Problem set 3°

Jakub Kuciński, Pratik Ghosal

Wrocław, March 14, 2020

Contents

1 Problem 9° **1**
1.1 a) 1
1.2 b) 1

1 Problem 9°

1.1 a)

As X has uniform distribution and $x \in [-2, 2]$ then we have $f_X(x) = \frac{1}{4}$ and consequently $F_X(s) = \int_{-2}^{s} \frac{1}{4}\, dx = \frac{s+2}{4}$.
$Y = |X|$
$t \in [0, 2]$
$F_Y(t) = P(Y < t) = P(|X| < t) = P(-t < X < t)$
$F_Y(t) = P(X < t) - P(X < -t) = F_X(t) - F_X(-t)$.
$F_Y(t) = F_X(t) - F_X(-t) = \int_{-2}^{t} \frac{1}{4}\, dx - \int_{-2}^{-t} \frac{1}{4}\, dx = \frac{t+2}{4} - \frac{-t+2}{4} = \frac{t}{2}$.
As $(F_Y(t))' = f_Y(t)$ we get $f_Y(t) = \left(\frac{t}{2}\right)' = \frac{1}{2}$.

1.2 b)

As X has uniform distribution and $x \in [-1, 1]$ then we have $f_X(x) = \frac{1}{2}$ and consequently $F_X(s) = \int_{-1}^{s} \frac{1}{2}\, dx = \frac{s+1}{2}$.
$Y = X^3$
$t \in [-1, 1]$
$F_Y(t) = P(Y < t) = P(X^3 < t) = P(X < \sqrt[3]{t}) = F_X(\sqrt[3]{t})$
$(F_Y(t))' = (F_X(\sqrt[3]{t}))'$
$f_Y(t) = f_X(\sqrt[3]{t})\frac{1}{3\sqrt[3]{t^2}} = \frac{1}{2} \cdot \frac{1}{3\sqrt[3]{t^2}} = \frac{1}{6\sqrt[3]{t^2}}$

$Z = X^2$
$t \in [0, 1]$
$F_Z(t) = P(Z < t) = P(X^2 < t) = P(-\sqrt{t} < X < \sqrt{t}) = P(X < \sqrt{t}) - P(X < -\sqrt{t}) =$
$= F_X(\sqrt{t}) - F_X(-\sqrt{t})$

$F_Z(t) = F_X(\sqrt{t}) - F_X(-\sqrt{t})$
$(F_Z(t))' = (F_X(\sqrt{t}) - F_X(-\sqrt{t}))'$
$f_Z(t) = f_X(\sqrt{t}) \cdot \frac{1}{2\sqrt{t}} + f_X(-\sqrt{t}) \cdot \frac{1}{2\sqrt{t}} = \frac{1}{2\sqrt{t}}$