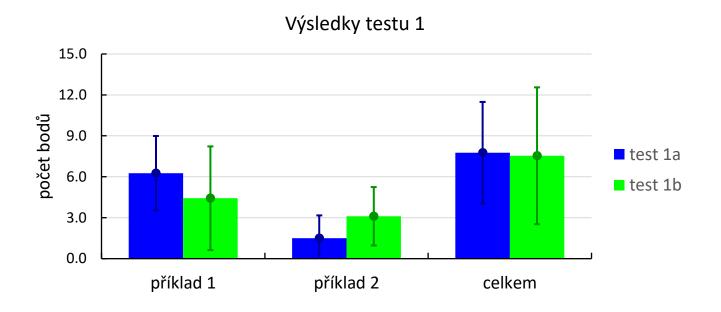
# Výsledky testu 1

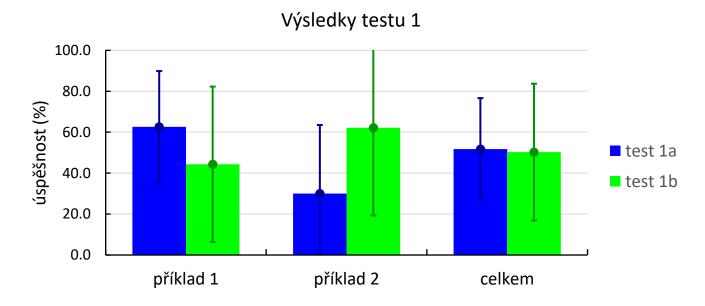
test 1a
 skupina 1 (29. 11., 9:00)

(8 ± 4) bodů

test 1b
 skupina 2 (30. 11., 8:10)

 $(8 \pm 5)$  bodů





- test 1a skupina 1 (29. 11., 9:00)
- x odhadovaný počet bodů

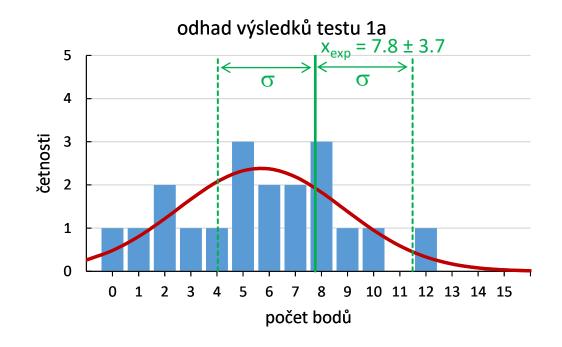
$$N = 19$$

$$\hat{\mu}_{x} = 5.7$$
  $\hat{\sigma}_{x} = 3.2$ 

histogram hodnot x

$$f(x) \approx \frac{N}{\sqrt{2\pi\hat{\sigma}^2}} \exp\left[-\frac{(x-\hat{\mu}_x)^2}{2\hat{\sigma}^2}\right]$$

$$\hat{\mu}_{exp} = 7.8 \qquad \hat{\sigma}_{exp} = 3.7$$



- test 1a skupina 1 (29. 11., 9:00)
- $\bar{x}$  odhadovaný průměrný počet bodů

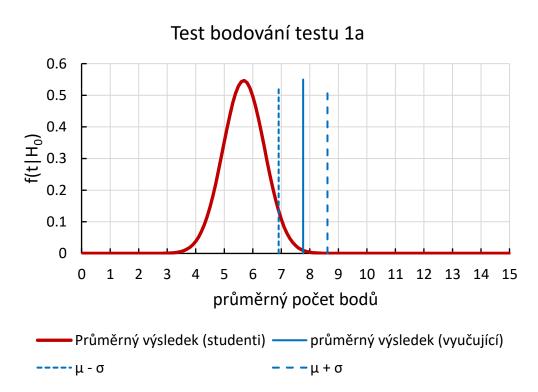
$$N = 19$$

$$\hat{\mu}_{\bar{x}} = 5.7$$
  $\hat{\sigma}_{\bar{x}} = \frac{\hat{\sigma}_x}{\sqrt{N}} = 0.7$ 

hustota pravděpodobnosti x̄

$$f(\bar{x}) \approx \frac{N}{\sqrt{2\pi\hat{\sigma}_{\bar{x}}^2}} \exp\left[-\frac{(\bar{x} - \hat{\mu}_{\bar{x}})^2}{2\hat{\sigma}_{\bar{x}}^2}\right]$$

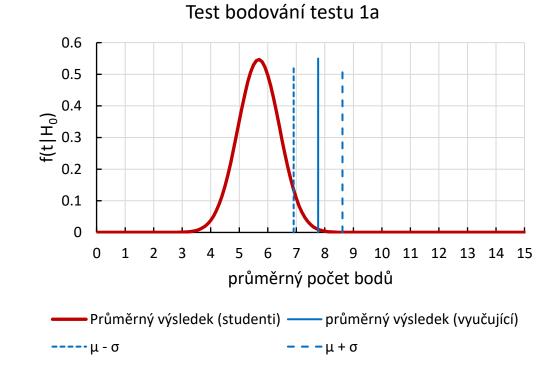
$$\hat{\mu}_{avg} = 7.8 \qquad \hat{\sigma}_{avg} = 0.9$$



- test 1a skupina 1 (29. 11., 9:00)
- Nulová hypotéza H<sub>0</sub>
   Hodnocení testu bylo spravedlivé.
- testovací statistika ( $\hat{\mu}_{\bar{x}} = 5.7, \, \hat{\sigma}_{\bar{x}} = 0.7$ )

$$f(\bar{x}|H_0) \approx \frac{N}{\sqrt{2\pi\hat{\sigma}_{\bar{x}}^2}} \exp\left[-\frac{(\bar{x}-\hat{\mu}_{\bar{x}})^2}{2\hat{\sigma}_{\bar{x}}^2}\right]$$

testovací proměnná (t-hodnota)



$$t_1 = \hat{\mu}_{avg} - \hat{\sigma}_{avg} = 6.9$$

$$P(\bar{x} > t_1) = 1 - F(\bar{x}|H_0) = \frac{1}{2} \left[ 1 - \text{erf}\left(\frac{t_1 - \hat{\mu}_{\bar{x}}}{\sqrt{2}\hat{\sigma}_{\bar{x}}}\right) \right] = 4.7\%$$

$$t_2 = \hat{\mu}_{avg} = 7.8$$

$$P(\bar{x} > t_2) = 1 - F(\bar{x}|H_0) = \frac{1}{2} \left[ 1 - \text{erf}\left(\frac{t_2 - \hat{\mu}_{\bar{x}}}{\sqrt{2}\hat{\sigma}_{\bar{x}}}\right) \right] = 0.2\%$$

$$t_3 = \hat{\mu}_{avg} + \hat{\sigma}_{avg} = 8.6$$

$$P(\bar{x} > t_3) = 1 - F(\bar{x}|H_0) = \frac{1}{2} \left[ 1 - \text{erf}\left(\frac{t_3 - \hat{\mu}_{\bar{x}}}{\sqrt{2}\hat{\sigma}_{\bar{x}}}\right) \right] = 0.003\%$$

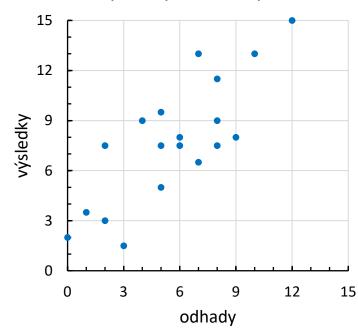
#### Příklad 2a – korelace veličin skupiny 1

x – počet bodů z testu (výsledek)

$$\hat{\mu}_x = 7.8$$
  $\hat{\sigma}_x = 3.7$ 

$$\hat{\sigma}_{x} = 3.7$$

výsledky vs odhady



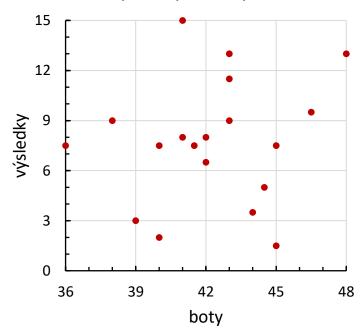
- korelace  $\hat{\rho}(x, y) = 0.8 \pm 0.1$
- Fisher  $(t = 4.021) \rightarrow P = 0.01\%$
- student  $(t = 4.878) \rightarrow P = 0.01\%$

y − počet bodů z testu (odhad)

$$\hat{\mu}_y = 5.7 \qquad \hat{\sigma}_y = 3.2$$

$$\hat{\sigma}_{v} = 3.2$$

výsledky vs boty



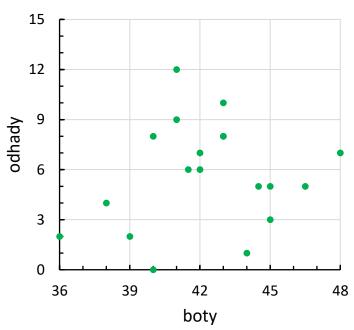
- korelace  $\hat{\rho}(x, y) = 0.1 \pm 0.2$
- Fisher  $(t = 0.578) \rightarrow P = 56.4\%$
- student  $(t = 0.598) \rightarrow P = 55.8\%$

z – číslo bot (EU)

$$\hat{\mu}_z = 42$$
  $\hat{\sigma}_z = 3$ 

$$\hat{\sigma}_z = 3$$

odhady vs boty



- korelace  $\hat{\rho}(x, y) = 0.2 \pm 0.2$
- Fisher  $(t = 0.664) \rightarrow P = 50.7\%$
- student (t = 0.688)  $\rightarrow P = 50.1\%$

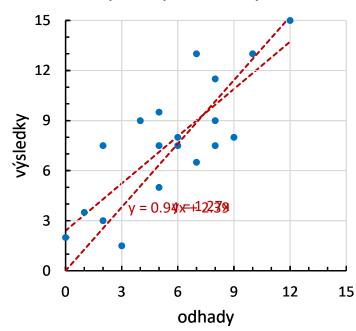
## Příklad 2a – korelace výsledků a odhadů

y − počet bodů z testu (odhad)

$$\hat{\mu}_y = 5.7 \qquad \hat{\sigma}_y = 3.2$$

$$\hat{\sigma}_{v} = 3.2$$

výsledky vs odhady



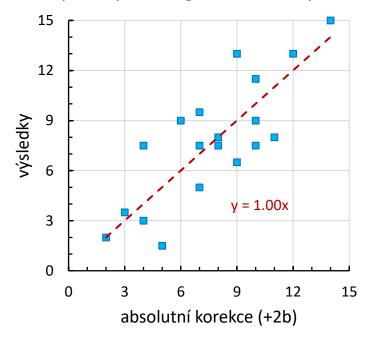
- korelace  $\hat{\rho}(x, y) = 0.8 \pm 0.1$
- Fisher  $(t = 4.021) \rightarrow P = 0.01\%$
- student  $(t = 4.878) \rightarrow P = 0.01\%$

 $y_1$  – korigovaný odhad (+2 body)

$$\hat{\mu}_{\nu_1} = 7.7$$

$$\hat{\mu}_{y_1} = 7.7$$
  $\hat{\sigma}_{y_1} = 3.2$ 

výsledky vs korigované odhady



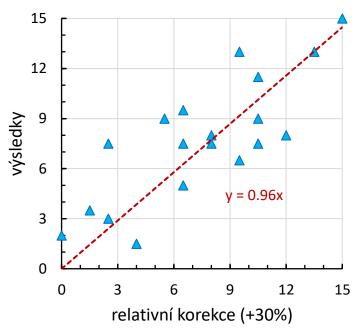
- korelace  $\hat{\rho}(x, y_1) = 0.8 \pm 0.1$
- Fisher  $(t = 4.021) \rightarrow P = 0.01\%$
- student  $(t = 4.878) \rightarrow P = 0.01\%$

 $y_2$  – korigovaný odhad (+30%)

$$\hat{\mu}_{y_2} = 7.5$$
  $\hat{\sigma}_{y_2} = 4.2$ 

$$\hat{\sigma}_{y_2} = 4.2$$

výsledky vs korigované odhady



- korelace  $\hat{\rho}(x, y_2) = 0.8 \pm 0.1$
- Fisher  $(t = 3.994) \rightarrow P = 0.01\%$
- student  $(t = 4.835) \rightarrow P = 0.02\%$

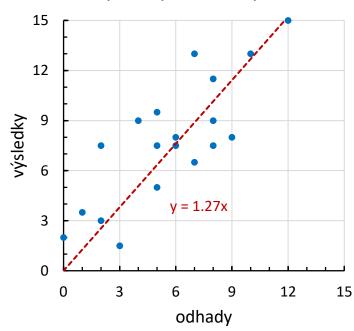
#### Příklad 2a – korelace výsledků a odhadů

y − počet bodů z testu (odhad)

$$\hat{\mu}_y = 5.7 \qquad \hat{\sigma}_y = 3.2$$

$$\hat{\sigma}_{v} = 3.2$$

výsledky vs odhady



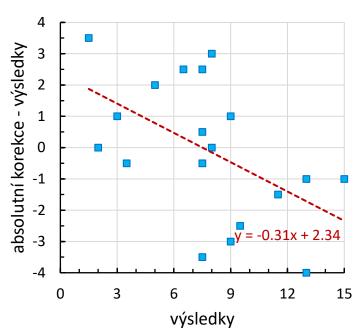
- korelace  $\hat{\rho}(x, y) = 0.8 \pm 0.1$
- Fisher  $(t = 4.021) \rightarrow P = 0.01\%$
- student  $(t = 4.878) \rightarrow P = 0.01\%$

•  $y_1$  – korigovaný odhad (+2 body)

$$\hat{\mu}_{\nu_1} = 7.7$$

$$\hat{\mu}_{y_1} = 7.7$$
  $\hat{\sigma}_{y_1} = 3.2$ 

absolutní korekce



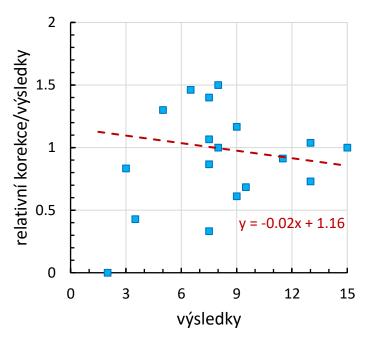
- korelace  $\hat{\rho}(x, y_1 x) = -0.5 \pm 0.2$
- Fisher  $(t = -2.188) \rightarrow P = 2.8\%$
- student  $(t = -2.370) \rightarrow P = 3.0\%$

 $y_2$  – korigovaný odhad (+30%)

$$\hat{\mu}_{y_2} = 7.5$$
  $\hat{\sigma}_{y_2} = 4.2$ 

$$\hat{\sigma}_{y_2} = 4.2$$

relativní korekce



- korelace  $\hat{\rho}(x, y_2/x) = -0.1 \pm 0.2$
- Fisher  $(t = -0.506) \rightarrow P = 61.3\%$
- student  $(t = -0.524) \rightarrow P = 60.7\%$

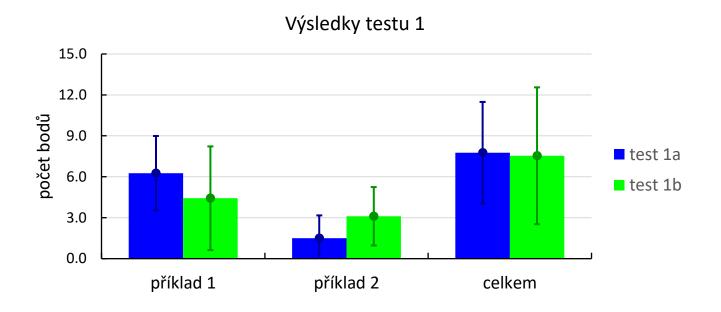
# Výsledky testu 1

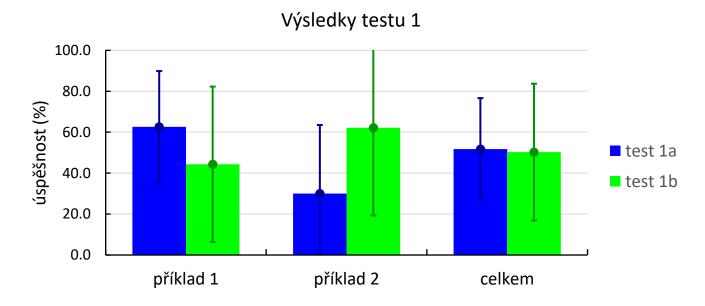
test 1a
 skupina 1 (29. 11., 9:00)

(8 ± 4) bodů

test 1b
 skupina 2 (30. 11., 8:10)

 $(8 \pm 5)$  bodů





- test 1b skupina 2 (30. 11., 8:10)
- x odhadovaný počet bodů

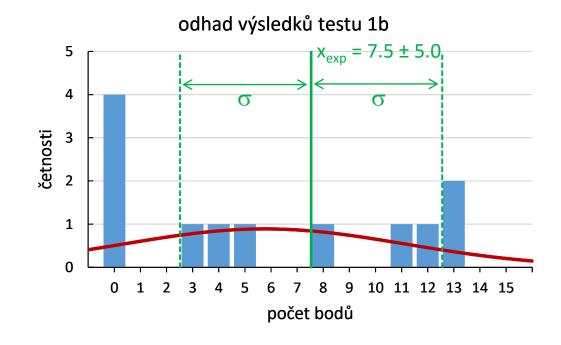
$$N = 12$$

$$\hat{\mu}_{x} = 5.8$$
  $\hat{\sigma}_{x} = 5.4$ 

histogram hodnot x

$$f(x) \approx \frac{N}{\sqrt{2\pi\hat{\sigma}^2}} \exp\left[-\frac{(x-\hat{\mu}_x)^2}{2\hat{\sigma}^2}\right]$$

$$\hat{\mu}_{exp} = 7.5 \qquad \hat{\sigma}_{exp} = 5.0$$



- test 1b skupina 2 (30. 11., 8:10)
- $\bar{x}$  odhadovaný průměrný počet bodů

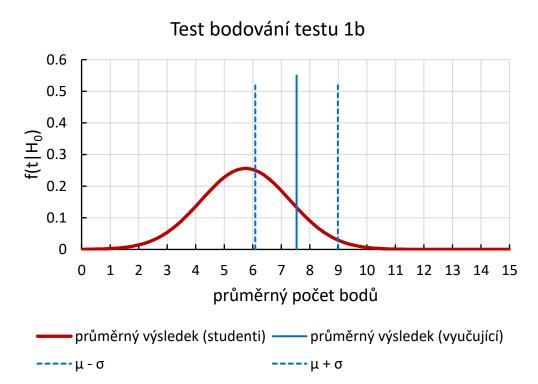
$$N = 12$$

$$\hat{\mu}_{\bar{x}} = 5.8 \qquad \hat{\sigma}_{\bar{x}} = \frac{\hat{\sigma}_{x}}{\sqrt{N}} = 1.6$$

hustota pravděpodobnosti x̄

$$f(\bar{x}) \approx \frac{N}{\sqrt{2\pi\hat{\sigma}_{\bar{x}}^2}} \exp\left[-\frac{(\bar{x} - \hat{\mu}_{\bar{x}})^2}{2\hat{\sigma}_{\bar{x}}^2}\right]$$

$$\hat{\mu}_{avg} = 7.5$$
  $\hat{\sigma}_{avg} = 1.5$ 

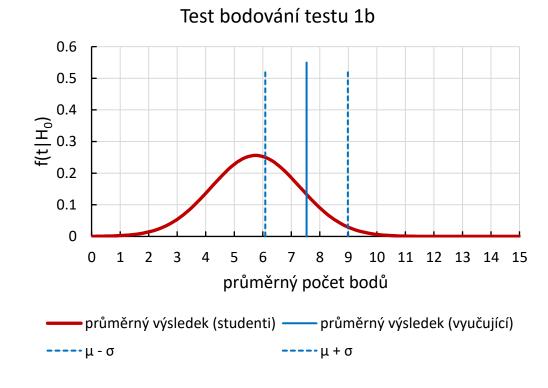


- test 1b skupina 2 (30. 11., 8:10)
- Nulová hypotéza H<sub>0</sub> Hodnocení testu bylo spravedlivé.
- testovací statistika ( $\hat{\mu}_{\bar{x}} = 5.8, \, \hat{\sigma}_{\bar{x}} = 1.6$ )

$$f(\bar{x}|H_0) \approx \frac{N}{\sqrt{2\pi\hat{\sigma}_{\bar{x}}^2}} \exp\left[-\frac{(\bar{x}-\hat{\mu}_{\bar{x}})^2}{2\hat{\sigma}_{\bar{x}}^2}\right]$$

testovací proměnná (t-hodnota)

 $t_3 = \hat{\mu}_{ava} + \hat{\sigma}_{ava} = 9.0$ 



$$t_{1} = \hat{\mu}_{avg} - \hat{\sigma}_{avg} = 6.1 \qquad P(\bar{x} > t_{1}) = 1 - F(\bar{x}|H_{0}) = \frac{1}{2} \left[ 1 - \text{erf}\left(\frac{t_{1} - \hat{\mu}_{\bar{x}}}{\sqrt{2}\hat{\sigma}_{\bar{x}}}\right) \right] = 41.4\%$$

$$t_{2} = \hat{\mu}_{avg} = 7.5 \qquad P(\bar{x} > t_{2}) = 1 - F(\bar{x}|H_{0}) = \frac{1}{2} \left[ 1 - \text{erf}\left(\frac{t_{2} - \hat{\mu}_{\bar{x}}}{\sqrt{2}\hat{\sigma}_{\bar{x}}}\right) \right] = 12.6\%$$

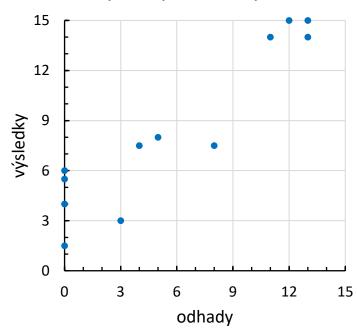
$$t_{3} = \hat{\mu}_{avg} + \hat{\sigma}_{avg} = 9.0 \qquad P(\bar{x} > t_{3}) = 1 - F(\bar{x}|H_{0}) = \frac{1}{2} \left[ 1 - \text{erf}\left(\frac{t_{3} - \hat{\mu}_{\bar{x}}}{\sqrt{2}\hat{\sigma}_{\bar{x}}}\right) \right] = 1.9\%$$

#### Příklad 2b – korelace veličin skupiny 1

x – počet bodů z testu (výsledek)

$$\hat{\mu}_{x} = 7.5 \qquad \hat{\sigma}_{x} = 5.0$$

výsledky vs odhady

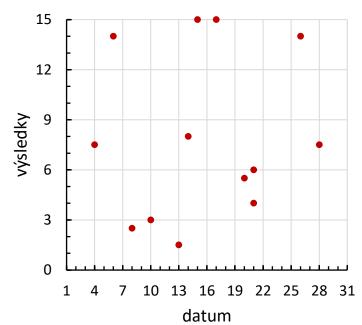


- korelace  $\hat{\rho}(x, y) = 0.85 \pm 0.08$
- Fisher  $(t = 3.998) \rightarrow P = 0.006\%$
- student  $(t = 5.402) \rightarrow P = 0.03\%$

y – počet bodů z testu (odhad)

$$\hat{\mu}_y = 5.8 \qquad \hat{\sigma}_y = 5.4$$

výsledky vs datum

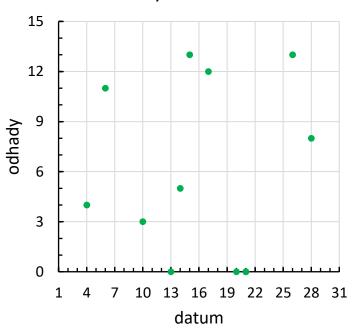


- korelace  $\hat{\rho}(x, y) = 0.1 \pm 0.3$
- Fisher  $(t = 0.353) \rightarrow P = 72.4\%$
- student  $(t = 0.371) \rightarrow P = 71.9\%$

• z – den narození

$$\hat{\mu}_z = 16$$
  $\hat{\sigma}_z = 7$ 

odhady vs datum



- korelace  $\hat{\rho}(x, y) = 0.0 \pm 0.3$
- Fisher  $(t = 0.060) \rightarrow P = 95.2\%$
- student  $(t = 0.063) \rightarrow P = 95.1\%$

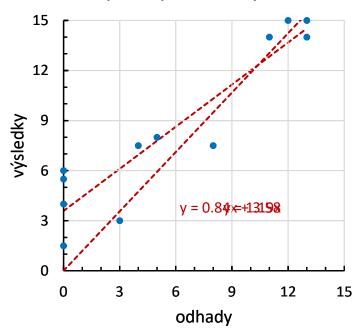
# Příklad 2b – korelace výsledků a odhadů

y − počet bodů z testu (odhad)

$$\hat{\mu}_{v} = 5.8 \qquad \hat{\sigma}_{v} = 5.4$$

$$\hat{\sigma}_{v} = 5.4$$

výsledky vs odhady



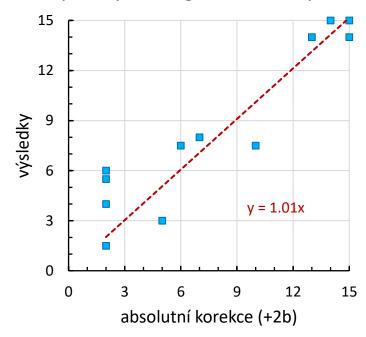
- korelace  $\hat{\rho}(x, y) = 0.85 \pm 0.08$
- Fisher  $(t = 3.998) \rightarrow P = 0.006\%$
- student  $(t = 5.402) \rightarrow P = 0.03\%$

•  $y_1$  – korigovaný odhad (+2 body)

$$\hat{\mu}_{\nu_1} = 7.8$$

$$\hat{\mu}_{y_1} = 7.8$$
  $\hat{\sigma}_{y_1} = 5.4$ 

výsledky vs korigované odhady



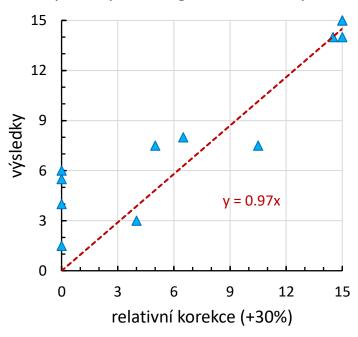
- korelace  $\hat{\rho}(x, y_1) = 0.85 \pm 0.08$
- Fisher  $(t = 3.998) \rightarrow P = 0.01\%$
- student  $(t = 5.402) \rightarrow P = 0.03\%$

 $y_2$  – korigovaný odhad (+30%)

$$\hat{\mu}_{y_2} = 7.1$$
  $\hat{\sigma}_{x_2} = 6.5$ 

$$\hat{\sigma}_{\chi_2} = 6.5$$

výsledky vs korigované odhady



- korelace  $\hat{\rho}(x, y_2) = 0.85 \pm 0.09$
- Fisher  $(t = 3.926) \rightarrow P = 0.01\%$
- student  $(t = 5.261) \rightarrow P = 0.04\%$

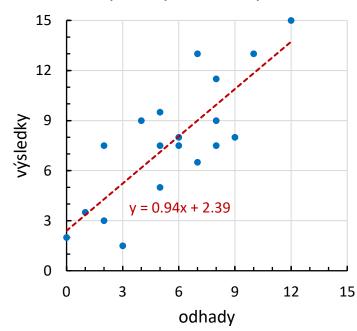
## Příklad 2b – korelace výsledků a odhadů

y − počet bodů z testu (odhad)

$$\hat{\mu}_{v} = 5.8 \qquad \hat{\sigma}_{v} = 5.4$$

$$\hat{\sigma}_{v} = 5.4$$

výsledky vs odhady



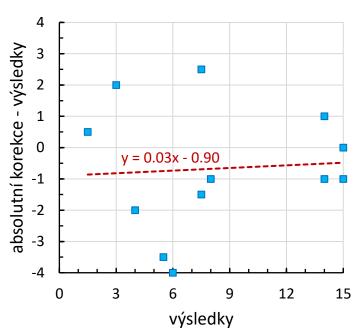
- korelace  $\hat{\rho}(x, y) = 0.85 \pm 0.08$
- Fisher  $(t = 3.998) \rightarrow P = 0.006\%$
- student  $(t = 5.402) \rightarrow P = 0.03\%$

•  $y_1$  – korigovaný odhad (+2 body)

$$\hat{\mu}_{\nu_1} = 7.8$$

$$\hat{\mu}_{y_1} = 7.8$$
  $\hat{\sigma}_{y_1} = 5.4$ 

absolutní korekce



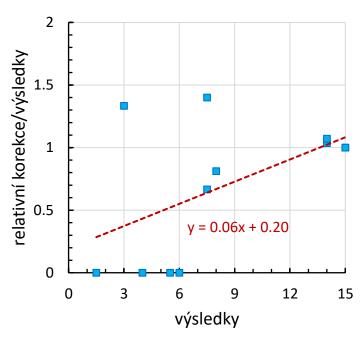
- korelace  $\hat{\rho}(x, y_1 x) = 0.4 \pm 0.3$
- Fisher  $(t = 1.346) \rightarrow P = 17.8\%$
- student  $(t = 1.455) \rightarrow P = 17.6\%$

 $y_2$  – korigovaný odhad (+30%)

$$\hat{\mu}_{y_2} = 7.1$$
  $\hat{\sigma}_{y_2} = 6.5$ 

$$\hat{\sigma}_{y_2} = 6.5$$

relativní korekce



- korelace  $\hat{\rho}(x, y_2/x) = 0.7 \pm 0.1$
- Fisher  $(t = 2.836) \rightarrow P = 0.5\%$
- student  $(t = 3.390) \rightarrow P = 0.07\%$