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In[774]:= ClearAll["Global`*"]

(* http://mini.pw.edu.pl/~porter/cc/psw/psw_cw2.pdf *)
(* System: Two bars and a cone *)

(* ----- Global Variables ----- *)
$Density := 1;

(* ----- Functions ----- *)

$I[$Integral_, x_, y_, z_] := {
  {$Integral[y^2 + z^2],
   -$Integral[x*y],
   -$Integral[x*z]},
  {-$Integral[x*y],
   $Integral[x^2 + z^2],
   -$Integral[y*z]},
  {-$Integral[x*z],
   -$Integral[y*z],
   $Integral[y^2 + x^2]}};

$IPointFun[x_, y_, z_, m_] :=
m*{
{y^2 + z^2, -x*y, -x*z},
{-x*y, x^2 + z^2, -y*z},
{-x*z, -y*z, x^2 + y^2}};

$PlotInertiaTensor[I_, a_] := Show[ContourPlot3D[
{{ix, iy, iz}.I.{ix, iy, iz} == 1}, {ix, -a, a}, {iy, -a, a}, {iz, -a, a}]]

$Angle = -30 °;
$RotationY = {{Cos[$Angle], 0, Sin[$Angle]}, {0, 1, 0}, {-Sin[$Angle], 0, Cos[$Angle]}};

(* ----- *)
(* Cone *)
$ConeR =  $\sqrt{3}$ ;
$ConeSlant =  $2\sqrt{3}$ ;
$ConeH =  $\sqrt{\text{ConeSlant}^2 - \text{ConeR}^2}$ ;

$xCone[r_, θ_, z_] := r * Cos[θ];
$yCone[r_, θ_, z_] := r * Sin[θ];
$zCone[r_, θ_, z_] := z;
$ConeParam[r_, θ_, z_] := {$xCone[r, θ, z], $yCone[r, θ, z], $zCone[r, θ, z]};

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$JacobianCone[r_, θ_, z_] :=
  
$$\left( \begin{array}{l} D[\$xCone[r, \theta, z], r] D[\$xCone[r, \theta, z], \theta] D[\$xCone[r, \theta, z], z] \\ D[\$yCone[r, \theta, z], r] D[\$yCone[r, \theta, z], \theta] D[\$yCone[r, \theta, z], z] \\ D[\$zCone[r, \theta, z], r] D[\$zCone[r, \theta, z], \theta] D[\$zCone[r, \theta, z], z] \end{array} \right);$$

$JacobianDetCone[r_, θ_, z_] := Abs[Det[$JacobianCone[r, θ, z]]];

$ConeIntegralVariables[R_, H_, a_] :=
  $Density * 
$$\int_0^R \int_0^{2\pi} \int_{1+\frac{H}{R}r}^{1+H} \$JacobianDetCone[r, \theta, z] * a \, dz \, d\theta \, dr;$$

$ConeIntegral[a_] := $ConeIntegralVariables[$ConeR, $ConeH, a];

$ConeMass = $ConeIntegral[1];
$ConeCenterOfMass := {
  $ConeIntegral[$xCone[r, θ, z]],
  $ConeIntegral[$yCone[r, θ, z]],
  $ConeIntegral[$zCone[r, θ, z]]} / $ConeMass;
$ICone = $I[$ConeIntegral,
  $xCone[r, θ, z],
  $yCone[r, θ, z],
  $zCone[r, θ, z]];

(* Bar Y *)
$BarYIntegral[a_] := $Density 
$$\int_{-1}^1 a \, dy;$$

$BarYMass = $BarYIntegral[1];
$BarYCenterOfMass :=
  {$BarYIntegral[0], $BarYIntegral[y], $BarYIntegral[0]} / $BarYMass;
$IBarY = $I[$BarYIntegral, 0, y, 0];

(* Bar Z *)
$BarZIntegral[a_] := $Density 
$$\int_0^1 a \, dz;$$

$BarZMass = $BarZIntegral[1];
$BarZCenterOfMass :=
  {$BarZIntegral[0], $BarZIntegral[0], $BarZIntegral[z]} / $BarZMass;
$IBarZ = $I[$BarZIntegral, 0, 0, z];

(* All *)
$MassAll = $ConeMass + $BarYMass + $BarZMass;
$CenterOfMassAll =
  ($ConeMass * $ConeCenterOfMass + $BarYMass * $BarYCenterOfMass +
   $BarZMass * $BarZCenterOfMass) / $MassAll;

$IAll = $ICone + $IBarY + $IBarZ;
$IAllPoint = $IPointFun[

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$CenterOfMassAll[[1]],
$CenterOfMassAll[[2]],
$CenterOfMassAll[[3]],
$MassAll];
$IAllCenter = $IAll - $IAllPoint;
$IAllCenterRotated = $RotationY.$IAllCenter.Transpose[$RotationY];

(* Around A *)
$A = {0, 1, 0};
$IAPoint = $IPointFun[
  $CenterOfMassAll[[1]] - $A[[1]],
  $CenterOfMassAll[[2]] - $A[[2]],
  $CenterOfMassAll[[3]] - $A[[3]],
  $MassAll];
$IA = $IAPoint + $IAllCenterRotated;

(* Prints *)
ParametricPlot3D[$ConeParam[r, \theta, 1 +  $\frac{\$ConeH}{\$ConeR} * r$ ], {r, 0, $ConeR}, {\theta, 0, 2\pi}]

Print["----- Properties -----"]
Print["Cone Mass: ", $ConeMass];

Print["Cone Center Of Mass: ", $ConeCenterOfMass];
Print["BarY Mass: ", $BarYMass];
Print["BarY Center Of Mass: ", $BarYCenterOfMass];
Print["BarZ Mass: ", $BarZMass];
Print["BarZ Center Of Mass: ", $BarZCenterOfMass];
Print["All Mass: ", $MassAll];
Print["All Center Of Mass: ", $CenterOfMassAll];

Print["----- Inertia Tensors around (0,0,0) -----"]
Print["Cone : ", N[MatrixForm[$ICone]]];
Print["BarY : ", N[MatrixForm[$IBarY]]];
Print["BarZ : ", N[MatrixForm[$IBarZ]]];

Print["----- Inertia Tensor All around (0,0,0) -----"]
Print[N[MatrixForm[$IAll]]];

Print["----- Inertia Tensors All Point -----"]
Print[N[MatrixForm[$IAllPoint]]];

Print["----- Inertia Tensors All Center -----"]
Print[N[MatrixForm[$IAllCenter]]];

Print["----- Inertia Tensors All Center Rotated -----"]
Print[N[MatrixForm[$IAllCenterRotated]]];

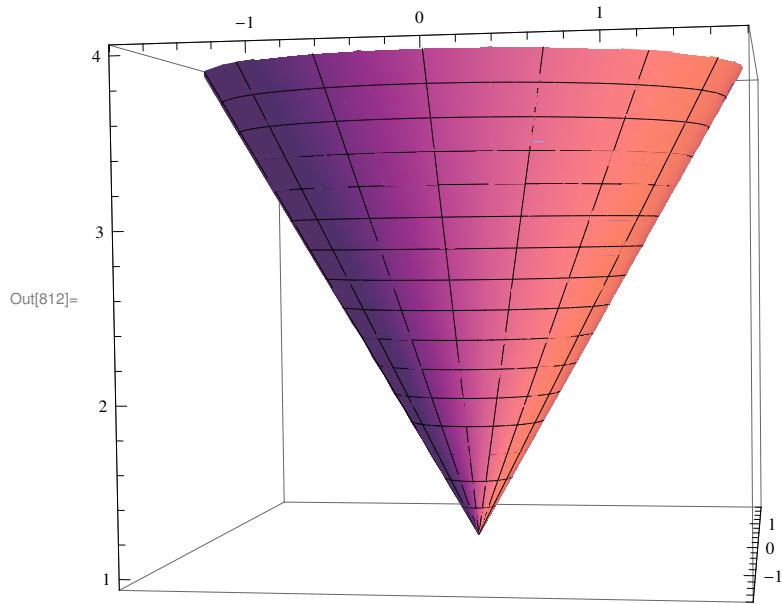
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Print["----- Inertia Tensors All Around A -----"]
Print[N[MatrixForm[$IA]]];

(* Plots *)
$a = 1/2 + 1/10;

Print["----- Inertia Tensors All around (0,0,0) -----"]
$PlotInertiaTensor[$IAll, $a]
Print["----- Inertia Tensors All Center -----"]
$PlotInertiaTensor[$IAllCenter, $a]

Print["-----"]
Print["----- Final Result -----"]
Print["-----"]
Print[" "]
Print["----- Inertia Tensors All Center Rotated -----"]
$PlotInertiaTensor[$IAllCenterRotated, $a]
Print["----- Inertia Tensors All Around A -----"]
$PlotInertiaTensor[$IA, $a]
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----- Properties -----

Cone Mass: 3π

Cone Center Of Mass: $\{0, 0, \frac{13}{4}\}$

BarY Mass: 2

BarY Center Of Mass: $\{0, 0, 0\}$

BarZ Mass: 1

BarZ Center Of Mass: $\{0, 0, \frac{1}{2}\}$

All Mass: $3 + 3\pi$

All Center Of Mass: $\left\{0, 0, \frac{\frac{1}{2} + \frac{39\pi}{4}}{3 + 3\pi}\right\}$

----- Inertia Tensors around (0,0,0) -----

Cone : $\begin{pmatrix} 106.971 & 0. & 0. \\ 0. & 106.971 & 0. \\ 0. & 0. & 8.4823 \end{pmatrix}$

BarY : $\begin{pmatrix} 0.666667 & 0. & 0. \\ 0. & 0. & 0. \\ 0. & 0. & 0.666667 \end{pmatrix}$

BarZ : $\begin{pmatrix} 0.333333 & 0. & 0. \\ 0. & 0.333333 & 0. \\ 0. & 0. & 0. \end{pmatrix}$

----- Inertia Tensor All around (0,0,0) -----

$\begin{pmatrix} 107.971 & 0. & 0. \\ 0. & 107.305 & 0. \\ 0. & 0. & 9.14897 \end{pmatrix}$

----- Inertia Tensors All Point -----

$\begin{pmatrix} 77.9982 & 0. & 0. \\ 0. & 77.9982 & 0. \\ 0. & 0. & 0. \end{pmatrix}$

----- Inertia Tensors All Center -----

$\begin{pmatrix} 29.9731 & 0. & 0. \\ 0. & 29.3064 & 0. \\ 0. & 0. & 9.14897 \end{pmatrix}$

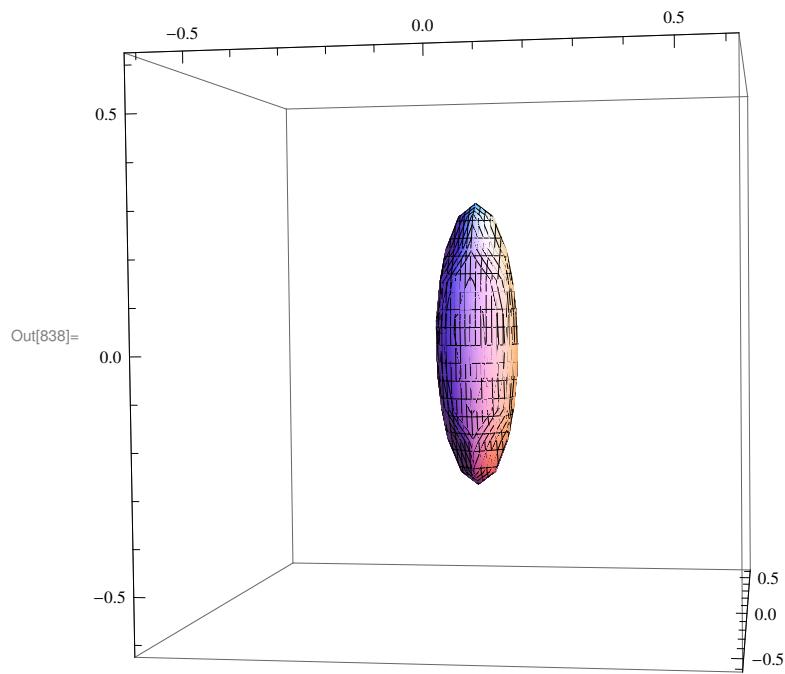
----- Inertia Tensors All Center Rotated -----

$\begin{pmatrix} 24.767 & 0. & 9.0171 \\ 0. & 29.3064 & 0. \\ 9.0171 & 0. & 14.355 \end{pmatrix}$

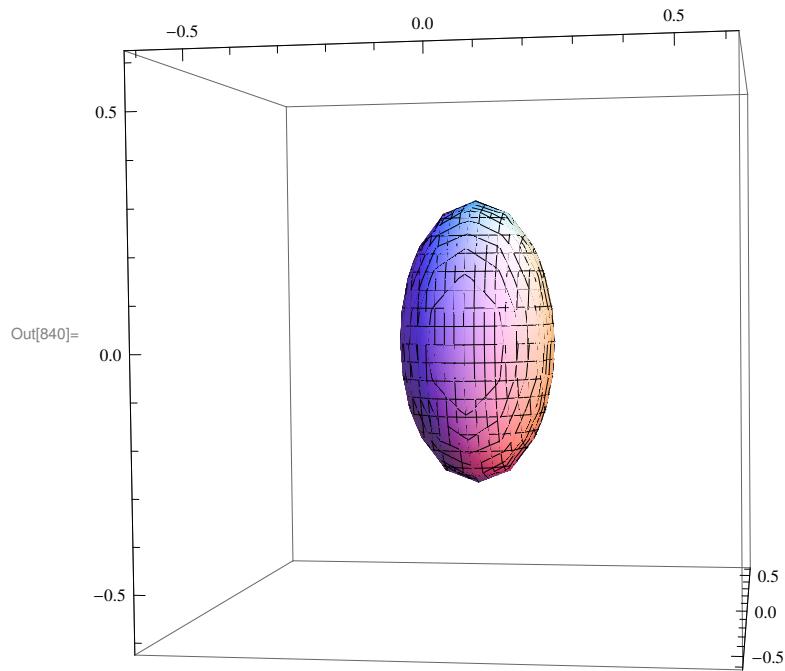
----- Inertia Tensors All Around A -----

$\begin{pmatrix} 115.19 & 0. & 9.0171 \\ 0. & 107.305 & 31.1305 \\ 9.0171 & 31.1305 & 26.7798 \end{pmatrix}$

----- Inertia Tensors All around (0,0,0) -----

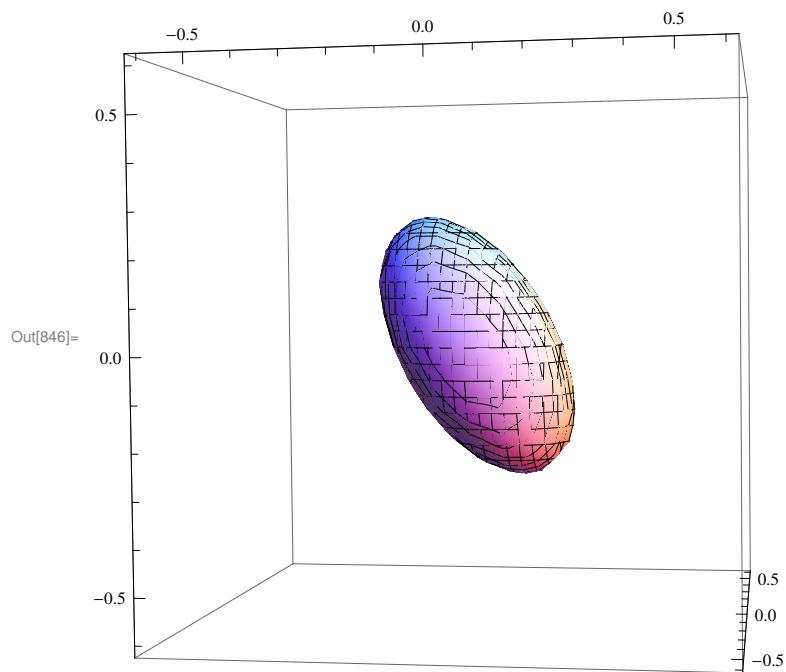


----- Inertia Tensors All Center -----

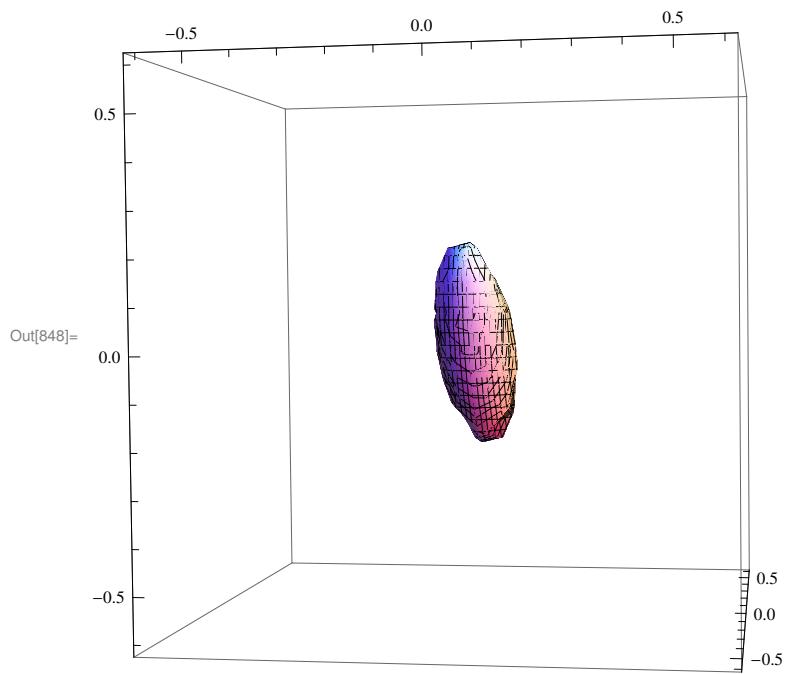


----- Final Result -----

----- Inertia Tensors All Center Rotated -----



----- Inertia Tensors All Around A -----



In[849]:=

In[850]:=

In[851]:=