Probability & Statistics

Problem set №12. June 2020

Symbol $z \equiv z_{(.)}$ means from now – until revocation – quantile of $N(0, 1)$ distribution. Additionally: exercises 1–10 are classes exercises (1 point each), while exercises 11–16 are of **(E1)** category each.

1. Rejection area is defined by $z > 2$. Significance level α equals: a) 0.2280 b) 0.0228 c) 0.0500 d) 0.1000

2. Significance level α which corresponds rejection area $|z| > 1.55$ equals: a) 0.5500 b) 0.0606 c) 0.1211 d) 0.1234

3. Level of significance $\alpha = 0.075$. Rejection area of the left-tailed hypothesis is given by: a) $z < -1.34$ b) $z < -1.38$ c) $z < -1.40$ d) $z < -1.44$.

4. Find p-value when $z = 2.34$ and $H_a : \mu \neq \mu_0$: a) 0.0096 b) 0.0101 c) 0.0193 d) 0.0202

5. Find p-value when $z = -3.05$, $H_a : \mu < \mu_0$: a) 0.0011 b) 0.0111 c) 0.0038 d) 0.0001

6. Give p-value when $z = 1.89$, $H_a : \mu > \mu_0$: a) 0.0588 b) 0.1234 c) 0.0249 d) 0.0669

7. Tested hypothesis $H_0 : \mu = 10$, alternative hypothesis $H_a : \mu \neq 10$, significance level $\alpha = 0.01$. For which of the following 99% confidence interval μ initial hypothesis is rejected? a) (12.1, 15.3) b) (8.8 12.5) c) (5.5, 15.5) d) (9.9 10.5)

8. Hypothesis about expected value is tested, based on a large sample.

```
Test of mu = 6 vs < 6
The assumed standard deviation = 0.476
                                          95%
                                          Upper
Variable N     Mean      StDev   SEMean   Bound      Z       P
Z         500  5.51912   0.47608 0.02129  5.55413  -22.59  0.000
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Which of the following are true? a) Hypothesis H_0 is one-tailed. b) Value of the test statistics equals -22.59. c) Sample size is $n = 500$. d) Tested value of μ_0 equals 5.51912.

9. During the experiment, the response time to the neurological stimulus was measured.

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One-Sample T: time
Test of mu = 1.5 vs not = 1.5
Variable N Mean     StDev   SEMean      95%CI            T     P
time     15 1.97087 0.55233 0.14261  (1.66500,2.27674)  3.30  0.005
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Which of the following are true? a) Tested hypothesis is two-tailed. b) Complement to 1 of $t(14)$ cdf in point $t = 3.30$ is equal 0.005. c) We have no reason to reject H_0 hypothesis, with significance level $\alpha = 0.05$, because 1.5 does not fit with 95% confidence interval (1.665, 2.277). d) 15 is the size of the sample.

10. Below are results of YES-NO question.

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             Test of p = 0.4 vs p not = 0.4

Sample  X   N    Sample p  95%CI                 Z-Value  P-Value
  1    180 400  0.450000 (0.401247,0.498753)  2.04     0.041
```

Which of the following are true? a) 400 persons were surveyed, 180 answers is YES. b) Tested hypothesis was one-sided. c) Significance level is $\alpha = 0.05$. d) cdf in point 2.04, has the value 0.041.

11. **(E1)** 500 people were questioned, the question was of YES-NO type.

	A	B	C	D	E	F
1	0.25	Frequency of YES answer	'=125/500			
2	0.0194	standard error	'=SQRT(C1*(1-C1)/500)			
3	1.96	quantile of N(0,1)	'=NORM.INV(0.975,0,1)			
4						
5	0.212	left side of confidence interva	'=C1-C2*C3			
6	0.288	right side of confidence interv	'=C1+C2*C3			
7						
8	2.7951	value of test statistics	'=(C1-0.2)/SQRT(0.2*0.8/500)			
9	0.0052	p-value	'=2*(1-NORM.DIST(C8,0,1, 1))			
10						

Which of the following are true? a) Tested parameters has the value $p_0 = 0.20$ b) 0.0052 is one-sided p-value. c) Tested p_0 does not fit in 95% confidence interval. d) Because n is large, $np_0 \geqslant 5, nq_0 \geqslant 5$, we can approximate binomial distribution by normal distribution.

12. **(E1)** We hypothesize that the standard deviation of the variable is less than 5.

	A	B	C	D	E	F
1	70	3.437758	Standard deviation	=STDEV(A1:A12)		
2	73	5.2	Value of test statistics	=11*B1^2/25		
3	70					
4	72	0.078905	p-value	=CHISQ.DIST(B2,11,1)		
5	74					
6	70	p-value > 0.05, do not reject H0 hypothesis				
7	74					
8	76					
9	75					
10	80					
11	74					
12	80					

Which of the following are true? a) Sample S^2 has the value 3.43776. b) If significance level is $\alpha = 0.05$ we accept hypothesis H_0. c) Area under density of $\chi^2(11)$ distribution is 0.078905. d) Test is right-tailed.

13. **(E1)** The speed of 100 cars was measured. The 95% confidence interval for standard deviation is determined below.
Which of the following are true? a) $S^2 = 10.64818$ b) Confidence interval of σ^2 is $(87.40710, 153.0102)$ c) Confidence interval of σ is equal $(9.349181, 12.36973)$ d) Area under the density function of $\chi^2(99)$ distribution on the interval $(73.3611, 128.4219)$ equals 0.99.

14. **(E1)** Significance level of tested hypothesis $\alpha = 0.05$. Which p-value results in rejecting initial hypothesis: a) p-value= 0.05 b) p-value= 0.14 c) p-value= 0.024 d) p-value= 0.34.

15. **(E1)** The goal of testing hypotheses is a) describing samples, b) describing the population, c) inferring about the population based on samples, d) inferring about samples based on the population.

	A	B	C	D	E
1					
2	10.64818	Standard deviation	=STDEV(A15:J24)		
3	113.3838	Variance	=A2^2		
4					
5	73.3611	Quantile 0.975	=CHISQ.INV(0.025,99)		
6	128.4219	Quantile 0.025	=CHISQ.INV(0.975,99)		
7					
8	87.40719	Left side of CI for variance	=99*A2/A6		
9	153.0102	Right side of CI for variance	=99*A2/A5		
10					
11	9.349181	Left side of st deviation	=sqrt(A8)		
12	12.36973	Right side of st deviation	=sqrt(A9)		
13					
14					

16. **(E1)** We perform t-test about two means. With this, we assume that: a) samples are independent b) samples come from population of normal distribution c) samples come from $t(n_1 + n_2 - 2)$ distribution d) samples are of the same distribution.

Hereby – I revoke the assumption that z means a quantile of $N(0,1)$ distribution.

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