

```
In[1180]:= ClearAll["Global`*"]
```

```
(* http://mini.pw.edu.pl/~porter/cc/psw/psw_cw1.pdf *)
```

```
(* Unit Circle with Spherical Coordinates *)
```

```
$density := 1;
```

```
$R := 1;
```

```
$xCircle[r_, u_, v_] := r * Sin[v] Cos[u];
```

```
$yCircle[r_, u_, v_] := r * Sin[v] Sin[u];
```

```
$zCircle[r_, u_, v_] := r * Cos[v];
```

```
$v =  $\pi/2$ ;
```

```
$Circle[r_, u_, v_] := {$xCircle[r, u, $v], $yCircle[r, u, $v], $zCircle[r, u, $v]};
```

```
ParametricPlot3D[$Circle[$R, u, $v], {u, 0, 2  $\pi$ }]
```

```
$JacobianMatrix[r_, u_, v_] :=  $\begin{pmatrix} D[\$xCircle[r, u, v], r] & D[\$xCircle[r, u, v], u] \\ D[\$yCircle[r, u, v], r] & D[\$yCircle[r, u, v], u] \end{pmatrix}$ ;
```

```
Print["Jacobian Matrix", MatrixForm[$JacobianMatrix[r, u, v]]]
```

```
$JacobianDet[r_, u_, v_] := Abs[Det[$JacobianMatrix[r, u, v]]];
```

```
$CircleIntegral[a_] := $density *  $\int_0^{2\pi} \int_0^1 \$JacobianDet[r, u, $v] a \, dr \, du$ ;
```

```
$CircleMass := $CircleIntegral[1];
```

```
$CircleCenterOfMass := {$CircleIntegral[$xCircle[r, u, $v]],  
$CircleIntegral[$yCircle[r, u, $v]], $CircleIntegral[0]} / $CircleMass;
```

```
$X := $xCircle[r, u, $v];
```

```
$Y := $yCircle[r, u, $v];
```

```
$Z := 0;
```

```
$ICircle = {  
{$CircleIntegral[$Y^2 + $Z^2],  
-$CircleIntegral[$X * $Y],  
-$CircleIntegral[$X * $Z]},  
{-$CircleIntegral[$X * $Y],  
$CircleIntegral[$X^2 + $Z^2],  
-$CircleIntegral[$Y * $Z]},  
{-$CircleIntegral[$X * $Z],  
-$CircleIntegral[$Y * $Z],  
$CircleIntegral[$Y^2 + $X^2]}};
```

```
Print["Circle Mass: ", $CircleMass]
```

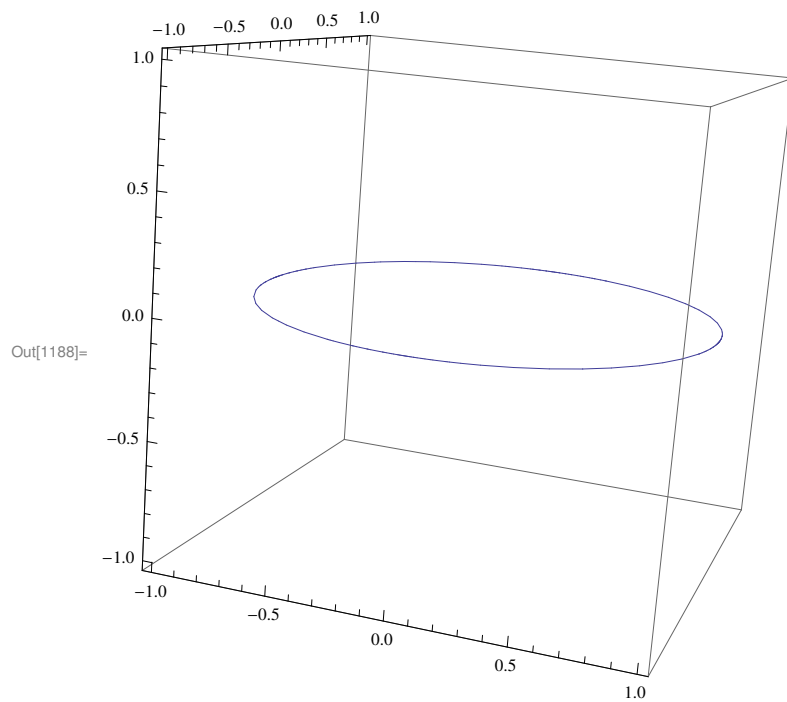
```
Print["Circle Center of Mass: ", MatrixForm[$CircleCenterOfMass]]
```

```
Print["Circle Tensor of Intertia: ", MatrixForm[$ICircle]]
```

```

$a = 3 / 2;
Show[ContourPlot3D[{{ix, iy, iz}.$ICircle.{ix, iy, iz} == 1},
  {ix, -$a, $a}, {iy, -$a, $a}, {iz, -$a, $a}]]

```

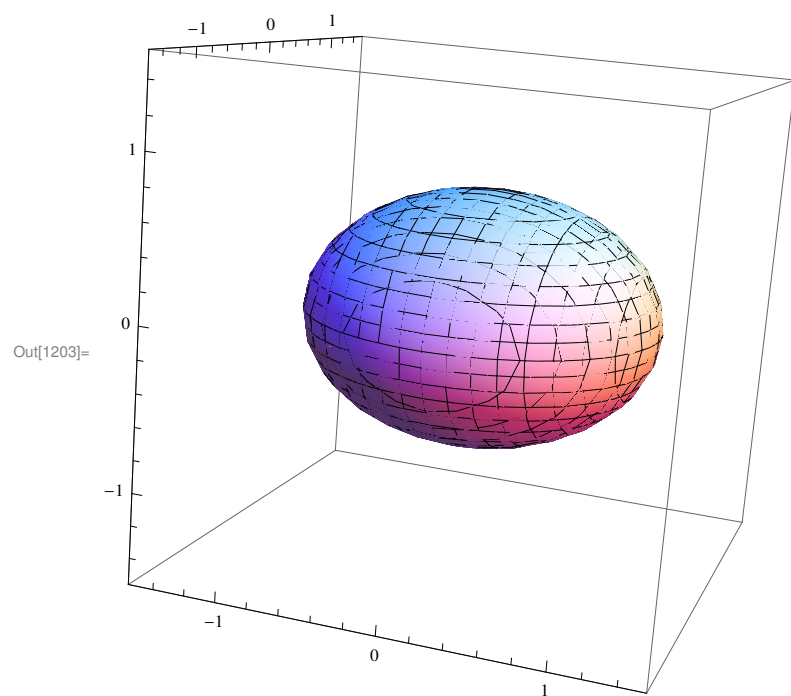


Jacobian Matrix $\begin{pmatrix} \cos[u] \sin[v] & -r \sin[u] \sin[v] \\ \sin[u] \sin[v] & r \cos[u] \sin[v] \end{pmatrix}$

Circle Mass: π

Circle Center of Mass: $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Circle Tensor of Intertia: $\begin{pmatrix} \frac{\pi}{4} & 0 & 0 \\ 0 & \frac{\pi}{4} & 0 \\ 0 & 0 & \frac{\pi}{2} \end{pmatrix}$



In[1204]:=