Social Networks & Recommendation Systems

V. Static random graphs.

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Project

Networks with given hamiltonian

Let us considet the space of every possible graph with N vertices i.e. set $M_N = \mathbb{M}^{N\times N}(\{0,1\})$. We want to define a probability distribution on it.

We maximize entropy:

$$-\sum_{G\in M_N} \mathcal{P}(G)\ln \mathcal{P}(G),$$

Under certain condition $f(\mathcal{P}(G)) = 0$,

Which leads us to Lagrange multipliers

$$\mathcal{L}[\mathcal{P}(G)] = -\sum_{G\in M_N} \mathcal{P}(G)\ln \mathcal{P}(G) + \lambda f(\mathcal{P}(G))$$

It *only* remains to solve this equation

$$\frac{\partial \mathcal{L}}{\partial \mathcal{P}(G)} = 0.$$

Example - Excercise 1.

Implementation of $G_{N,p}$ version of ER graph – case study

Ecercise 2.
Implement a function that returns the adjacency matrix of one realization of the ER graph with given values of N and p. Watch out for the trap!

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Excercise 5.

What degree of vertex distribution do we expect?

ER model - comparison of theoretical results and simulations

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Plot both the simulation results and analytically obtained distributions on one graph. Test appropriate hypotheses.

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Excercises 8.
Checkdependence of the results of the previous excercise for various values of p and N.

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Attention!

Excercises 1-8 in total are worth 20% points for the project.

Non-physicality of ER graphs

Assuming the Poisson approximation, we calculate the variance

$$\mathbb{E}(K) = \sum_{k=0}^{\infty} \frac{k e^{-\langle k \rangle} \langle k \rangle^k}{k!} = \cdots = \langle k \rangle,$$

$$\mathbb{E}(K^2) = \sum_{k=0}^{\infty} \frac{k^2 e^{-\langle k \rangle} \langle k \rangle^k}{k!} = \cdots = \langle k \rangle + \langle k \rangle^2.$$

$$\mathrm{Var}(K) = \mathbb{E}(K^2) - [\mathbb{E}(K)]^2 = \langle k \rangle^2$$

P5.1 Complete the missing calculations.[10%]

Non-physicality of ER graphs

Clustering coefficient

$$\langle C \rangle = p.$$

P5.2 Check the above analytical result by simulation. [20%]

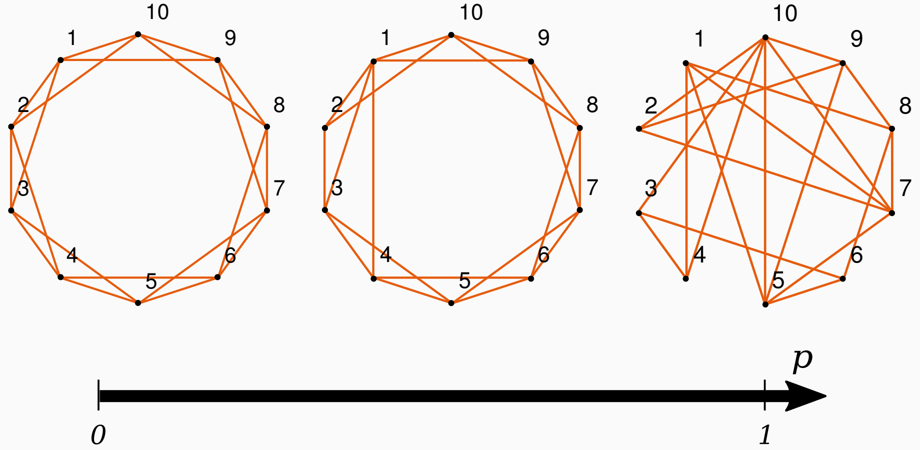
Stochastic block model

ER model generalization

$$\begin{bmatrix} [p_{11}] & [p_{12}] & \dots & [p_{1N}] \\ [p_{21}] & [p_{22}] & \dots & [p_{2N}] \\ \dots & \dots & \ddots & \dots \\ [p_{N1}] & [p_{N2}] & \dots & [p_{NN}] \end{bmatrix}$$

P5.3 Generate and draw a graph consisting of 4 community each with $N = 20$ nodes and the probability of connection within the community higher than between them. Draw the result. How it depends on the parameter values? [40%]

Watts-Strogatz model



P5.4 Draw a graph of the averaged coefficient of clustering of the WS network against its parameter *p*. [30%]

Other projects

P5.5 With (or without) Mathematica solve ER model in the case of $G_{N,E}$. [50%]

P5.6 Implement configuration model and test when the procedure converge. [50%]

P5.7 Compute partition function and distribution of the network with given hamiltonian for the case with fixed number of edges. [50%]

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