Probability & Statistics

Problem set №10. May 2020

[Ex. 1–6] The data is the observations $x_1, \dots, x_n$ from the following below distributions. Find MLE estimator of listed parameters:

1. Geometric distribution Geom(p), parameter p.
2. Pareto distribution, $f(x; a, k) = \dfrac{ka^k}{x^{k+1}}$, $x \in (a, \infty)$, k known, parameter a.
3. Pareto distribution, $f(x; a, k) = \dfrac{ka^k}{x^{k+1}}$, a known, parameter k.
4. Exponential distribution, $f(x; \lambda) = \lambda \exp(-\lambda x)$, for $x \in (0, \infty)$. Parameter λ.
5. Weibull distribution, $f(x; k, \lambda) = \dfrac{k}{\lambda} \cdot \left(\dfrac{x}{\lambda}\right)^{k-1} \exp\left\{-\left(\dfrac{x}{\lambda}\right)^k\right\}$, where $x \in (0, \infty)$. k known, parameter λ.

[Ex. 6–7] Independent random variables X, Y are of distribution: $\chi^2(n), \chi^2(k)$ respectively.

6. (0.5p) Find 2-dimensional density of random variable (X, Y).
7. (3p.) Find density of random variable $F = \dfrac{X}{Y} \cdot \dfrac{k}{n}$.
8. Points $(x_1, y_1), \dots, (x_n, y_n)$ are given. We are looking for a regression coefficients in the form of $y = a + bx + cx^2$. Justify that the parameters a, b, c are solution of the system of linear equations:

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \end{bmatrix}.$$

9. Given are points $(x_1, y_1, z_1), \dots, (x_n, y_n, z_n)$. Consider regression coefficients in the form of $z = a + bx + cy$. Justify that the parameters a, b, c are solution of the system of linear equations:

$$\begin{bmatrix} n & \sum x_i & \sum y_i \\ \sum x_i & \sum x_i^2 & \sum x_i y_i \\ \sum y_i & \sum x_i y_i & \sum y_i^2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum z_i \\ \sum x_i z_i \\ \sum y_i z_i \end{bmatrix}.$$

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10. **E2** Find the regression coefficients Y with respect to$X$:

x_k	1	3	4	6	8	9	11	14
y_k	1	2	4	4	5	7	8	9

11. **E2** The table below contains data on the pressure of P and the volume V of a certain constant mass of gas. The equation that combines these two values is $PV^k = C$, where k, C are some constants.

Volume V	54.3	61.8	72.4	88.7	118.6	194.0
Pressure P	61.2	49.5	37.6	28.4	19.2	10.1

Using linear regression find values of C and k. What is the predicted value of P when $V = 100$?

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