Complex Physics — Midterm Exam 2

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Hand-in date: 23:59, Tuesday, Oct 10, 2023
It is allowed to work in groups, but each student must hand in separately (without copying) and include information about collaborators. A maximum of three students are allowed per group.
For the hand-in, please include your code in a file separate from your report. We will not grade the quality of your code (only what is written in the report), but you *must* submit it.
In total, **10 points** can be achieved by solving problems 1-4 below (see annotation), which map directly onto your grade (in percent of the total course grade). 10 other points were allocated to Midterm exam 1. The remaining 80 percent can be obtained from the oral exam.
Note: 2 extra points can optionally be achieved by also solving problem 5.

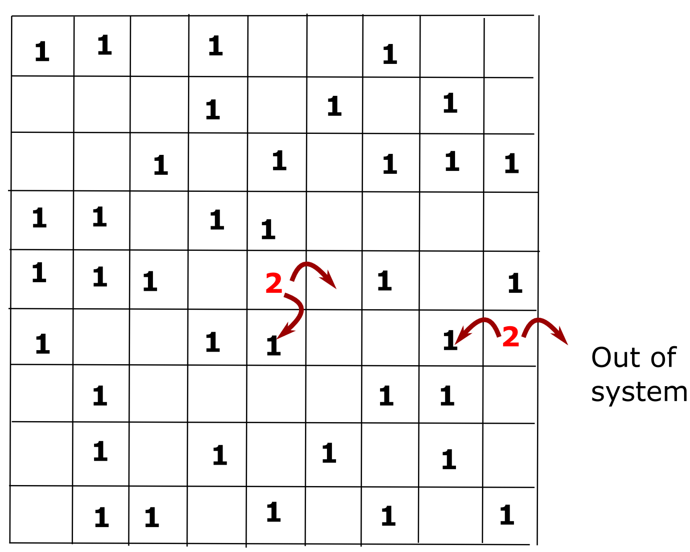

Figure 1: Lattice with two active sites that each relaxes by moving "grains" to their neighbors. Sites on the boundary of the $L \times L$ system can relax by putting "grains" out of the system.

Self organized critical dynamics

Consider a model on the 2-d lattice with size length L in Fig. 1. Each lattice site x, y can take value $h_{x,y}$ =0, 1, or more. Sites with 1 or less grains are relaxed. The system is excited by adding a grain to some random position x, y. Grains are only added after the entire system is relaxed, i.e. all $h_{x,h}$ less than 2. When the system is excited it is relaxed in a sequence of relaxation moves where sites where $h_i > 1$ relax. This is done by moving two grains randomly to the four nearest neighbors of the excited site (grains are moved out of the system when the excited site is on the boundary of the lattice). Each grain is assigned to these new neighbor positions randomly and independently of each other. We recommend synchronous toppling, where all grains with h above 1 topple at the same time (hint: define a current lattice and a lattice of active changes).

1. Implement the above model for a $L \times L$ lattice with $L = 25$ and plot the sequence of avalanches as a function of time. The avalanche size is defined as the number of single-site relaxations until the new relaxed state is reached. Convince yourself when steady state dynamics is reached (this critical attractor is characterized by avalanches that do not anymore increase in size) **2pt**.

2. Consider system sizes $L = 25$, $L = 50$, $L = 100$ and $L = 200$ and simulate each system until a steady state is reached. In steady state, you should extract the size distribution of the avalanches, and estimate the exponent for the size distribution and for the scaling of the avalanche cut off with system size L (dimension of avalanches) **4pt**.

3. For a big lattice, then plot the size of avalanches (in number of relaxation events) as a function of the largest linear dimension of each avalanche (longest extension of avalanche along either the x-axis or y-axis). Deduce the dimension of the avalanche **2pt**.

4. Simulate the system where grains are only added on the edge of the system and record the avalanche size distribution with this constraint. Deduce the corresponding exponent for these avalanches **2pt**.

5. Additional points: Can you understand the scaling exponent of the boundary-driven avalanche in terms of the dimension of these avalanches? **2pt**