Task No. 8

May 17, 2020

Base of subspace: $\{1, \mathbb{X}, \mathbb{X}^2\}$. The best approximation of Y in that subspace is $w^*(x) = a + bx + cx^2$ such that a, b, c satisfy the equation:

$$\begin{bmatrix} \langle 1, 1\rangle & \langle 1, \mathbb{X}\rangle & \langle 1, \mathbb{X}^2\rangle \\ \langle \mathbb{X}, 1\rangle & \langle \mathbb{X}, \mathbb{X}\rangle & \langle \mathbb{X}, \mathbb{X}^2\rangle \\ \langle \mathbb{X}^2, 1\rangle & \langle \mathbb{X}^2, \mathbb{X}\rangle & \langle \mathbb{X}^2, \mathbb{X}^2\rangle \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \langle 1, \mathbb{Y}\rangle \\ \langle \mathbb{X}, \mathbb{Y}\rangle \\ \langle \mathbb{X}^2, \mathbb{Y}\rangle \end{bmatrix}$$

Now by expanding scalar products we get:

$$\begin{bmatrix} \sum 1 \cdot 1 & \sum 1 \cdot x_i & \sum 1 \cdot x_i^2 \\ \sum x_i \cdot 1 & \sum x_i \cdot x_i & \sum x_i \cdot x_i^2 \\ \sum x_i^2 \cdot 1 & \sum x_i^2 \cdot x_i & \sum x_i^2 \cdot x_i^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum 1 \cdot y_i \\ \sum x_i \cdot y_i \\ \sum x_i^2 \cdot y_i \end{bmatrix}$$

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \end{bmatrix}$$