Quantitative Macroeconomics Homework 3

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Exercise 1

Infinite number of household maximize their utility function:

$$\begin{aligned} \underset{c_t}{\text{maximize}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t \\ \text{subject to} \quad & c_t + i_t = y_t, \\ & y_t = k_t^{1-\theta}(zh_t)^{\theta}, \\ & i_t = k_{t+1} - (1-\delta)k_t \end{aligned} \tag{1}$$

We also know that $\theta = .067$ and $h_t = 0.31$ for all t. Additionally, in point a) we have to assume that annual capital-to-output ratio is 4 and investment-to-output ratio is .25. Our role is to calculate transition paths under productivity factor shocks z.

a)

Steady state is computed by the VFI method. In the standard neoclassical growth model (such as the one discussed in this exercise) the only state variable is capital k_t. Setting capital grid $\{0, 0.05, 0.10, ..., 50\}$ and starting from $k_{init} = 25$, after 81 iterations we obtain following results, shown in Table 1:

k_{ss}	y_{ss}	i_{ss}	c_{ss}
4	1	0.25	0.75

Table 1: Steady states in the neoclassical growth model with $z = 1.63$

Indeed, we can see that the assumed boundaries for capital are slack, which means that our computations are valid. Moreover, time path of capital is presented in Figure 1.

b)

k_{ss}	y_{ss}	i_{ss}	c_{ss}
8	2	0.5	1.5

Table 2: Steady states in the neoclassical growth model with $z = 3.26$

As presented in Table 2, choosing different productivity factor z changes nominal values of the steady state, but in equal proportions for all variables (they are simply upscaled). Figure 2 shows the updated capital path.

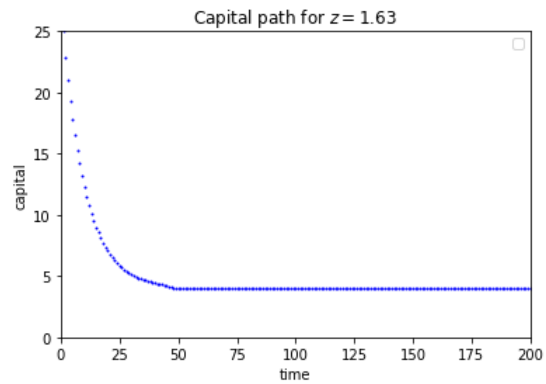


Figure 1: Neoclassical growth model capital path

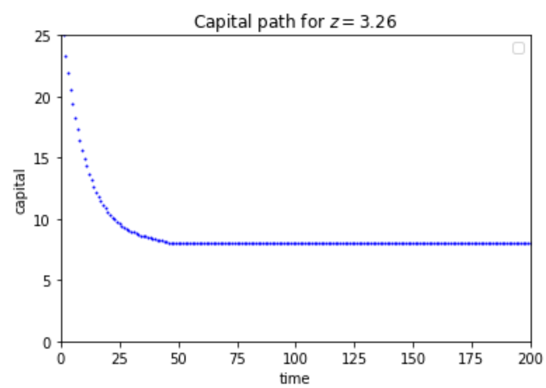


Figure 2: Neoclassical growth model capital path

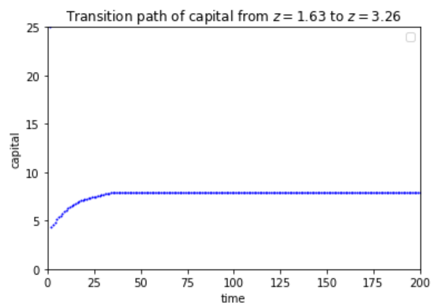
c)

Setting $k_{init} = 4$, that is the low-productivity steady state value of capital in the high-productivity environment produces convergence paths shown in Figure 2. Since labour supply is fixed, path of labour is omitted (it is flat).

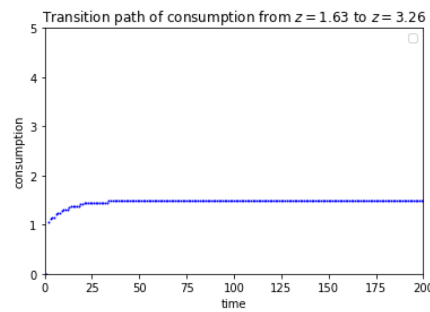
d)

Transition paths presented in Figure 4 are calculated under assumption that the initial capital level is $k_{init} = 25$. For the first 10 periods agents are allowed to optimize under productivity factor $z = 3.26$, but from 11 period on they start using decision rule $g(k_t)$[1] calculated under $z = 1.63$.

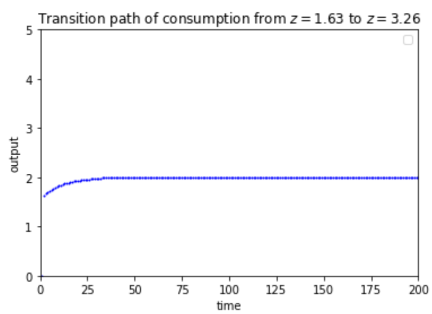
[1] Vector $g(k_t)$ stores optimal choice of k_{t+1} given k_t. More technically, it consists of indices of the arg max of the i^{th} verse of χ matrix.



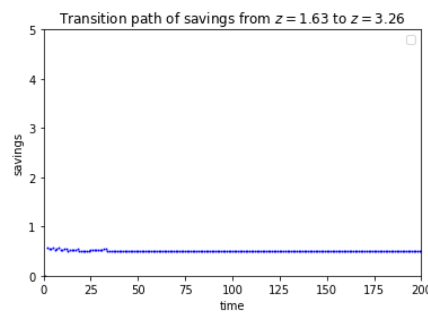
(a) Capital



(b) Consumption



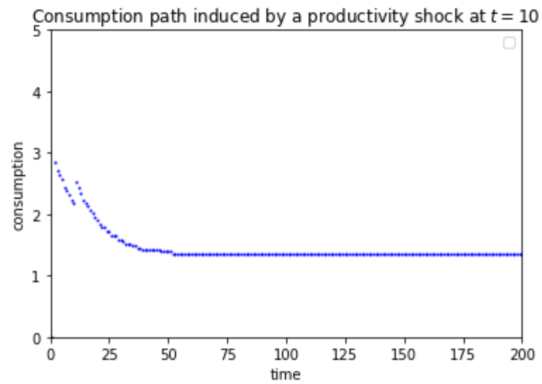
(c) Output



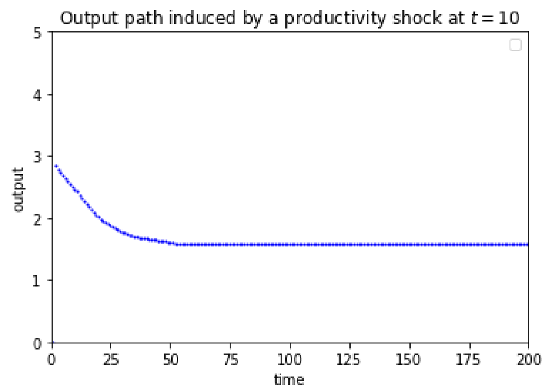
(d) Savings

Figure 3: Transition paths from $z = 1.63$ to $z = 3.26$

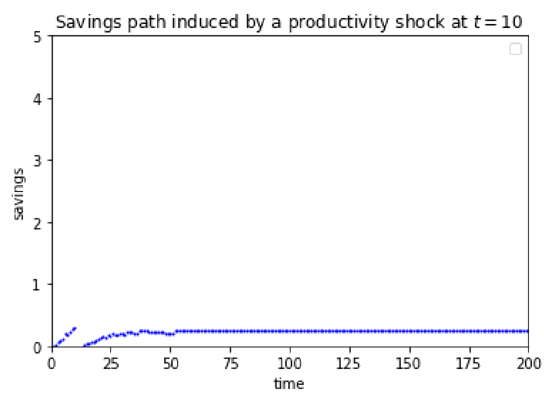
Interestingly, we can observe that control variables (called also *jumpers*) indeed do not need to be continuous and as a result of the shock they abruptly change their values. In the short run negative productivity shock increases consumption and decreases savings. It affects also slope of the output convergence curve.



(a) Consumption



(b) Output



(c) Savings

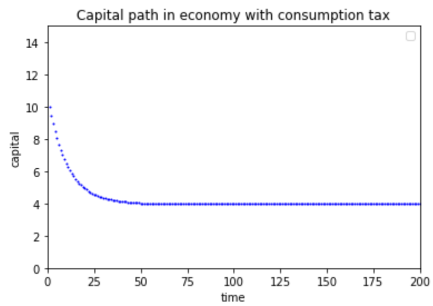
Figure 4: Transition paths under negative productivity shock at $t = 11$

Exercise 1 - bonus tasks

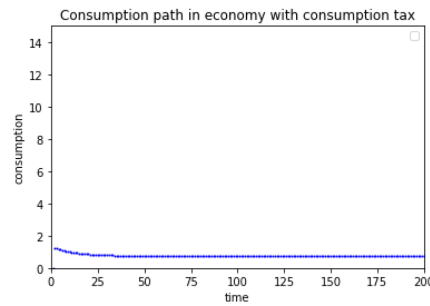
e)

Figures 5, 6, and 7 show optimal paths for capital, consumption, output, and savings in an economy with, respectively, consumption tax, capital tax, and both.

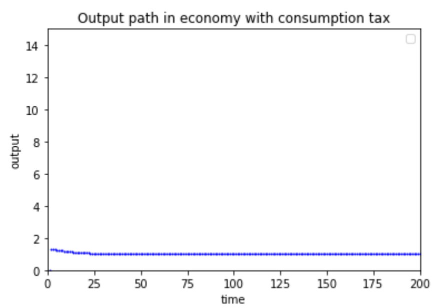
Tax rate in both cases has ben set to 20%. Initial point for capital is 10.



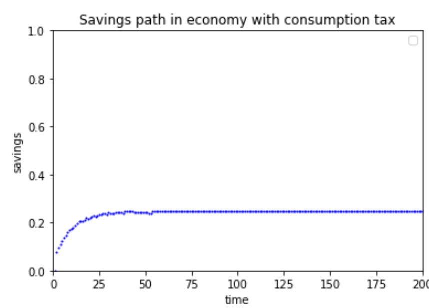
(a) Capital



(b) Consumption



(c) Output



(d) Savings

Figure 5: Transition paths in economy with consumption tax $\tau_c = 20\%$

Exercise 2

a)

Household's maximization problem:

$$\begin{aligned} \underset{c_l, k_l, h_l}{\text{maximize}} \quad & \frac{c_l^{1-\sigma}}{1-\sigma} - \kappa \frac{h_l^{1+1/v}}{1+1/v} \\ \text{subject to} \quad & c = \lambda (w_l(h_l)\eta_l)^{1-\phi_l} + r_l k_l^\eta + r_{-l}(\tilde{k}_l - k_l) \end{aligned} \tag{2}$$

In closed economy foreign investments are equal to zero: $\tilde{k}_l - k_l = 0$.
FOCs:

$$\frac{\partial(...)}{\partial c} : \frac{1}{c_l^\sigma} = \lambda_L \tag{3}$$

$$\frac{\partial(...)}{\partial h} : \kappa h_l^{1/v} = \lambda_L \lambda (w\eta)^{1-\phi_l}(1-\phi_l)h^{-\phi_l} \tag{4}$$

Finally we receive Euler equation:

$$\kappa h^{1/v} = \frac{\lambda(1-\phi)(w\eta)^{1-\phi}}{h^\eta c^\sigma} \tag{5}$$

We assume that in our economy we have two types of people : low and high productive. So we end up with 2 Euler equations,each for one type.
Firms problem:

$$\underset{K_l^d, H_l^d}{\text{maximize}} \quad Z(K_l^d)^{1-\theta}(H_l^d)^\theta - w_l H_l^d - r_l K_l^d \tag{6}$$

FOCs:

$$r = (1-\theta)Z(K^d)^{-\theta}(H^d)^\theta \tag{7}$$

$$w = \theta Z K^{1-\theta} H^{\theta-1} \tag{8}$$

Together with market clearing conditions:

- $K = k_h + k_l$
- $H = \eta_h h_h + \eta_l h_l$

Algorithm:

- Solve firms and households maximization problems.
- Plug market clearing conditions into the obtained first order conditions.
- Add budget constraints to obtain set of six equations.
- Solve the system.

Solving: Solved in Python using 'fsolve' from scipy.optimize package. Initial capital is standardized to unity for each type in each country.

Equilibrium of country A:

r	w	h_l	h_h	c_l	c_h
0.63	0.44	0.16	0.76	0.69	2.18

Table 3: Equilibrium for country A

Equilibrium of country B:

r	w	h_l	h_h	c_l	c_h
0.57	0.47	0.56	0.65	1.25	1.57

Table 4: Equilibrium for country A

0.1 b)