Machine Learning

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Index

- Supervised Learning
 - Decision Trees
 - Bayesian Learning
- Unsupervised learning
 - Hierarchical clustering
 - K-Means clustering

Sources of this document

- Programming Collective Intelligence, Toby Segaran.
- http://iie.fing.edu.uy/ense/asign/recpat/material/tema3_00-01/node25.html

Index

- Supervised Learning
 - Decision Trees
 - Bayesian Learning
- Unsupervised learning
 - Hierarchical clustering
 - K-Means clustering

Ex: Predicting Sign-ups

Objective: Detect potential clients.

We have a new Internet service. To enhance its visibility we offer free trials.

After some weeks, the users can choose to upgrade their account to a *basic* or *premium* service, or cancel the service.

During trial time, we have collected information on the users activity and the final decision.

This information is collected passively through user interaction with the system.

Data: user interaction and final decision

Referrer	Location	Read FAQ	Pages viewed	Service
slashdot	USA	yes	18	None
google	France	yes	23	Premium
digg	USA	yes	24	Basic
kiwitobes	France	yes	23	Basic
google	UK	no	21	Premium
(direct)	New Zealand	no	12	None
(direct)	UK	no	21	Basic
google	USA	no	24	Premium
slashdot	France	yes	19	None
digg	USA	no	18	None
google	UK	no	18	None
kiwitobes	UK	no	19	None
digg	New Zealand	yes	12	Basic
slashdot	UK	no	21	None
google	UK	yes	18	Basic
kiwitobes	France	yes	19	Basic

Lets program ...

- t1 Download the data file *descision_tree_example.txt* from the virtual campus at folder */lab/learning*.
- t2 Create a file named treepredict.py
- t3 Define a function to load the data into a bidimensional list named data

```
def read(data, file_name):
```

The problem of classifying data

- Our goal is to obtain automatically a classifier
- We have a set of classes or categories
- We also have a set of objects defined as a function of attributes to values
- Objective: depending of the values of the attributes of a given object we want to classify it, i.e., we want to determine the category (special attribute) the object belongs to, named goal attribute
- In particular, we want to build a function that takes as arguments the values of the attributes of an object and returns the value of the goal attribute
- We initially have a training set or prototype set where we know the value of all the attributes, even the goal attribute



- Decision trees are one of the most simple methods of supervised learning
- The classifier looks like a set of if-then statements organized as a directed tree
- Every internal node contains a question on a particular attribute (one child per possible answer). Every arc corresponds to an answer and every leaf node refers to a decision (classification).
- The classification of the objects is performed through a sequence of questions on the attribute values. It starts at the root node, and it follows the path determined by the answers to the questions of the internal nodes, till a leaf node is reached. The label assigned to a leaf corresponds to the value assigned to the goal attribute

• Lets imagine that our root node contains initially the objects in our training set. Through every arc (n_1, n_2) we send a copy of the original set containing those objects that match the answer to the question of n_1 . At the leaves, the remaining objects share the value of the goal attribute

- Classifiers based on decision trees (ID3, C4, C4.5, Bayesian trees, etc.)
- We will implement CART (Classification And Regression Trees)
- The main differences have to do with the pruning strategies and the way nodes are split
- CART applies binary partitions and uses a pruning strategy based on a cost-complexity criterion

The methodology we will follow, can be described into two steps:

- Learning. Consist in the construction of a decision tree.
 This is the more complex step and determines the final result
- Classification. Consist in the labelling of an object, not into the training set. We basically answer to all the questions associated to the internal nodes, following the path to a leaf node

A classification tree T represents a recursive partition of the space representation, P, based on a set of prototypes, S.

- Every node of T has assigned a subset of prototypes in S.
- The root node has assigned S.
- Every leave, t, has assigned a region, R_t , in P. If leaves(T) is the set of leaf nodes in T, then, $\bigcup_{t \in leaves(T)} R_t = P$, i.e, the sets of prototypes assigned to the leave nodes form a partition of P
- Every internal node n has assigned a region in P, which is the union of the regions assigned to the leaf nodes of the subtree with root node n
- The union of the sets of prototypes assigned to the nodes at the same level is equal to S.

Building the classification tree

We follow this recursive scheme:

- Advance: Partition a node according to some rule.
 - (ex: evaluate a condition on an attribute).
 - The prototypes that verify (falsify) the condition are assigned to the left (right). Obsv: these subsets are disjoint
- Stop condition: stops the partition of nodes.
 - The nodes that satisfy this condition are leaf nodes.

Building the classification tree

There are still some questions to answer:

- How do we perform the partitions and how do we select the best?
- When do we declare a node to be a leaf?
- How do we assign a label to a leaf?

Depending on the answers we get different algorithms.

Selecting the partitions

- In CART partitions are binary. The answer is always yes or not
- The objective of a partition is to increase the homogeneity (class-based) of the resulting subsets
- We assign a purity measure to each partition, that we will use:
 - To select the best partition.
 - As stop criterion.

Formulating the partition rule

In CART we define the standard set of questions (Q) in the following way:

- Each partition depends on a unique attribute.
- If x_i is a categorical attribute, s.t., $x_i \in C = \{c_1, c_2, \dots, c_k\}$, Q includes the questions:

$$\dot{c}_i x_i$$
 ∈ C' ?, where C' ⊂ C

If x_i is a continuous attribute, Q includes questions as:
 ¿x_i ≤ v?, where v is a real number or value.
 In CART, v is the middle point of two consecutive values x_i.

Partitioning criterion

Impurity (purity) measure:

Given a function of impurity ϕ , we define the measure of impurity of a node t, named i(t), as:

$$i(t) = \phi(p(1|t), p(2|t), ..., p(k|t))$$

where p(j|t) is the probability that a prototype assigned to t belongs to class j, i.e.,

$$p(j|t) = \frac{N_j(t)}{N(t)}$$

N(t) is the number of prototypes in t, and $N_j(t)$ is the number of prototypes of class j in t

Lets program ...

t4 Define a function unique_counts that counts the number of prototypes of a given class in a partition part. We assume that the goal attribute is the last one. We need to return a dictionary that has as keys the different classes in part and as values the number of prototypes of the given class

```
# Create counts of possible results
# (the last column of each row is
# the result)
def unique_counts(part):
    results={}
    ...
    return results
```

Partitioning criterion

Lets observe that:

- $p(j|t) \geq 0$
- $\sum_{k} p(j|t) = \sum_{k} \frac{N_{j}(t)}{N(t)} = \frac{1}{N(t)} \sum_{k} N_{j}(t) = 1$
- The maximum impurity is obtained when all the classes are equally represented in *t*.
- the minimum impurity is obtained when in *t* all prototypes belong to the same class (maximum homogeneity).

Partitioning criteria

Criteria to measure impurity:

 Gini index: measures the expected error if a class is randomly assigned to one the prototypes in the node:

$$i(t) = \sum_{\substack{j_1, j_2 = 1 \\ j_1 \neq j_2}}^{K} \rho(j_1|t) \cdot \rho(j_2|t) = 1 - \sum_{j=1}^{K} \rho(j|t)^2$$

 Entropy: measures the amount of disorder or chaos in a set:

$$i(t) = -\sum_{i=1}^{K} p(j|t) \cdot \log p(j|t)$$

Lets program ...

t5 Define a function that computes the Gini index of a node:

```
def gini_impurity(part):
  total = len(part)
  results = unique_counts(part)
  imp = 0
  ...
  return imp
```

Lets keep programming ...

t6 Define a function that computes the entropy of a node:

```
Entropy is the sum of p(x)\log(p(x))
  across all the different possible
# results
def entropy (rows):
   from math import log
   log2 = lambda x: log(x)/log(2)
   results = unique_counts(rows)
   # Now calculate the entropy
   imp = 0.0
   return imp
```

Partitioning criteria

Goodness of a partition: The goodness of a partition s in a node t has to be related to the impurity of the node, and the impurity of the resulting nodes after the partition, t_L y t_R .

The goodness of a partition s in a node t, $\Phi(s, t)$, is defined as the decrease of impurity achieved:

$$\Phi(s,t) = \Delta i(s,t) = i(t) - p_L \cdot i(t_L) - p_R \cdot i(t_R)$$

where, p_L y p_R is the proportion of prototypes of t in t_L and t_R , respectively.

As we know how to compute i(t), we can compute each partition s and select the best partition, i.e., the one that maximizes the drop in impurity.

Lets program ...

t7 Define a function that partitions a previous partition, taking into account the values of a given attribute (column). column is the index of the column and value is the value of the partition criterion.

```
# Divides a set on a specific column. Can handle
# numeric or categorical values
def divideset(part, column, value):
    isplit_function = None

if isinstance(value, int) or isinstance(value, float):
        split_function = lambda prot: prot[column]>=value
    else:
    ...
    return (set1, set2)
```

Stop criterion

We set a value $\beta > 0$. A node *t* is terminal iff:

$$\max_{s} \Delta i(s,t) < \beta$$

- If β is low, it is complicated to stop the building process since the impurity has to be low. This can result into very large trees
- Si β is high, the height of the trees is low. Although, we may find nodes where $\max_s \Delta i(s,t)$ is low, it may be possible to find a partition of its descendants that results in a bigger decrease of the impurity.

How to label leaves

The objective is to assign a class, j, to every terminal node $t \in leaves(T)$.

The simplest way is to select the class j where p(j|t) is maximum.

We can break ties randomly selecting a class.

Lets program ...

t8 Define a new class *decisionnode*, which represents a node in the tree, using the following constructor:

```
def __init__(self,col=-1,value=None,results=None,tb=None,fb=None)
```

We have five member variables.

- col is the column index which represents the attribute we use to split the node
- value corresponds to the answer that satisfies the question
- tb y fb are internal nodes, representing the positive and negative answers, respectively.
- results is a dictionary that stores the results for this branch.
 Is None except for the leaves.

Lets keep programming ...

t9 Define a new function *buildtree*. This is a recursive function that builds a decision tree using any of the impurity measures we have seen. The stop criterion is $\max_s \Delta i(s,t) < \beta$:

```
def buildtree(part, scoref=entropy, beta=0):
    if len(part)==0: return decisionnode()
    current_score = scoref(part)

#Set up some variables to track the best criteria
    best_gain = 0
    best_criteria = None
    best_sets = None
    ...
    else:
        return decisionnode(results=unique_counts(part))
```

t10 Define the iterative version of the previous function.

Drawing the tree ...

t11 Include the following function:

```
def printtree(tree, indent=''):
   # Is this a leaf node?
   if tree results!=None:
      print str(tree.results)
   else:
      # Print the criteria
      print str(tree.col) +':'+str(tree.value) +'? '
      # Print the branches
      print indent+'T->',
      printtree (tree.tb, indent+'
      print indent+'F->',
      printtree (tree.fb, indent+'
```

Lets program ...

t12 Build a function *classify* that allows to classify new objects. It must return the dictionary that represents the partition of the leave node where the object is classified.

```
def classify(obj, tree):
```

Evaluating a learning algorithm

A learning algorithm can be considered good if it is capable to classify accurately objects that have never seen before.

A basic methodology to evaluate a learning algorithm is the following:

- 1 Retrieve a wide set of examples.
- 2 Divide this set into two sets: the training set and the test set.
- 3 Build the classifier with the training set.
- 4 Measure the percentage of examples of the test set that are correctly classified.
- 5 Repeat steps 2 and 4 for different sizes of training and test sets chosen randomly.

We can plot the measure of prediction quality as a function of the size of the training set.

Lets program ...

t13 Define a function test that takes a test set and a training set and computes the percentage of examples correctly classified.

```
def test_performance(testset, trainingset):
```

t14 Show the quality of the classifier increasing by a 20% the training set. You can retrieve data from the following database: http://archive.ics.uci.edu/ml/

Missing data

- An advantage of the decision trees on is their ability to treat missing data
- In our working example, a user's geolocazation through the IP may not be possible
- Suppose we want to compare the variation of impurity as a function of attribute A. However, for attribute A some of the examples have no value
- Solutions:
 - We assign to the example the most frequent value of attribute A
 - t15 Suggest other solutions ...



Stop criterion: prunning strategy

- The use of the threshold β to stop the process of partition presents difficulties, as we have seen above
- The procedure is more complex and will be based on a pruning strategy
- Idea: build a very large tree and prune towards the root those subtrees that produce subtrees small benefits into the decrease of impurity
- Obsv: it is more efficient to prune a tree than stopping its growth. Pruning allows a subtree of a node to remain and the other disappear, while stopping the growth prunes all branches simultaneously

General prunning strategy

- Build the maximum tree (T_{max}) partitioning nodes till any of these conditions is reached:
 - The node is pure
 - $N(t) < N_{min}$ (usually, $N_{min} = 5$)
- Prune T_{max} obtaining a decreasing sequence of nested trees. if T' is obtained by prunning T, then T ≻ T'.

$$T_{max} \succ T1 \succ T2 \succ \ldots \succ \{t\}$$

where $\{t\}$ is a tree of a single node.

- Assign an error measure to every tree and choose the least mistake
 - It is expected that pruned trees will have more capacity of generalization since they are not so adjusted to the training set (the problem of overfitting)

A particular prunning strategy ...

- Build completely the decision tree.
- For every pair of leaves with a common father check if their union increases the entropy below a given threshold. If that is the case, delete those leaves by joining their prototypes in the father.
- Repeat till it is not possible to delete more leaves.
- t16 Define a function that implements the above prunning strategy:

```
def prune(tree, threshold):
```

Index

- Supervised Learning
 - Decision Trees
 - Bayesian Learning
- Unsupervised learning
 - Hierarchical clustering
 - K-Means clustering

Ej:Spam filtering

Objective: Classify documents according to their content. One of the best known applications is *spam* filtering.

Spam not only affects email, it also affects webs that have become more interactive: requesting user comments and content creation.

Whenever you create an application to request the general public to contribute, you need an strategy to eliminate spam.

The idea is to apply a learning algorithm capable of classifying documents into categories, one of which is *spam*

Document classification

- We classify documents according to their characteristics
- An item is a document
- We use as characteristics (features) the words that appear in the document. We can also use combinations of words, phrases, or anything that can be classified as absent or present in the document
- We classify as bad | good an item depending whether it is spam or not

Recognizing the words in a document

t1 Create a file docclass.py. Incorporate the function getwords. This function splits the text into words using as delimiters any character which is not a letter. All words are converted to lower case.

```
import re
import math
def getwords (doc):
  splitter=re.compile('\\W*')
  print doc
  # Split the words by non-alpha characters
  words=[s.lower() for s in splitter.split(doc)
          if len(s) > 2 and len(s) < 201
  # Return the unique set of words only
  return dict([(w,1) for w in words])
```

Recognizing the words in a document

Test the program.

```
>>> import docclass
>>> docclass.getwords('quick money at Cayman-Casino')
quick money at Cayman-Casino
{'quick': 1, 'money': 1, 'casino': 1, 'cayman': 1}
```

Which are the good features

- To determine which features to use is crucial and difficult
- They must be common enough to appear frequently, but not appearing in all documents
- Extreme cases are: the whole document or the individual characters
- If we use words, we must decide how to split them, which punctuation should be included, ...

We need a class that represents our classifier. That will allow us to instantiate multiple classifiers for different users, groups, queries, and train them according to our particular needs

t2 Incorporate the definition of the class *classifier*:

```
class classifier:
    def __init__(self, getfeatures, filename=None):
        # Counts of feature/category combinations
        self.fc = {}
        # Counts of documents in each category
        self.cc = {}
        self.getfeatures = getfeatures
```

Dictionary *fc* contains the number of appearances by category of a given *feature*. Ej:

```
{'python': {'bad': 0, 'good': 6}, 'the': {'bad':3, 'good': 3}}
```

Dictionary *cc* indicates how many times a classification has been used.

t3 Complete the following functions:

```
# Increase the count of a feature/category pair
def incf(self,f,cat):
    self.fc.setdefault(f,{})
    self.fc[f].setdefault(cat,0)
    ...
# Increase the count of a category
def incc(self,cat):
    self.cc.setdefault(cat,0)
    ...
```

```
# The number of times a feature is in a category
def fcount (self, f, cat):
  if f in self.fc and cat in self.fc[f]:
    . . .
  return 0.0
# The number of items in a category
def catcount(self, cat):
  if cat in self.cc:
  return 0.0
# The total number of items
def totalcount(self):
  . . .
# The list of all categories
def categories (self):
  return self.cc.keys()
```

t4 Incorporate a function train which takes an item and a category. This function uses getfeatures in order to obtain the features of the item, and calls appropriately functions incf and incc.

Lets check the program ...

Test the previous program.

```
>>> import docclass
>>> cl = docclass.classifier(docclass.getwords)
>>> cl.train('the quick brown fox jumps over the lazy dog','good'
the quick brown fox jumps over the lazy dog
>>> cl.train('make quick money in the online casino','bad')
make quick money in the online casino
>>> cl.fcount('quick','good')
1.0
>>> cl.fcount('quick','bad')
1.0
```

t5 Incorporate the following function in order to test your program:

```
def sampletrain(cl):
    cl.train('Nobody owns the water.','good')
    cl.train('the quick rabbit jumps fences','good')
    cl.train('buy pharmaceuticals now','bad')
    cl.train('make quick money at the online casino','bad')
    cl.train('the quick brown fox jumps','good')
```

Computing probabilities

- We are interested on computing the probability that a feature appears in a given category. In probability theory, a conditional probability is the probability that an event would have, conditional that another occurs
- The probability of *A* given *B*, p(A|B) is, taking into account the worlds where *B* holds, the fraction in which also *A* holds, i.e., $p(A|B) = \frac{p(A \cap B)}{p(B)}$
- t6 Define and incorporate a function which computes the probability that a given feature appears into a given category

```
def fprob(self, f, cat):
   if self.catcount(cat) == 0: return 0
# The total number of times this feature appeared
# in this category divided by the total number of
# items in this category
```

Computing probabilities

Test the program:

- fprob is particularly sensitive in the early stages of training and for words that do not appear so often. Ej:
 The word 'money' only appears into one document classified as 'bad'. Therefore, its fprob for class 'good' is 0
- It is more realistic that fprob('money','good') gradually approaches 0 as 'money' is found in more and more documents of category 'bad'
- One approach to avoid that, is to think about an assumed probability. For example, we can start with 0.5
- We must establish which weight has this assumed probability.
 For example, a weight of 1 means that the assumed probability is weighted the same as one word. The weighted probability returns the weighted average between the basic probability and the assumed probability

 In the example of 'money', the weighted probability for the word money starts at 0.5 for all categories. When the classifier detects 'money' into a 'bad' document, the probability of ('money','bad') becomes a 0.75, i.e.,:

$$(1*0.5+1*1)/(1+1)=0.75$$

where *count* is the number of times this *feature* has appeared in all the classes.

t7 Which is the weighted probability for ('money','good')?

return wp

t8 Define a function that computes the weighted probability.

```
def weightedprob(self,f,cat,prf,weight=1.0,ap=0.5):
    # Calculate current probability
    basicprob = prf(f,cat)

# Count the number of times this feature has appeared in
# all categories
    totals = ...

# Calculate the weighted average
    wp = ...
```

 Load twice our training set, and analyze the result of the weighted probability for ('money','good') after each load:

Naive Bayesian Classifier

- We need to combine all the previous individual probabilities in order to obtain the probability that a given item belongs to a class
- Naive: because it is assumed that the probabilities that we have to combine are independent
- Although the probability corresponding to an item does not represent the true probability of belonging to a class, you can compare the results for various classes and choose the biggest probability
- Bayes: because we will use the Bayes' Theorem in order to invert the conditional probabilities
- The previous strategy has been shown to be effective as a classification criterion for documents



Probability of an item given a category

- In the first place, we must determine the probability of an item given a category.
- Assuming that the probabilities are independent we just need to multiply the probabilities of all the *features* that appear into the item. Ex:
 - If p('python'|'bad') = 0.2 and p('casino'|'bad') = 0.8, we say that the probability that both appear into an *item* of class *bad* (assuming independence) is p('python'|'assino'|'bad') = 0.8, 0.2 = 0.16
 - $p('python\&casino'|'bad') = 0.8 \cdot 0.2 = 0.16.$
- Therefore, we can define p(item|category) in the following way:
 - $p(item|category) = p(feature_1|category) \cdot ... \cdot p(feature_1|category)$

Probability of an item given a category

t9 Define and incorporate a subclass of classifier called naivebayes, and create a member function docpro which analyzes the features of an item and returns the probability of this item given the category cat.

```
class naivebayes(classifier):
  def docprob(self,item,cat):
    features=self.getfeatures(item)
    # Multiply the probabilities of all the features together
    p=1
    for f in features:
        ...
    return p
```

Probability of an item given a category

Test the previous program:

```
>>> import docclass
>>> cl = docclass.naivebayes(docclass.getwords)
>>> docclass.sampletrain(cl)
>>> cl.docprob('quick rabbit','good')
quick rabbit
0.260416666666666663
>>> cl.docprob('quick rabbit','bad')
quick rabbit
0.125
```

Bayes' Theorem

- Previously, we have seen how to compute
 p(item|category). However, we actually need to compute
 p(category|item)
- Bayes' Theorem:

$$p(A|B) = p(B|A) \cdot \frac{p(A)}{p(B)}$$

in our example,

$$p(category|item) = p(item|category) \cdot \frac{p(category)}{p(item)}$$

t10 Do we really need to compute p(item)?



Bayes' Theorem: Máximo A Posteriori (MAP)

 This name is given to the most probable hypothesis using Bayes's theorem

$$h_{MAP} = \arg \max_{h \in H} p(h|D)$$

$$= \arg \max_{h \in H} \frac{p(D|h) \cdot p(h)}{p(D)}$$

$$= \arg \max_{h \in H} p(D|h) \cdot p(h)$$

 Sometimes, these hypothesis are equiprobable. In that case, we refer to maximum likelihood:

$$h_{ML} = \arg \max_{h \in H} p(D|h)$$



t11 Define a function *prob* that computes p(category|item).

```
def prob(self, item, cat):
  catprob = ...
  docprob = ...
  return ...
```

Compute p('quick rabbit','good') and p('quick rabbit','bad')

t12 Which class would you assign to 'quick rabbit'?



- We can choose the highest probability
- In some applications it is better for the classifier to admit it does not know how to classify a document than assigning it to a class due to marginally superior probability. Ex: it is better to do not classify a document as spam if it is not clear enough
- In order to avoid this problem we can establish a minimum threshold
- In order to classify an item in a category its probability has to be at least threshold times greater than any other probability

- Ex: In order to filter *spam*, the threshold for 'bad' could be 3, while for 'good' could be 1.
- t13 Incorporate in the class *naivebayes*:

```
def __init__(self,getfeatures):
    classifier.__init__(self,getfeatures)
    self.thresholds={}

def setthreshold(self,cat,t):
    self.thresholds[cat]=t

def getthreshold(self,cat):
    if cat not in self.thresholds: return 1.0
    return self.thresholds[cat]
```

t14 Define a function classify that classifies an item returning a category or a default value when the class can not be determined

```
def classify (self, item, default=None):
  probs={}
  # Find the category with the highest probability
  max = 0.0
  best = None
  for cat in self.categories():
    probs[cat]= ...
    if probs[cat]>max:
      max = ...
      best = ...
  # Make sure the probability exceeds threshold*next best
  for cat in probs:
    if cat == best: continue
    if ...:
      return default
  return best
```

Test your classifier:

```
>>> import docclass
>>> cl = docclass.naivebayes(docclass.getwords)
>>> docclass.sampletrain(cl)
>>> cl.classify('quick rabbit', default='unknown')
'aood'
>>> cl.classify('quick money', default='unknown')
'bad'
>>> cl.setthreshold('bad', 3.0)
>>> cl.classify('quick money', default='unknown')
'unknown'
>>> for i in range(10): docclass.sampletrain(cl)
. . .
>>> cl.classify('quick money', default='unknown')
'bad'
```