Quantitative Macroeconomics - Homework 2

Jakub Bławat

September 2019

1 Exercise 1: Function Approximation: Univariate

1.1 Taylor approximation of the exponential function

Question: Approximate f(x) = $x^{0.321}$ with a Taylor series around x = 1. Compare your approximation over the domain (0,4). Compare when you use up to 1; 2; 5 and 20 order approximations. Discuss your results.

Plot:

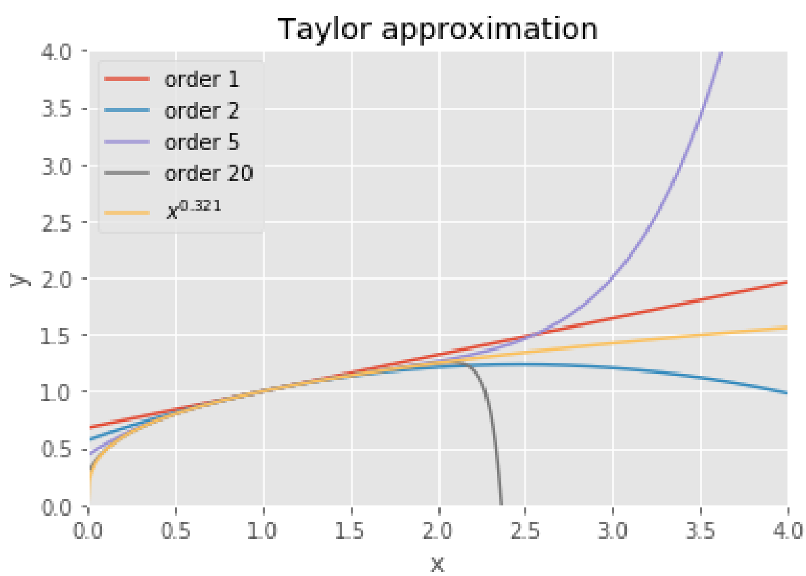

Figure 1: Taylor approximation of the exponential function

Comment: As we can see our approximation is quite good around x = 1. However if we go further away from x our approximations are becoming less accurate. We can also notice that higher order Taylor functions are giving worse results than lower order Taylor functions, which could be surprising.

1.2 Taylor approximation of the ramp function

Question: Approximate the ramp function f(x) = $\frac{x+|x|}{2}$ with a Taylor series around x = 2. Compare your approximation over the domain (-2,6). Compare when you use up to 1; 2; 5 and 20 order approximations. Discuss your results.

$$f(x) = \frac{x + |x|}{2} \tag{1}$$

Plot:

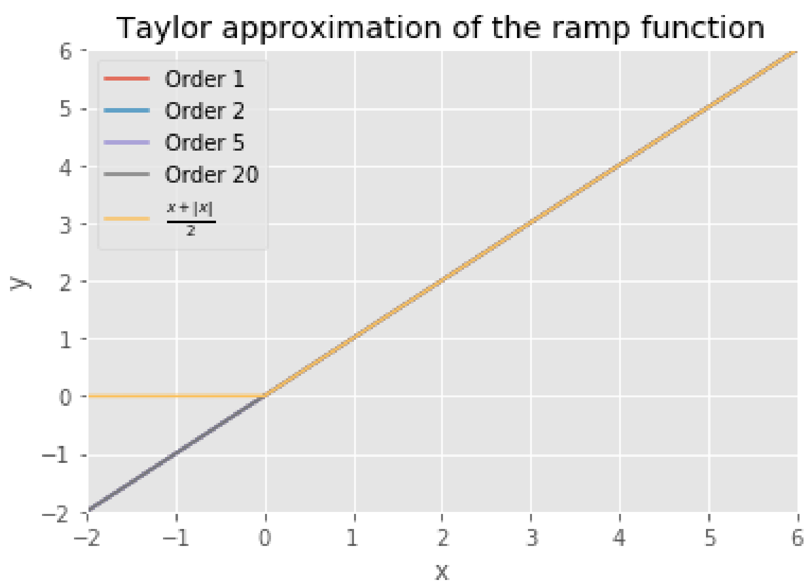

Figure 2: Taylor approximation of the ramp function

Again our approximation is accurate around x = 2, but it this case it is also accurate for all 'x' that are greater than 0. However, in point (0,0) we have a kink because of an absolute value, that is why for x $\leq$ 0 our approximation fails completely. Our Taylor series approximation just can not deal with ramp functions.

1.3

Approximate these three functions: $e^{\frac{1}{e}}$, the runge function $\frac{1}{1+25x^2}$, and the ramp function $\frac{x+|x|}{2}$ for the domain x[-1; 1] with:

- Evenly spaced interpolation nodes and a cubic polynomial. Redo with monomials of order 5 and 10. Plot the exact function and the three approximations in the same graph. Provide an additional plot that reports the errors as the distance between the exact function and the approximand.

- Chebyshev interpolation nodes and a cubic polynomial. Redo with monomials of order 5 and 10. Plot the exact function and the three approximations in the same graph. Provide an additional plot that reports the errors as the distance between the exact function and the approximand.

- Chebyshev interpolation nodes and Chebyshev polynomial of order 3, 5 and 10. How does it compare to the previous results? Report your approximation and errors.

1.3.1 a) Evenly spaced interpolation nodes and a cubic polynomial

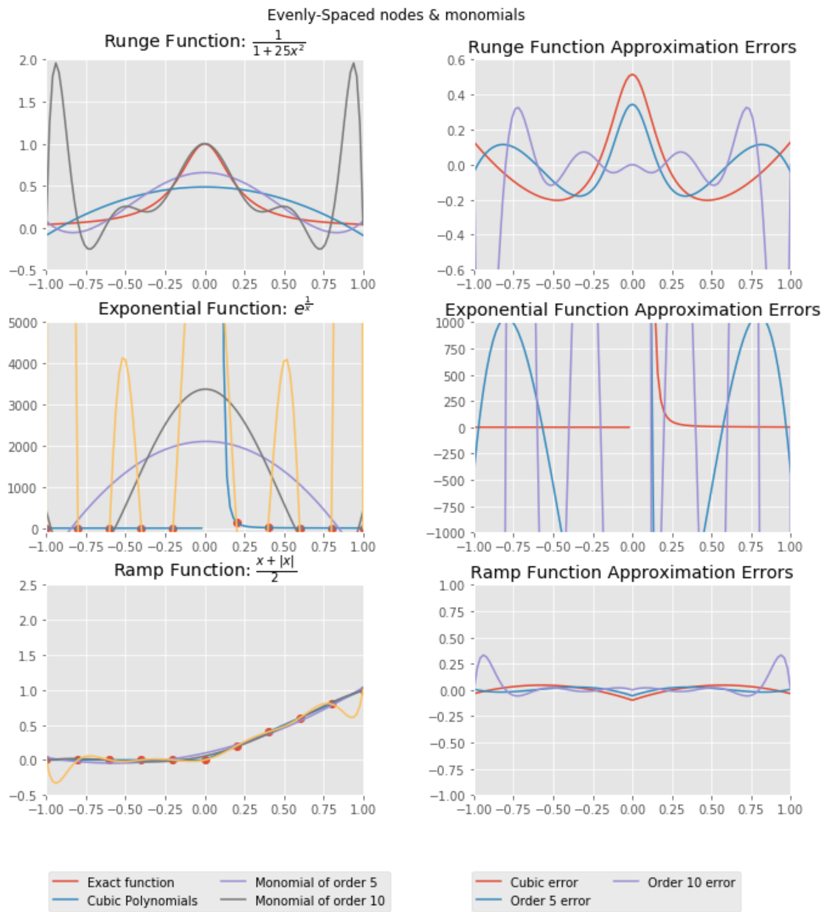

Figure 3: Function approximation

In our first approximation we use evenly-spaced interpolation nodes and monomials of order 3, 5 and 10.

1.3.2 b) Czebyszow interpolation nodes and a cubic polynomial

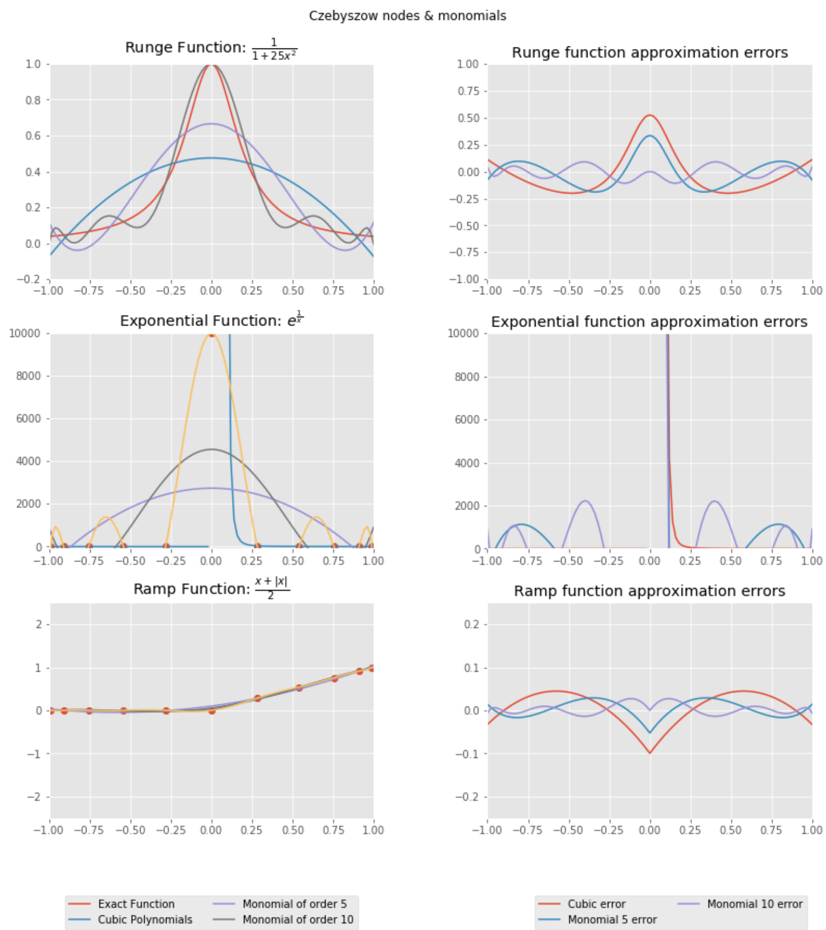

Figure 4: Function approximation

1.3.3 c) Czebyszow interpolation nodes and Czebyszow polynomials

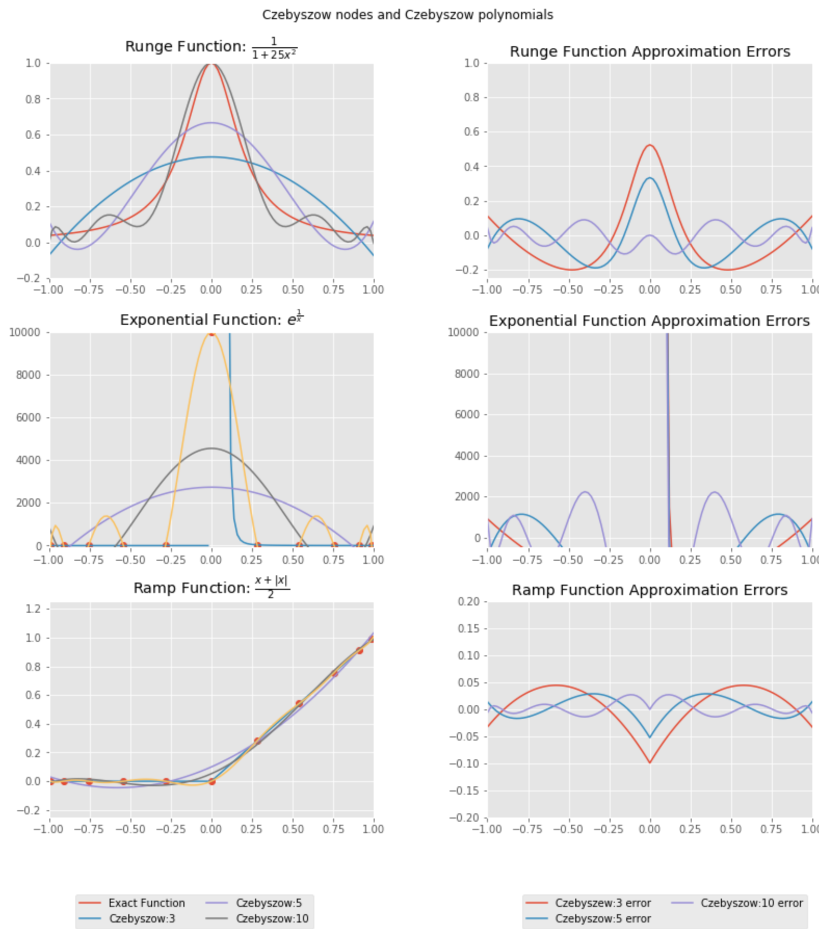

Figure 5: Function approximation

2 Question 2: Function Approximation: Multivariate

2.1 a)

Show that σ is the ES (hint: show this analytically).

$$f(k,h) = ((1-\alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \tag{2}$$

Now we calculate marginal productivity of labour:

$$MPL = \frac{\partial f(k,h)}{\partial h} = \alpha h^{\frac{-1}{\sigma}}((1-\alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}} \tag{3}$$

And marginal productivity of capital:

$$MPK = \frac{\partial f(k,h)}{\partial k} = (1-\alpha)k^{\frac{-1}{\sigma}}((1-\alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}} \tag{4}$$

We divide both marginal productivities:

$$\frac{MPL}{MPK} = \frac{\alpha h^{\frac{-1}{\sigma}}}{(1-\alpha)k^{\frac{-1}{\sigma}}} \tag{5}$$

Now we take a log:

$$log(\frac{MPL}{MPK}) = log(\frac{\alpha}{1-\alpha}) + \frac{1}{\sigma}log(\frac{k}{h}) \tag{6}$$

To receive elasticity of substitution we need to take derivative of the above equation with respect to $log(\frac{h}{k})$. Hence we get:

$$ES = \sigma \tag{7}$$

2.2 b)

We know that labour share is defined by:

$$LS = \frac{hw}{f(k,h)} \tag{8}$$

On competitive markets wage in this case is our MPH from previous subpoint:

$$w = MPH = (1-\alpha)k^{\frac{-1}{\sigma}}((1-\alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}} \tag{9}$$

Finally we get:

$$LS = \frac{\alpha h^{\frac{\sigma-1}{\sigma}}}{((1-\alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}})^{\sigma}} \tag{10}$$

2.3 The rest TBA

This document should be updated till Friday (04/10/2019), as decided on TA session.

Contents