

# Social Networks & Recommendation Systems

## VI. Evolving networks.

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**European  
Funds**  
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MSc program in Data Science has been developed  
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# Project

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## Attention!

Exercices 1-8 in total are worth 40% points for the project.

## Random connections (A model)

P6.1 Fill the gaps in the following derivation [20%]

$$\Pi(k_i) = \frac{1}{t + m_0} \approx \frac{1}{t}.$$

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**Solution:**

Differential equation of the form

$$\frac{dk_i}{dt} = \frac{m}{t},$$

has solution

$$k_i(t) = m \ln \left( \frac{t}{t_i} \right) + m,$$

which leads to

$$\mathcal{P}(k) = \frac{e}{m} e^{-k/m}.$$

# Network with fixed size (B model)

P6.2 Fill the gaps in the following derivation [30%]

With the mean-field approach (as far as possible!) determine the degree distribution of the network in which

- The number of vertices is from start constant and equal to  $N$ .
- The edges are distributed with preferential attachment rule.

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**Solution:**

Following differential equation

$$\frac{dk_i}{dt} = \frac{N-1}{N} \frac{k_i}{2t} + \frac{1}{N},$$

has the solution of the form

$$k_i(t) = \frac{2(N-1)}{N(N-2)} t \approx \frac{2}{N} t,$$

P6.3 Check with simulations result obtained for model A. [30%]

P6.4 Check with simulations result obtained for model B. [30%]

P6.5 How to derive distribution in model B? [30%]



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Thank you for your attention!