

# Probability & Statistics

## Problem set №5. Week starting April 6<sup>th</sup>

1. Continuous random variable  $(X, Y)$  has density function  $f(x, y)$ . Prove that

$$E(X + Y) = E(X) + E(Y).$$

2.  $X$  is a discrete random variable. Sequences  $\{x_i\}, \{p_i\}$  – values and probabilities of the r.v. Prove that if  $Y = aX + b$  then  $V(Y) = a^2V(X)$ ,  $(a, b \in \mathbb{R})$ .

3. Random variable  $X$  has standard normal distribution, i.e.  $f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$ ,  $x \in \mathbb{R}$ .  
(In short:  $X \sim N(0, 1)$ ). Find distribution (density  $f_Y(y) \equiv g(y)$ ) of variable  $Y = X^2$ .

4. Check that  $\Gamma(1/2) = \sqrt{\pi}$ . (HINT: In exercise 1.3 substitution  $t = x^2/2$  and compare with exercise 1.6)

5. R.v.  $X$  has Gamma distribution with parameters  $b, p > 0$  (Abb.:  $X \sim \text{Gamma}(b, p)$ ) if and only if  $f(x) = \frac{b^p}{\Gamma(p)} x^{p-1} \exp(-bx)$ , where  $x \in (0, \infty)$ . Does  $Y$  from exercise 3. has Gamma distribution? If answer is positive give the values of parameters  $b, p$ .

6. Random variable  $X$  has standard normal distribution  $X \sim N(0, 1)$ . Let  $\sigma > 0, \mu \in \mathbb{R}$ . Find distribution (density) of variable  $Y = \sigma X + \mu$ .

7. **2p.** Variable  $(X, Y)$  has density fncion of the form  $f(x, y) = xy$ , in area  $[0, 2] \times [0, 1]$ . Find cummulative distribution function of this variable. In other words find formula(e) of the integral

$$F_{XY}(s, t) = \int_{-\infty}^s \int_{-\infty}^t xy \, dy \, dx.$$

8. **2p.**  $(X, Y)$  like in exercise 7. Find distribution of  $Z = X + Y$ .
9. Variable  $(X, Y)$  is of continuous type, variables  $X, Y$  are independent. Prove that  $\text{Cov}(X, Y) = 0$ .

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