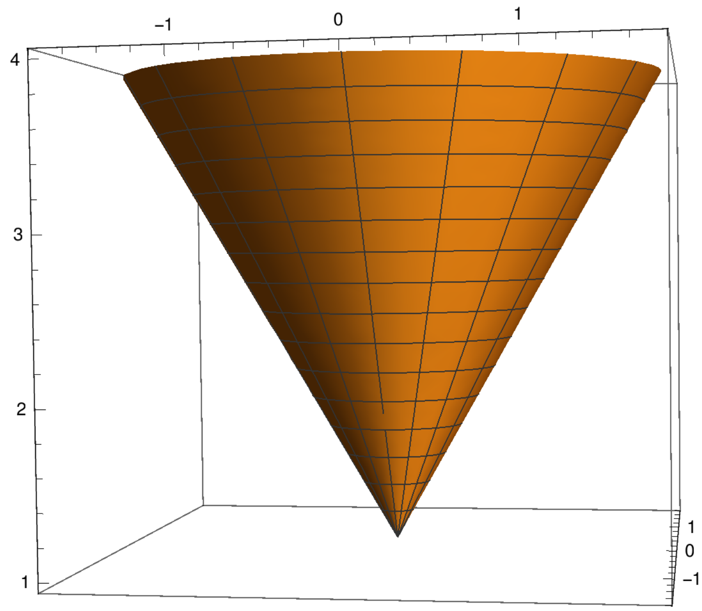
```
In[2804]:= ClearAll["Global`*"]

(* http://mini.pw.edu.pl/~porter/cc/psw/psw_cw2.pdf *)
(* System: Two bars and a cone *)

(* ------------- Global Variables -------------- *)
$Density := 1;

(* ------------- Functions -------------- *)

$I[$Integral_, x_, y_, z_] := {
   {$Integral[y^2 + z^2],
    -$Integral[x*y],
    -$Integral[x*z]},
   {-$Integral[x* y],
    $Integral[x^2 + z^2],
    -$Integral[y*z]},
   {-$Integral[x*z ],
    -$Integral[y*z],
    $Integral[y^2+x^2]}};

$IPointFun[x_, y_, z_, m_] :=
  m* {
    {y^2+z^2, -x*y, -x*z},
    {-x*y, x^2 + z^2, -y*z},
    {-x*z, -y*z, x^2+y^2}};

$PlotInertiaTensor[I_, a_] := Show[ContourPlot3D[
   {{ix, iy, iz}.I.{ix, iy, iz} == 1}, {ix, -a, a}, {iy, -a, a}, {iz, -a, a}]]

$Angle = -30°;
$RotationY = ( Cos[$Angle]   0  Sin[$Angle]
                   0         1      0
              -Sin[$Angle]   0  Cos[$Angle] );

(* --------------------------------------- *)
(* Cone *)
$ConeR = √3;
$ConeSlant = 2 √3;
$ConeH = √($ConeSlant^2 - $ConeR^2);

$xCone[r_, θ_, z_] := r*Cos[θ];
$yCone[r_, θ_, z_] := r*Sin[θ];
$zCone[r_, θ_, z_] := z;
```

```
$ConeParam[r_, θ_, z_] := {$xCone[r, θ, z], $yCone[r, θ, z], $zCone[r, θ, z]};

$JacobianCone[r_, θ_, z_] :=
  ( D[$xCone[r, θ, z], r]  D[$xCone[r, θ, z], θ]  D[$xCone[r, θ, z], z]
    D[$yCone[r, θ, z], r]  D[$yCone[r, θ, z], θ]  D[$yCone[r, θ, z], z]
    D[$zCone[r, θ, z], r]  D[$zCone[r, θ, z], θ]  D[$zCone[r, θ, z], z] );
$JacobianDetCone[r_, θ_, z_] := Abs[Det[$JacobianCone[r, θ, z]]];

$ConeIntegralVariables[R_, H_, a_] :=
  $Density * ∫_0^R ∫_0^(2 π) ∫_(1+H/R r)^(1+H) $JacobianDetCone[r, θ, z] * a dz dθ dr;
$ConeIntegral[a_] := $ConeIntegralVariables[$ConeR, $ConeH, a];


$ConeMass = $ConeIntegral[1];
$ConeCenterOfMass := {
     $ConeIntegral[$xCone[r, θ, z]],
     $ConeIntegral[$yCone[r, θ, z]],
     $ConeIntegral[$zCone[r, θ, z]]} / $ConeMass;
$ICone = $I[$ConeIntegral,
    $xCone[r, θ, z],
    $yCone[r, θ, z],
    $zCone[r, θ, z]];

(* Bar Y *)
$BarYIntegral[a_] := $Density ∫_-1^1 a dy;
$BarYMass = $BarYIntegral[1];
$BarYCenterOfMass :=
  {$BarYIntegral[0], $BarYIntegral[y], $BarYIntegral[0]} / $BarYMass;
$IBarY = $I[$BarYIntegral, 0, y, 0];

(* Bar Z *)
$BarZIntegral[a_] := $Density ∫_0^1 a dz;
$BarZMass = $BarZIntegral[1];
$BarZCenterOfMass :=
  {$BarZIntegral[0], $BarZIntegral[0], $BarZIntegral[z]} / $BarZMass;
$IBarZ = $I[$BarZIntegral, 0, 0, z];

(* All *)
$MassAll = $ConeMass + $BarYMass + $BarZMass;
$CenterOfMassAll = ($ConeMass * $ConeCenterOfMass +
      $BarYMass * $BarYCenterOfMass + $BarZMass * $BarZCenterOfMass) / $MassAll;
```

```
$IAll = $ICone + $IBarY  + $IBarZ;
$IAllPoint = $IPointFun[
    $CenterOfMassAll[[1]],
    $CenterOfMassAll[[2]],
    $CenterOfMassAll[[3]],
    $MassAll];
$IAllCenter = $IAll - $IAllPoint;
$IAllCenterRotated = $RotationY.$IAllCenter.Transpose[$RotationY];

(* Around A *)
$A = {0, 1, 0};
$IAPoint = $IPointFun[
    $CenterOfMassAll[[1]] - $A[[1]],
    $CenterOfMassAll[[2]] - $A[[2]],
    $CenterOfMassAll[[3]] - $A[[3]],
    $MassAll];
$IA = $IAPoint + $IAllCenter;

(* Prints *)
ParametricPlot3D[$ConeParam[r, θ, 1 + $ConeH/$ConeR * r], {r, 0, $ConeR}, {θ, 0, 2 π}]

Print["----------- Properties -----------"]
Print["Cone Mass: ", $ConeMass];

Print["Cone Center Of Mass: ", $ConeCenterOfMass];
Print["BarY Mass: ", $BarYMass];
Print["BarY Center Of Mass: ", $BarYCenterOfMass];
Print["BarZ Mass: ", $BarZMass];
Print["BarZ Center Of Mass: ", $BarZCenterOfMass];
Print["All Mass: ", $MassAll];
Print["All Center Of Mass: ", $CenterOfMassAll];

Print["----------- Inertia Tensors around (0,0,0) -----------"]
Print["Cone : " , N[MatrixForm[$ICone]]];
Print["BarY : " , N[MatrixForm[$IBarY]]];
Print["BarZ : " , N[MatrixForm[$IBarZ]]];

Print["----------- Inertia Tensor All around (0,0,0) -----------"]
Print[N[MatrixForm[$IAll]]];

Print["----------- Inertia Tensors All Point -----------"]
Print[N[MatrixForm[$IAllPoint]]];
```

```
Print["----------- Inertia Tensors All Center -----------"]
Print[N[MatrixForm[$IAllCenter]]];

Print["----------- Inertia Tensors All Center Rotated -----------"]
Print[N[MatrixForm[$IAllCenterRotated]]];

Print["----------- Inertia Tensors All Around A -----------"]
Print[N[MatrixForm[$IA]]];

(* Plots *)
$a = 1 / 2 + 1 / 10;

Print["----------- Inertia Tensors All around (0,0,0) -----------"]
$PlotInertiaTensor[$IAll, $a]
Print["----------- Inertia Tensors All Center -----------"]
$PlotInertiaTensor[$IAllCenter, $a]

Print["------------------------------------"]
Print["----------- Final Result ------------"]
Print["------------------------------------"]
Print[" "]
Print["----------- Inertia Tensors All Center Rotated -----------"]
$PlotInertiaTensor[$IAllCenterRotated, $a]
Print["----------- Inertia Tensors All Around A -----------"]
$PlotInertiaTensor[$IA, $a]
```

Out[2842]=



----------- Properties -----------

Cone Mass: 3π

Cone Center Of Mass: $\left\{0, 0, \frac{13}{4}\right\}$

BarY Mass: 2

BarY Center Of Mass: $\{0, 0, 0\}$

BarZ Mass: 1

BarZ Center Of Mass: $\left\{0, 0, \frac{1}{2}\right\}$

All Mass: $3 + 3\pi$

All Center Of Mass: $\left\{0, 0, \frac{\frac{1}{2} + \frac{39\pi}{4}}{3 + 3\pi}\right\}$

----------- Inertia Tensors around (0,0,0) -----------

Cone : $\begin{pmatrix} 106.971 & 0. & 0. \\ 0. & 106.971 & 0. \\ 0. & 0. & 8.4823 \end{pmatrix}$

BarY : $\begin{pmatrix} 0.666667 & 0. & 0. \\ 0. & 0. & 0. \\ 0. & 0. & 0.666667 \end{pmatrix}$

BarZ : $\begin{pmatrix} 0.333333 & 0. & 0. \\ 0. & 0.333333 & 0. \\ 0. & 0. & 0. \end{pmatrix}$

----------- Inertia Tensor All around (0,0,0) -----------

$$\begin{pmatrix} 107.971 & 0. & 0. \\ 0. & 107.305 & 0. \\ 0. & 0. & 9.14897 \end{pmatrix}$$

----------- Inertia Tensors All Point -----------

$$\begin{pmatrix} 77.9982 & 0. & 0. \\ 0. & 77.9982 & 0. \\ 0. & 0. & 0. \end{pmatrix}$$

----------- Inertia Tensors All Center -----------

$$\begin{pmatrix} 29.9731 & 0. & 0. \\ 0. & 29.3064 & 0. \\ 0. & 0. & 9.14897 \end{pmatrix}$$

----------- Inertia Tensors All Center Rotated -----------

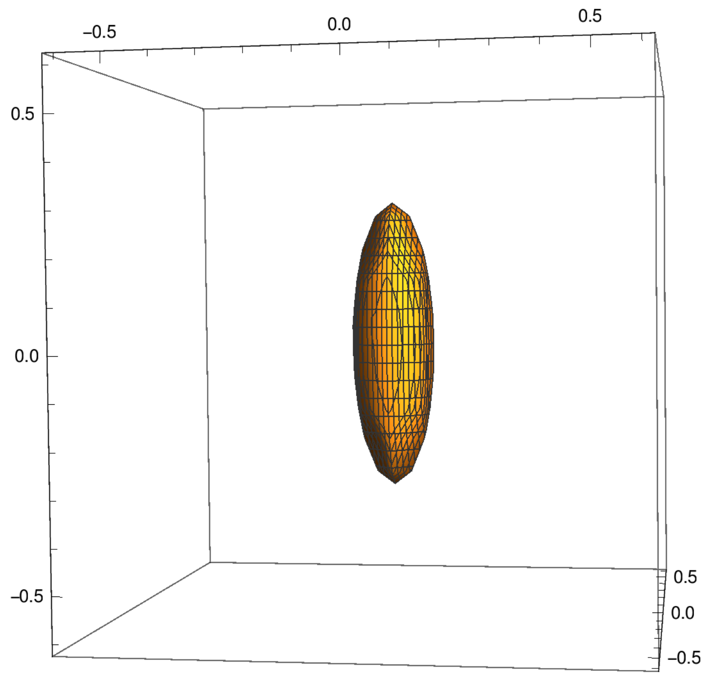
$$\begin{pmatrix} 24.767 & 0. & 9.0171 \\ 0. & 29.3064 & 0. \\ 9.0171 & 0. & 14.355 \end{pmatrix}$$

----------- Inertia Tensors All Around A -----------

$$\begin{pmatrix} 120.396 & 0. & 0. \\ 0. & 107.305 & 31.1305 \\ 0. & 31.1305 & 21.5737 \end{pmatrix}$$

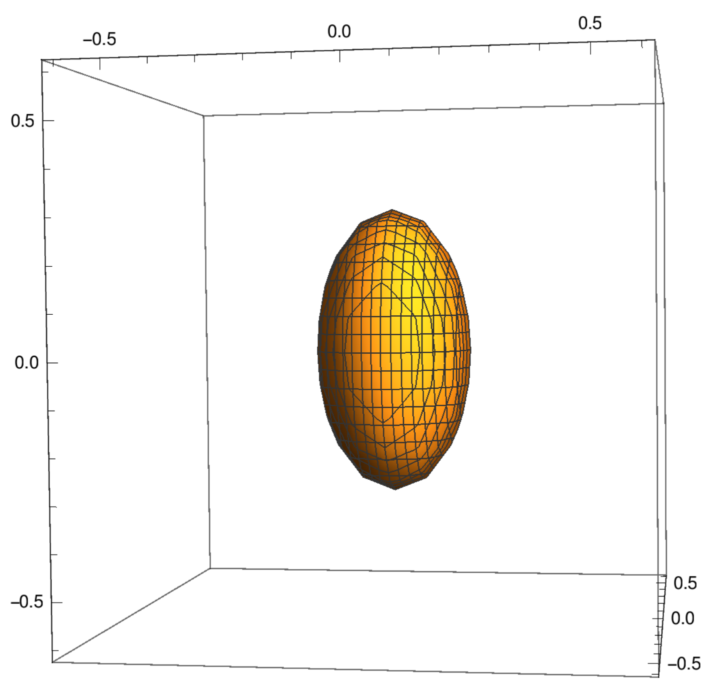
----------- Inertia Tensors All around (0,0,0) -----------

Out[2868]=



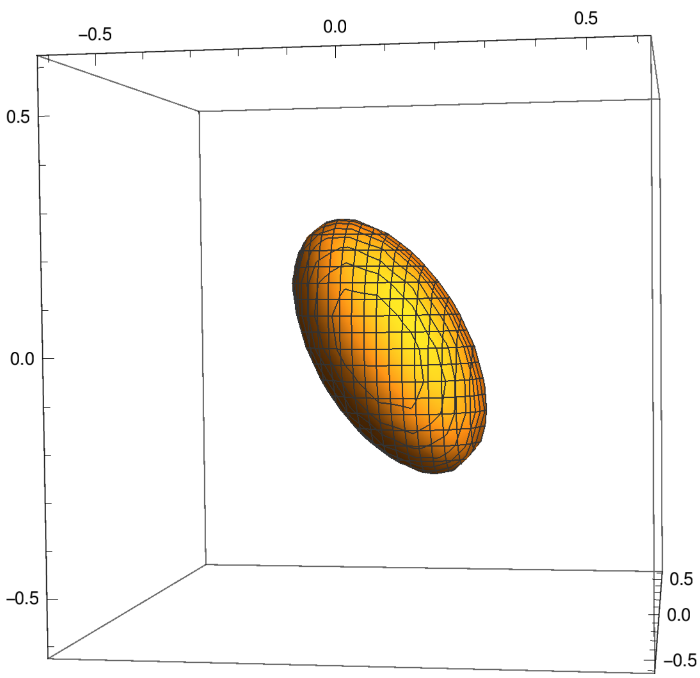
----------- Inertia Tensors All Center -----------

Out[2870]=



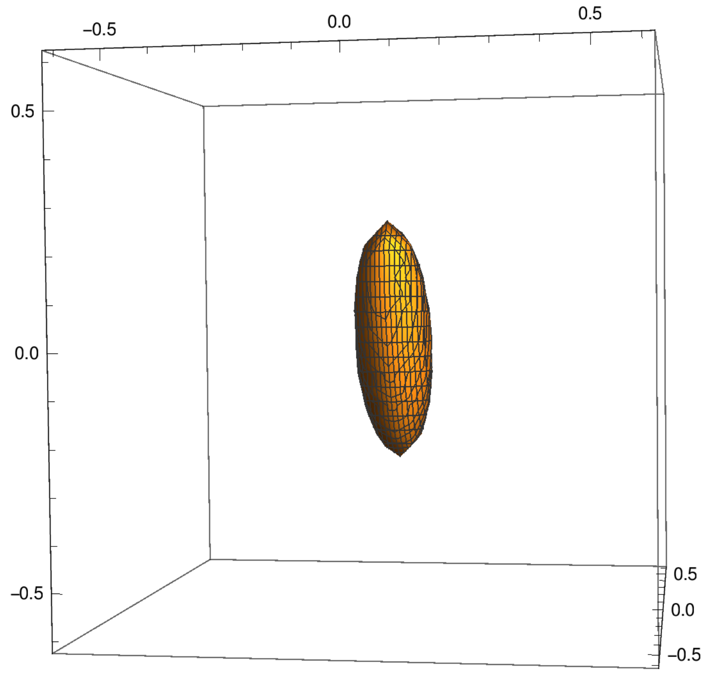
```
-------------------------------------

----------- Final Result ------------

-------------------------------------

----------- Inertia Tensors All Center Rotated -----------
```

Out[2876]=




```
----------- Inertia Tensors All Around A -----------
```

Out[2878]=



In[2879]:=

_

In[2880]:=

In[2881]:=