Corrigendum to “Geometric motives and the h-topology” [Sch12]

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Abstract

I would like to thank Tom Bachmann for pointing out that the proof of Proposition 2.2 in [Sch12] relies on a wrong claim. This corrigendum provides a replacement. It does not affect the validity of the statement of Proposition 2.2 or those in the rest of the paper.

In the proof of [Sch12, Proposition 2.2] it is claimed that for any presheaf G with transfers, there is a canonical isomorphism

$$G(X) \cong \mathrm{Hom}_{\mathbf{PSh(Sm)}}(\mathbb{Z}_{\mathrm{tr}}(X), G),$$

where $\mathbb{Z}_{\mathrm{tr}}(X)$ denotes the representable presheaf with transfer of $X \in \mathbf{Sm}$. This claim is wrong. We now correct this issue.

Proposition 2.2. There is an equivalence of categories of h-sheaves with and without transfers:

$$\mathbf{Shv}_{\mathrm{h,tr}}(\mathbf{Sm}) \cong \mathbf{Shv}_{\mathrm{h}}(\mathbf{Sm}).$$

Proof (corrected version): In order to show that for any presheaf F with transfers, the map $F \to F_{\mathrm{h}}$ is a map of presheaves with transfer (where F_{h} is endowed with the canonical transfer structure), consider the following commutative cube,[1] where $\mathrm{Hom} := \mathrm{Hom}_{\mathbf{PSh}(\mathbf{Sm})}$ and $\mathrm{Hom}_{\mathrm{tr}} := \mathrm{Hom}_{\mathbf{PSh}(\mathbf{SmCor})}$:

$$
\begin{tikzcd}
 & F(X) \arrow[rr] \arrow[dd] & & F_{\mathrm{h}}(X) \arrow[dd] \\
\mathrm{Hom}_{\mathrm{tr}}(\mathbb{Z}_{\mathrm{tr}}(X), F) \arrow[ur, "\cong"] \arrow[rr] \arrow[dd] & & \mathrm{Hom}(\mathbb{Z}_{\mathrm{tr}}(X), F_{\mathrm{h}}) \arrow[ur, "\cong"] \arrow[dd] & \\
 & F(Y) \arrow[rr] & & F_{\mathrm{h}}(Y) \\
\mathrm{Hom}_{\mathrm{tr}}(\mathbb{Z}_{\mathrm{tr}}(Y), F) \arrow[ur, "\cong"] \arrow[rr] & & \mathrm{Hom}(\mathbb{Z}_{\mathrm{tr}}(Y), F_{\mathrm{h}}) \arrow[ur, "\cong"] &
\end{tikzcd}
$$

The vertical maps are induced by the map of presheaves (with transfers) $\mathbb{Z}_{\mathrm{tr}}(Y) \to \mathbb{Z}_{\mathrm{tr}}(X)$ induced by a correspondence W. The horizontal maps are induced by restriction along $\mathbf{Sm} \to \mathbf{SmCor}$ and the h-sheafification map (in $\mathbf{PSh}(\mathbf{Sm})$) $F \to F_{\mathrm{h}}$. The maps in the diagonal direction are evaluation maps such as

$$
\begin{array}{rcl}
\mathrm{Hom}(\mathbb{Z}_{\mathrm{tr}}(X), F) & \to & F(X) \\
f & \mapsto & f(\mathrm{id}_X) \in F(X).
\end{array}
$$

The left hand diagonal maps are isomorphisms by the Yoneda lemma (applied to presheaves with transfers). The right face of the box is commutative by definition of the canonical transfer structure on F_{h}. By [Sch12, Lemma 2.1], the rightmost diagonal maps are isomorphisms. Thus the commutativity of the back face is implied by the one of the front face, which is clear by the above description of the involved maps.

This finishes the proof of the above-mentioned statement, which is part 1 in the proof of [Sch12, Proposition 2.2]. The remaining two statements carry over in the same vein.

References

[Sch12] Jakob Scholbach, *Geometric motives and the h-topology*, Math. Zeit. **272** (2012).

[1] This diagram replaces the one at [Sch12, p. 972].