

In[1997]:=

```
In[1998]:= Clear["Global`*"];
(* Q - parameters Q = (q1 == alpha, q2 == beta) *)
(* Configuration *)
X := {
  l * Cos[q1[t]], (* x1 *)
  l * Sin[q1[t]], (* y1 *)
  2 * l * Cos[q1[t]], (* x2 *)
  0, (* y2 *)
  (2 * l * Cos[q1[t]]) + (2 * l * Cos[q2[t]]), (* x3 == x2 + ... *)
  0 + (2 * l * Sin[q2[t]]) (* y3 == y2 + .... *)
};
```

```
dx := Simplify[D[X, t]];
```

```
m := 
$$\begin{pmatrix} m1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m1 & 0 & 0 & 0 & 0 \\ 0 & 0 & m2 & 0 & 0 & 0 \\ 0 & 0 & 0 & m2 & 0 & 0 \\ 0 & 0 & 0 & 0 & m3 & 0 \\ 0 & 0 & 0 & 0 & 0 & m3 \end{pmatrix};$$

```

```
G := {
  0,
  -g,
  0,
  -g,
  0,
  -g
};
(* Trig→False disabled trig. simplifications *)
T := Simplify[(m.dx.dx) / 2, Trig→False];
U := m.G.X;
```

```
(* Lagrange *)
L = T - U;
dtdq1 = D[D[L, q1'[t]], t];
dq1 = D[L, q1[t]];
```

```
dtdq2 = D[D[L, q2'[t]], t];
dq2 = D[L, q2[t]];
```

```
LangrageDiff1 = Simplify[dtdq1 - dq1];
LangrageDiff2 = Simplify[dtdq2 - dq2];
```

```
r1 = LangrageDiff1;
r2 = LangrageDiff2;
```

```

(* Hamilton *)
H = T + U;

p1 = Simplify[D[L, q1'[t]]];
p2 = Simplify[D[L, q2'[t]]];

dp1 = -Simplify[D[H, q1[t]]];
dp2 = -Simplify[D[H, q2[t]]];

(* H = pq' - L *)
dq1 = (H - L) / p1;
dq2 = (H - L) / p2;

(* Prints *)

Print["X"]
MatrixForm[X]

Print["dX"]
MatrixForm[dX]

Print["m"]
MatrixForm[m]

Print["T"]
MatrixForm[T]

Print["U"]
MatrixForm[U]

Print["L"]
MatrixForm[L]

Print["dtdq1"]
MatrixForm[dtdq1]
Print["dtdq2"]
MatrixForm[dtdq2]

Print["dq1"]
MatrixForm[dq1]
Print["dq2"]
MatrixForm[dq2]

```

```
Print["LangrageDiff1"]
MatrixForm[LangrageDiff1]
```

```
Print["LangrageDiff2"]
MatrixForm[LangrageDiff2]
```

```
Print["H"]
MatrixForm[H]
```

```
Print["p1"]
MatrixForm[p1]
Print["p2"]
MatrixForm[p2]
```

```
Print["dp1"]
MatrixForm[dp1]
Print["dp2"]
MatrixForm[dp2]
```

```
Print["dq1"]
MatrixForm[dq1]
Print["dq2"]
MatrixForm[dq2]
```

```
(* Example *)
```

```
m1 = 1;
m2 = 1;
m3 = 1;
g = 9.81;
l = 1;
T = 100;
```

```
sol = NDSolve[{
  r1 == 0, r2 == 0,
  q1[0] == Pi/2, q1'[0] == 3,
  q2[0] == Pi/2, q2'[0] == 0
},
  {q1, q2}, {t, 0, T}];
```

```
q1 = First[q1 /. sol];
q2 = First[q2 /. sol];
```

```
Plot[q1[t], {t, 0, T}]
Plot[q2[t], {t, 0, T}]
```

```

Animate[Graphics3D[{Line[{{-1.5, 0, 0}, {1.5, 0, 0}}], Line[{{0, -1.5, 0}, {0, 1.5, 0}}],
  Line[{{0, 0, -1.5}, {0, 0, 1.5}}], Arrow[{0, 0, 0}, {
    (2 * l * Cos[q1[t]]) + (2 * l * Cos[q2[t]]),
    0 + (2 * l * Sin[q2[t]]),
    0}
  ]}], {t, 0, T}]

```

X

Out[2022]//MatrixForm=

$$\begin{pmatrix} l \cos[q_1[t]] \\ l \sin[q_1[t]] \\ 2 l \cos[q_1[t]] \\ 0 \\ 2 l \cos[q_1[t]] + 2 l \cos[q_2[t]] \\ 2 l \sin[q_2[t]] \end{pmatrix}$$

dX

Out[2024]//MatrixForm=

$$\begin{pmatrix} -l \sin[q_1[t]] q_1'[t] \\ l \cos[q_1[t]] q_1'[t] \\ -2 l \sin[q_1[t]] q_1'[t] \\ 0 \\ -2 l (\sin[q_1[t]] q_1'[t] + \sin[q_2[t]] q_2'[t]) \\ 2 l \cos[q_2[t]] q_2'[t] \end{pmatrix}$$

m

Out[2026]//MatrixForm=

$$\begin{pmatrix} m1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m1 & 0 & 0 & 0 & 0 \\ 0 & 0 & m2 & 0 & 0 & 0 \\ 0 & 0 & 0 & m2 & 0 & 0 \\ 0 & 0 & 0 & 0 & m3 & 0 \\ 0 & 0 & 0 & 0 & 0 & m3 \end{pmatrix}$$

T

Out[2028]//MatrixForm=

$$\frac{1}{2} l^2 \left((m1 \cos[q_1[t]]^2 + (m1 + 4 (m2 + m3)) \sin[q_1[t]]^2) q_1'[t]^2 + 8 m3 \sin[q_1[t]] \sin[q_2[t]] q_1'[t] q_2'[t] + 4 m3 (\cos[q_2[t]]^2 + \sin[q_2[t]]^2) q_2'[t]^2 \right)$$

U

Out[2030]//MatrixForm=

$$-g l m1 \sin[q_1[t]] - 2 g l m3 \sin[q_2[t]]$$

L

Out[2032]//MatrixForm=

$$g l m1 \sin[q_1[t]] + 2 g l m3 \sin[q_2[t]] + \frac{1}{2} l^2 \left((m1 \cos[q_1[t]]^2 + (m1 + 4 (m2 + m3)) \sin[q_1[t]]^2) q_1'[t]^2 + 8 m3 \sin[q_1[t]] \sin[q_2[t]] q_1'[t] q_2'[t] + 4 m3 (\cos[q_2[t]]^2 + \sin[q_2[t]]^2) q_2'[t]^2 \right)$$

dtdq1

Out[2034]//MatrixForm=

$$\frac{1}{2} l^2 \left(2 q_1' [t] \right. \\ \left. (-2 m_1 \cos[q_1[t]] \sin[q_1[t]] q_1' [t] + 2 (m_1 + 4 (m_2 + m_3)) \cos[q_1[t]] \sin[q_1[t]] q_1' [t]) + \right. \\ \left. 8 m_3 \cos[q_1[t]] \sin[q_2[t]] q_1' [t] q_2' [t] + 8 m_3 \cos[q_2[t]] \sin[q_1[t]] q_2' [t]^2 + \right. \\ \left. 2 (m_1 \cos[q_1[t]]^2 + (m_1 + 4 (m_2 + m_3)) \sin[q_1[t]]^2) q_1'' [t] + \right. \\ \left. 8 m_3 \sin[q_1[t]] \sin[q_2[t]] q_2'' [t] \right)$$

dtdq2

Out[2036]//MatrixForm=

$$\frac{1}{2} l^2 \left(8 m_3 \cos[q_1[t]] \sin[q_2[t]] q_1' [t]^2 + 8 m_3 \cos[q_2[t]] \sin[q_1[t]] q_1' [t] q_2' [t] + \right. \\ \left. 8 m_3 \sin[q_1[t]] \sin[q_2[t]] q_1'' [t] + 8 m_3 (\cos[q_2[t]]^2 + \sin[q_2[t]]^2) q_2'' [t] \right)$$

dq1

Out[2038]//MatrixForm=

$$\frac{2 (-2 g l m_1 \sin[q_1[t]] - 4 g l m_3 \sin[q_2[t]])}{l^2 (2 (m_1 + 2 (m_2 + m_3) - 2 (m_2 + m_3) \cos[2 q_1[t]]) q_1' [t] + 8 m_3 \sin[q_1[t]] \sin[q_2[t]] q_2' [t])}$$

dq2

Out[2040]//MatrixForm=

$$\frac{-2 g l m_1 \sin[q_1[t]] - 4 g l m_3 \sin[q_2[t]]}{4 l^2 m_3 (\sin[q_1[t]] \sin[q_2[t]] q_1' [t] + q_2' [t])}$$

LangrageDiff1

Out[2042]//MatrixForm=

$$l \left(-g m_1 \cos[q_1[t]] + 2 l (m_2 + m_3) \sin[2 q_1[t]] q_1' [t]^2 + 4 l m_3 \cos[q_2[t]] \sin[q_1[t]] q_2' [t]^2 + \right. \\ \left. l m_1 \cos[q_1[t]]^2 q_1'' [t] + l m_1 \sin[q_1[t]]^2 q_1'' [t] + 4 l m_2 \sin[q_1[t]]^2 q_1'' [t] + \right. \\ \left. 4 l m_3 \sin[q_1[t]]^2 q_1'' [t] + 4 l m_3 \sin[q_1[t]] \sin[q_2[t]] q_2'' [t] \right)$$

LangrageDiff2

Out[2044]//MatrixForm=

$$2 l m_3 \left(-g \cos[q_2[t]] + 2 l \cos[q_1[t]] \sin[q_2[t]] q_1' [t]^2 + \right. \\ \left. 2 l \sin[q_1[t]] \sin[q_2[t]] q_1'' [t] + 2 l q_2'' [t] \right)$$

H

Out[2046]//MatrixForm=

$$-g l m_1 \sin[q_1[t]] - 2 g l m_3 \sin[q_2[t]] + \\ \frac{1}{2} l^2 \left((m_1 \cos[q_1[t]]^2 + (m_1 + 4 (m_2 + m_3)) \sin[q_1[t]]^2) q_1' [t]^2 + \right. \\ \left. 8 m_3 \sin[q_1[t]] \sin[q_2[t]] q_1' [t] q_2' [t] + 4 m_3 (\cos[q_2[t]]^2 + \sin[q_2[t]]^2) q_2' [t]^2 \right)$$

p1

Out[2048]//MatrixForm=

$$\frac{1}{2} l^2 \left(2 (m_1 + 2 (m_2 + m_3) - 2 (m_2 + m_3) \cos[2 q_1[t]]) q_1' [t] + 8 m_3 \sin[q_1[t]] \sin[q_2[t]] q_2' [t] \right)$$

p2

Out[2050]//MatrixForm=

$$4 l^2 m_3 (\sin[q_1[t]] \sin[q_2[t]] q_1' [t] + q_2' [t])$$

dp1

Out[2052]//MatrixForm=

$$-l \cos[q_1[t]] \left(-g m_1 + 4 l (m_2 + m_3) \sin[q_1[t]] q_1' [t]^2 + 4 l m_3 \sin[q_2[t]] q_1' [t] q_2' [t] \right)$$

dp2

Out[2054]//MatrixForm=

$$-2 \, l \, m_3 \cos[q_2[t]] \left(-g + 2 \, l \sin[q_1[t]] \, q_1'[t] \, q_2'[t] \right)$$

dq1

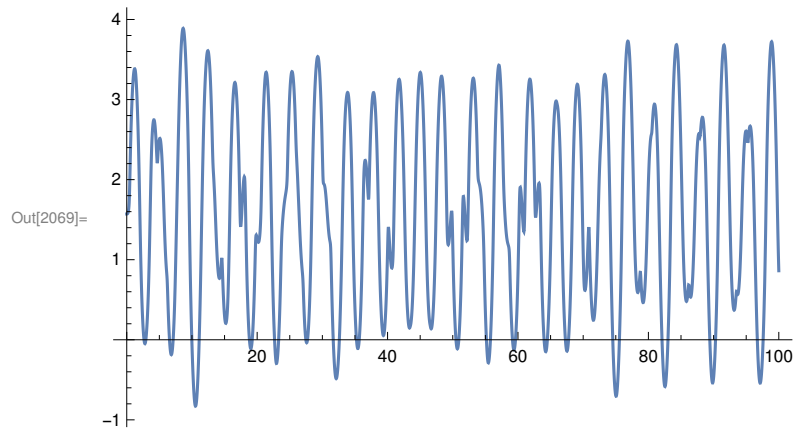
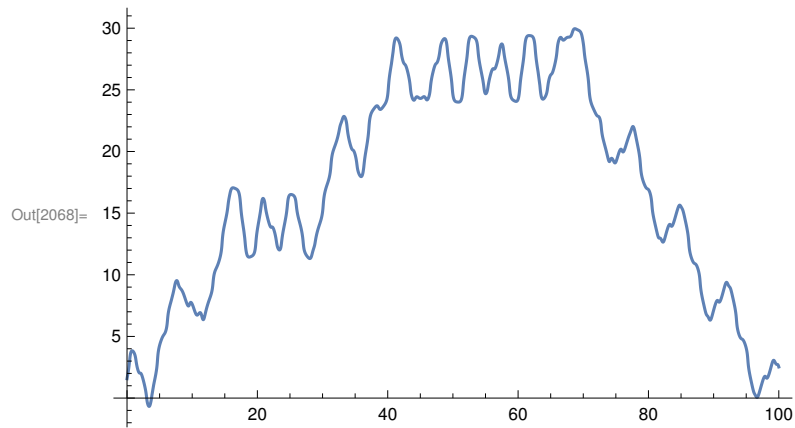
Out[2056]//MatrixForm=

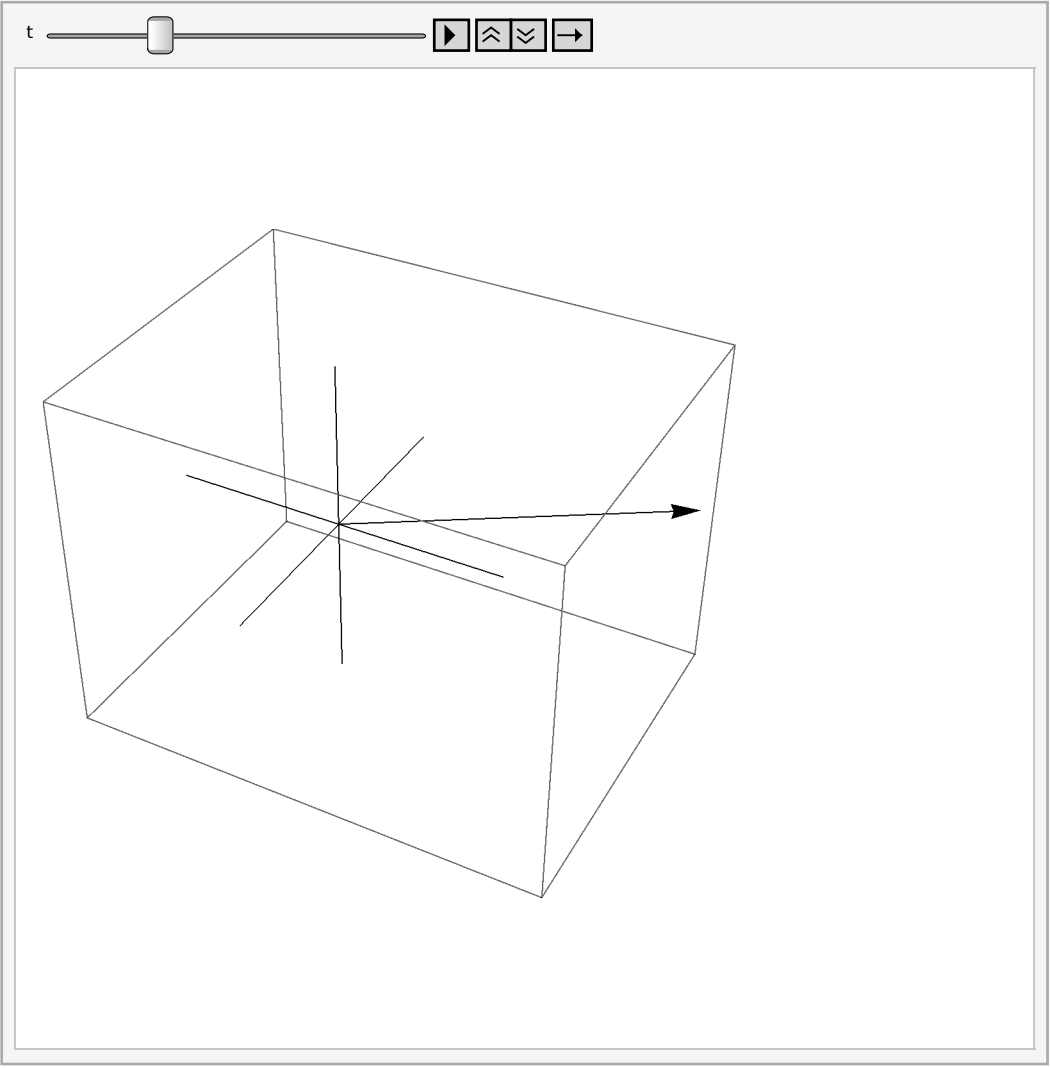
$$\frac{\left(2 \left(-2 \, g \, l \, m_1 \sin[q_1[t]] - 4 \, g \, l \, m_3 \sin[q_2[t]] \right) \right)}{\left(l^2 \left(2 \left(m_1 + 2 \left(m_2 + m_3 \right) - 2 \left(m_2 + m_3 \right) \cos[2 \, q_1[t]] \right) q_1'[t] + 8 \, m_3 \sin[q_1[t]] \sin[q_2[t]] \, q_2'[t] \right) \right)}$$

dq2

Out[2058]//MatrixForm=

$$\frac{-2 \, g \, l \, m_1 \sin[q_1[t]] - 4 \, g \, l \, m_3 \sin[q_2[t]]}{4 \, l^2 \, m_3 \left(\sin[q_1[t]] \sin[q_2[t]] \, q_1'[t] + q_2'[t] \right)}$$





Out[2070]=

In[2071]:=

In[2072]:=