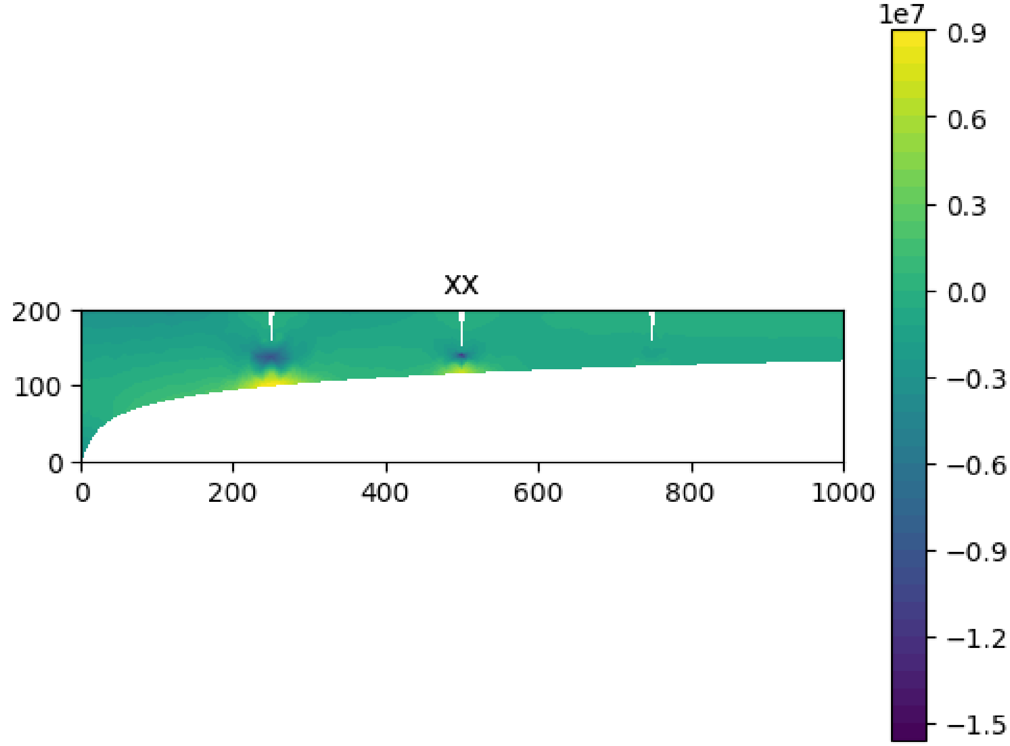

Figure 8: Plot for τ_{xx}

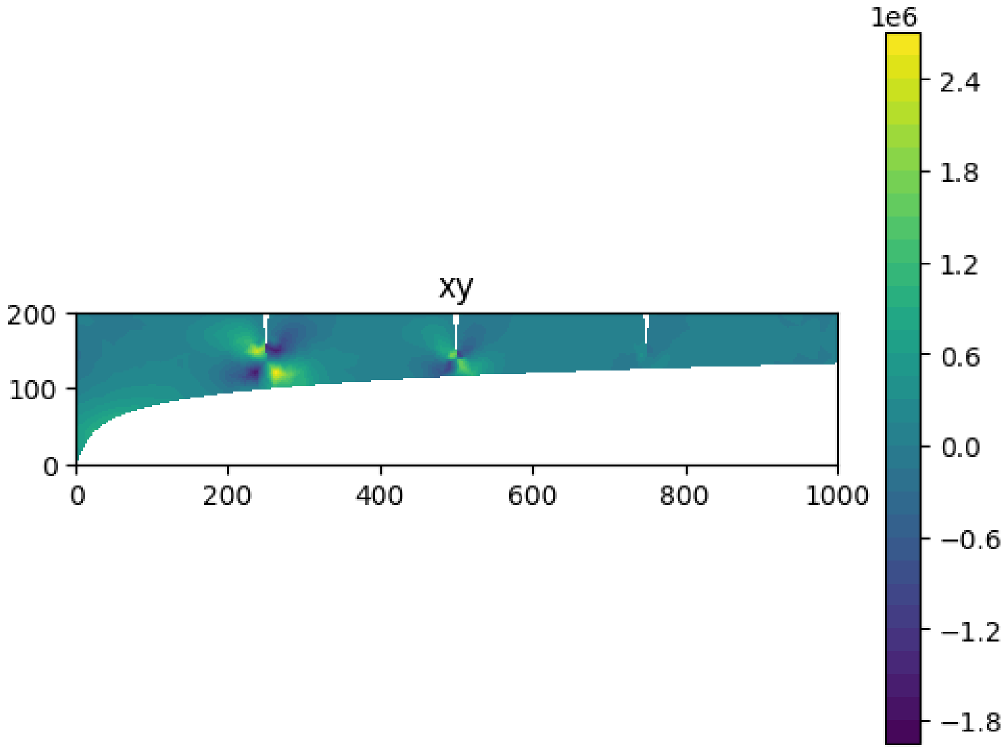

Figure 9: Plot for τ_{xy} (where we know that xy = yx)

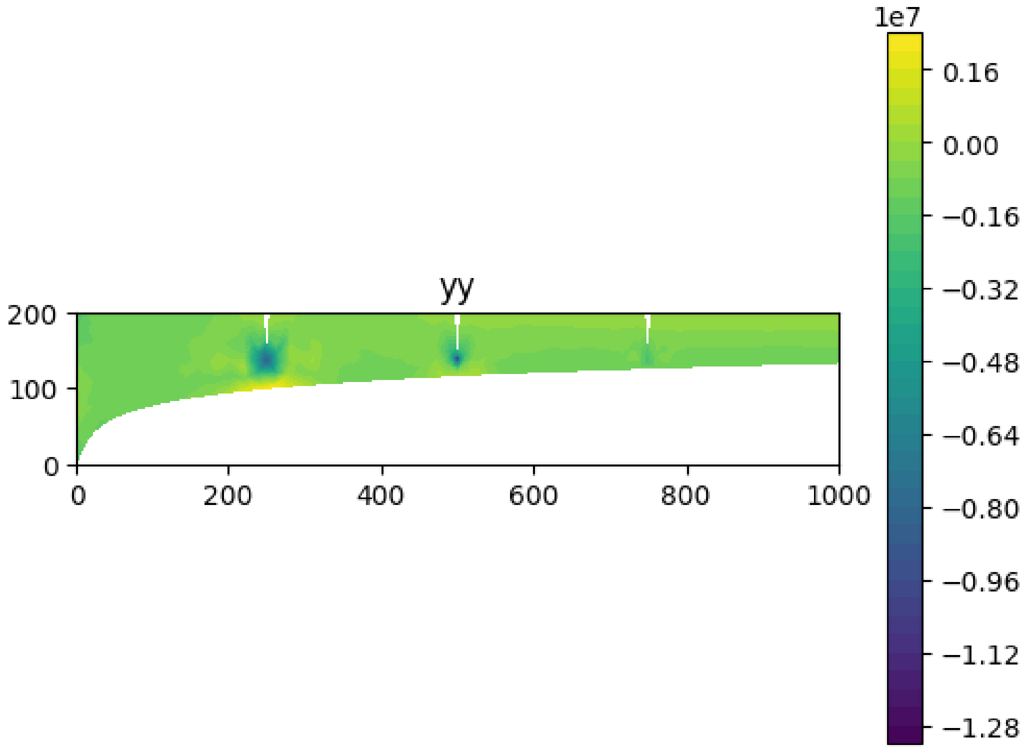

Figure 10: Plot for τ_{yy}

2.8

How does the reduced shelf respond to tides compared to the longer shelf?

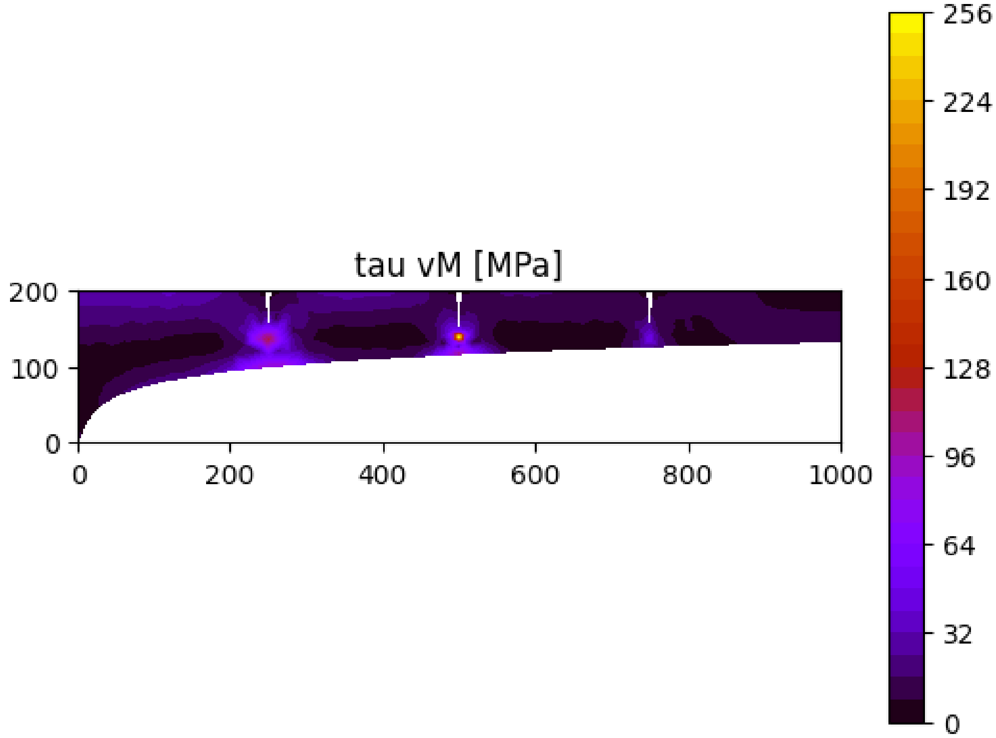

Figure 11: Caption

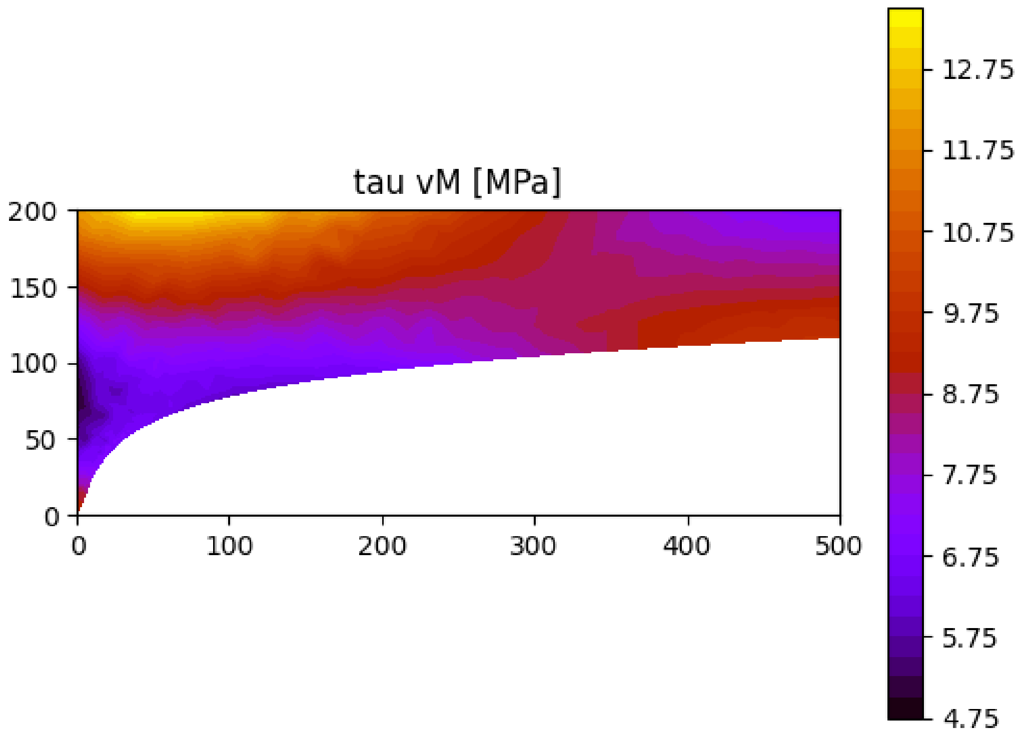

Figure 12: Caption

Von Mises stress is 20 times higher in the broken but uncalved.