In[736]:=
```
ClearAll["Global`*"]

(* http://mini.pw.edu.pl/~porter/cc/psw/psw_cw1.pdf *)

(* Half of a unit sphere Spherical Coordinates *)
$density := 1;
$R := 1;

$x[r_, u_, v_] := r * Sin[v] Cos[u];
$y[r_, u_, v_] := r * Sin[v] Sin[u];
$z[r_, u_, v_] := r * Cos[v];

$Body[r_, u_, v_] := {$x[r, u, v], $y[r, u, v], $z[r, u, v]};
ParametricPlot3D[$Body[$R, u, v], {u, 0, 2 π}, {v, 0, π / 2}]
```

$$\$JacobianMatrix[r\_, u\_, v\_] := \begin{pmatrix} D[\$x[r,u,v],r] & D[\$x[r,u,v],v] & D[\$x[r,u,v],u] \\ D[\$y[r,u,v],r] & D[\$y[r,u,v],v] & D[\$y[r,u,v],u] \\ D[\$z[r,u,v],r] & D[\$z[r,u,v],v] & D[\$z[r,u,v],u] \end{pmatrix};$$

```
$JacobianDet[r_, u_, v_] := Abs[Det[$JacobianMatrix[r, u, v]]];
Print["Jacobian Matrix", MatrixForm[$JacobianMatrix[r, u, v]]]
Print["Jacobian Det", MatrixForm[$JacobianDet[r, u, v]]]
```

$$\$Integral[a\_] := \$density * \int_0^{\pi/2}\int_0^{2\pi}\int_0^{1} a\ \$JacobianDet[r,u,v]\ dr\,du\,dv;$$

```
$Mass := $Integral[1];
$CenterOfMass :=
  {$Integral[$x[r, u, v]], $Integral[$y[r, u, v]], $Integral[$z[r, u, v]]} / $Mass;

Print["Mass: ", $Mass]
Print["Center of Mass: ", MatrixForm[$CenterOfMass]]

$X := $x[r, u, v];
$Y := $y[r, u, v];
$Z := $z[r, u, v];
$I = {
   {$Integral[$Y^2 + $Z^2],
    -$Integral[$X * $Y],
    -$Integral[$X * $Z]},
   {-$Integral[$X * $Y],
    $Integral[$X^2 + $Z^2],
    -$Integral[$Y * $Z]},
```

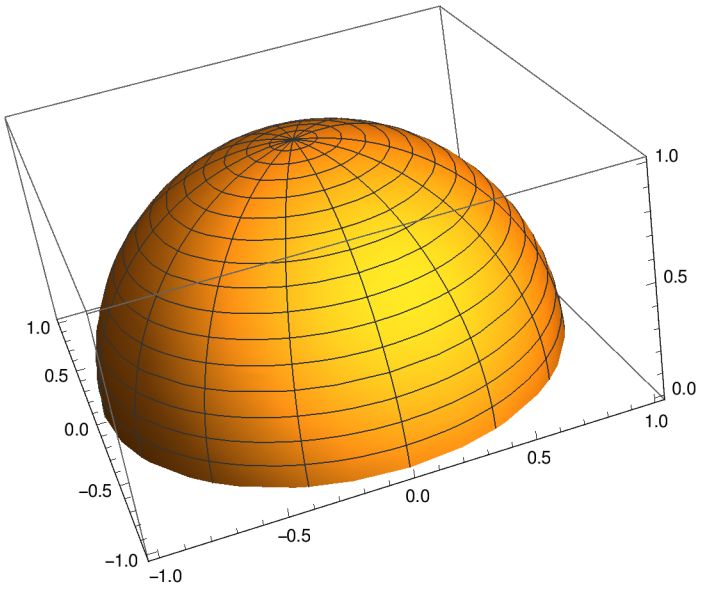
```
    {-$Integral[$X * $Z ],
     -$Integral[$Y * $Z],
     $Integral[$Y^2 + $X^2]}};

$IPointFun[x_, y_, z_, m_] :=
  m * {
    {y^2 + z^2, -x * y, -x * z},
    {-x * y, x^2 + z^2, -y * z},
    {-x * z, -y * z, x^2 + y^2}};
$IPoint =
 $IPointFun[$CenterOfMass[[1]], $CenterOfMass[[2]], $CenterOfMass[[3]], $Mass]
$ICenter = $I - $IPoint;

Print["Tensor of Intertia around 0,0,0: ", MatrixForm[$I]]
Print["Tensor of Intertia around Point ", MatrixForm[$IPoint]]
Print["Tensor of Intertia around Center (Result): ", MatrixForm[$ICenter]]

$a = 2;
Show[ContourPlot3D[{{ix, iy, iz}.$ICenter.{ix, iy, iz} == 1},
  {ix, -$a, $a}, {iy, -$a, $a}, {iz, -$a, $a}]]
```

Out[743]=



Jacobian Matrix $\begin{pmatrix} \text{Cos}[u]\,\text{Sin}[v] & r\,\text{Cos}[u]\,\text{Cos}[v] & -r\,\text{Sin}[u]\,\text{Sin}[v] \\ \text{Sin}[u]\,\text{Sin}[v] & r\,\text{Cos}[v]\,\text{Sin}[u] & r\,\text{Cos}[u]\,\text{Sin}[v] \\ \text{Cos}[v] & -r\,\text{Sin}[v] & 0 \end{pmatrix}$

Jacobian DetAbs$\Big[$

$r^2\,\text{Cos}[u]^2\,\text{Cos}[v]^2\,\text{Sin}[v] + r^2\,\text{Cos}[v]^2\,\text{Sin}[u]^2\,\text{Sin}[v] + r^2\,\text{Cos}[u]^2\,\text{Sin}[v]^3 + r^2\,\text{Sin}[u]^2\,\text{Sin}[v]^3\Big]$

Mass: $\frac{2\,\pi}{3}$

Center of Mass: $\begin{pmatrix} 0 \\ 0 \\ \frac{3}{8} \end{pmatrix}$

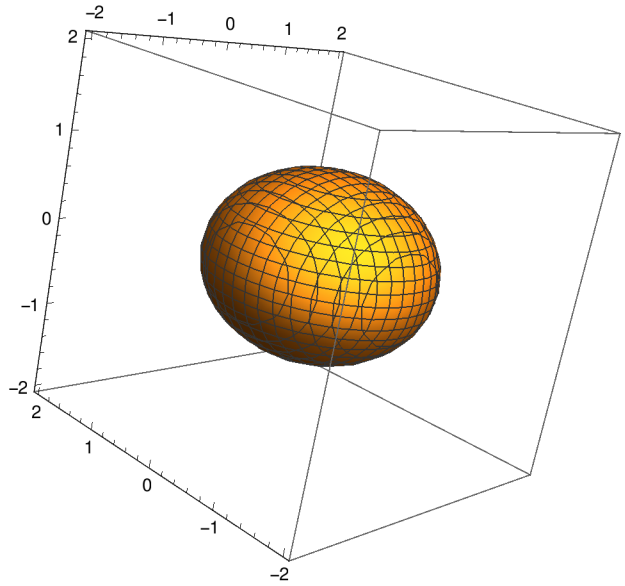
Out[758]= $\left\{\left\{\frac{3\,\pi}{32}, 0, 0\right\}, \left\{0, \frac{3\,\pi}{32}, 0\right\}, \{0, 0, 0\}\right\}$

Tensor of Intertia around 0,0,0: $\begin{pmatrix} \frac{4\,\pi}{15} & 0 & 0 \\ 0 & \frac{4\,\pi}{15} & 0 \\ 0 & 0 & \frac{4\,\pi}{15} \end{pmatrix}$

Tensor of Intertia around Point $\begin{pmatrix} \frac{3\,\pi}{32} & 0 & 0 \\ 0 & \frac{3\,\pi}{32} & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Tensor of Intertia around Center (Result): $\begin{pmatrix} \frac{83\,\pi}{480} & 0 & 0 \\ 0 & \frac{83\,\pi}{480} & 0 \\ 0 & 0 & \frac{4\,\pi}{15} \end{pmatrix}$

Out[764]=



In[765]:=