# Statistical Learning

Adaptive LASSO and SLOPE

1. Consider again the Problem 5 from list 2 and the model

$$Y = X\beta^{k_{ID}} + \epsilon \ ,$$

with $\beta_1 = \ldots = \beta_{k_{ID}} = 20$.

a) Find the value of the tuning parameter $\lambda$ for which LASSO MSE is minimal and use $\beta^L$ to denote the corresponding LASSO estimate of $\beta$.

b) Calculate the number of true and false discoveries for this selection of $\lambda$.

c) Run the adaptive LASSO with weights $w_i = \frac{1}{|\beta_i^L|+0.000001}$. Select the tuning parameter so as MSE is minimal and compare this value with the optimal value of MSE for LASSO (from point a)).

d) Calculate the number of true and false discoveries for the above selection of $\lambda$ for adaptive LASSO and compare to the values obtained in point b).

2. Generate the matrix $X$ of dimension $100 \times 150$ such that its rows come from the multivariate normal distribution with the covariance $\Sigma$ with $\Sigma[i,i] = 1$ and for $i \neq j$, $\Sigma[i,j] = 0.8$. Then generate the response vector $Y$ such that

$$Y = X\beta + \epsilon \ ,$$

with $\beta_1 = \ldots = \beta_{50} = 50$ and $\sigma \sim N(0, I)$.

Find the value of the tuning parameter for LASSO such that MSE is optimal. Do the same for SLOPE with the 'BH' sequence of the tuning parameters at the FDR level $fdr = 0.8$ an with the tuning parameter $\alpha$ adjusted to obtain the minimal MSE. Compare this MSE to the MSE obtained by LASSO.

3. Generate $X_1, \ldots, X_{100}$ random vectors from the multivariate normal distribution $N(0, \Sigma)$, with the covariance matrix $Sigma_{30\times 30}$ consisting of 3 diagonal blocks of the dimension $10 \times 10$. In each block the diagonal elements are all equal to 1 and off-diagonal elements are all equal to 0.7. Estimate the precision matrix using

a) Inverse of the sample covariance matrix.

b) gLASSO adjusted such that the MSE is optimal.

Compare MSE for these two approaches.

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