Continuum Mechanics - hand in 2

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1 Exercise: Gravity Wave

We want the group velocity. My thoughts can also be read in **2.4**, but summarized: Waves as we think of them in the real world is an emergent phenomenon. When asking a layman, the group velocity would be the wave they think of. The phase speed could even be the going the wrong way. (damn, it would be easier if I could insert an animated gif into this pdf)

$$1 \approx \lambda \ll 2\pi d \approx 60 \tag{1}$$

I approximate the wavelength to be around $1[m]$ so I would call it deep water

$$c_g = 1/2c = 1/2\sqrt{g\lambda/2\pi} \approx 1/2\sqrt{10 \cdot 1/6} \approx 1/2\sqrt{3/2} \approx 0.6[m/s] \tag{2}$$

time taken:

$$2 \cdot 100[m]/0.6[m/s] \approx 333[s] \tag{3}$$

ie. approximately 5,5 minutes

2 Exercise 2: Tsunami Model

2.1 How does the wavelength of the Tsunami change when approaching the coast?

I wrote an algorithm that looks at the two resulting, trying to estimate the wavelegnth numerically:

To see how it was made, You can see three random snapshots from the simulation:

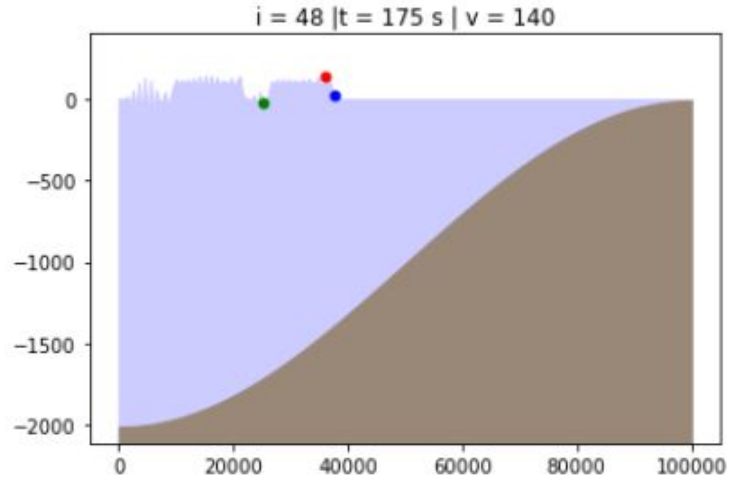

Figure 1: Snapshot 1

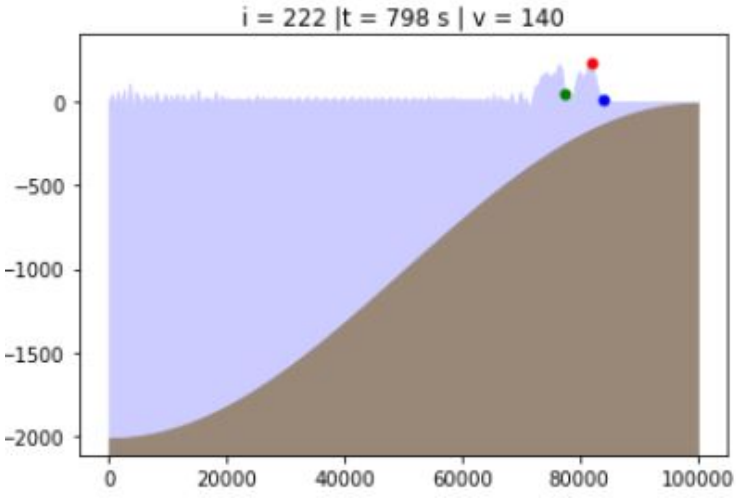

Figure 2: Snapshot 2

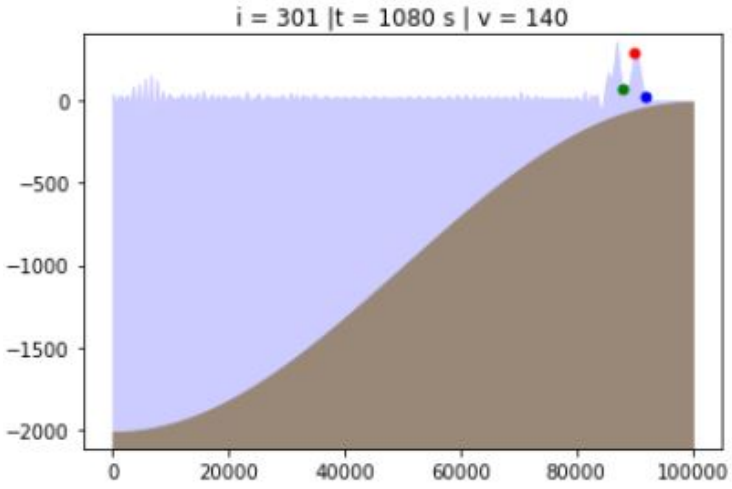

Figure 3: Snapshot 3

Looking at it as a plot over time, I find the following:

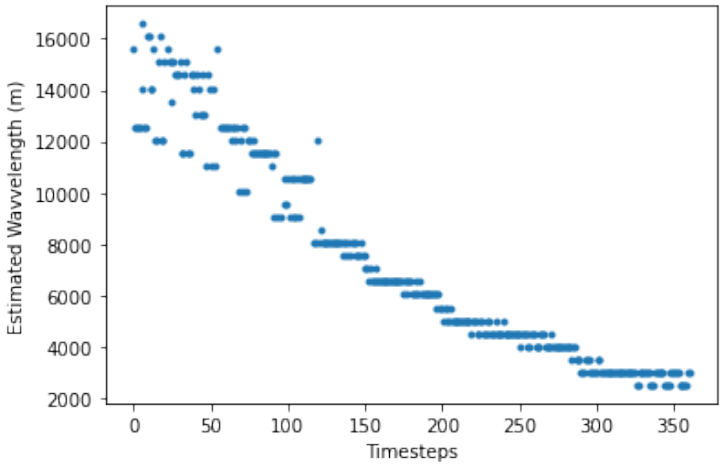

Figure 4: Wavelength over time

I can see that the wavelength falls over time, but i cannot re-find the *cos* I used for the ocean floor, so I am unsure what you are looking for.

2.2 How fast does the Tsunami travel in the open ocean? Do you think this is a realistic value?

I wrote a peak-finding-algorithm and used this to estimate the velocity.

I get a pretty consistent 140 [m/s] which seems super quick, but is comparable to the tsunami described in example 25.1 in the book which moves with 360 [km/h] or 100 [m/s]

2.3 How fast do individual water particles travel in the crest of the Tsunami in the open ocean?

Here, I am unsure how we defined the crest on a wave on the open ocean, so I choose the average velocity of the front part of the wave (from found peak and 100 m forward).

Over time this developed as such:

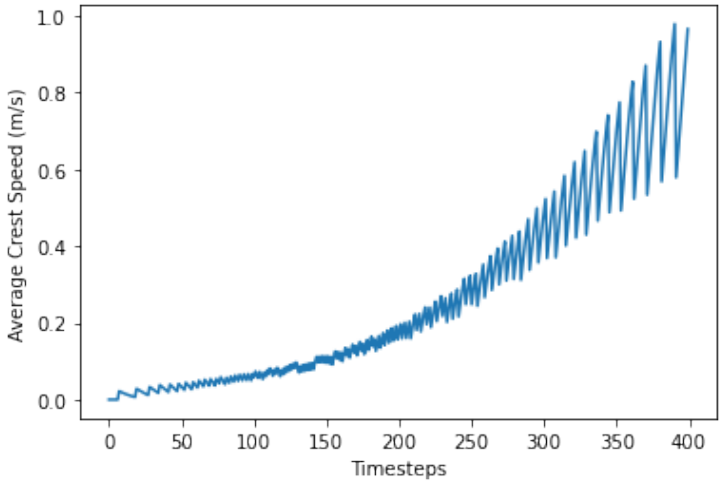

Figure 5: Average Crest Speed over time.

Here it can be seen that the individual particles in the wave only moves at about 0.2 [m/s]!

2.4 Should you use group or phase speed to characterize the velocity of the approaching Tsunami?

Group velocity undoubtedly! The phase speed simply defines the speed of the individual particles which can be much lower or even in the wrong direction!

Waves are an emergent behaviour. You are crushed by the group speed, not the individual particles - looking at the phase speed would be misleading. Think of the 0.2 [m/s] we found in the last question.

2.5 Why can we model a Tsunami that is generated in the open ocean (where the water column is, say, 4000m deep) with a "shallow water" model?

"Shallow water" does not refer to the actual depth of the water - it refers to the depth compared to the characteristics of the wave.

$\lambda \ll 2 \cdot \pi d$

having measured (on the open seas aka. having a completely flat floor) a wave length of around 20000 (see figure 6). Inserting this:

$$20000 \gg 2 \cdot \pi \cdot 2000 \approx 12000 \tag{4}$$

While this is not very convincing, I assume this is either the answer you are looking for or I have estimated the wavelength wrongly.

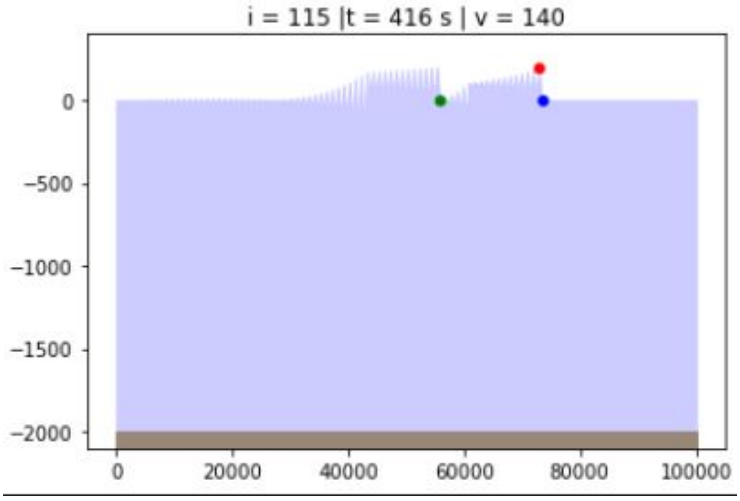

Figure 6: A completely flat ocean floor