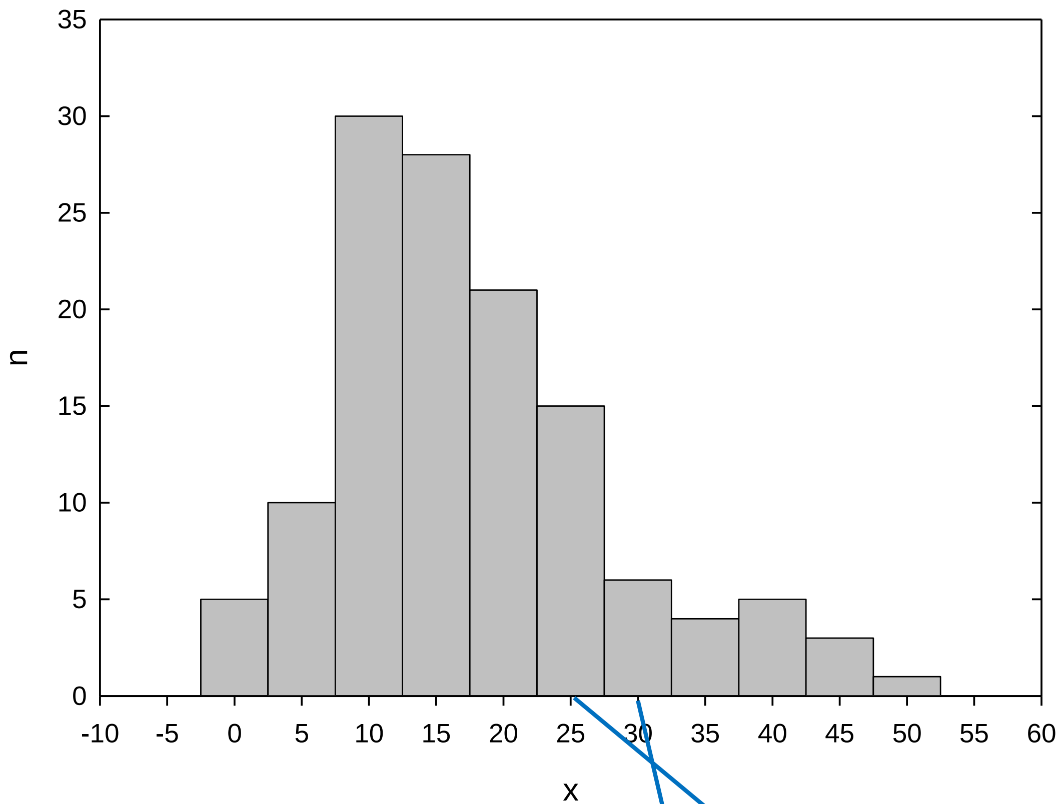
Histogram

- **histogram** – způsob, jak zjistit hustotu pravděpodobnosti z experimentálních dat



šířka binu: $\Delta_i = x_{i+1} - x_i$

plocha histogramu: $\sum_{i=1}^{m} n_i \Delta_i$

normalizovaný histogram: $n_i \rightarrow \xi_i$

$$\xi_i = \frac{n_i}{\Delta_i N} \qquad N = \sum_{i=1}^{m} n_i$$

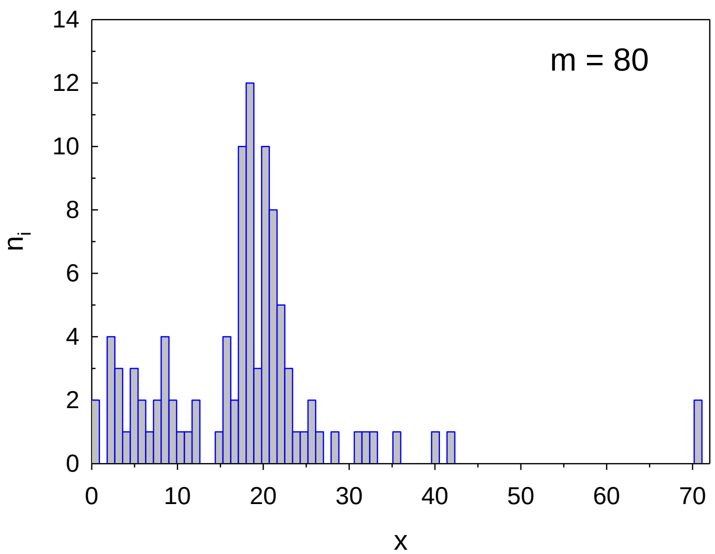
plocha normalizovaného histogramu:

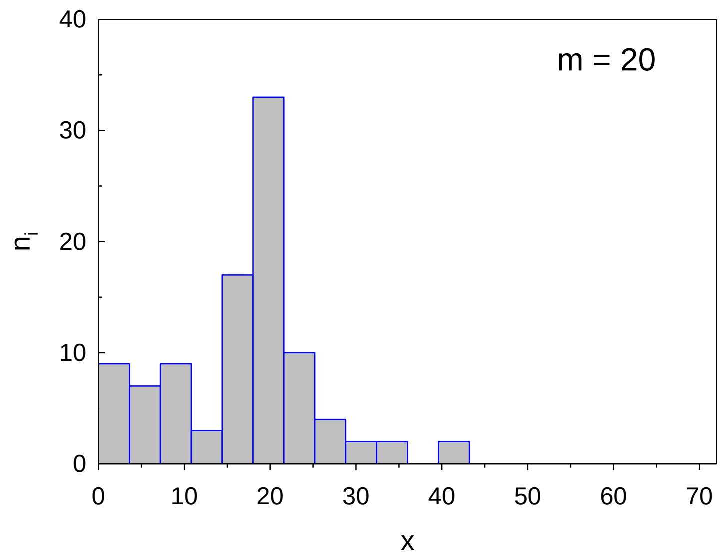
$$\sum_{i=1}^{m} \xi_i \Delta_i = 1$$

hustota pravděpodobnosti:

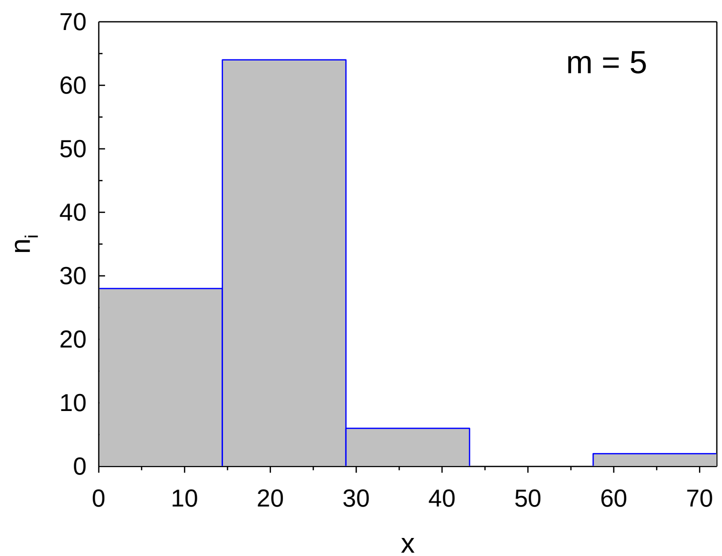
$$\boxed{f(x) = \lim_{\substack{\Delta_i \to 0 \\ N \to \infty}} \xi_i = \lim_{\substack{\Delta_i \to 0 \\ N \to \infty}} \frac{n_i}{\Delta_i N}}$$

Histogram – šířka binu





šířka binu $$\Delta = \frac{x_{\max} - x_{\min}}{m}$$



H. A. Sturges, J. American Statistical Association, 65–66 (1926).

$$m_{opt} = \frac{\log N}{\log 2} + 1$$

16.17759	18.75516	2.10586
22.36831	41.95208	8.60201
3.29369	19.23135	3.18462
17.96900	8.88075	15.66299
18.52658	32.60371	21.10663
17.63568	4.27135	2.28124
17.79473	18.43469	16.14332
39.80907	23.99716	35.88762
18.25682	18.94920	28.72841
20.63264	8.22661	0.17358
25.89910	17.88642	31.91945
17.57289	17.96704	70.80681
18.74632	20.07927	9.47664
8.46536	7.04639	23.20253
21.63599	12.39286	6.16414
31.43157	18.06331	15.65710
2.71104	17.36080	7.47195
9.89574	17.95492	20.18533
18.16503	7.71726	1.98676
20.18927	20.49528	17.71942
11.27086	21.00411	21.70207
2.49163	25.37069	21.28737
11.77613	21.77872	16.99344
0.25810	24.99534	18.19663
4.53349	21.43774	19.87326
21.22557	10.56477	5.84716
20.04356	4.50194	71.01371
18.79175	23.01736	18.09185
20.86614	20.48741	21.75327
17.80408	20.42592	17.09857
18.29748	20.22713	15.19833
18.08830	21.35032	19.04226
4.65786	26.23743	22.89348
21.52645		

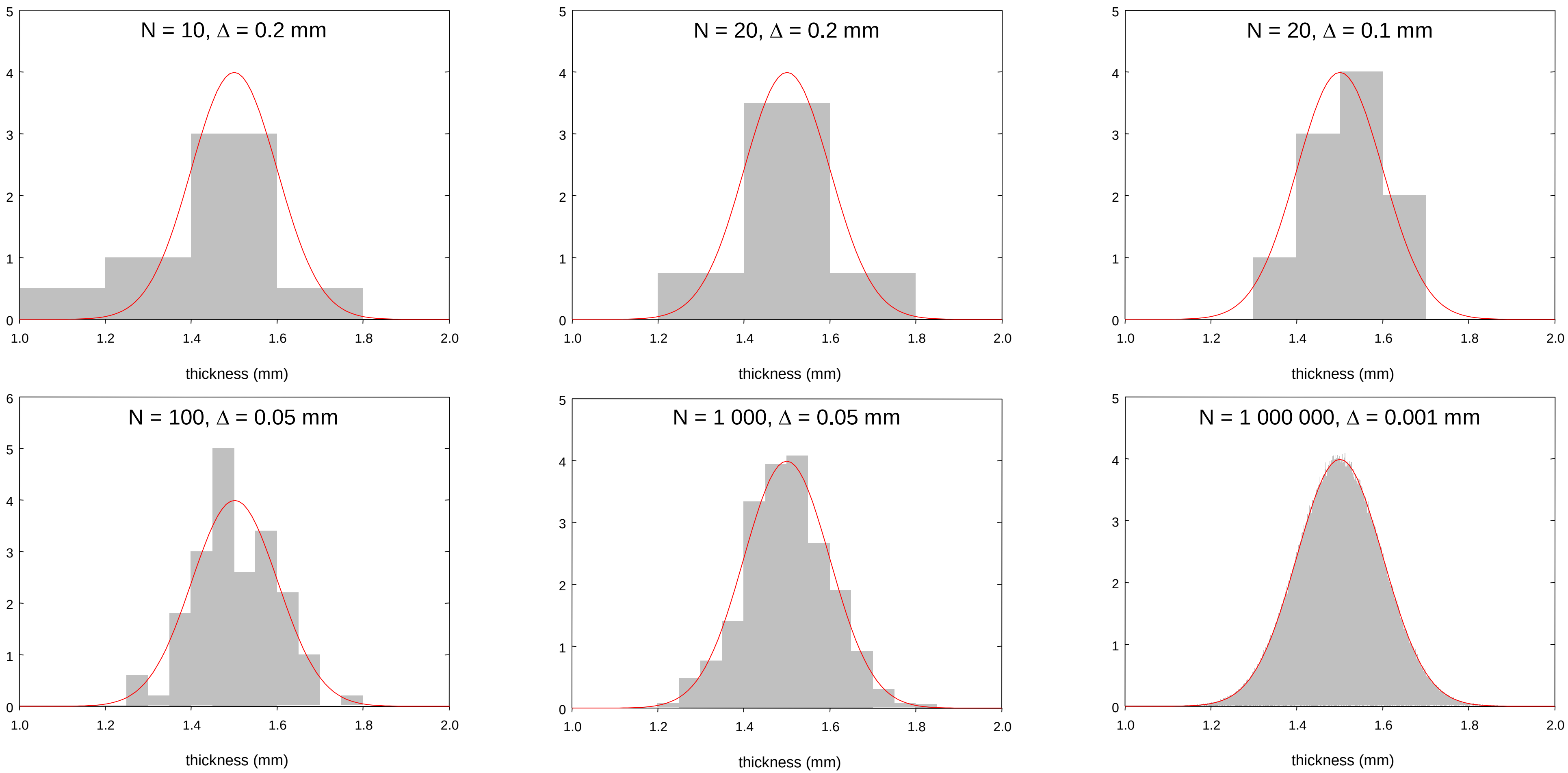
Hustota pravděpodobnosti – měření tloušťky vzorku

- hustota pravděpodobnosti
- počet naměřených hodnot *N*
- šířka binu Δ

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

μ = 1.5 mm, σ = 0.1 mm

$$f(x_i) \approx \frac{n_i}{N\Delta} \qquad N \to \infty, \Delta \to 0$$



Momenty

- operátor **střední (očekávané) hodnoty** $\mu = E[x]$

  diskrétní náhodná proměnná: $$\boxed{\mu \equiv E[x] = \sum_i x_i P_i}$$

  spojitá náhodná proměnná: $$\boxed{\mu \equiv E[x] = \int_{-\infty}^{\infty} x f(x) \mathrm{d}x}$$

- operátor **rozptylu (variance)** $\sigma^2 = V[x]$

  standardní odchylka: $\sigma = \sqrt{V[x]}$

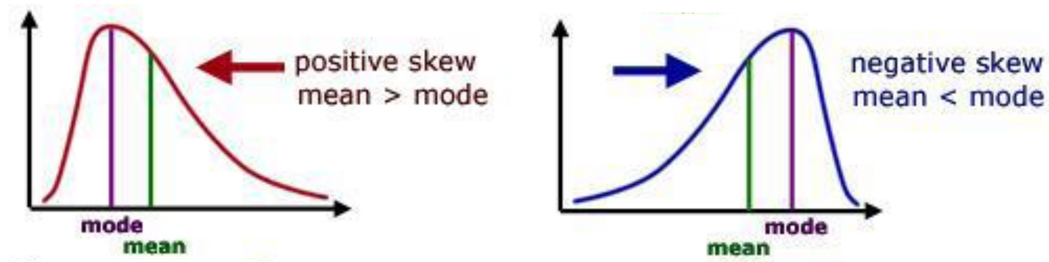
  druhý centrální moment: $$\boxed{\begin{aligned}\sigma^2 \equiv V[x] = \mu_2' &= E\left[(x-\mu)^2\right] \\ &= E\left[x^2\right] - (E[x])^2\end{aligned}}$$

- **n-tý moment** $\boxed{\mu_n = E\left[x^n\right]}$ $\mu_1 = \mu$
- **n-tý centrální moment** $\boxed{\mu_n' = E\left[(x-\mu)^n\right]}$ $\mu_1' = 0$ $\mu_2' = \sigma^2$

Momenty vyšších řádů

- **šikmost** (skewness)

$$\gamma_3 = \frac{\mu_3'}{\sigma^3} = \frac{E\left[(x-\mu)^3\right]}{\left(E\left[(x-\mu)^2\right]\right)^{\frac{3}{2}}}$$



- **špičatost** (kurtosis)

$$\gamma_4 = \frac{\mu_4'}{\sigma^4} - 3 = \frac{E\left[(x-\mu)^4\right]}{\left(E\left[(x-\mu)^2\right]\right)^2} - 3$$