

Probability & Statistics

Problem set №3. Week starting March 23th

1. A and B are events such that $P(A \cap B) = 1/4$, $P(A^C) = 1/3$, $P(B) = 1/2$. Find $P(A \cup B)$.
2. Is it true that 13. day of the month is connected with Friday? (January 1, 1601 – December 31, 2000)
EXPLANATION: Year n is a leap year if $n \equiv_4 0$, with the exception of years divisible by 100 ($n \not\equiv_{100} 0$); unless $n \equiv_{400} 0$ (i.e. year 2000). How many times in 400-year cycle 13. day of the month was Monday, Tuesday, ..., Sunday?

Random variables X, Y are independent, iff, in discrete case, condition $P(X = x_i, Y = y_k) = P(X = x_i) \cdot P(Y = y_k)$ holds.

3. R.v. X has binomial distribution $B(n_1, p)$ and r.v. Y $B(n_2, p)$ distribution. X, Y are independent. Prove that $Z = X + Y$ has $B(n_1 + n_2, p)$ distribution.
4. Independent r.v.s. X, Y have Poisson distribution with parameters λ_1 i λ_2 . Prove that r.v. $Z = X + Y$ has Poisson distribution with parameter $\lambda_1 + \lambda_2$.

Density of r.v. (X, Y) has form $f(x, y) = 3xy$ on area bounded by $y = 0$, $y = x$, $y = 2 - x$.

5. Find marginal densities $f_1(x), f_2(y)$.
6. Calculate expected value of Y . Check if r.v. X, Y are independent.
7. Probability of success in independent trials equals p . We perform trials until 3 successes occur. R.v. X is equal to number of performed trials. Find distribution of X , i.e. find density function (probabilities) and expected value X .
8. Readable and thoroughly - without using the notes - write upper and lower Greek letters: alpha α , beta β , zeta ζ , eta η , lambda λ , chi χ , xi ξ , phi ϕ , rho ρ .
9. (a) Let $X \sim U[-2, 2]$. Find distribution of $Y = |X|$.
(b) Given $X \sim U[-1, 1]$ find distributions of $Y = X^3$, $Z = X^2$.
10. Let X be r.v. with geometric distribution ($X \sim \text{Geom}(p)$). Check that $V(X) = \frac{1-p}{p^2}$.
11. Cardinality of sets A_1, \dots, A_4 is equal – respectively – 40, 32, 20, 50. An element (from set of 142 elements) is randomly chosen. Cardinality of the set from which chosen element was taken is the value of random variable X . Next a set is randomly chosen. Cardinality of the chosen set is the value of random variable Y . Find $E(X)$ i $E(Y)$.

Witold Karczewski