Social Networks & Recommendation Systems

II. Historical overview of the complex network science. Examples of the real-life networks.

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MSc program in Data Science has been developed
as a part of task 10 of the project
„NERW PW. Science - Education - Development - Cooperation”
co-funded by European Union from European Social Fund.

Before classes

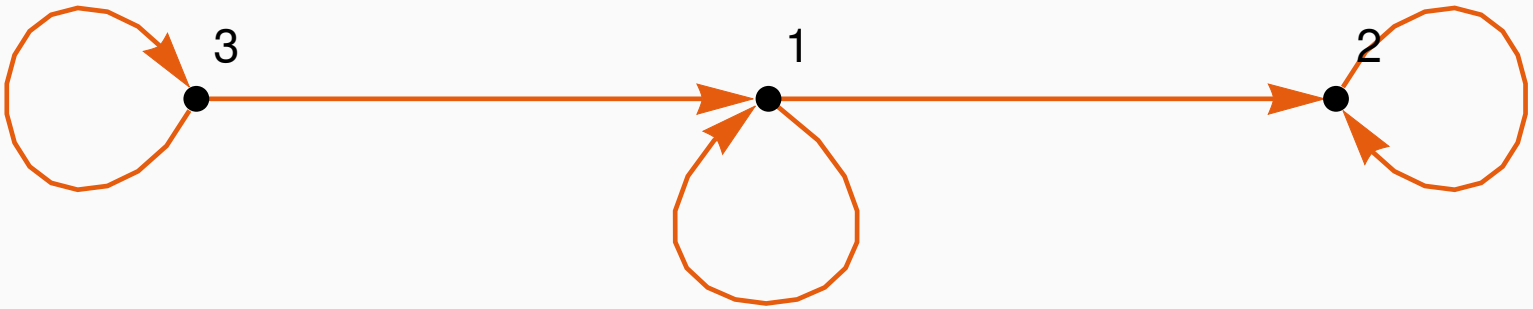
Remember: graph representation methods

Adjacency matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

Adjacency list

$$L = \{\{1,2\},\{2\},\{1,3\}\}.$$



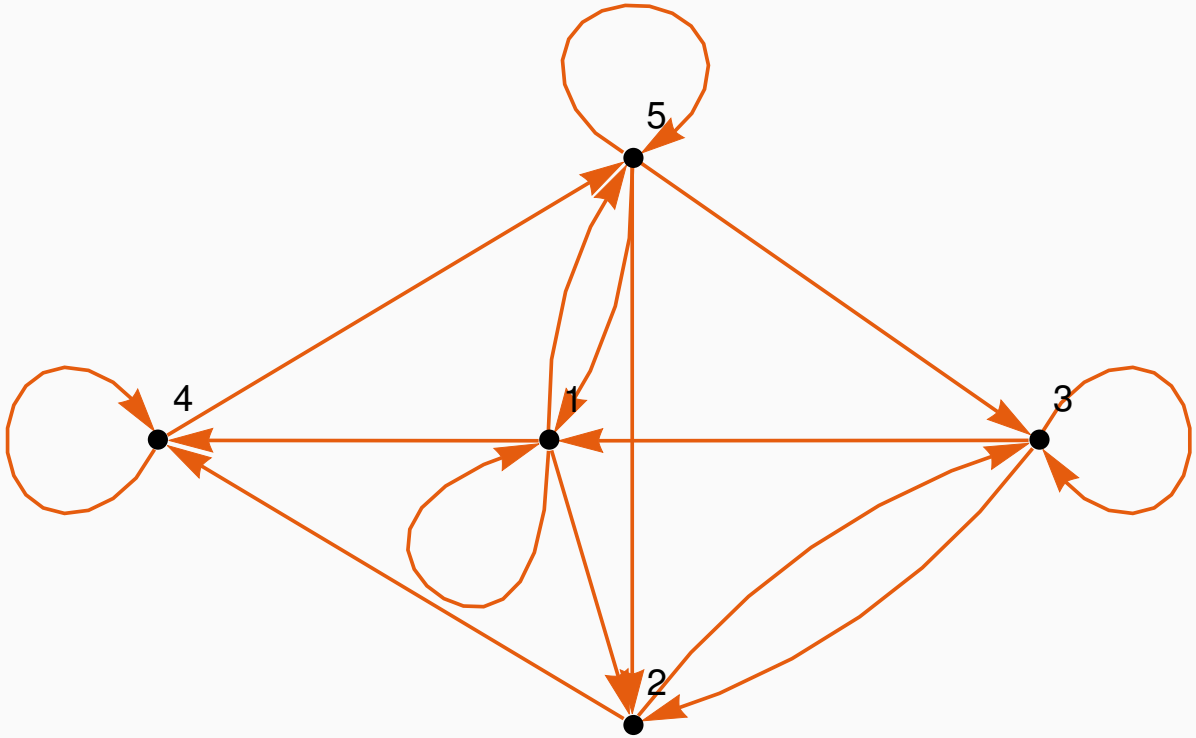
Remember: vertex degree

Vertex degree

Number of incoming or outcoming edges.

$$k_{out} = \{4, 2, 3, 2, 4\},$$

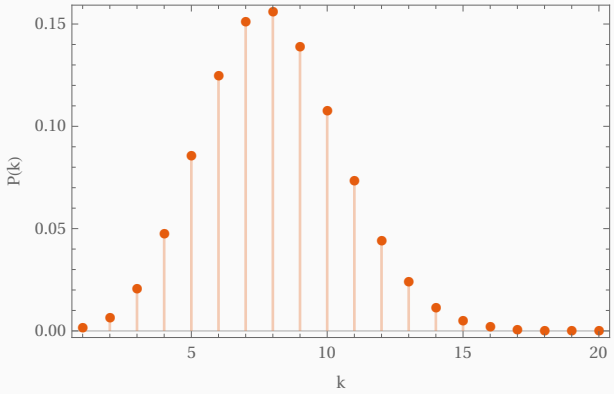
$$k_{in} = \{3, 3, 3, 3, 3\}.$$

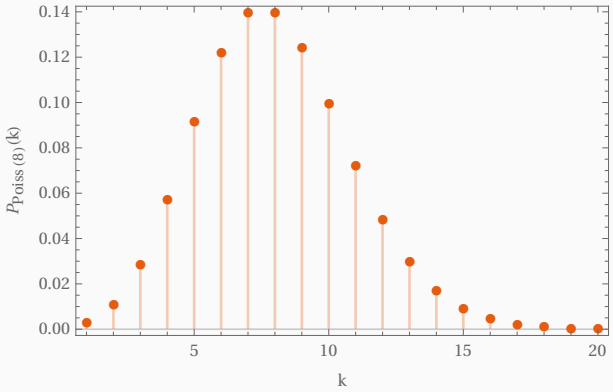


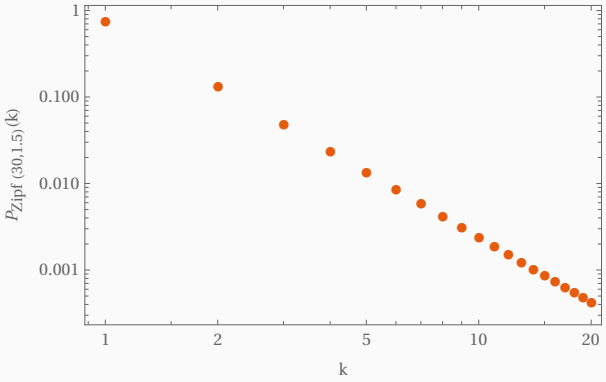
Remember: discrete probability distributions

Discrete probability distributions – examples

- binomial distribution,
- Poissona distribution,
- Zipf distribution.







$$P(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$P(k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$P(k) = \frac{1/k^s}{H_{N,s}}$$

Reminder

$$\text{Binom}(N, p) \to \text{Poiss}(Np)$$

Exercise to think about - continuous distributions

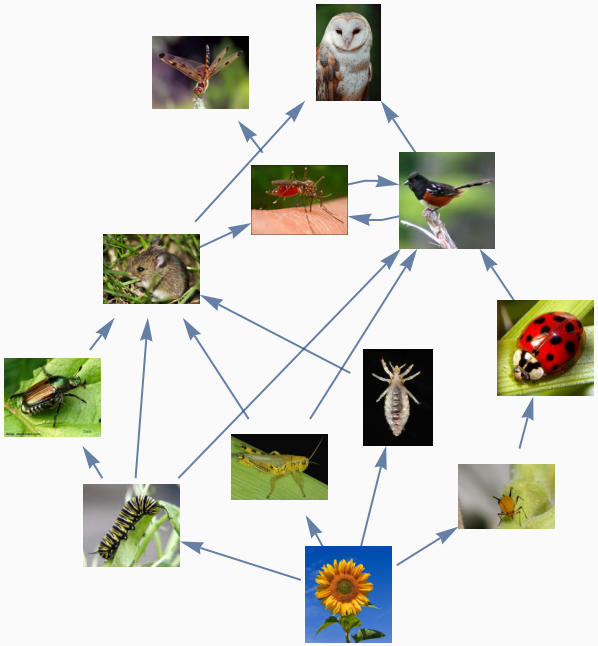
In complex networks, you often replace discrete distributions with continuous ones (this is the way we'll think during class).
Find continuous analogs of distributions from the previous slide.

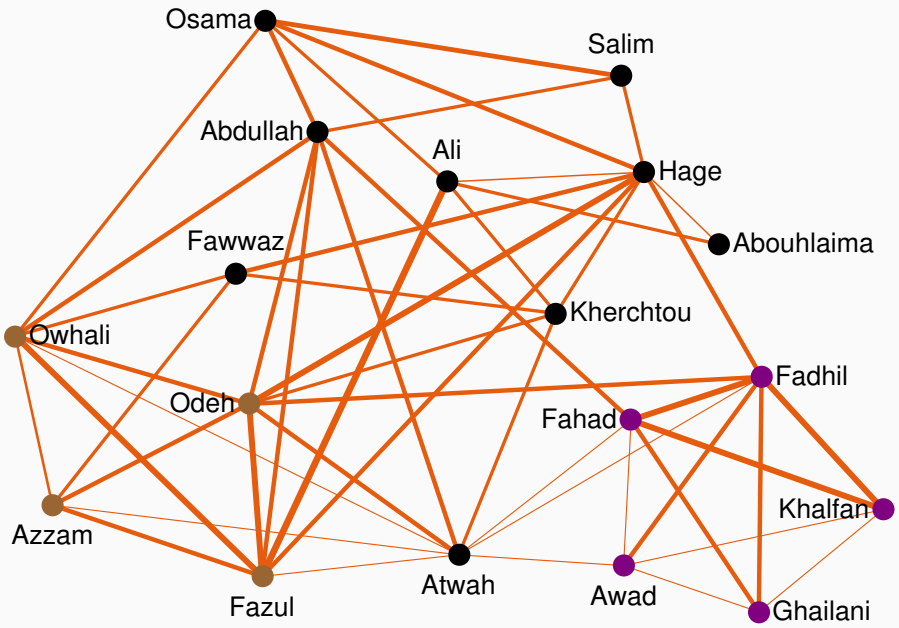
Lecture

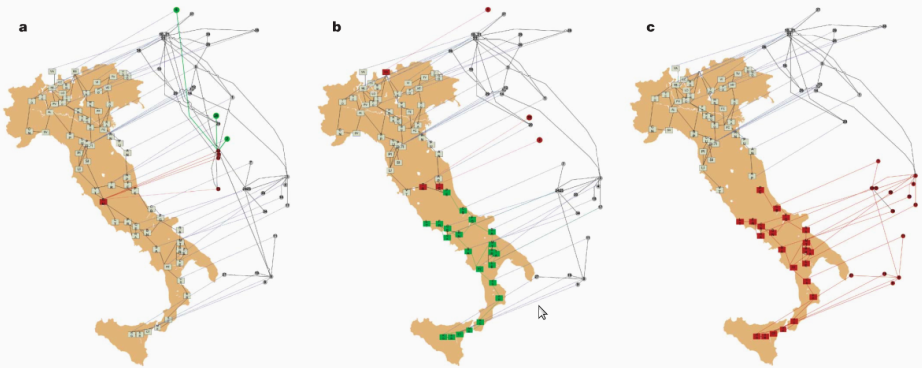
Why are networks/graphs so useful?

Graphs are an illustration of a relationship

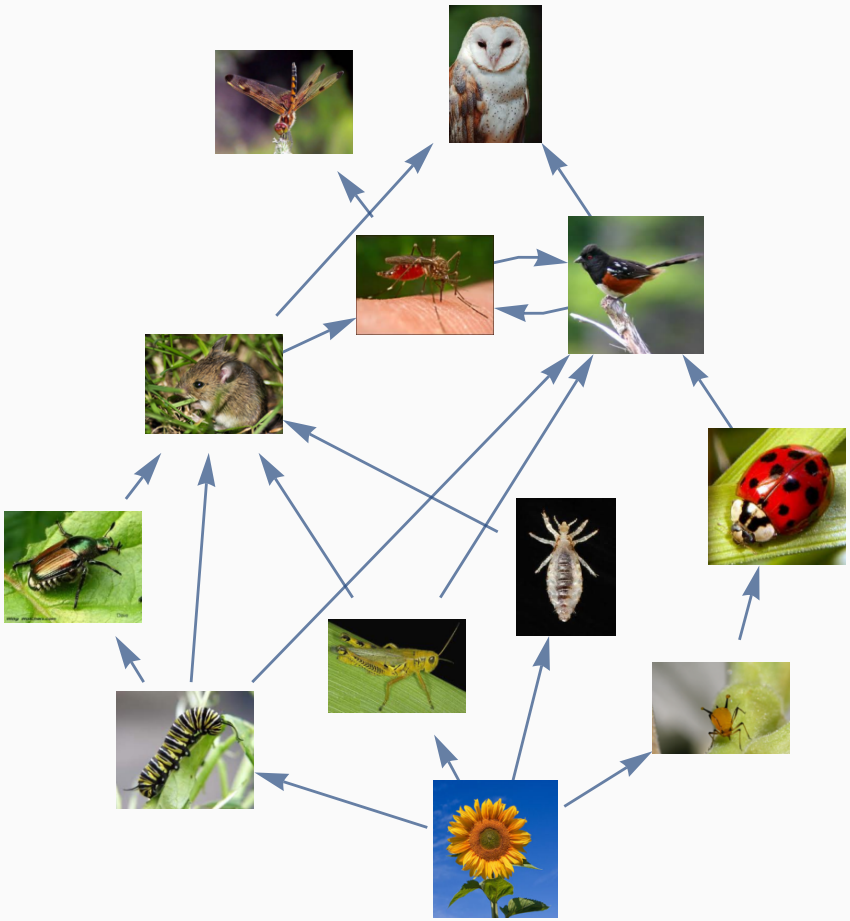
What relations are illustrated in the following graphs?

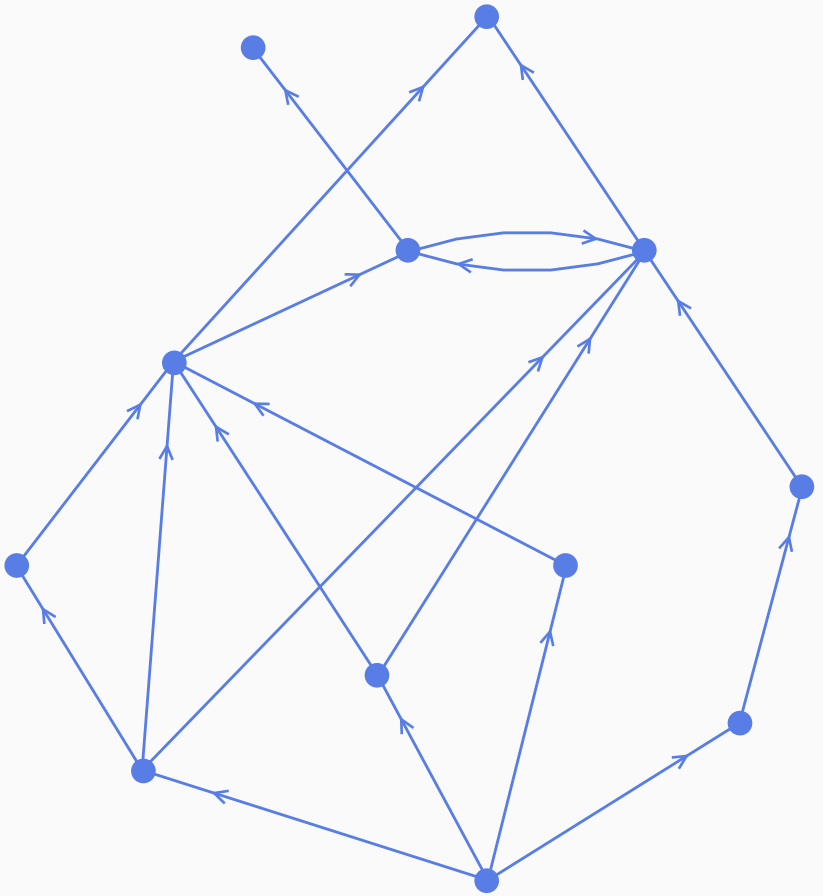






What is the difference between graphs and networks?





Why networks are complex?

Properties of the real networks

- distributions with fat tails.

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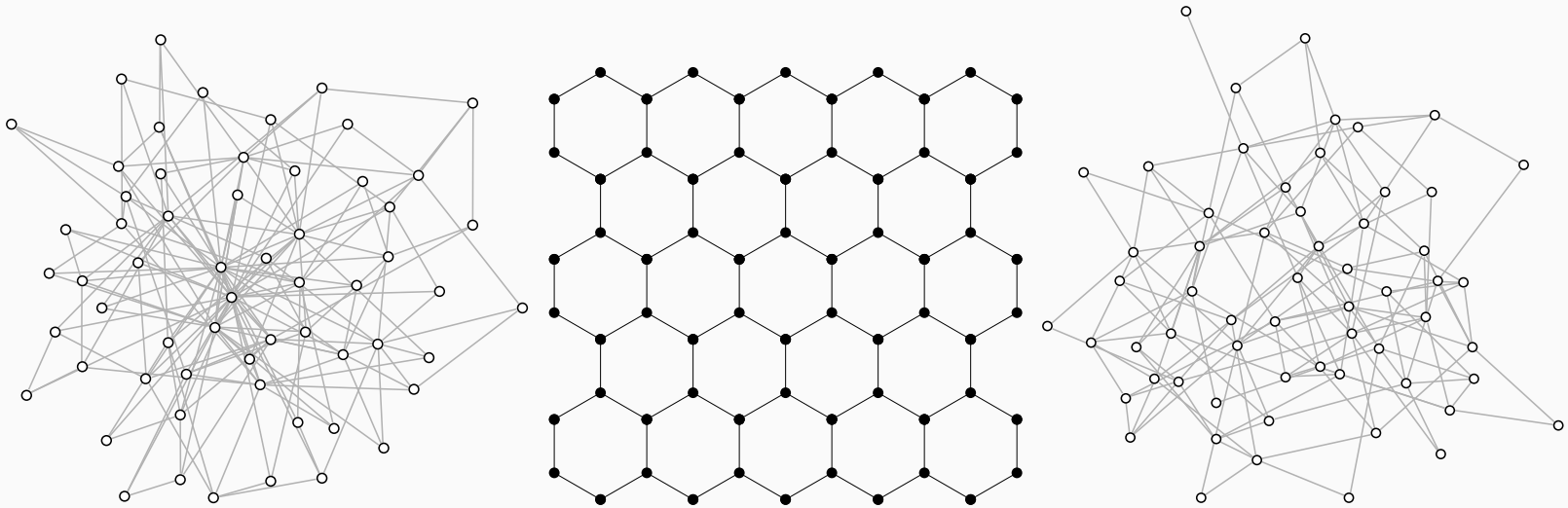

Which of these graphs represents the real network?

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- physics (statistical physics methods, phase transitions),
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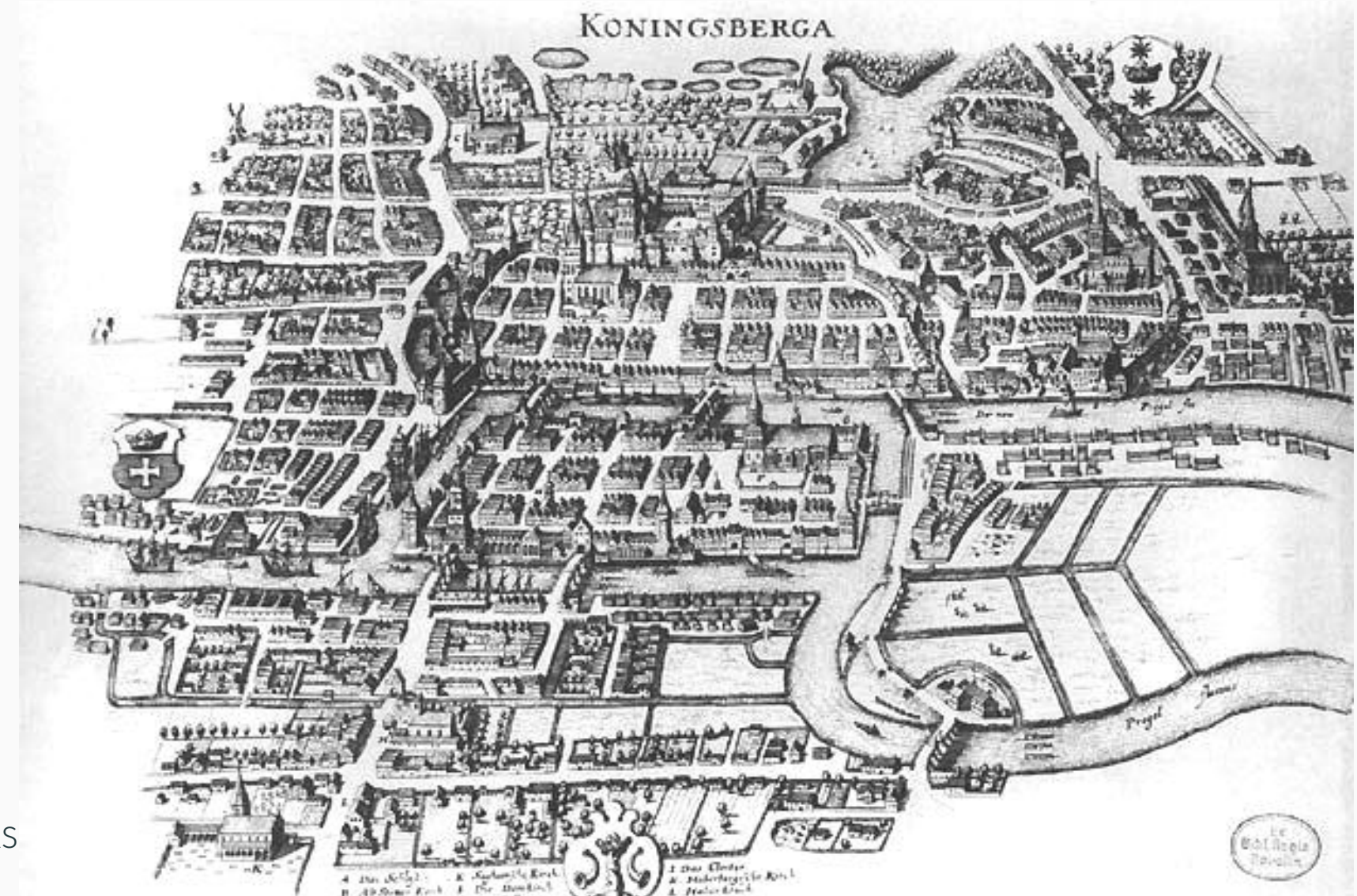
- data science,

with expert knowledge from

- sociology,
- economics,
- biology,
- medicine,
- engineering,
- and many other...

The story of network science – Seven Bridges of Königsberg (1736)



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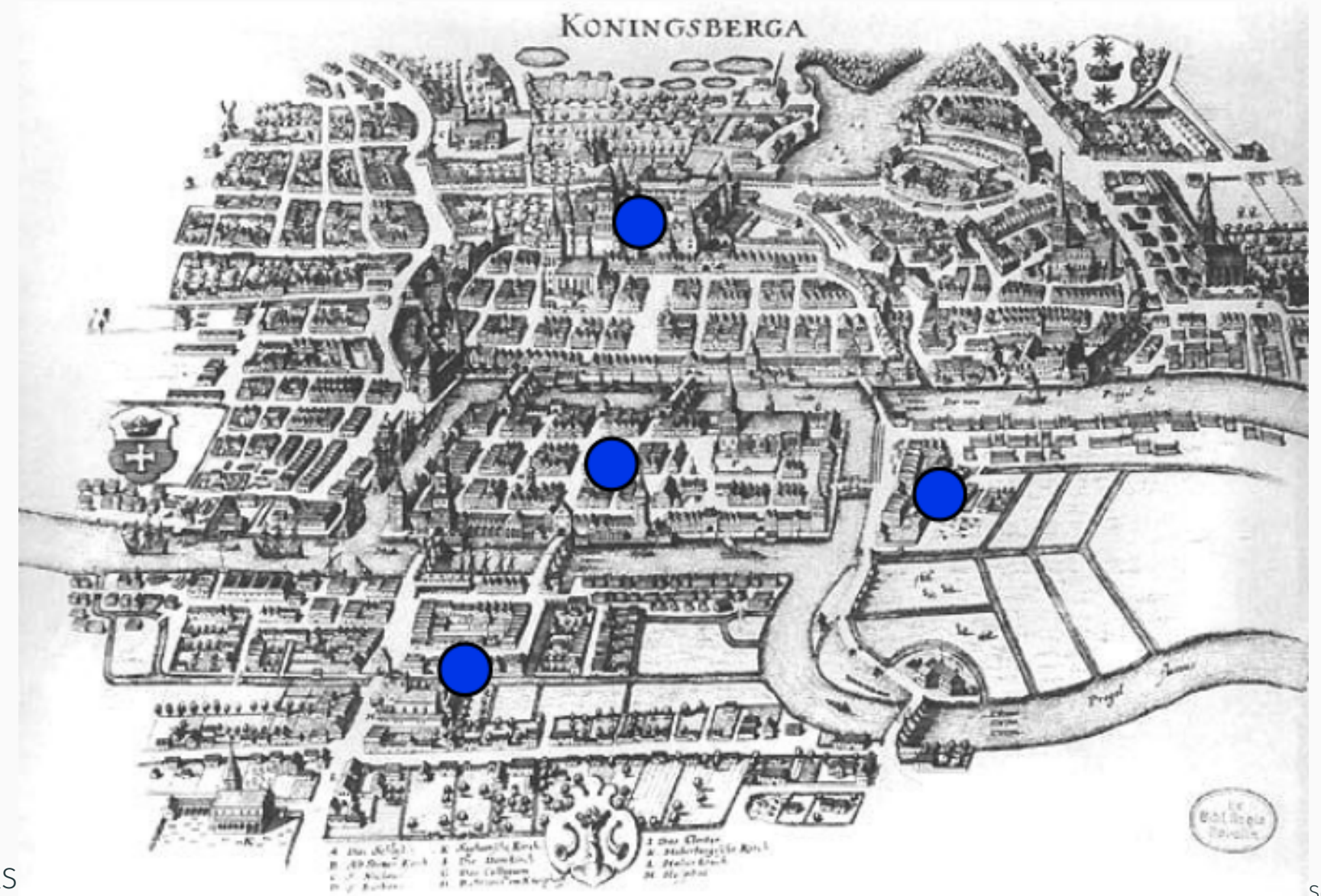

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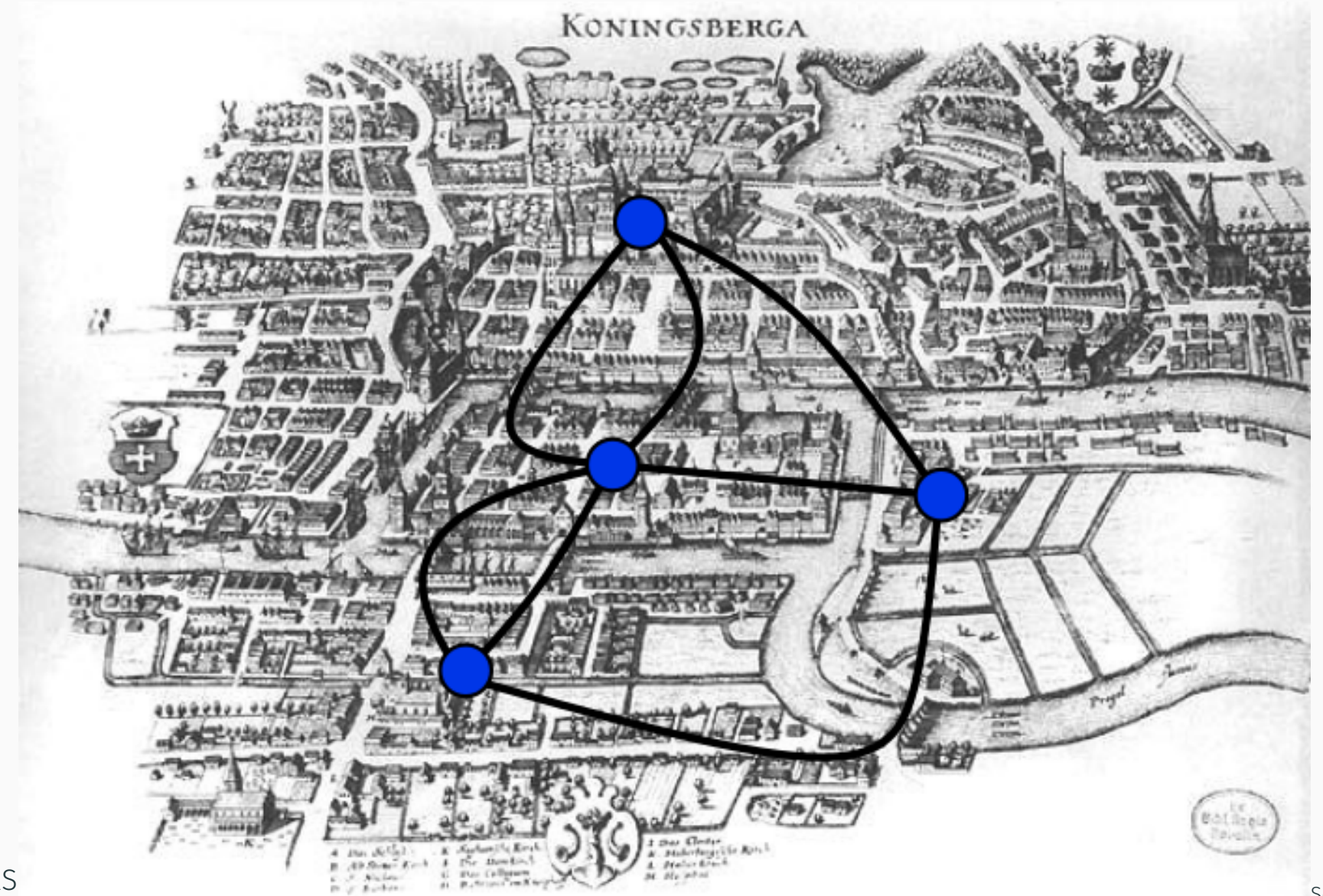

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Great interest of sociologists in researching social networks

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- we do not focus on this line of research (with one exception!).

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More details on Lecture 5.

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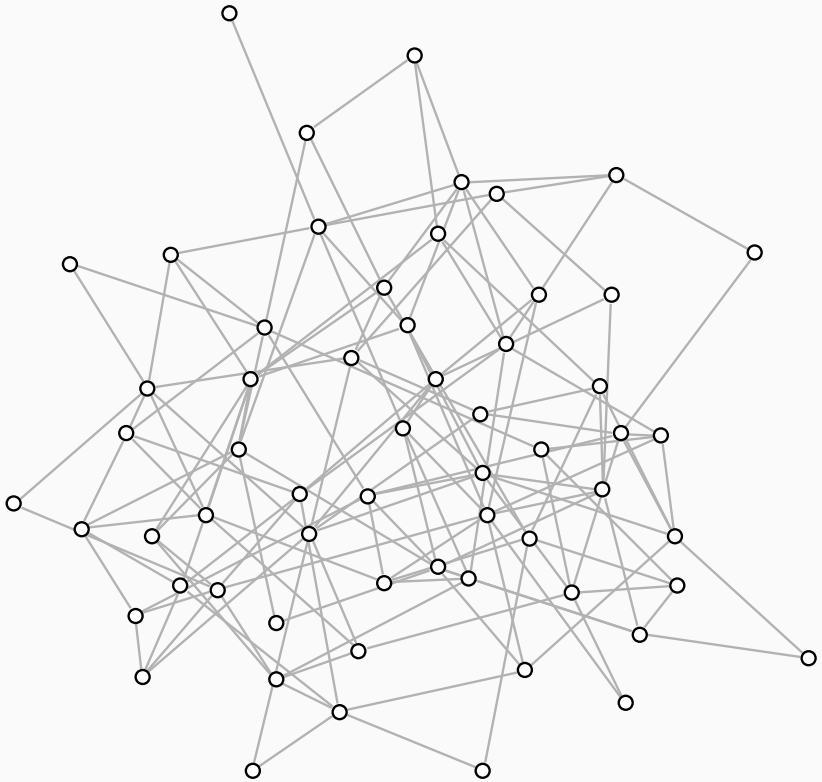
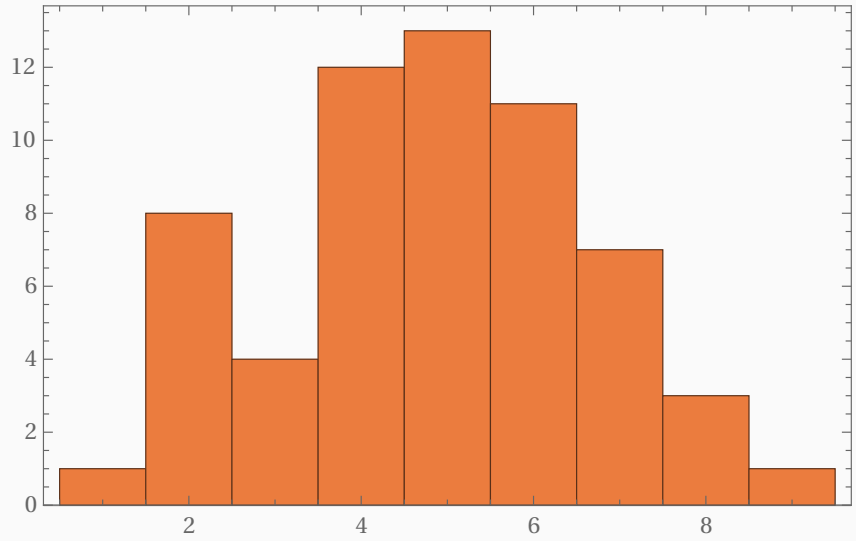
Degree distribution

$$P(k) = \text{fraction of vertices with degree } k$$





Is this a real network?

The story of network science – Milgram experiment (1967)

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Do not confuse the it correspondence experiment with the famous experiment of obedience to authorities!

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More details during Lecture 4.

The story of network science – Watts-Strogatz model (1998)

Problem:

Poissonian graphs do not have small world property.

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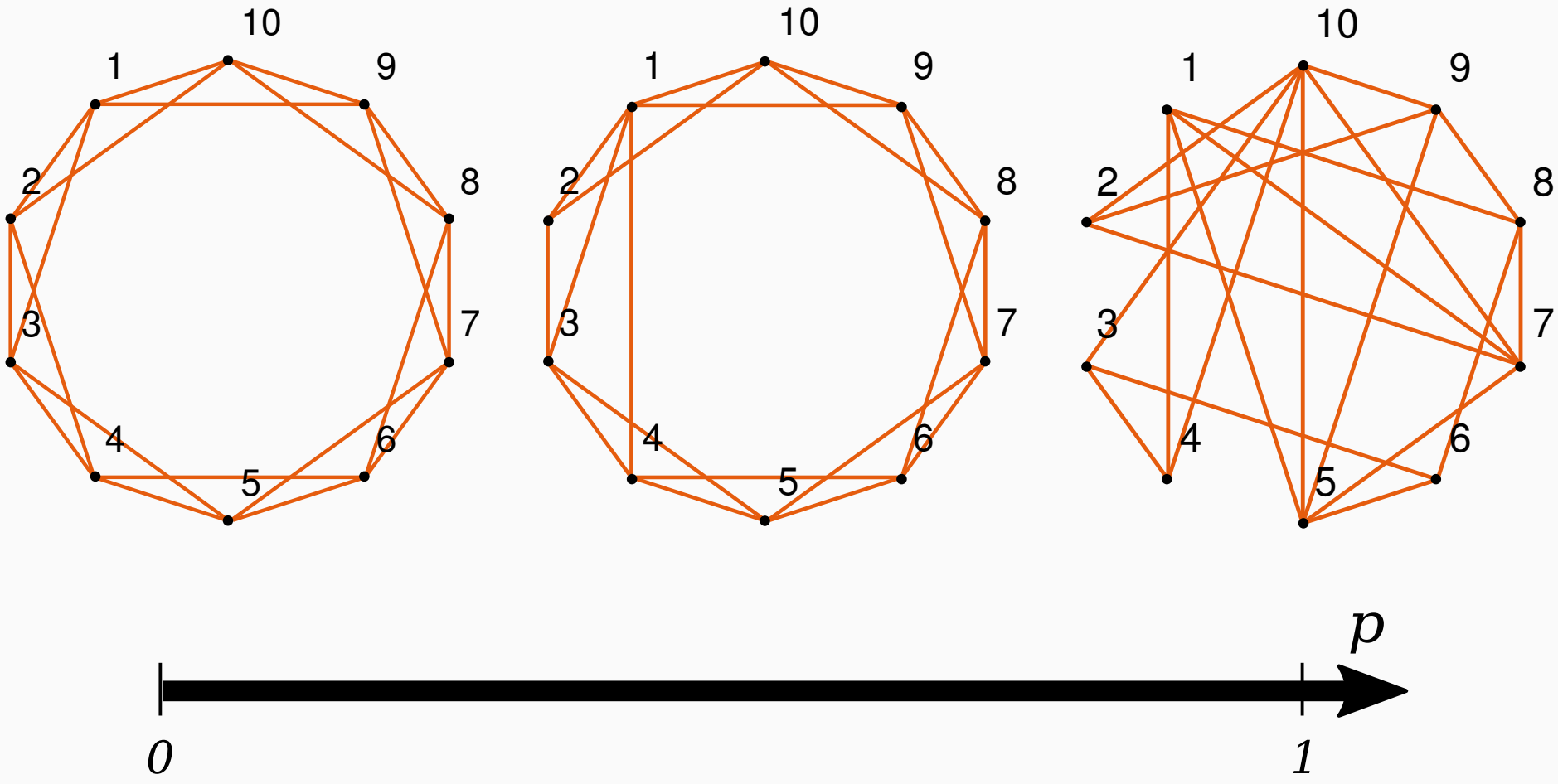
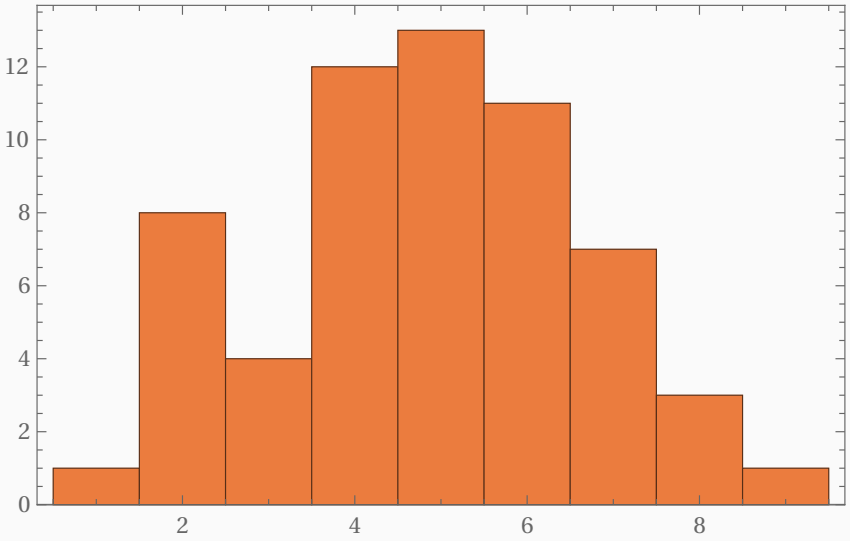
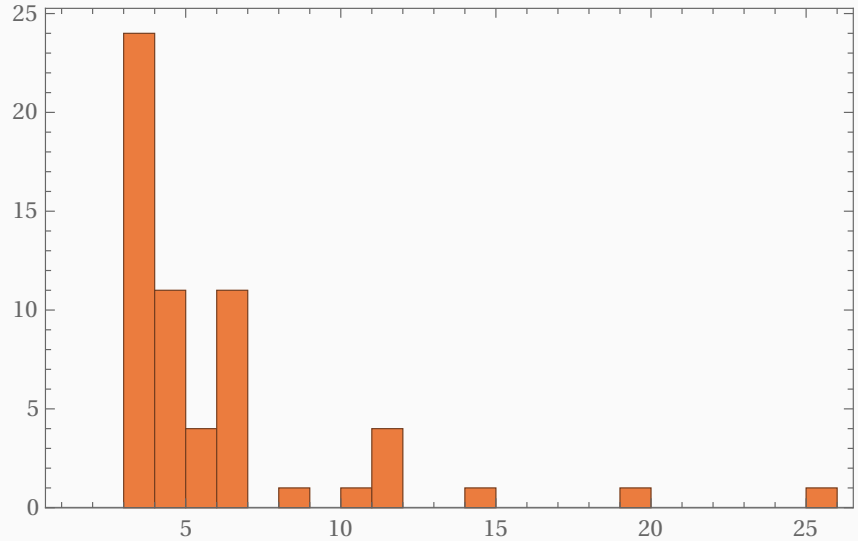


The story of network science – Barabási-Albert model (1999)

Problem:

Real-world networks usually have power law like distributions.





What does it means?

- No typical scale.
- Fat tails.
- Fast spreading epidemics...

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BA model

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Both assumptions are necessary!

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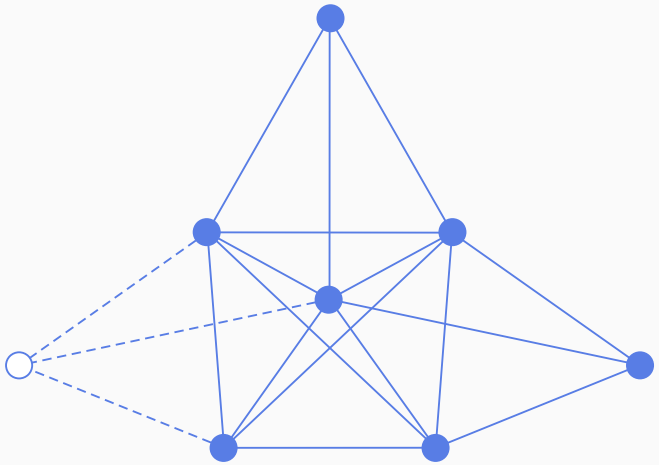

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More about BA model on Lecture 5.

The story of network science – Barabási-Albert model (1999)

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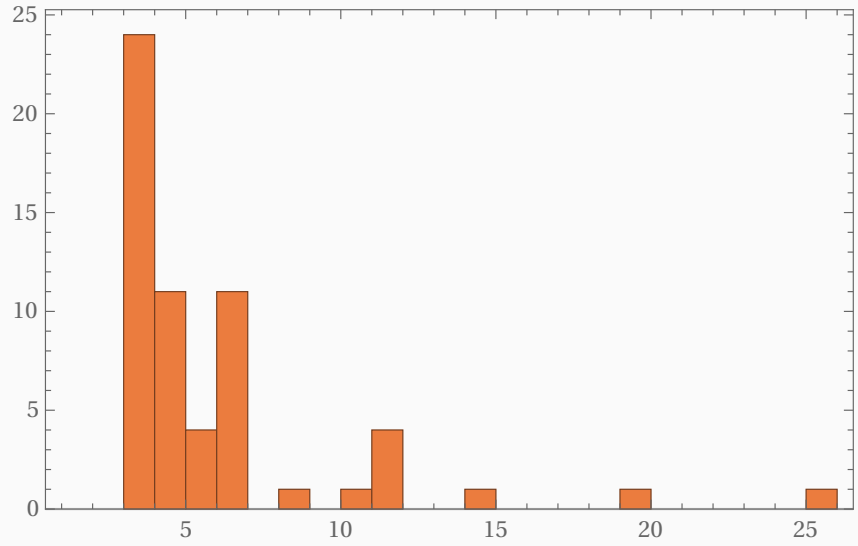
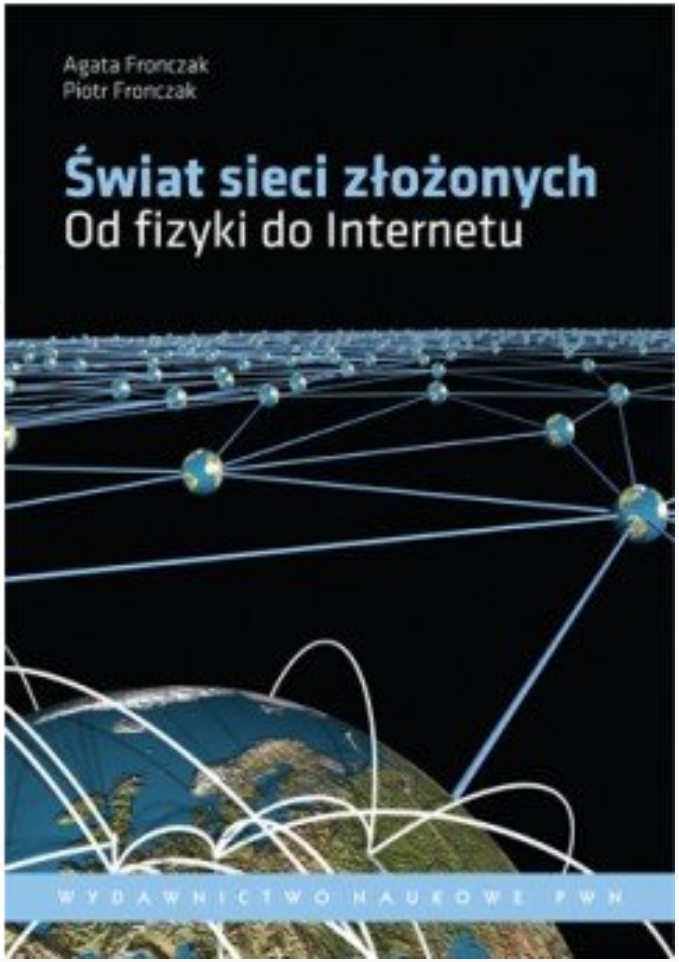
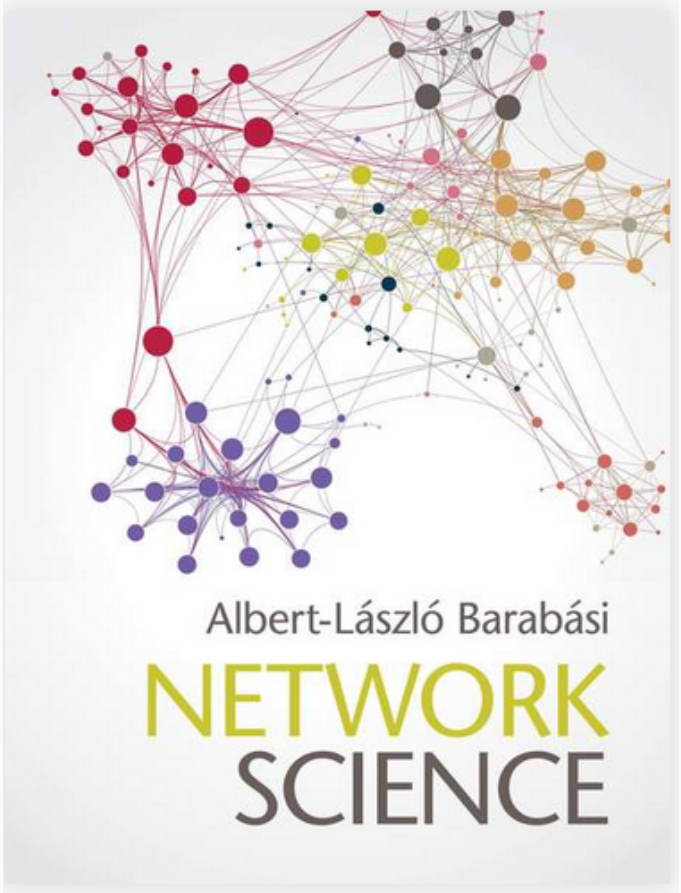


More on Lecture 5.

If you are interested in the history of complex networks read





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  - Desciption of the dynamical processes within real networks (e.g. epidemics, news spreading, etc.) (Lecture 8,11-15).

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  - Desciption of the dynamical processes within real networks (e.g. epidemics, news spreading, etc.) (Lecture 8,11-15).
- Theoretical complex networks models (Lectures 5,6,9,10).

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- Quantitative description of the network, including
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  - discussion of their properties (Lectures 4,8).
  - measuring with them real networks (Lectures 4,8, 13-15).
  - Desciption of the dynamical processes within real networks (e.g. epidemics, news spreading, etc.) (Lecture 8,11-15).
- Theoretical complex networks models (Lectures 5,6,9,10).
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Main topics of complex network science

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- Visualization of networks (both theoretical and real) (Lecture 3).

Thank you for your attention!

Warsaw University of Technology

MSc program in Data Science has been developed
as a part of task 10 of the project
„NERW PW. Science - Education - Development - Cooperation”
co-funded by European Union from European Social Fund.