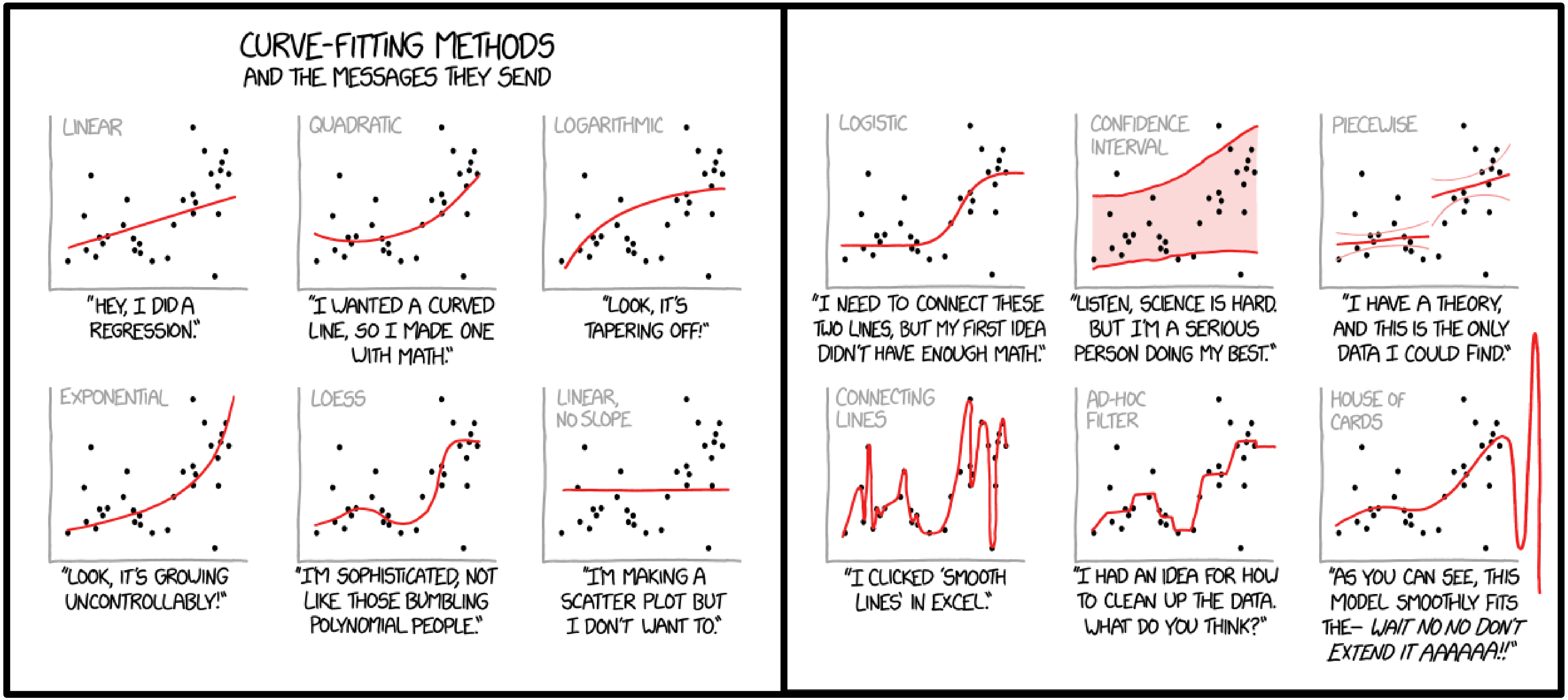
χ^2 test kvality fitu



xkcd.com/2048/

χ^2 test kvality fitu

- sada naměřených hodnot $\boldsymbol{x} = (x_1, x_2, \dots, x_n)$ (nezávislé proměnné)

  $\boldsymbol{y} = (y_1, y_2, \dots, y_n)$ (závislé proměnné) $y_i \in N(\mu_i, \sigma_i)$

- modelová funkce $\lambda(x|\boldsymbol{\theta})$ (modelujeme závislost $y(x)$)

  $\boldsymbol{\theta} = (\theta_1, \theta_2, \dots, \theta_m)$ (parametry modelové závislosti)

- náhodná proměnná χ^2
$$\chi^2(\boldsymbol{\theta}|\boldsymbol{y}, \boldsymbol{\sigma}, \boldsymbol{x}) = \sum_{i=1}^{n} \frac{\left(y_i - \lambda(x_i|\boldsymbol{\theta})\right)^2}{\sigma_i^2}$$

- testovací statistika $\chi^2 \in f_{\chi^2}(\nu)$ χ^2 rozdělení o $\nu = n - m$ stupních volnosti

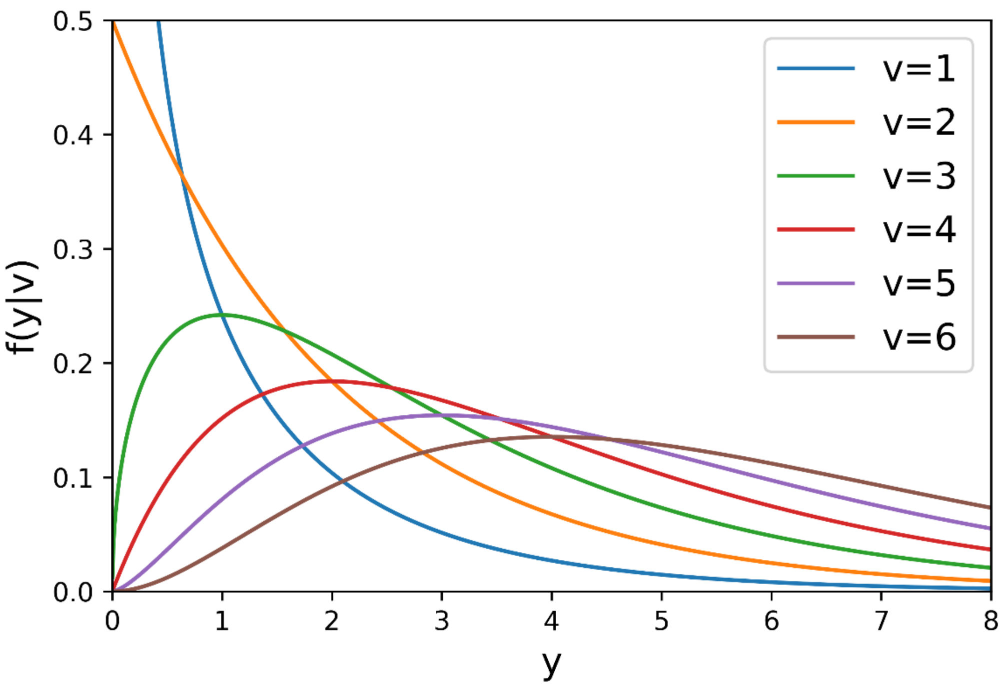
χ^2 rozdělení

- hustota pravděpodobnosti

$$\boxed{f(y|\nu) = \frac{1}{2^{\frac{\nu}{2}}\Gamma\left(\frac{\nu}{2}\right)} y^{\frac{\nu}{2}-1} e^{-\frac{y}{2}} \qquad y \in [0, \infty) \quad \nu = 1,2, \ldots}$$

počet stupňů volnosti ν

gama funkce $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} \mathrm{d}t$



- χ^2 rozdělení

$$z_i \in N(0,1)$$

$$y = \sum_{i=1}^{\nu} z_i \rightarrow y \in f(\nu) \text{ resp. } \chi^2(\nu)$$

$$x_i \in N(\mu_i, \sigma_i)$$

$$y = \sum_{i=1}^{\nu} \frac{(x_i - \mu_i)^2}{\sigma_i^2} \rightarrow y \in f(\nu) \text{ resp. } \chi^2(\nu)$$

- momenty χ^2 rozdělení

$$E[y] = \nu$$

$$V[y] = 2\nu$$

χ^2 test kvality fitu

- χ^2 rozdělení $\quad f(y|n-m) = \dfrac{1}{2^{\frac{n-m}{2}}\Gamma\left(\frac{n-m}{2}\right)} y^{\frac{n-m}{2}-1} e^{-\frac{y}{2}}$

$$\boxed{P[y \geq \chi_0^2] = \int_{\chi_0^2}^{\infty} f(y|n-m)\mathrm{d}y}$$

- počet stupňů volnosti $\nu = n - m$

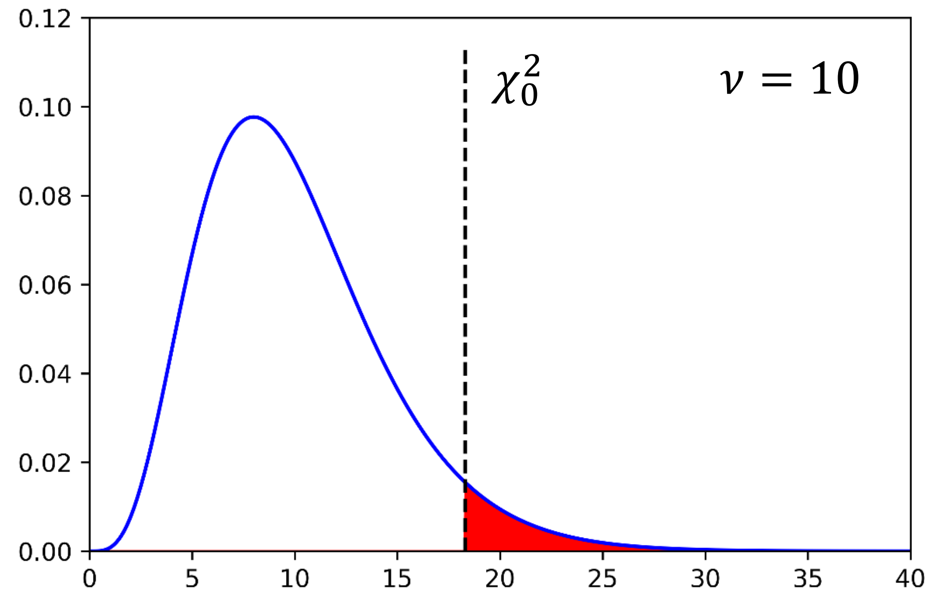
  $E[\chi^2] = n - m$

  $V[\chi^2] = 2(n - m\ )$

- χ^2 na počet stupňů volnosti $\chi^2/(n-m)$

  $E[\chi^2/(n-m)] = 1$

  $V[\chi^2/(n-m)] = 2/(n-m\ )$



- nulová hypotéza H_0 $\qquad$ naměřené hodnoty $y_i \in N(\mu_i, \sigma_i)$ jsou navzájem nezávislé a modelová funkce $\lambda(x, \boldsymbol{\theta})$ správně vystihuje závislost $y(x)$

- pokud je pravděpodobnost $P[y \geq \chi_0^2] < \alpha$, potom zamítneme nulovou hypotézu

- α hladina signifikance (typicky 0.05 nebo 0.01)

χ^2 test kvality fitu

- tabulka hodnot $P[y \geq \chi_0^2]$ pro počet stupňů volnosti $\nu = 1 - 10$

Počet stupňů volnosti ν	χ_0^2										
1	0.004	0.02	0.06	0.15	0.46	1.07	1.64	2.71	3.84	6.64	10.83
2	0.10	0.21	0.45	0.71	1.39	2.41	3.22	4.60	5.99	9.21	13.82
3	0.35	0.58	1.01	1.42	2.37	3.66	4.64	6.25	7.82	11.34	16.27
4	0.71	1.06	1.65	2.20	3.36	4.88	5.99	7.78	9.49	13.28	18.47
5	1.14	1.61	2.34	3.00	4.35	6.06	7.29	9.24	11.07	15.09	20.52
6	1.63	2.20	3.07	3.83	5.35	7.23	8.56	10.64	12.59	16.81	22.46
7	2.17	2.83	3.82	4.67	6.35	8.38	9.80	12.02	14.07	18.48	24.32
8	2.73	3.49	4.59	5.53	7.34	9.52	11.03	13.36	15.51	20.09	26.12
9	3.32	4.17	5.38	6.39	8.34	10.66	12.24	14.68	16.92	21.67	27.88
10	3.94	4.87	6.18	7.27	9.34	11.78	13.44	15.99	18.31	23.21	29.59
$P[y \geq \chi_0^2]$	0.95	0.90	0.80	0.70	0.50	0.30	0.20	0.10	0.05	0.01	0.001

Pro počet stupňů volnosti $\nu > 10$ rozdělení $\chi^2(\nu)$ konverguje k normálnímu rozdělení $N\left(\nu, \sqrt{2\nu}\right)$.

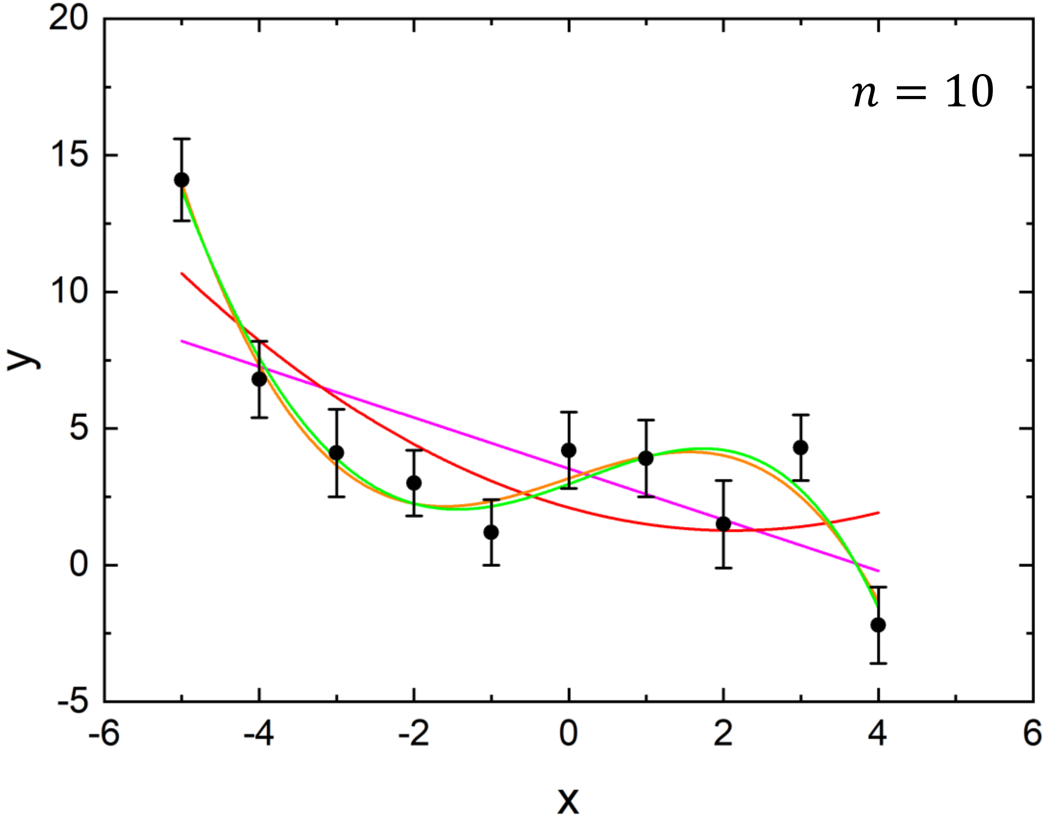
χ^2 test kvality fitu – polynom

- $\boldsymbol{m} = \mathbf{2}$ $(\nu = 8)$
- $\chi^2 = 40.916$
- $\chi^2/\nu = 5.114$
- $0.001 < P$

- $\boldsymbol{m} = \mathbf{3}$ $(\nu = 7)$
- $\chi^2 = 31.362$
- $\chi^2/\nu = 4.480$
- $0.001 < P$

- $\boldsymbol{m} = \mathbf{4}$ $(\nu = 6)$
- $\chi^2 = 7.174$
- $\chi^2/\nu = 1.196$
- $0.3 < P < 0.5$

- $\boldsymbol{m} = \mathbf{5}$ $(\nu = 5)$
- $\chi^2 = 6.939$
- $\chi^2/\nu = 1.388$
- $0.2 < P < 0.3$



χ^2 test kvality fitu – polynom

- $\boldsymbol{m = 2}$ $(\nu = 8)$
- $\chi^2 = 40.916$
- $\chi^2/\nu = 5.114$
- $0.001 < P$

- $\boldsymbol{m = 3}$ $(\nu = 7)$
- $\chi^2 = 31.362$
- $\chi^2/\nu = 4.480$
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- $\boldsymbol{m = 4}$ $(\nu = 6)$
- $\chi^2 = 7.174$
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- $\chi^2 = 6.939$
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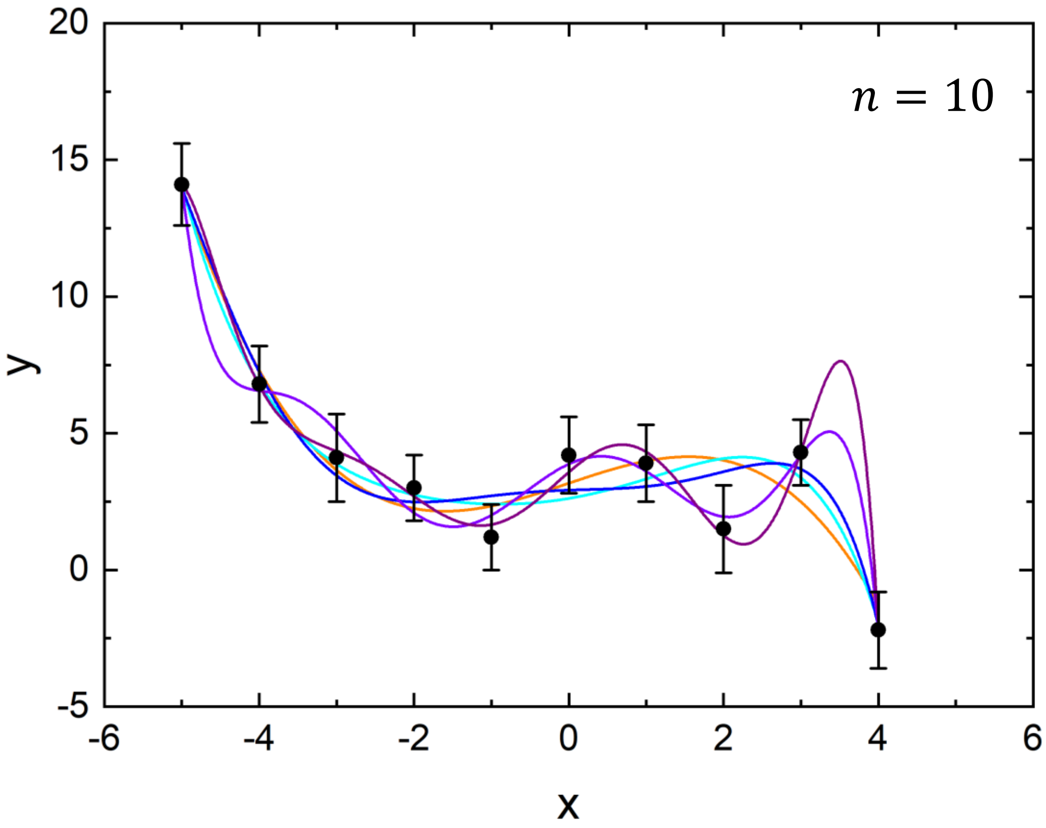

- $\boldsymbol{m = 6}$ $(\nu = 4)$
- $\chi^2 = 5.756$
- $\chi^2/\nu = 1.439$
- $0.2 < P < 0.3$

- $\boldsymbol{m = 7}$ $(\nu = 3)$
- $\chi^2 = 5.230$
- $\chi^2/\nu = 1.743$
- $0.1 < P < 0.2$

- $\boldsymbol{m = 8}$ $(\nu = 2)$
- $\chi^2 = 1.616$
- $\chi^2/\nu = 0.808$
- $0.3 < P < 0.5$

- $\boldsymbol{m = 9}$ $(\nu = 1)$
- $\chi^2 = 0.545$
- $\chi^2/\nu = 0.545$
- $0.3 < P < 0.5$