Algorithmic Causality with Applications

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Exercise Sheet 2 (Deadline 27.01.2023)

Sheet Objectives

- Getting familiar with interventional distributions.
- Reaching familiarity with the causal identification task.
- Understanding the notion of CPDAGs, i.e. concise graphical representations of Markov equivalence classes.
- Getting to know algorithms for the learning of causal structures from conditional independence statements.

1. Validity of the interventional distribution, hard, 4 points

Prove that the interventional distribution $P(v_1, ..., v_n \mid do(x))$ (see definition, lecture slides on the topic *Causal BNs and Structural Causal Models*) is correctly defined as a probability distribution, i.e. show that:

$$\sum_{v_1,...,v_n} P(v_1, ..., v_n \mid do(x)) = 1.$$

To simplify matters, you may restrict yourself to the case that X is a single variable.

2. Identification with non-observed variables, easy, 4 points

Recall the definition of the identification problem. Obviously, if we have in a DAG $G = (\mathbf{V}, \mathbf{E})$ that $\mathbf{R} = \mathbf{V}$, then for every $\mathbf{X}, \mathbf{Y} \subseteq \mathbf{V}$ the causal effect of $\mathbf{X}$ on $\mathbf{Y}$ in G is identifiable. Show that this is also true in the case that the non-observed variables, i.e. the variables $\mathbf{V} \setminus \mathbf{R}$ contain only sink nodes in G.

3. The class of CPDAGs, easy, 2 points

Give a partially directed acyclic graph (i.e., a graph which may contain undirected and directed edges, but no directed cycle) with three nodes which is *not* a CPDAG.

4. The PC-algorithm and Meek's rules, easy, 4 points

Let P be a Markovian and faithful (to some underlying true DAG) probability distribution over variables $\mathbf{X} = \{X_1, X_2, X_3, X_4\}$ which entails *only* the conditional independence $(X_2 \perp\!\!\!\perp X_3 \mid X_4)$ – meaning all other statements are dependencies.

1. Find the CPDAG G representing the corresponding Markov equivalence class for P using the PC-algorithm.

   It suffices to give the graphs obtained after each of the three phases *Finding the Skeleton*, *Finding v-Structures* and *Propagate Orientations* together with appropriate explanations.

2. List all DAGs which are in the Markov equivalence class represented by the CPDAG G from part 1 of this task.

3. Using your insights from the previous tasks, prove the soundness of Meek's third rule.