In[1293]:=
```
ClearAll["Global`*"]

(* http://mini.pw.edu.pl/~porter/cc/psw/psw_cw2.pdf *)
(* System: Two bars and a cone *)

(* ------------- Global Variables -------------- *)
$Density := 1;

(* ------------- Functions -------------- *)

$I[$Integral_, x_, y_, z_] := {
   {$Integral[y^2 + z^2],
    -$Integral[x * y],
    -$Integral[x * z]},
   {-$Integral[x * y],
    $Integral[x^2 + z^2],
    -$Integral[y * z]},
   {-$Integral[x * z ],
    -$Integral[y * z],
    $Integral[y^2 + x^2]}};

$IPointFun[x_, y_, z_, m_] :=
  m * {
    {y^2 + z^2, -x * y, -x * z},
    {-x * y, x^2 + z^2, -y * z},
    {-x * z, -y * z, x^2 + y^2}};

$PlotInertiaTensor[I_, a_] := Show[ContourPlot3D[
   {{ix, iy, iz}.I.{ix, iy, iz} == 1}, {ix, -a, a}, {iy, -a, a}, {iz, -a, a}]]

$Angle = -30 °;
$RotationY = ( Cos[$Angle]   0  Sin[$Angle]
                   0         1       0
               -Sin[$Angle]  0  Cos[$Angle] );

(* ---------------------------------------- *)
(* Cone *)
$ConeR = √3 ;
$ConeSlant = 2 √3 ;
$ConeH = √($ConeSlant^2 - $ConeR^2 ) ;

(*r→x, v→z, u→y*)
$xCone[r_, h_, v_, u_] := (h - v)/h r * Cos[u];
```

```
$yCone[r_, h_, v_, u_] := (h - v)/h r * Sin[u];
$zCone[r_, h_, v_, u_] := h - v;
$ConeParam[r_, h_, v_, u_] :=
  {$xCone[r, h, v, u], $yCone[r, h, v, u], $zCone[r, h, v, u]};

$JacobianCone[r_, h_, v_, u_] :=
  ( D[$xCone[r, h, v, u], r]  D[$xCone[r, h, v, u], v]  D[$xCone[r, h, v, u], u]
    D[$yCone[r, h, v, u], r]  D[$yCone[r, h, v, u], v]  D[$yCone[r, h, v, u], u]
    D[$zCone[r, h, v, u], r]  D[$zCone[r, h, v, u], v]  D[$zCone[r, h, v, u], u] );
$JacobianDetCone[r_, h_, v_, u_] := Abs[Det[$JacobianCone[r, h, v, u]]];

$ConeIntegralVariables[R_, H_, a_] :=
  $Density * ∫_1^(H+1) ∫_0^(2 π) ∫_0^R $JacobianDetCone[x, H, z, y] * a dx dy dz;
$ConeIntegral[a_] := $ConeIntegralVariables[$ConeR, $ConeH, a];

$ConeMass = $ConeIntegral[1];
$ConeCenterOfMass := {
    $ConeIntegral[$xCone[x, $ConeH, z, y]],
    $ConeIntegral[$yCone[x, $ConeH, z, y]],
    $ConeIntegral[$zCone[x, $ConeH, z, y]]} / $ConeMass;
$ICone = $I[$ConeIntegral,
   $xCone[x, $ConeH, z, y],
   $yCone[y, $ConeH, z, y],
   $zCone[x, $ConeH, z, y]];

(* Bar Y *)
$BarYIntegral[a_] := $Density ∫_-1^1 a dy;
$BarYMass = $BarYIntegral[1];
$BarYCenterOfMass :=
  {$BarYIntegral[0], $BarYIntegral[y], $BarYIntegral[0]} / $BarYMass;
$IBarY = $I[$BarYIntegral, 0, y, 0];

(* Bar Z *)
$BarZIntegral[a_] := $Density ∫_0^1 a dz;
$BarZMass = $BarZIntegral[1];
$BarZCenterOfMass :=
  {$BarZIntegral[0], $BarZIntegral[0], $BarZIntegral[z]} / $BarZMass;
$IBarZ = $I[$BarZIntegral, 0, 0, z];

(* All *)
$MassAll = $ConeMass + $BarYMass + $BarZMass;
```

```
$CenterOfMassAll = ($ConeMass * $ConeCenterOfMass +
      $BarYMass * $BarYCenterOfMass + $BarZMass * $BarZCenterOfMass) / $MassAll;

$IAll = $ICone + $IBarY  + $IBarZ;
$IAllPoint = $IPointFun[
   $CenterOfMassAll[[1]],
   $CenterOfMassAll[[2]],
   $CenterOfMassAll[[3]],
   $MassAll];
$IAllCenter = $IAll - $IAllPoint;
$IAllCenterRotated = $RotationY.$IAllCenter.Transpose[$RotationY];

(* Around A *)
$A = {0, 1, 0};
$IAPoint = $IPointFun[
   $CenterOfMassAll[[1]] - $A[[1]],
   $CenterOfMassAll[[2]] - $A[[2]],
   $CenterOfMassAll[[3]] - $A[[3]],
   $MassAll];
$IA = $IAPoint + $IAllCenter;

(* Prints *)
ParametricPlot3D[$ConeParam[$ConeR, $ConeH, v, u], {v, 0, $ConeH}, {u, 0, 2 π}]

Print["----------- Properties -----------"]
Print["Cone Mass: ", $ConeMass];

Print["Cone Center Of Mass: ", $ConeCenterOfMass];
Print["BarY Mass: ", $BarYMass];
Print["BarY Center Of Mass: ", $BarYCenterOfMass];
Print["BarZ Mass: ", $BarZMass];
Print["BarZ Center Of Mass: ", $BarZCenterOfMass];
Print["All Mass: ", $MassAll];
Print["All Center Of Mass: ", $CenterOfMassAll];

Print["----------- Inertia Tensors around (0,0,0) -----------"]
Print["Cone : " , N[MatrixForm[$ICone]]];
Print["BarY : " , N[MatrixForm[$IBarY]]];
Print["BarZ : " , N[MatrixForm[$IBarZ]]];

Print["----------- Inertia Tensor All around (0,0,0) -----------"]
Print[N[MatrixForm[$IAll]]];

Print["----------- Inertia Tensors All Point -----------"]
```

```
Print[N[MatrixForm[$IAllPoint]]];

Print["----------- Inertia Tensors All Center -----------"]
Print[N[MatrixForm[$IAllCenter]]];

Print["----------- Inertia Tensors All Center Rotated -----------"]
Print[N[MatrixForm[$IAllCenterRotated]]];

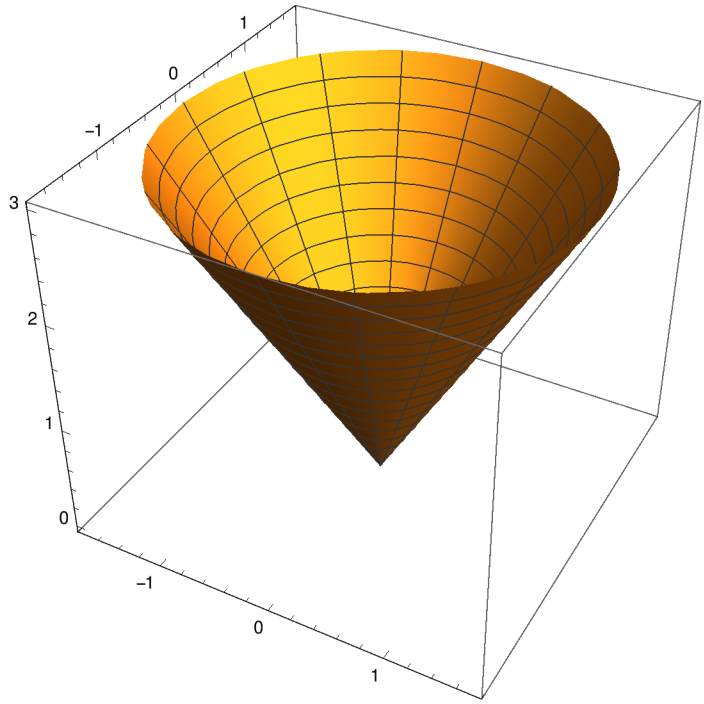
Print["----------- Inertia Tensors All Around A -----------"]
Print[N[MatrixForm[$IA]]];

(* Plots *)
$a = 1 / 2 + 1 / 10;

Print["----------- Inertia Tensors All around (0,0,0) -----------"]
$PlotInertiaTensor[$IAll, $a]
Print["----------- Inertia Tensors All Center -----------"]
$PlotInertiaTensor[$IAllCenter, $a]

Print["-------------------------------------"]
Print["----------- Final Result ------------"]
Print["-------------------------------------"]
Print[" "]
Print["----------- Inertia Tensors All Center Rotated -----------"]
$PlotInertiaTensor[$IAllCenterRotated, $a]
Print["----------- Inertia Tensors All Around A -----------"]
$PlotInertiaTensor[$IA, $a]
```

Out[1331]=



---------- Properties ----------

Cone Mass: π

Cone Center Of Mass: $\left\{0, 0, \frac{5}{4}\right\}$

BarY Mass: 2

BarY Center Of Mass: {0, 0, 0}

BarZ Mass: 1

BarZ Center Of Mass: $\left\{0, 0, \frac{1}{2}\right\}$

All Mass: $3 + \pi$

All Center Of Mass: $\left\{0, 0, \frac{\frac{1}{2} + \frac{5\pi}{4}}{3 + \pi}\right\}$

---------- Inertia Tensors around (0,0,0) ----------

Cone : $\begin{pmatrix} 11.7724 & 0.221687 & 0. \\ 0.221687 & 7.48746 & 2.30383 \\ 0. & 2.30383 & 5.43685 \end{pmatrix}$

BarY : $\begin{pmatrix} 0.666667 & 0. & 0. \\ 0. & 0. & 0. \\ 0. & 0. & 0.666667 \end{pmatrix}$

BarZ : $\begin{pmatrix} 0.333333 & 0. & 0. \\ 0. & 0.333333 & 0. \\ 0. & 0. & 0. \end{pmatrix}$

---------- Inertia Tensor All around (0,0,0) ----------

$\begin{pmatrix} 12.7724 & 0.221687 & 0. \\ 0.221687 & 7.8208 & 2.30383 \\ 0. & 2.30383 & 6.10351 \end{pmatrix}$

---------- Inertia Tensors All Point ----------

$\begin{pmatrix} 3.19107 & 0. & 0. \\ 0. & 3.19107 & 0. \\ 0. & 0. & 0. \end{pmatrix}$

---------- Inertia Tensors All Center ----------

$\begin{pmatrix} 9.58132 & 0.221687 & 0. \\ 0.221687 & 4.62973 & 2.30383 \\ 0. & 2.30383 & 6.10351 \end{pmatrix}$

---------- Inertia Tensors All Center Rotated ----------

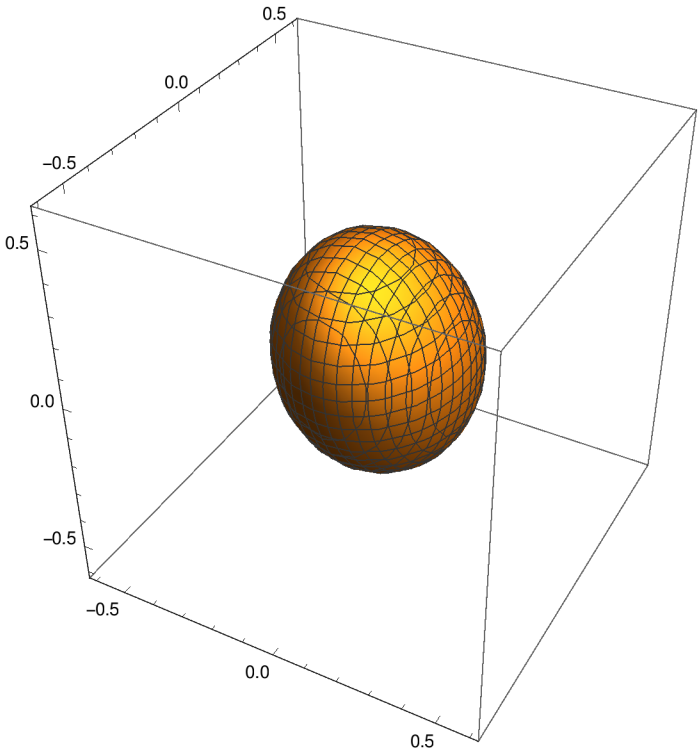
$\begin{pmatrix} 8.71187 & -0.959931 & 1.50594 \\ -0.959931 & 4.62973 & 2.10602 \\ 1.50594 & 2.10602 & 6.97297 \end{pmatrix}$

---------- Inertia Tensors All Around A ----------

$\begin{pmatrix} 18.914 & 0.221687 & 0. \\ 0.221687 & 7.8208 & 6.73083 \\ 0. & 6.73083 & 12.2451 \end{pmatrix}$

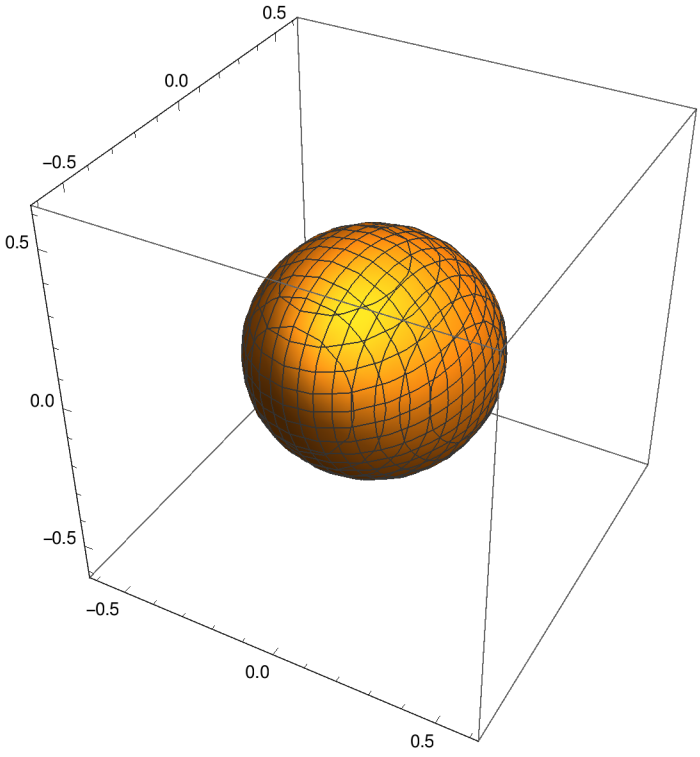
---------- Inertia Tensors All around (0,0,0) ----------

Out[1357]=



---------- Inertia Tensors All Center ----------

Out[1359]=



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----------- Final Result ------------

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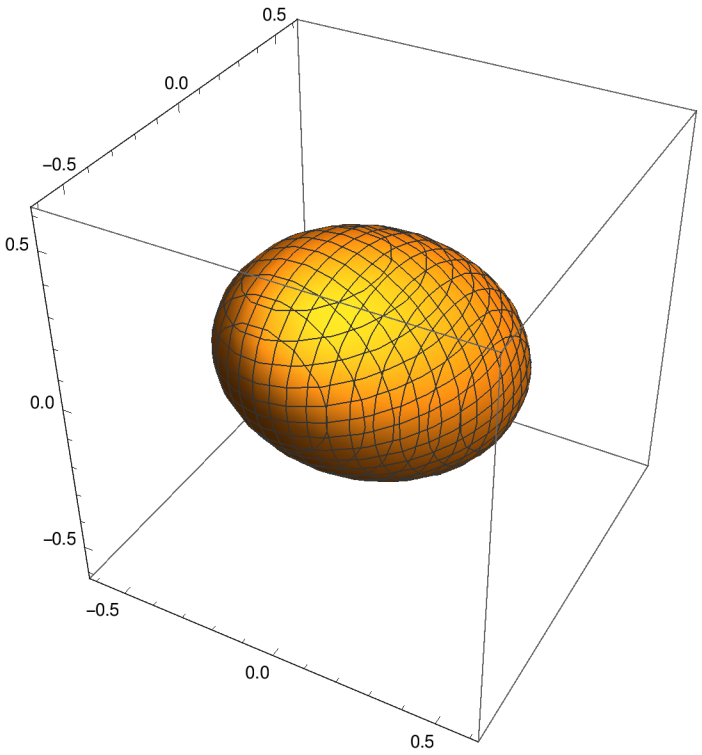
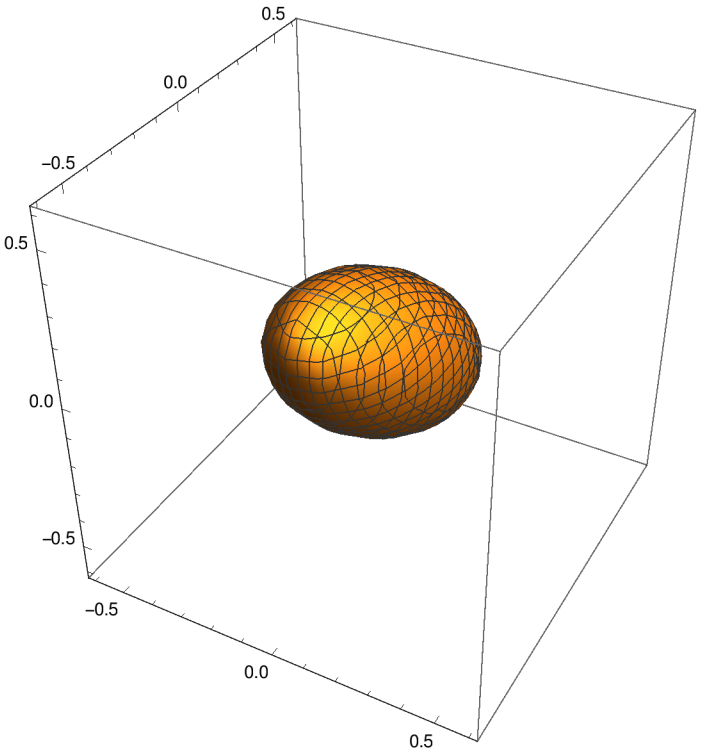
----------- Inertia Tensors All Center Rotated -----------
```

Out[1365]=




```
----------- Inertia Tensors All Around A -----------
```

Out[1367]=



In[1368]:=