CausalitySolving for Why

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University of Lübeck

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Algorithmic Causality

The goals of the mini-course

- providing a gentle introduction to causal inference,
- presenting recent achievements in algorithmic causality, and
- describing new research directions, applications, new techniques, and challenging open problems.

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The major contributors to the field

- Judea Pearl (Turing Award, 2011)
- Peter Spirtes
- Clark Glymour
- Donald Rubin
- Jamie Robins
- Peter Bühlman
- Elias Bareinboim
- Joshua D. Angrist, Guido W. Imbens (Nobel in Economic Sciences, 2021)
- and many others...

What causal relationships can be established? In ...







... health?



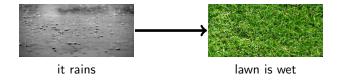
... Al-based systems?

Predict effect of

- ... the tax increase on economic growth
- ... a new drug have on the recovery
- ... the decision of an autonomous vehicle

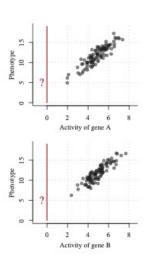






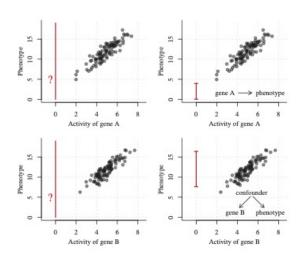
correlation is not causation

Example: Gene Perturbation¹



 $¹_{
m J.}$ Peters, D. Janzing, and B. Schölkopf. Elements of causal inference: foundations and learning algorithms. MIT press, 2017

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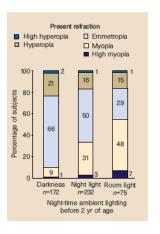


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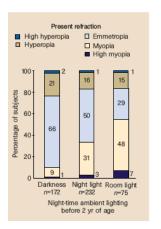
Central Problems

- Useful graphical representations for causal models
- Learning causal structure from data
- Mow to compute interventions?
- How can we deal with the problem of hidden variables?
- Oan we test counterfactual statements?
- Is causality useful?

Myopia and ambient lighting at night



Myopia and ambient lighting at night



"... the strength of the association ... does suggest that the absence of a daily period of darkness during childhood is a potential precipitating factor in the development of myopia"

Quinn, Shin, Maguire, Stone: Myopia and ambient lighting at night, Nature 1999

Myopia and ambient lighting at night

Patents

Night light with sleep timer US20050007889A1, United States

Abstract A timer a light and an optional music source is located on or in a housing of a nightlight assembly. When this assembly is plugged into a source of electric power, the timer is set to a selected time for the light and optional music to remain on. After this selected time has elapsed, the light and music automatically turns off, allowing for sleep in appropriate darkness and silence.

II BACKGROUND OF THE INVENTION: Many young children prefer the comfort of a nightlight when they go to bed, but their caregivers would prefer them to sleep in darkness. A University of Pennsylvania study, Quinn, G. E., Shin, C. H., Maguire, M. G. and Stone, R. A. Myopia and ambient lighting at night. Nature, 399: 113-114, 1999 (May 13,1999), has shown that children who sleep with a light on may have a higher risk of developing nearsightedness as they get older.

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Q: Does the night light with sleep timer decreases the risk of myopia?

Kidney stone treatments

- Effectiveness of kidney stone treatments
- Relevant factors: Treatment (A or B), Recovery (Y or N)

| Stones | Treatment A | Treatment B |
|---------|---------------------------|---------------------------|
| Overall | $\frac{273}{350} = 0.780$ | $\frac{289}{350} = 0.826$ |
| | | |
| | | |

[Charig et al. British Medical Journal, 1986]

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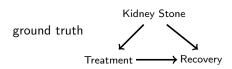
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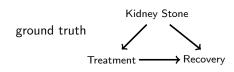


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[Charig et al. British Medical Journal, 1986]



Q: What is the expected recovery if all get treatment B, independently on size?

Smoking

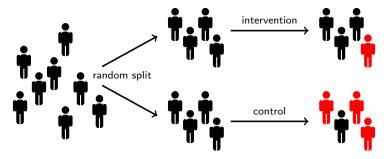
Richard Doll and A. Bradford Hill, *Smoking and Carcinoma of the Lung*, Br Med J. 1950 Sep 30; 2(4682): 739–748

TABLE VII.—Estimate of Total Amount of Tobacco Ever Consumed by Smokers; Lung-carcinoma Patients and Control Patients with Disease Other Than Cancer

| | No. Who have Smoked Altogether | | | | n | |
|--|--------------------------------|----------------|-----------------|-----------------|--------------------|---|
| Disease Group | 365 Cigs | 50,000 Cigs | 150,000 Cigs | 250,000 Cigs | 500,000 Cigs. + | Probability Test |
| Males: Lung-carcinoma patients (647) Control patients | (2-9%) | 145 (22·4%) | 183 (28·3%) | 225 (34-8%) | 75 (11·6%) | χ ² = 30·60; n = 4; P < 0·001 |
| with diseases other than cancer (622). | 36 (5·8%) | 190 (30-5%) | 182 (29-3%) | 179 (28·9%) | 35 (5·6%) | |
| Females: Lung-carcinoma patients (41). | 10 (24-4%) | 19 (46·3%) | (12.2%) | (17-1%) | (0-0%) | χ ² =12-97; n=2; 0-001 < P < |
| Control patients with diseases other than cancer (28) | 19 (67-9%) | (17-9%) | (10-7%) | (3.6%) | (0.0%) | 0.01 (Women smoking 15 or more cig arettes a day grouped to gether) |

- Causal relationships in complex systems:
 - Does smoking cause lung cancer?
 - Did drug X cure the patient?
 - Predict effect of the tax increase on economic growth
 - Predict effect of the decision of an autonomous vehicle

- Causal relationships in complex systems:
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- Direct experimentation

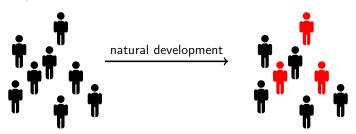


- Causal relationships in complex systems:
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 - ethically problematic
 - expensive
 - impossible

- Causal relationships in complex systems:
 - Does smoking cause lung cancer?
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 - Predict effect of the decision of an autonomous vehicle
- Direct experimentation
 - ethically problematic
 - expensive
 - impossible
- On the other hand, there are often available large amounts of observed data that can provide relevant information about these issues

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58
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                                                                   14
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L198V
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M216L
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L225V
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                                                                                                         6,8
                               3.6
                                                            1.6
                                                                  8.9
                                                                                     [Aoto et al., Scientific reports 6 (2016)]
```

- Causal relationships in complex systems:
 - Does smoking cause lung cancer?
 - Did drug X cure the patient?
 - Predict effect of the tax increase on economic growth
 - Predict effect of the decision of an autonomous vehicle
- Direct experimentation
 - ethically problematic
 - expensive
 - impossible
- Discovering of causal effects from observed data

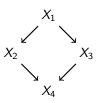


Structural Equations with Noise Distribution

• SCMs model observational distributions over X_1, \ldots, X_n

$$X_1 := F_1(N_1)$$

 $X_2 := F_2(X_1, N_2)$
 $X_3 := F_3(X_1, N_3)$
 $X_4 := F_4(X_2, X_3, N_4)$

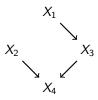


Structural Equations with Noise Distribution

• SCMs can model interventions, as e.g.

$$X_1 := F_1(N_1)$$

 $X_2 := 1$
 $X_3 := F_3(X_1, N_3)$
 $X_4 := F_4(X_2, X_3, N_4)$

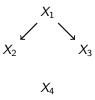


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$$X_1 := F_1(N_1)$$

 $X_2 := F_2(X_1, N_2)$
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 $X_4 := 11$

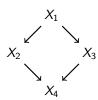


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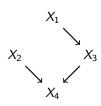


Structural Equations with Noise Distribution

• SCMs can model interventions, denoted as $P_{do(X_2:=1)}$

$$X_1 := F_1(N_1)$$

 $X_2 := 1$
 $X_3 := F_3(X_1, N_3)$
 $X_4 := F_4(X_2, X_3, N_4)$



Structural Equations with Noise Distribution

• Note that in general, $P(\ldots \mid X_4=11) \neq P_{do(X_4:=11)}(\ldots)$

$$X_1 := F_1(N_1)$$
 $X_2 := F_2(X_1, N_2)$
 $X_3 := F_3(X_1, N_3)$
 $X_4 := 11$
 $X_1 := X_1$
 $X_2 := X_3$
 $X_4 := X_4$

Graph is a DAG

Identification of Causal Effects From Observed Data: Kidney Stones

| Stones | Treatment A | Treatment B |
|---|---------------------------|---------------------------|
| Overall | $\frac{273}{350} = 0.780$ | $\frac{289}{350} = 0.826$ |
| Small $\left(\frac{357}{700} = 0.51\right)$ | $\frac{81}{87} = 0.931$ | $\frac{234}{270} = 0.867$ |
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Q: What is the expected recovery if all get treatment B, independently on stone size?

• SCM models observational distributions P over S, T, R

$$S := F_1(N_1)$$

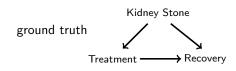
 $T := F_2(S, N_2)$
 $R := F_3(S, T, N_3)$



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Q: What is the expected recovery if all get treatment B, independently on stone size?

• Task: compute interventional distribution $P_{do(T:=B)}(R=1)$

$$S := F_1(N_1)$$

 $T := B$
 $R := F_3(S, T, N_3)$



Identification of Causal Effects From Observed Data: Kidney Stones

$$S := F_1(N_1)$$

$$T := B$$

$$R := F_3(S, T, N_3)$$

$$T \longrightarrow R$$

$$P_{do(T:=B)}(R = 1)$$

$$= \sum_{s} P_{do(T:=B)}(R = 1, S = s, T = B)$$

$$= \sum_{s} P_{do(T:=B)}(R = 1 \mid S = s, T = B)P_{do(T:=B)}(S = s, T = B)$$

$$= \sum_{s} P_{do(T:=B)}(R = 1 \mid S = s, T = B)P_{do(T:=B)}(S = s)$$

$$= \sum_{s} P(R = 1 \mid S = s, T = B)P(S = s)$$

$$= 0.782$$

$$< 0.832$$

$$= ...$$

$$P_{do(T:=A)}(R = 1)$$

Identification of Causal Effects From Observed Data: Smoking

- Does smoking (S) cause lung cancer (C)?
- Assume that the variables are binary, taking on true (1) or false (0) values
- Moreover, assume the following (hypothetical) data set from a study on the relations among cancer and cigarette smoking

| | Group Type | % of Population | % of Cancer cases |
|-------|------------|-----------------|-------------------|
| S=0 | Nonsmokers | 50 | 9.75 |
| S = 1 | Smokers | 50 | 85.25 |

• Task: compute the causal effects $P_{do(S:=1)}(C=1)$ from data in this model

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$$S \longrightarrow C$$
 $P_{do(S:=s)}(C=c) = P(c \mid s)$ Smoking Cancer

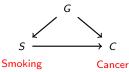
- $P_{do(S:=0)}(C=1) = 0.0975$
- $P_{do(S:=1)}(C=1) = 0.8525$

Identification of Causal Effects From Observed Data: Smoking

- Does smoking (S) cause lung cancer (C)?
- Assume, to forestall antismoking legislation, the tobacco industry has argued that the
 observed correlation between smoking and lung cancer could be explained by some sort of
 carcinogenic genotype that involves inborn craving for nicotine
- Thus, consider in our model the relevant factor: Genotype (G)
- Unfortunately, the feature is not measurable (called also unobserved)
- Can the causal effects $P_{do(S:=1)}(C=1)$ be estimated from data in this model?

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Genotype (unobserved)

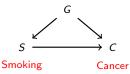


Identification of Causal Effects From Observed Data: Smoking

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- Thus, consider in our model the relevant factor: Genotype (G)
- Unfortunately, the feature is not measurable (called also unobserved)
- Can the causal effects $P_{do(S;=1)}(C=1)$ be estimated from data in this model?

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Genotype (unobserved)



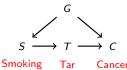
$$P_{do(S:=s)}(C=c)$$
 non-computable :(

Identification of Causal Effects From Observed Data: Smoking

ullet Task: compute the causal effects $P_{do(S:=s)}(C=1)$ from data:

| | | | P(s,t) | $P(C=1 \mid s,t)$ |
|-------|-------|--------------------|-----------------|-------------------|
| | | Group Type | % of Population | % of Cancer cases |
| S=0 | T=0 | Nonsmokers, No tar | 47.5 | 10 |
| S = 1 | T = 0 | Smokers, No tar | 2.5 | 90 |
| S = 0 | T = 1 | Nonsmokers, Tar | 2.5 | 5 |
| S = 1 | T = 1 | Smokers, Tar | 47.5 | 85 |

Genotype (unobserved)

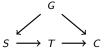


Identification of Causal Effects From Observed Data: Smoking

• Task: compute the causal effects $P(C = 1 \mid do(s))$ from data:

| | | | P(s,t) | $P(C=1 \mid s,t)$ |
|-------|-------|--------------------|-----------------|-------------------|
| | | Group Type | % of Population | % of Cancer cases |
| S=0 | T=0 | Nonsmokers, No tar | 47.5 | 10 |
| S = 1 | T = 0 | Smokers, No tar | 2.5 | 90 |
| S=0 | T = 1 | Nonsmokers, Tar | 2.5 | 5 |
| S=1 | T=1 | Smokers, Tar | 47.5 | 85 |

Genotype (unobserved)



Smoking Tar Cancer

In our course we will show that the following formula can be used:

$$P_{do(s)}(c) = \sum_t P(t \mid s) \sum_{s'} P(c \mid s', t) P(s')$$

•
$$P_{do(S:=0)}(C=1) = .95(.10 \times .50 + .90 \times .50) + .05(.05 \times .50 + .85 \times .50)$$

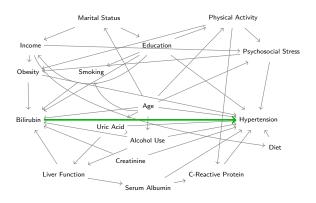
= $.95 \times .50 + .05 \times .45 = .4975$

•
$$P_{do(S:=1)}(C=1) = .05(.10 \times .50 + .90 \times .50) + .95(.05 \times .50 + .85 \times .50)$$

= .05 × .50 + .95 × .45 = .4525

Causal inference

Serum bilirubin and the risk of hypertension analysis with dagitty



Conclusions: High serum bilirubin may decrease the risk of hypertension [Wang et al., Int J Epidemiol 2015]

Causal inference

More motivations

- Al-controlled robots need to know the effects of their actions before they act.
- Autonomous vehicles can be trained faster and more effectively if they distinguish cause and effect instead of acting randomly.
- Exploratory scientific studies try to derive the causes of effects from observations.

Causal inference

Challenging problems

- Estimation of causal effects
- Identification problem
- Learning causal structures
- ...

Algorithmic Causality with Applications

- Backgrounds conditional independence, marginalization, covariance, regression
- Causal structures, causal models
 DAGs, d-separation, d-paths, colliders, CI and d-separation, Markov equivalence, latent / hidden variables
- Causal effects: direct and total effects Intervention
 modelling of controlled experiments, do-operator, do-calculus: calculus of intervention,
 Simpson's paradox
- Estimation of causal effects
 covariate adjustment, Pearl's back door criterion, complete back door criterion, front-door
 criterion classes of Markov equivalent structures (CPDAGS, PAGS, chain graphs, etc.)
- Causal structural learning learning from lists of Cls (Verma/Pearl, Meek's), learning from Cl tests (IC algorithm, PC algorithm, FCl,...
- Structural equations, linear structural equation models (SEMs)
 inference in linear systems, parameter identification, estimation of causal effects in SEMs
- Counterfactual inference