## An upper triangular dense matrix is split by columns on N processors. Implement the backward substitution. Analyze the speed-up.

Parallel Processing Project 4

## 1 Algorithm description

## Algorithm 1 Sequential Backwards substitution

```
 \begin{tabular}{lll} \hline \textbf{Data:} & \textbf{Ax=b} \\ \textbf{for} & i=n\text{-}1 & to & 1 & \textbf{do} \\ & & x[i] = b[i]/A[i,i] \\ & \textbf{for} & j=0 & to & i\text{-}1 & \textbf{do} \\ & & & b[j] = b[j] - x[i]*A[j,i] \\ & \textbf{end} \\ & \\ \hline \end \\ \hline \end \\ \hline \end \\ \end \\ \hline \end \\ \hline \end \\ \end \\ \hline \end \\ \end \\ \end \\ \end \\ \hline \end \\ \
```

At i-th iteration, the process which has i-th column, computes  $x_i$  and updates the b vector. When worker is done computing, he sends the next worker updated vectors b and x. Since each i-th iteration requires data from the previous iteration, column-wise parallel backward substitution is scarcely a parallel algorithm and actually works faster sequentially as I will show in the next section.

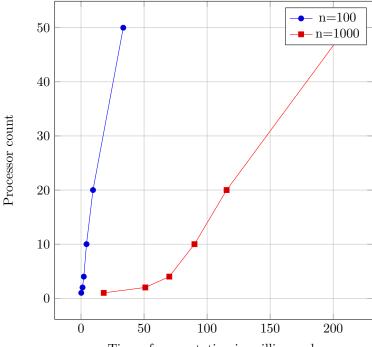
## 2 Analysis of speed up

I created upper triangular dense matrices using a simple formula:

```
Algorithm 2 Column major order Matrix
```

```
 \begin{aligned} \mathbf{Data} &: \mathbf{n} = \text{Dimension of A; val} = 1; \ \mathbf{c} = \mathbf{r} = 0 \\ \mathbf{while} \ c > n \ \mathbf{do} \\ & | \ \mathbf{while} \ r > n \ \mathbf{do} \\ & | \ \mathbf{if} \ r > c \ \mathbf{then} \\ & | \ A[c*n+r] = 0 \\ & \mathbf{else} \\ & | \ A[c*n+r] = val \\ & | \ val + + \\ & \mathbf{end} \\ & \mathbf{end} \end{aligned}
```

with various dimension size and processors count.



Time of computation in milliseconds

The plot clearly shows that the algorithm is not really parallel. In fact the computations are the fastest with one process.