# Exercise 3:
# Stationary gravity waves and downslope windstorm

When a stable, stratified atmosphere with constant basic state zonal wind is perturbed, for instance by the upward-forcing at the windward side of a mountain, internal gravity waves arise. The waves can be described by

$$\left(\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x}\right)^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) w + N^2 \left(\frac{\partial^2}{\partial x^2}\right) w = 0,$$

where N is the Brunt-Väisälä frequency and $w$ the perturbation of the vertical velocity (cf. Durran 1990). With the harmonic function

$$w = w_0 exp^{i(kx+mz-\omega t)}$$

we get the dispersion relation for internal gravity waves,

$$(-\omega + u_0 k)^2 = \frac{N^2 k^2}{k^2 + m^2}$$

which gives the angular frequency $\omega$ as a function of the horizontal ($k$), the vertical wavenumber ($m$) and the basic state wind velocity ($u_0$).

## 3.1 Relaxed lateral boundaries and absorbing boundary at the top

After having implemented the prognostic time step and the subsequent diagnostic step, we are only a few steps away from studying the development of stationary gravity waves above the ridge. First, we have to switch lateral boundary conditions from periodic to relaxed (refer to Exercise 1, Section 3).

- Perform a simulation of 24 hours with the current setup of the model.
- Now switch to relaxed lateral boundaries by setting **irelax = 1** in your namelist. Perform a new simulation and compare it to the previous. What do you observe in the simulated flow? What is the advantage of these boundary conditions? You may want to increase the duration of the simulation.

As a next step, we have to make the uppermost levels absorbing by increasing their diffusion coefficients. Thereby we can avoid the reflection of upward-propagating waves and finally obtain an undistorted atmosphere at lower levels. Note that the transition into the absorbing "sponge" region should be as smooth as possible. If we had some abrupt increase of the diffusion coefficient from one model layer to the next, we would get new reflections at that interface. We therefore choose a smooth $sin^2$-type transition from $D_0$ (diff) to $D_{abs}$ (diffabs)

$$\tau(k) = D_0 + (D_{abs} - D_0) sin^2\left(\frac{\pi}{2}\frac{k-(nz-nab-1)}{nab}\right), \qquad (nz - nab) \le k \le nz$$

where $k$ is index for the vertical layer and starts at 0 in Python.

- Add the height dependence of $\tau$ to **solver.py** at *Exercise 3.1 height-dependent diffusion coefficient*. For the implementation of the absorbing boundary condition, you may use the following additional variables and functions:

| | |
|---|---|
| tau[0:nz] | height-dependent diffusion coefficient $\tau$ |
| diff | default diffusion coefficient $D_0$ |
| diffabs | maximum diffusion coefficient in absorber $D_{abs}$ |
| nab | number of grid points in absorber |
| np.pi | $\pi$ |
| np.sin() | *sin* |
| (x)**2 | $(x)^2$ |

  These variables are already defined in the source code.
- To activate the absorbing layer, set diffabs to 1.0. The thickness of the absorbing region has to be about 1.5 times the vertical wavelength of the hydrostatic gravity wave. Therefore, a good choice for the thickness of the absorbing layer is nab=30 (for nz = 60).
- Compare a simulation with open top (nab=0) with a simulation with absorbing top layers. Note that the magnitude of the diffusion coefficient, diff, controls the smoothing of the waves (the higher diff the more smoothing). Run the model over a long period (e.g. simulation of 60 hours), such that the reflected waves (if nab=0) can spread all over the domain.

## 3.2 Hovmöller diagrams

Hovmöller diagrams (Hovmöller, 1949) are a good method to visualize wave propagation. It is a velocity contour plot on a two-dimensional plane where one dimension is time (e.g. $(x,t)$, $(t,z)$).
There are two python-routines you can use for plotting Hovmöller diagrams:

| | |
|---|---|
| python hovx_vel.py output.npz zlev | for (x,t)-diagrams on level zlev |
| python hovz_vel.py output.npz xcoord | for (t,z)-diagrams at position xcoord |

- Do you see effects of the different boundary conditions (reflection / periodicity)?
- After around 100 hours of simulation time, the model will reach some steady state with a gravity wave standing right above the mountain ridge. Make an animation of the results and plot horizontal and vertical Hovmöller diagrams. Why is the steady state only reached after such a long integration time?

## 3.3 Downslope windstorm

How is the vertical propagation affected by the presence of a critical layer (i.e. where $u_0 = \omega/k$)? Argue by making reference to the dispersion relation equation.

Adapt the source code such that you can run the simulation with the following initial velocity profile u0[0:nz]:

- velocity u00_sh between the surface and the lower boundary of a shear zone (k=k_shl)
- a linearly decreasing horizontal velocity in the shear zone (between k=k_shl and k=k_sht)
- velocity u00 between the top of the shear layer (k=k_sht) and the model top

Implement the initial velocity shear zone with k_shl, k_sht, u00 and u00_sh in **makesetup.py** at the section *Downslope windstorm*. These parameters should be specified in the **namelist**. The initialization of the shear profile should be activated only with namelist parameter ishear=1.

**Hint:** You can use np.linspace(a,b,n) to linearly interpolate between a and b on n points.

Perform a simulation with 10 $ms^{-1}$ incoming flow velocity below the shear zone and 0 $ms^{-1}$ above. Set *N* to 0.01 $s^{-1}$ and the ridge height to 1400 m with a half width of maximum 25 km. You will need to adapt the timestep and the diffusion coefficient such that the CFL-criterion is not violated.

Compare the following cases:

a) shear zone between 5 and 7 km height,
b) shear zone between 9 and 11 km height,
c) no shear zone.

Since you still don't know which k-level corresponds to which height in [km], load the initial state from your output (output = np.load("output.npz")) and have a look at the variable output["z"] to obtain the corresponding model levels. Note that the gravity wave cannot propagate through the shear zone. Therefore, if the shear zone lies low enough, the flow becomes resonant and its velocity will increase strongly (downslope windstorm). For further information refer to Durran and Klemp (1987). The parameters given here are taken from this publication.

## References


Durran (1990): Mountain Waves and Downslope Winds, in Atmospheric Processes over Complex Terrain. B. Blumen (Ed), American Meteorological Society, Boston

Durran and Klemp (1987): Another look at downslope winds. Part II: Nonlinear amplification beneath wave-overturning layers, Journal of the Atmospheric Sciences, Vol. 44, pp. 3402-3412

Hovmöller, E. (1949): The Trough-and-Ridge diagram, Tellus 1:62-66