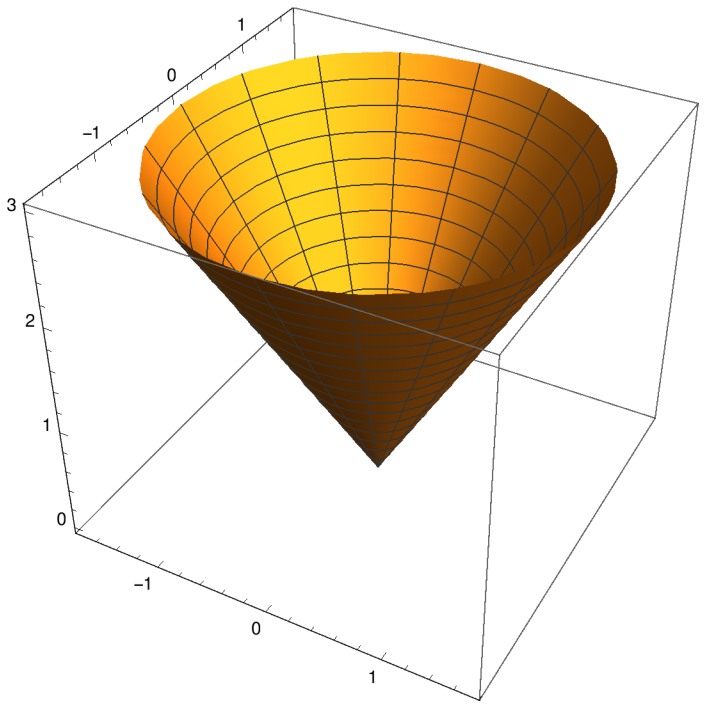
```
ClearAll["Global`*"]

(* http://mini.pw.edu.pl/~porter/cc/psw/psw_cw2.pdf *)
(* System: Two bars and a cone *)

(* ------------- Global Variables -------------- *)
$Density := 1;

(* ------------- Functions -------------- *)

$I[$Integral_, x_, y_, z_] := {
   {$Integral[y^2 + z^2],
    -$Integral[x * y],
    -$Integral[x * z]},
   {-$Integral[x * y],
    $Integral[x^2 + z^2],
    -$Integral[y * z]},
   {-$Integral[x * z ],
    -$Integral[y * z],
    $Integral[y^2 + x^2]}};

$IPointFun[x_, y_, z_, m_] :=
  m * {
    {y^2 + z^2, -x * y, -x * z},
    {-x * y, x^2 + z^2, -y * z},
    {-x * z, -y * z, x^2 + y^2}};

$PlotInertiaTensor[I_, a_] := Show[ContourPlot3D[
   {{ix, iy, iz}.I.{ix, iy, iz} == 1}, {ix, -a, a}, {iy, -a, a}, {iz, -a, a}]]

$Angle = -30 °;
$RotationY = {{Cos[$Angle], 0, Sin[$Angle]},
              {0, 1, 0},
              {-Sin[$Angle], 0, Cos[$Angle]}};

(* ---------------------------------------- *)
(* Cone *)
$ConeR = √3 ;
$ConeSlant = 2 √3 ;
$ConeH = √($ConeSlant^2 - $ConeR^2 );

$x[r_, h_, v_, u_] := (h - v)/h r * Cos[u];
$y[r_, h_, v_, u_] := (h - v)/h r * Sin[u];
```

```
$z[r_, h_, v_, u_] := h - v;
$ConeParam[r_, h_, v_, u_] := {$x[r, h, v, u], $y[r, h, v, u], $z[r, h, v, u]};

$JacobianCone[r_, h_, v_, u_] :=
  ( D[$x[r, h, v, u], r]  D[$x[r, h, v, u], v]  D[$x[r, h, v, u], u]
    D[$y[r, h, v, u], r]  D[$y[r, h, v, u], v]  D[$y[r, h, v, u], u]
    D[$z[r, h, v, u], r]  D[$z[r, h, v, u], v]  D[$z[r, h, v, u], u] );
$JacobianDetCone[r_, h_, v_, u_] := Abs[Det[$JacobianCone[r, h, v, u]]];

$ConeIntegralVariables[R_, H_, a_] :=
  $Density * ∫_0^H ∫_0^(2 π) ∫_0^R $JacobianDetCone[x, H, z, y] * a dx dy dz;
$ConeIntegral[a_] := $ConeIntegralVariables[$ConeR, $ConeH, a];

$ConeMass = $ConeIntegral[1];
$ConeCenterOfMass :=
  {$ConeIntegral[x], $ConeIntegral[y], $ConeIntegral[z]} / $ConeMass;
$ICone = $I[$ConeIntegral, x, y, z];

(* Bar Y *)
$BarYIntegral[a_] := $Density ∫_-1^1 a dy;
$BarYMass = $BarYIntegral[1];
$BarYCenterOfMass :=
  {$BarYIntegral[0], $BarYIntegral[y], $BarYIntegral[0]} / $BarYMass;
$IBarY = $I[$BarYIntegral, 0, y, 0];

(* Bar Z *)
$BarZIntegral[a_] := $Density ∫_0^1 a dz;
$BarZMass = $BarZIntegral[1];
$BarZCenterOfMass :=
  {$BarZIntegral[0], $BarZIntegral[0], $BarZIntegral[z]} / $BarZMass;
$IBarZ = $I[$BarZIntegral, 0, 0, z];

(* All *)
$MassAll = $ConeMass + $BarYMass + $BarZMass;
$CenterOfMassAll = ($ConeMass * $ConeCenterOfMass +
     $BarYMass * $BarYCenterOfMass + $BarZMass * $BarZCenterOfMass) / $MassAll;

$IAll = $ICone + $IBarY + $IBarZ;
$IAllPoint = $IPointFun[
   $CenterOfMassAll[[1]],
   $CenterOfMassAll[[2]],
   $CenterOfMassAll[[3]],
```

```
   $MassAll];
$IAllCenter = $IAll - $IAllPoint;
$IAllCenterRotated = $RotationY.$IAllCenter.Transpose[$RotationY];

(* Around A *)
$A = {0, 1, 0};
$IAPoint = $IPointFun[
   $CenterOfMassAll[[1]] - $A[[1]],
   $CenterOfMassAll[[2]] - $A[[2]],
   $CenterOfMassAll[[3]] - $A[[3]],
   $MassAll];
$IA = $IAPoint + $IAllCenter;

(* Prints *)
ParametricPlot3D[$ConeParam[$ConeR, $ConeH, v, u], {v, 0, $ConeH}, {u, 0, 2 π}]

Print["----------- Properties -----------"]
Print["Cone Mass: ", $ConeMass];

Print["Cone Center Of Mass: ", $ConeCenterOfMass];
Print["BarY Mass: ", $BarYMass];
Print["BarY Center Of Mass: ", $BarYCenterOfMass];
Print["BarZ Mass: ", $BarZMass];
Print["BarZ Center Of Mass: ", $BarZCenterOfMass];
Print["All Mass: ", $MassAll];
Print["All Center Of Mass: ", $CenterOfMassAll];

Print["----------- Inertia Tensors around (0,0,0) -----------"]
Print["Cone : " , N[MatrixForm[$ICone]]];
Print["BarY : " , N[MatrixForm[$IBarY]]];
Print["BarZ : " , N[MatrixForm[$IBarZ]]];

Print["----------- Inertia Tensor All around (0,0,0) -----------"]
Print[N[MatrixForm[$IAll]]];

Print["----------- Inertia Tensors All Point -----------"]
Print[N[MatrixForm[$IAllPoint]]];

Print["----------- Inertia Tensors All Center -----------"]
Print[N[MatrixForm[$IAllCenter]]];

Print["----------- Inertia Tensors All Center Rotated -----------"]
Print[N[MatrixForm[$IAllCenterRotated]]];
```

```
Print["----------- Inertia Tensors All Around A -----------"]
Print[N[MatrixForm[$IA]]];

(* Plots *)
$a = 1 / 2;

Print["----------- Inertia Tensors All around (0,0,0) -----------"]
$PlotInertiaTensor[$IAll, $a]
Print["----------- Inertia Tensors All Center -----------"]
$PlotInertiaTensor[$IAllCenter, $a]

Print["---------------------------------------"]
Print["----------- Final Result ------------"]
Print["---------------------------------------"]
Print[" "]
Print["----------- Inertia Tensors All Center Rotated -----------"]
$PlotInertiaTensor[$IAllCenterRotated, $a]
Print["----------- Inertia Tensors All Around A -----------"]
$PlotInertiaTensor[$IA, $a]
```



---------- Properties ----------

Cone Mass: 3π

Cone Center Of Mass: $\left\{\frac{2}{\sqrt{3}}, \pi, \frac{3}{4}\right\}$

BarY Mass: 2

BarY Center Of Mass: {0, 0, 0}

BarZ Mass: 1

BarZ Center Of Mass: $\left\{0, 0, \frac{1}{2}\right\}$

All Mass: $3 + 3\pi$

All Center Of Mass: $\left\{\frac{2\sqrt{3}\,\pi}{3+3\pi}, \frac{3\pi^2}{3+3\pi}, \frac{\frac{1}{2}+\frac{9\pi}{4}}{3+3\pi}\right\}$

---------- Inertia Tensors around (0,0,0) ----------

Cone :
$$\begin{pmatrix} 132.507 & -34.1893 & -8.1621 \\ -34.1893 & 22.6195 & -22.2066 \\ -8.1621 & -22.2066 & 138.162 \end{pmatrix}$$

BarY :
$$\begin{pmatrix} 0.666667 & 0. & 0. \\ 0. & 0. & 0. \\ 0. & 0. & 0.666667 \end{pmatrix}$$

BarZ :
$$\begin{pmatrix} 0.333333 & 0. & 0. \\ 0. & 0.333333 & 0. \\ 0. & 0. & 0. \end{pmatrix}$$

---------- Inertia Tensor All around (0,0,0) ----------

$$\begin{pmatrix} 133.507 & -34.1893 & -8.1621 \\ -34.1893 & 22.9528 & -22.2066 \\ -8.1621 & -22.2066 & 138.829 \end{pmatrix}$$

---------- Inertia Tensors All Point ----------

$$\begin{pmatrix} 75.1696 & -25.9342 & -6.62928 \\ -25.9342 & 14.1426 & -18.0363 \\ -6.62928 & -18.0363 & 80.0913 \end{pmatrix}$$

---------- Inertia Tensors All Center ----------

$$\begin{pmatrix} 58.3378 & -8.25511 & -1.53282 \\ -8.25511 & 8.8102 & -4.17033 \\ -1.53282 & -4.17033 & 58.7376 \end{pmatrix}$$

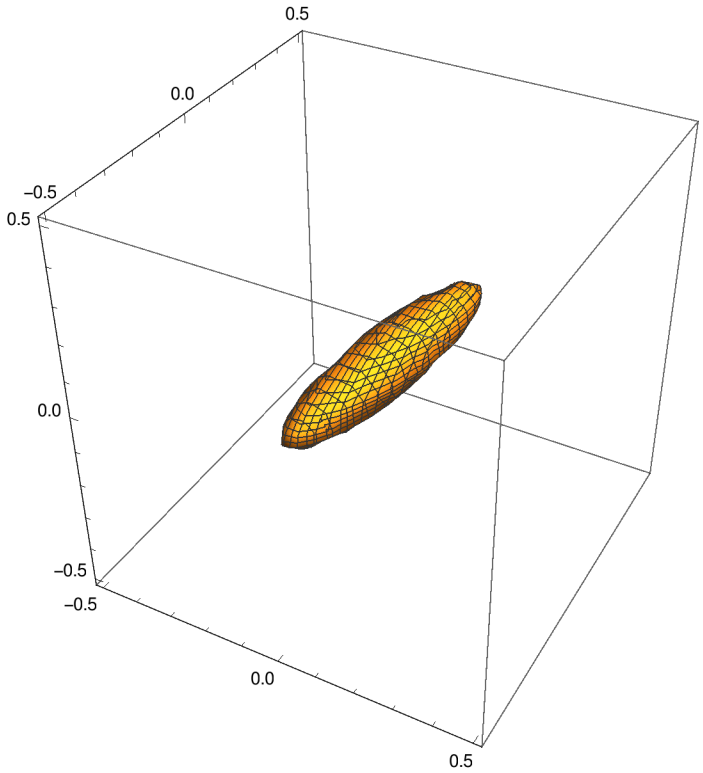
---------- Inertia Tensors All Center Rotated ----------

$$\begin{pmatrix} 59.7652 & -5.06397 & -0.939514 \\ -5.06397 & 8.8102 & -7.73917 \\ -0.939514 & -7.73917 & 57.3102 \end{pmatrix}$$

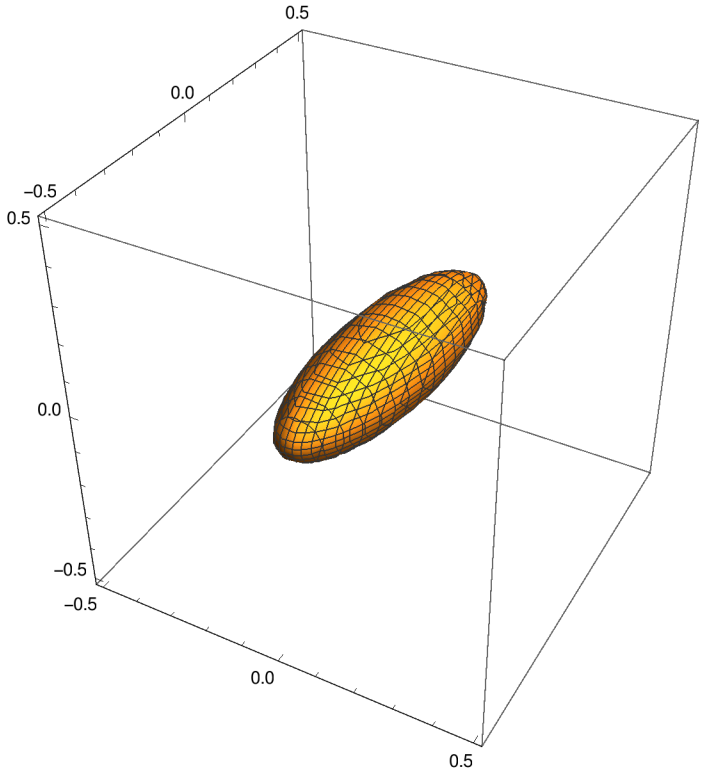
---------- Inertia Tensors All Around A ----------

$$\begin{pmatrix} 86.7146 & -23.3065 & -8.1621 \\ -23.3065 & 22.9528 & -14.638 \\ -8.1621 & -14.638 & 92.0361 \end{pmatrix}$$

---------- Inertia Tensors All around (0,0,0) ----------

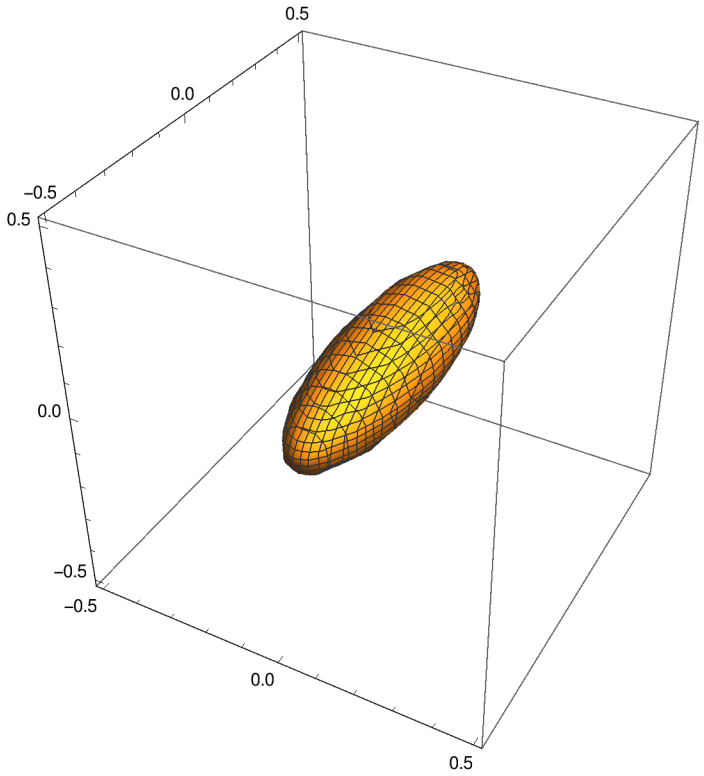


---------- Inertia Tensors All Center ----------

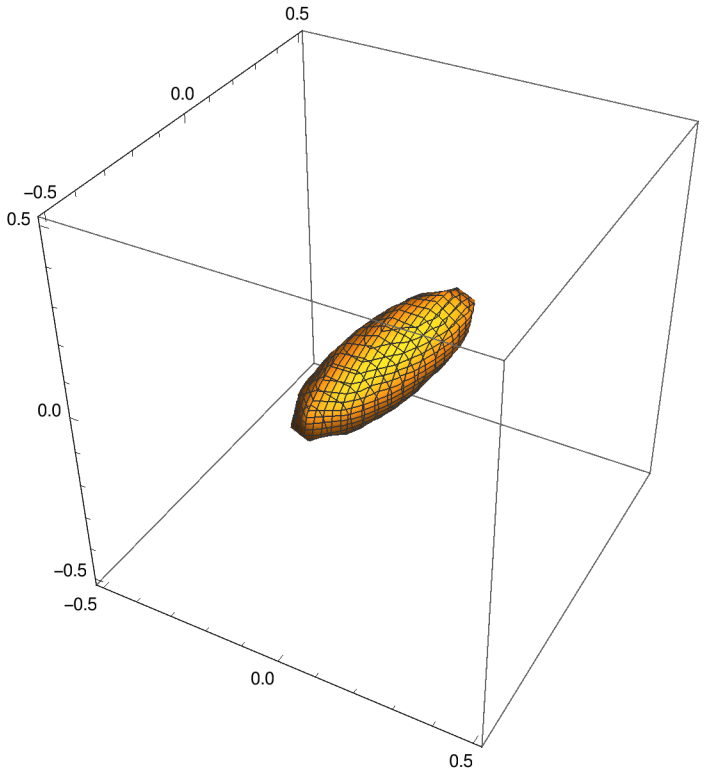


---------- Final Result ----------

---------- Inertia Tensors All Center Rotated ----------



---------- Inertia Tensors All Around A ----------


0.5
0.0
−0.5
0.5
0.0
−0.5
−0.5
0.0
0.5