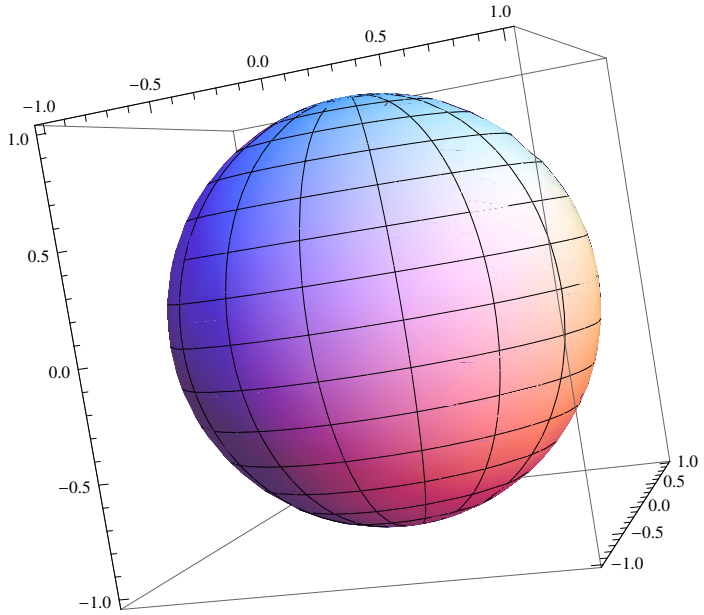
In[256]:=
```
ClearAll["Global`*"]

(* Sphere Equation *)
$R = 1;
$x[ r_, u_, v_] = r * Sin[v] * Cos[u];
$y[ r_, u_, v_] = r * Sin[v] * Sin[u];
$z[ r_, u_, v_] = r * Cos[v];
$Sphere[r_, u_, v_] = {$x[r, u, v], $y[r, u, v], $z[r, u, v]};

(* Plot Sphere *)
ParametricPlot3D[$Sphere[$R, u, v], {u, 0, 2 π}, {v, 0, π}]

(* Compute derivatives *)
$xu[r_, u_, v_] = D[$x[r, u, v], u];
$yu[r_, u_, v_] = D[$y[r, u, v], u];
$zu[r_, u_, v_] = D[$z[r, u, v], u];

$xv[r_, u_, v_] = D[$x[r, u, v], v];
$yv[r_, u_, v_] = D[$y[r, u, v], v];
$zv[r_, u_, v_] = D[$z[r, u, v], v];

$xvu[r_, u_, v_] = D[$xv[r, u, v], u];
$yvu[r_, u_, v_] = D[$yv[r, u, v], u];
$zvu[r_, u_, v_] = D[$zv[r, u, v], u];

$xuv[r_, u_, v_] = D[$xu[r, u, v], v];
$yuv[r_, u_, v_] = D[$yu[r, u, v], v];
$zuv[r_, u_, v_] = D[$zu[r, u, v], v];

$xvv[r_, u_, v_] = D[$xv[r, u, v], v];
$yvv[r_, u_, v_] = D[$yv[r, u, v], v];
$zvv[r_, u_, v_] = D[$zv[r, u, v], v];

$xuu[r_, u_, v_] = D[$xu[r, u, v], u];
$yuu[r_, u_, v_] = D[$yu[r, u, v], u];
$zuu[r_, u_, v_] = D[$zu[r, u, v], u];

(* Solve first system of linear equations *)

$Equations[r_, u_, v_] =
  {$fxTMP[r, u, v] * $xu[r, u, v] + $fyTMP[r, u, v] * $yu[r, u, v] == $zu[r, u, v],
   $fxTMP[r, u, v] * $xv[r, u, v] + $fyTMP[r, u, v] * $yv[r, u, v] == $zv[r, u, v]};
$Solution = Simplify[Solve[$Equations[r, u, v],
    {$fxTMP[r, u, v], $fyTMP[r, u, v]}]];

$fx[r_, u_, v_] = $Solution[[All, 1, 2]];
$fy[r_, u_, v_] = $Solution[[All, 2, 2]];
```

```
(* Solve second system of linear equations *)
$Equations2[r_, u_, v_] = {
    ($fxxTMP[r, u, v] * $xu[r, u, v] + $fyxTMP[r, u, v] * $yu[r, u, v]) * $xu[r, u, v] +
      ($fxyTMP[r, u, v] * $xu[r, u, v] + $fyyTMP[r, u, v] * $yu[r, u, v]) * $yu[r, u, v] +
      $fx[r, u, v] * $xuu[r, u, v] + $fy[r, u, v] * $yuu[r, u, v] == $zuu[r, u, v],
    ($fxxTMP[r, u, v] * $xu[r, u, v] + $fyxTMP[r, u, v] * $yu[r, u, v]) * $xv[r, u, v] +
      ($fxyTMP[r, u, v] * $xu[r, u, v] + $fyyTMP[r, u, v] * $yu[r, u, v]) * $yv[r, u, v] +
      $fx[r, u, v] * $xuv[r, u, v] + $fy[r, u, v] * $yuv[r, u, v] == $zvu[r, u, v],
    ($fxxTMP[r, u, v] * $xv[r, u, v] + $fyxTMP[r, u, v] * $yv[r, u, v]) * $xu[r, u, v] +
      ($fxyTMP[r, u, v] * $xv[r, u, v] + $fyyTMP[r, u, v] * $yv[r, u, v]) * $yu[r, u, v] +
      $fx[r, u, v] * $xvu[r, u, v] + $fy[r, u, v] * $yvu[r, u, v] == $zuv[r, u, v],
    ($fxxTMP[r, u, v] * $xv[r, u, v] + $fyxTMP[r, u, v] * $yv[r, u, v]) * $xv[r, u, v] +
      ($fxyTMP[r, u, v] * $xv[r, u, v] + $fyyTMP[r, u, v] * $yv[r, u, v]) * $yv[r, u, v] +
      $fx[r, u, v] * $xvv[r, u, v] + $fy[r, u, v] * $yvv[r, u, v] == $zvv[r, u, v]
  };
$Solution2 = Simplify[Solve[$Equations2[r, u, v],
     {$fxxTMP[r, u, v], $fxyTMP[r, u, v], $fyxTMP[r, u, v], $fyyTMP[r, u, v]}]];

$fxx[r_, u_, v_] = $Solution2[[All, 1, 2]];
$fxy[r_, u_, v_] = $Solution2[[All, 2, 2]];
$fyx[r_, u_, v_] = $Solution2[[All, 3, 2]];
$fyy[r_, u_, v_] = $Solution2[[All, 4, 2]];

Print["fx = ", $fx[r, u, v]]
Print["fy = ", $fy[r, u, v]]
Print["fxx = ", $fxx[r, u, v]]
Print["fxy = ", $fxy[r, u, v]]
Print["fyx = ", $fyx[r, u, v]]
Print["fyy = ", $fyy[r, u, v]]

$u0 = π;
$v0 = π;
(*
$f[x_,y_] := $fx[$R,$u0,$v0]x + $fy[$R,$u0,$v0]y +
    ($fxx[$R,$u0,$v0]x^2 + 2$fxy[$R,$u0,$v0]x*y + $fyy[$R,$u0,$v0]y^2)/2;
*)
$f[r_, x_, y_] := $fx[r, $u0, $v0] x + $fy[r, $u0, $v0] y +
    ($fxx[r, $u0, $v0] x^2 + 2 $fxy[r, $u0, $v0] x * y + $fyy[r, $u0, $v0] y^2) / 2;
Print["f = ", $f[r, x, y]]

$a = 1;
Plot3D[$f[$R, x, y], {x, -$a, $a}, {y, -$a, $a}]
```

Out[262]=



$$fx = \{-\text{Cos}[u]\ \text{Tan}[v]\}$$

$$fy = \{-\text{Sin}[u]\ \text{Tan}[v]\}$$

$$fxx = \left\{\frac{(-6 - 2\ \text{Cos}[2\,u] + \text{Cos}[2\,(u-v)] - 2\ \text{Cos}[2\,v] + \text{Cos}[2\,(u+v)])\ \text{Sec}[v]^3}{8\,r}\right\}$$

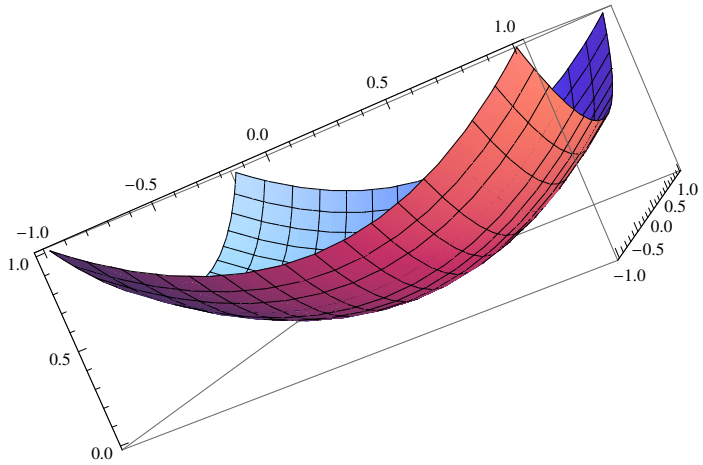
$$fxy = \left\{-\frac{\text{Cos}[u]\ \text{Sec}[v]\ \text{Sin}[u]\ \text{Tan}[v]^2}{r}\right\}$$

$$fyx = \left\{-\frac{\text{Cos}[u]\ \text{Sec}[v]\ \text{Sin}[u]\ \text{Tan}[v]^2}{r}\right\}$$

$$fyy = \left\{-\frac{(6 - 2\ \text{Cos}[2\,u] + \text{Cos}[2\,(u-v)] + 2\ \text{Cos}[2\,v] + \text{Cos}[2\,(u+v)])\ \text{Sec}[v]^3}{8\,r}\right\}$$

$$f = \left\{\frac{1}{2}\left(\frac{x^2}{r} + \frac{y^2}{r}\right)\right\}$$

Out[302]=



In[303]:=

In[304]:=

In[305]:=

In[306]:=

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In[308]:=

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In[316]:=

In[317]:=

In[318]:=

In[319]:=

In[320]:=

In[321]:=