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Project 4.1: Ordinary Differential Equations.

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Abstract

This project contains one subproject concerning applications of ordinary differential equations. For the subproject, you need for each assignment to perform the simulations using (i) a forward Euler's method that you have implemented (a copy of the program has to be presented as documentation) and (ii) a fourth order Runge-Kutta method. You should compare the results that you obtain from the two approaches.

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I. SUBPROJECT: THE INITIAL SPREAD OF THE HIV VIRUS

The HIV virus that eventually leads to AIDS is spread mainly through (i) sexual contact, (ii) transfer of blood through transfusions or (iii) the sharing of needles by drug users. We start out by considering the spread of the HIV virus through sexual contact where the initial group is homosexual. We suppose that at time t there are in a group of individuals: (i) p_1 homosexual males of which $x_1(t)$ are infected, (ii) p_2 bisexual males of which $x_2(t)$ are infected, (iii) q heterosexual females of which $y(t)$ are infected and (iv) r heterosexual males of which $z(t)$ are infected. We assume that the rate of infection is proportional to the number of sexual contacts and therefore we have the following set of equations:

$$\frac{dx_1}{dt} = a_1 x_1(p_1 - x_1) + a_2 x_2(p_1 - x_1) \tag{1}$$

$$\frac{dx_2}{dt} = b_1 x_1(p_2 - x_2) + b_2 x_2(p_2 - x_2) + b_3 y(p_2 - x_2) \tag{2}$$

$$\frac{dy}{dt} = c_1 x_2(q - y) + c_2 z(q - y) \tag{3}$$

$$\frac{dz}{dt} = d_1 y(r - z) \tag{4}$$

The parameters are larger than zero. Concerning the values of the parameters, we note that only the relative values of the parameters are important but we expect that a_1 is the largest since the largest number of contacts belongs to the first group. Then we expect that a_2 and b_1 follow in terms of magnitude. You should as a start select the following values: $a_1 = 10$, $a_2 = b_1 = 5$ and set remaining parameters equal to one. The populations should be scaled but as a start use $p_1 = p_2 = 5$ and $q = r = 100$. Initially, you should try $x_1 = 0.01$ and $x_2 = y = z = 0$ and then you have to simulate how the virus will spread in the four groups.

We introduce the effects of blood transfusions by adding the following terms

$$e(p_1 - x_1) \tag{5}$$

to the equation for $\frac{dx_1}{dt}$,

$$e(p_2 - x_2) \tag{6}$$

to the equation for $\frac{dx_2}{dt}$,

$$e(q - y) \tag{7}$$

to the equation for $\frac{dy}{dt}$ and

$$e(r - z) \tag{8}$$

to the equation for $\frac{dz}{dt}$. The parameter e is small and you should start with $e = 0.001$ and slowly increase the value. What happens? How does the virus spread this time? Are you able to find the equilibria for this system? You can also use the following expression fore

$$e = 0.01 * frac \tag{9}$$

where $frac$ is given by:

$$frac = \frac{x_1 + x_2 + y + z}{p_1 + p_2 + q + r} \tag{10}$$

Finally, you should simulate the effects of removal/death by subtracting the terms

$$r_1 x_1 \tag{11}$$

to the equation for $\frac{dx_1}{dt}$,

$$r_2 x_2 \tag{12}$$

to the equation for $\frac{dx_2}{dt}$,

$$r_3 y \tag{13}$$

to the equation for $\frac{dy}{dt}$ and

$$r_4 z \tag{14}$$

to the equation for $\frac{dz}{dt}$. The parameters r_1, r_2, r_3, r_4 are larger than e but smaller than the other parameters. You should start with values for the removal parameters of 0.05 and slowly increase or decrease the values. What happens? How does the virus spread this time? You should experiment with the effects to simulate the spread of the virus.