Task 2

May 11, 2020

$$y_{k+1} = y_k + x_{k+1} \wedge y_k \geqslant 0 \wedge x_{k+1} \in [0, \infty) \;\Rightarrow\; y_k \in [0, y_{k+1}]$$

$$\begin{aligned}
f_{Y_n}(y_n) &= \int_0^{y_n} \int_0^{y_{n-1}} \cdots \int_0^{y_2} \lambda^n e^{-\lambda y_n} \, dy_1 \, dy_2 \ldots dy_{n-1} = \\
&= \lambda^n e^{-\lambda y_n} \int_0^{y_n} \int_0^{y_{n-1}} \cdots \int_0^{y_2} 1 \, dy_1 \, dy_2 \ldots dy_{n-1} = \\
&= \lambda^n e^{-\lambda y_n} \int_0^{y_n} \int_0^{y_{n-1}} \cdots \int_0^{y_3} y_2 \, dy_2 \ldots dy_{n-1} = \\
&= \lambda^n e^{-\lambda y_n} \int_0^{y_n} \int_0^{y_{n-1}} \cdots \int_0^{y_4} \frac{y_3^2}{2} \, dy_3 \ldots dy_{n-1} = \\
&= \lambda^n e^{-\lambda y_n} \int_0^{y_n} \int_0^{y_{n-1}} \cdots \int_0^{y_5} \frac{y_4^3}{3!} \, dy_4 \ldots dy_{n-1} = \\
&= \ldots = \lambda^n e^{-\lambda y_n} \int_0^{y_n} \frac{y_{n-1}^{n-2}}{(n-2)!} \, dy_{n-1} = \\
&= \lambda^n e^{-\lambda y_n} \frac{y_{n-1}^{n-1}}{(n-1)!} \bigg|_0^{y_n} = \lambda^n e^{-\lambda y_n} \frac{y_n^{n-1}}{(n-1)!}
\end{aligned}$$