1) Quantum assignment 2. pwn274, Team Laurie

1) $H' = kr^2(1-3\cos^2\theta)$

$H^0 = -\frac{\hbar}{2m}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0}\frac{1}{r}$

$$E' = \langle 100|H'|100\rangle = \int_0^\infty dr \int_0^\pi d\theta \int_0^{2\pi} d\phi \, (r^2\sin\theta)\, kr^2(1-\cos^2\theta)\, 4 a^{-3} e^{-2r/a} \frac{1}{4\pi}$$

$$= \frac{k}{\pi a^3}\int_0^\infty dr \int_0^\pi d\theta \int_0^{2\pi} d\phi \left(r^4 e^{-2r/a}\sin\theta + \frac{r^4}{3} e^{-2r/a} \frac{d}{d\theta}\cos^3\theta\right)$$

$$= \frac{k2}{a^3}\int_0^\infty dr \left(-2-\frac{2}{3}\right) r^4 e^{-2r/a} = -\frac{k2}{a^3}\frac{8}{3}\, 4!\left(\frac{a}{2}\right)^5 = -\frac{ka^2\cdot 2\cdot 2\cdot 2\cdot 4\cdot 3\cdot 2}{3\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2} = \underline{-2ka^2}$$

2)

E_2^0 is the eigenvalue of $|200\rangle$, $|21-1\rangle$, $|210\rangle$ and $|211\rangle$

ϕ-part will be $\int_0^{2\pi} d\phi\, e^{-im\phi} e^{im'\phi} = \int_0^{2\pi} d\phi\, e^{i(m'-m)\phi}$

this will for all $m \neq m'$ be an integral over Δm periods of $e^{i\phi}$ which, by symmetri, always be 0.

Only for $m = m'$ we will have $\int_0^{2\pi} d\phi\, e^{i0\phi} = \int_0^{2\pi} d\phi = 2\pi$

3) The pertubation-matrix is diagonal in the basis of $|21m\rangle$, as H' effectively is $-Y_2^0(\theta,\phi)$, meaning $\langle n1m|H'|n00\rangle = \langle n1m|n20\rangle = 0$ as Y_l^m are orthogonal

The matrix is then:

ℓ m	$\|1\ -1\rangle$	$\|1\ 0\rangle$	$\|1\ 1\rangle$	$\|0\ 0\rangle$
$\|1-1\rangle$	$\langle 1-1\|1-1\rangle$	0	0	0
$\|10\rangle$	0	$\langle 10\|10\rangle$	0	0
$\|11\rangle$	0	0	$\langle 11\|11\rangle$	0
$\|00\rangle$	0	0	0	$\langle 00\|00\rangle$

As I now have the pertubation matrix in diagonal shape, I can see the first-order pertubation being:

$$E_2^1 = \langle 00|H'|00\rangle, \langle 1\,{-1}|H'|1\,{-1}\rangle, \langle 10|H'|10\rangle, \langle 11|H'|11\rangle$$

I have no room on this page, but having used wolfram-Alpha to solve the integrals I get

$\langle 00|H'|00\rangle = \langle 10|H'|10\rangle = 0$ and $\langle 11|H'|11\rangle = \frac{45}{16}$ $\langle 1\,{-1}|H'|1\,{-1}\rangle = -\frac{45}{16}$

This means the total pertubation is either:

$$\{0, 0, \tfrac{45}{16}, -\tfrac{45}{16}\}$$

2.1 We have the formula from lab:

$$\Delta E = \gamma B_0 \hbar$$

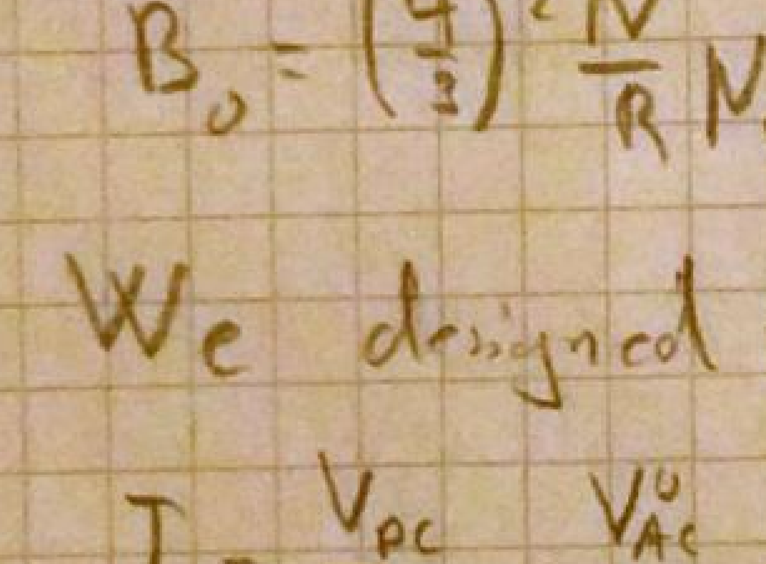
In our experiment B_0 was given by:

$$B_0 = \left(\frac{4}{5}\right)^{3/2} \frac{N}{R} \mu_0 I$$

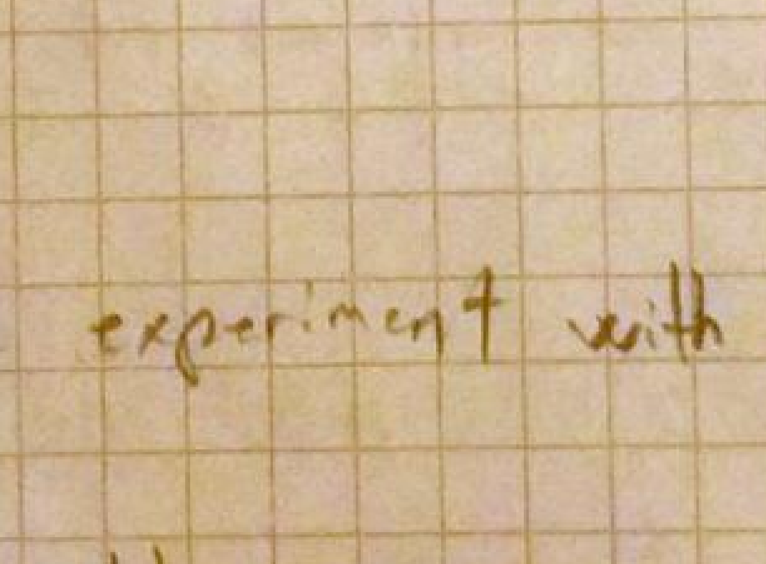
We designed the experiment with $I = (I_{DC} + I_{AC})$:

$$I = \frac{V_{DC}}{R} + \frac{V_{AC}^0}{R}\cos(wt)$$

This gives $(w=1)$:



For low V_{AC}



For high V_{AC}

It is clear, that E hits the right ΔE four times per period when looking at a high enough V_{AC}

3. Kicked Rotor

1) The eigenfunctions are Y_1^m as they are simultaneous eigenfunctions for $\hat{L}^2$ and $\hat{L}_z$.

The eigenenergies are therefore $\frac{\hbar^2}{2I} l(l+1) + a\hbar m$

with the ground-state at $Y_1^{-1} \Rightarrow \frac{\hbar^2}{I} - a\hbar$

2) $c_a^0 = 1 \quad c_b^0 = 0$

$\dot{c}_a^1 = 0 \Rightarrow c_a^1 = 1 \qquad \omega_0 = \frac{E_1 - E_0}{\hbar} = \frac{a\hbar}{\hbar} = a$

$$\dot{c}_b^1 = -\frac{i}{\hbar} \langle Y_1^0 | H' | Y_1^{-1} \rangle e^{i\omega_0 t} = -\frac{i}{\hbar} \langle Y_1^0 | b \hat{L}_x \sum_n^N \delta(t - n\,dt) | Y_1^{-1} \rangle e^{i\omega_0 t}$$

$$= -\frac{ib}{2\hbar} \langle Y_1^0 | \hat{L}_+ + \hat{L}_- | Y_1^{-1} \rangle \sum_n^N \delta(t - n\,dt) e^{i\omega_0 t}$$

$$= -\frac{ib}{2\hbar} \hbar\sqrt{2} \langle Y_1^0 | Y_1^0 \rangle \sum_n^N \delta(t - n\,dt) e^{i\omega_0 t}$$

$$= -i\frac{\sqrt{2}}{2} b \sum_n^N \delta(t - n\,dt) e^{i\omega_0 t}$$

$$c_b^1 = -i\frac{\sqrt{2}}{2} b \int_0^\infty dt' \sum_n^N \delta(t' - n\,dt) e^{i\omega_0 t'} = -i\frac{\sqrt{2}}{2} b \sum_n^N e^{i\omega_0 n\,dt} = -i\frac{\sqrt{2}}{2} b \sum_{n=1}^N e^{ian\,dt}$$

The probability is then the square of the absolute value:

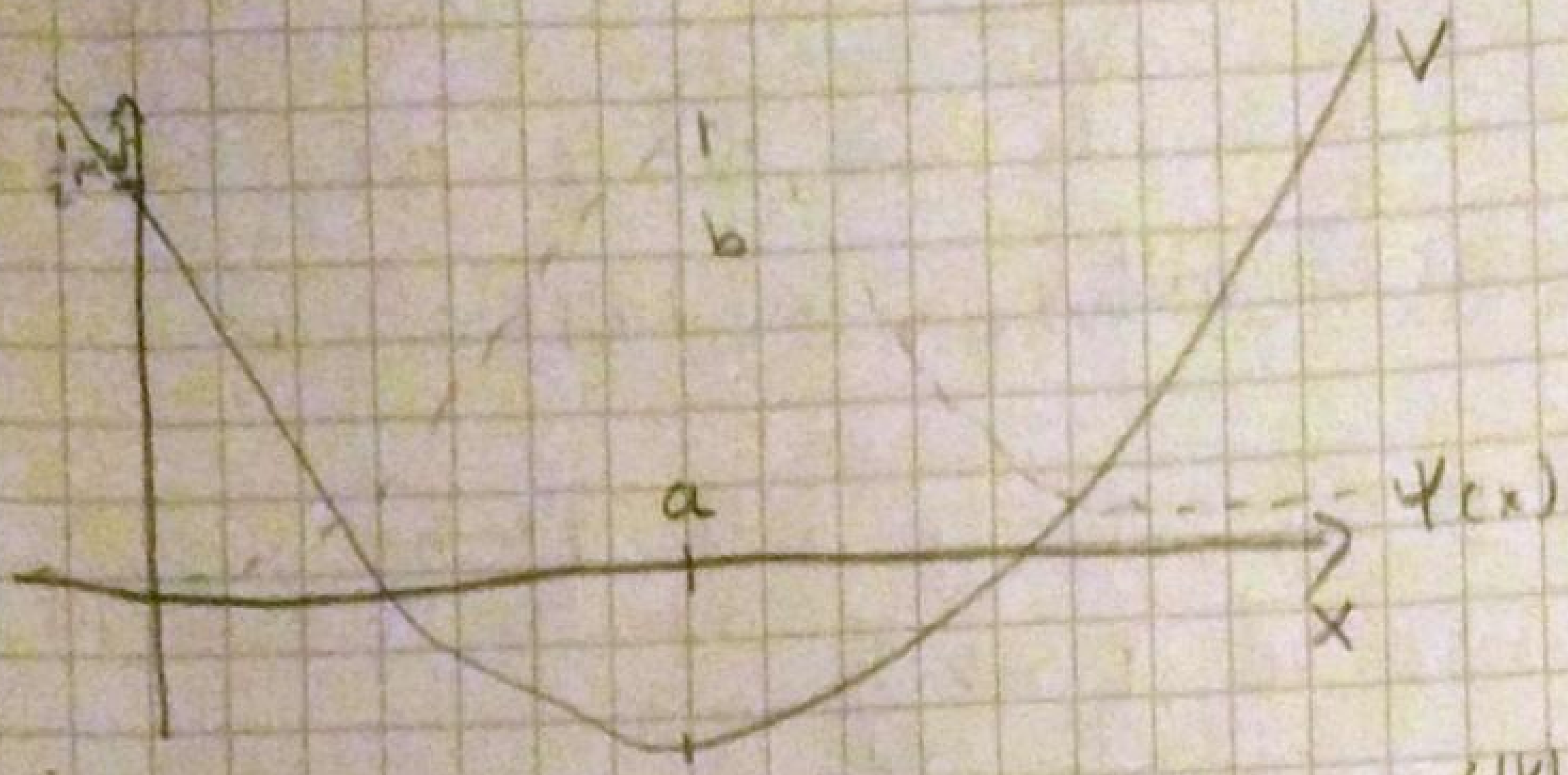
$$P = |c_b^1|^2 = 2^{-1} b^2 \left(\sum_{n=1}^N e^{ian\,dt} \right) \left(\sum_{n=1}^N e^{-iam\,dt} \right) = 2^{-1} b^2 \sum_{n,m=1}^N e^{i(n-m)a\,dt}$$

3) This can be seen as adding N unit-vectors, the length of this is of course maximized when they all point in the same direction.

At $dt = \frac{2\pi}{a}$ all $(n-m)a\,dt$ will do this exactly:

$$P = 2^{-1} b^2 \sum_{n,m=1}^N e^{i2\pi(n-m)} = 2^{-1} b^2 \sum_{n,m=1}^N e^0 = 2^{-1} N^2 b^2$$

4) $H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2(x^2-2ax)$



1) My guess is $b=a$ minimizes $\langle\psi|H|\psi\rangle$

2) As the pertubation looks like $H' = -ma\omega^2 x$, and x can be written as a raising and a lowering operator it is trivial to see, the first order pertubation is zero.

To find the second order correction I write:

$$E_0^2 = \sum_{m\neq 0} \frac{|\langle\psi_m^0| -ma\omega^2\sqrt{\frac{\hbar}{2m\omega}}(\hat{a}_+ + \hat{a}_-)|\psi_0^0\rangle|^2}{E_0 - E_m}$$

As it is only the raising operator that will hit ψ_1^0 I get:

$$E_0^2 = \frac{|-ma\omega^2\sqrt{\frac{\hbar}{2m\omega}}|^2}{\frac{1}{2}\hbar\omega - \frac{3}{2}\hbar\omega} = \frac{m^2a^2\omega^4\frac{\hbar}{2m\omega}}{-\hbar\omega} = -\frac{m\omega^2a^2}{2}$$

3) From the hint I can see the potential is simply a harmonic oscillator lowered by a constant. this constant is $\frac{m\omega^2a^2}{2}$, just as we found in 2.