Omnidirectional Mobile Robot with four Mecanum Wheels

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Abstract: *The aim of this research paper is to approach the issue of mobile robotics from the perspective of solving kinematic models. The creation of an exemplary kinematic model is based on the construction of an omnidirectional mobile robot with four mecanum wheels.*

Keywords: *mobile robotics, wheeled mobile robots, omnidirectional mobility, kinematic model*

1 Introduction

The introduction of this research paper describes mobile robotics in terms of mobility, advantages and disadvantages that must be taken into account when designing a robot with respect to the operating environment. Classification of an important structural element, which is the wheel of a mobile robot, and how its construction affects the overall movement properties of the mobile robot.

Demonstration of different types of mobile robots with a detailed view of an omnidirectional mobile robot with four mecanum wheels, including a description of the process of creating a kinematic model of this mobile robot.

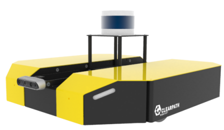
2 Wheeled Mobile Robots

In general wheeled mobile robots are widely used to achieve mobility in many applications because there are many advantages such as lower energy consumption, faster movement, lower control requirements, owing to their simple mechanisms and reduced stability problems in comparison with other locomotion mechanisms (e.g., legged robots or tracked vehicles). Although it is difficult to overcome rough terrain or uneven ground conditions, wheeled mobile robots are suitable for a large class of target environments in practical applications.

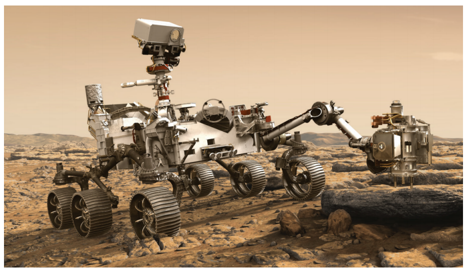
The issue of mobile robotics is mainly associated with driving on uneven surfaces along with the requirements of the mobile robot for a given robotic application because it is necessary to maintain wheel contact with the ground to achieve traction, stability, maneuverability and control.

Therefore, it is necessary to choose suitable materials (e.g., stiffness of the wheels), mechanisms (e.g.,suspension allows the wheels to maintain contact with the ground despite disturbances from the uneven ground surface, which allows generation of traction, braking, and cornering forces) and wheel configuration (e.q., standard wheels, special wheels, number of wheels) of the mobile robot to achieve the desired goal in the environment. [1] [2]

(a) HUSKY [4]



(b) DINGO [4]



(c) PERSEVERANCE [5]

Figure 1: Demonstration of various robotic solutions (constructions) with regard to the operating environment (a) outdoor, (b) indoor (c) Mars

2.1 Mobility of Wheeled Robots

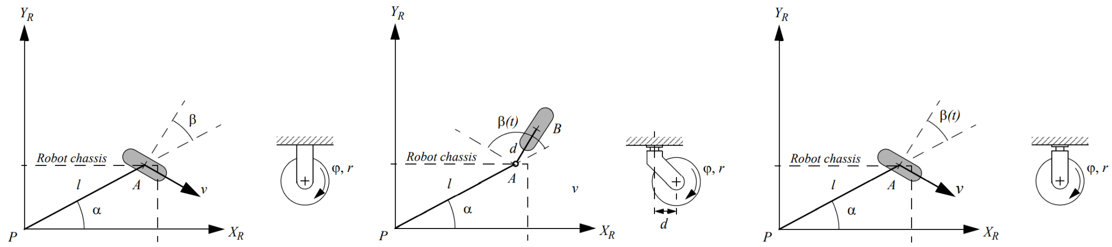
Wheeled mobile robots may be classified in two major categories, omnidirectional and nonholonomic. Classification depends in part on the type of wheels it employs. The term holonomic has broad applicability to several mathematical areas, including differential equations, functions, and constraint expressions. In mobile robotics, the term refers specifically to the kinematic constraints of the robot chassis. A holonomic robot is a robot that has zero nonholonomic kinematic constraints (controllable degrees of freedom are equal to the total degrees of freedom). Conversely, a nonholonomic robot is a robot with one or more nonholonomic kinematic constraints. [2] [3]

2.1.1 Types of Wheels

As mentioned, the motion characteristics and capabilities of the robot depend on the design of the robot wheel. If we look at the design of wheels in detail we can consider two main groups, standard wheel or a special wheel. A standard wheel can be understood as a conventional tire. Special wheels possess unique mechanical structures including rollers or spheres.

Although standard wheels are advantageous because of their simple structure and good reliability, the nonholonomic velocity constraint (i. e., no side-slip condition) limit its robot motion. On the other hand, special wheels can be employed in order to obtain omnidirectional motion of a mobile robot (omnidirectional mobile robot), i. e., to ensure three degrees of freedom for plane motion. We consider two typical designs of special wheels: the Swedish wheel and the spherical wheel.

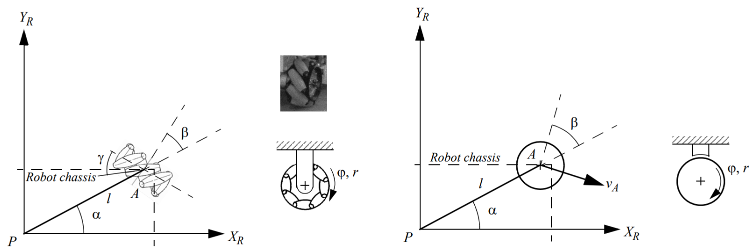
In summary, four types of standard wheels are commonly used. First is a passively driven wheel with a fixed steering axis. Second is a passive caster wheel with offset d. Third is an active caster wheel with offset d, where the steering and driving motions are controlled by actuators. The fourth is an active orientable wheel with zero offset d, where steering and driving motions are driven by actuators. [1]



(a) (b) (c)

Figure 2: Structures of standard wheels. (a) Passive fixed wheel, (b) passive or active, off-centered orientable wheel, and (c) active orientable wheel without offsets [2]

Swedish wheel consists of small passive free rollers are located along the outer rim of the wheel. Free rollers are employed in order to eliminate the nonholonomic velocity constraint. A spherical wheel. The rotation of the sphere is constrained by rollers that make rolling contact with the sphere. The rollers can be divided into driving and supporting rollers. The sphere is driven by the actuation of the driving rollers, whereas the rolling contacts provide nonholonomic constraints, and the resultant motion of the sphere module becomes holonomic. [1]



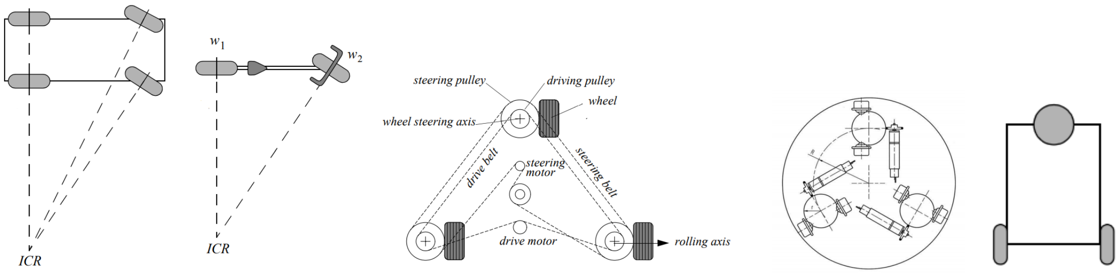
(a) (b)

Figure 3: Structures of special wheels. (a) Swedish wheel, (b) Spherical wheel [2]

2.1.2 Types of Wheeled Robots

Although it is possible to design different types of mobile wheeled robots with different motion characteristics that correspond to combinations of the selected wheel design and wheel chassis configuration, only some types are used, an example may be that we usually meet single-track or two-track vehicles on the road, although the design of these robots differs from model to model, but they have the same kinematic model within their group.

This research paper is focused on omnidirectional wheeled mobile robots with Mecanum/Swedish wheels, so other designs with their kinematic models are not involved in. In case of further interest in another type of mobile robot, including kinematic descriptions and so on, I recommend the materials, which can be found in the chapter references.



(a) (b) (c) (d) (e)

Figure 4: Some types of wheeled mobile robots (a) Ackermann (b) Bicycle (c) Synchro-drive (d) Omnidirectional locomotion with three spherical wheels (e) Differential drive [2]

3 Omnidirectional Wheeled Mobile Robots

In this chapter We will deal in detail with a group of omnidirectional mobile wheeled robots with special wheels, Swedish wheels.

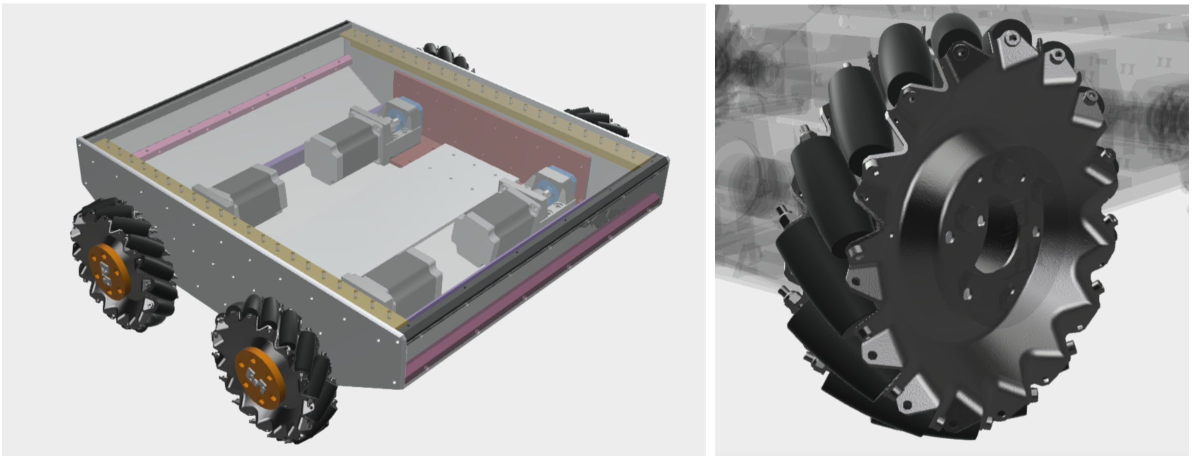
At least three Swedish wheels are required to build a holonomic omnidirectional robot. A major advantage of using the Swedish wheel is that omnidirectional mobile robots can be easily constructed. At least three Swedish wheels are required to build a holonomic omnidirectional robot. Since omnidirectional robots can be built without using active steering of wheel modules, the mechanical structures of actuating parts can have simple structures. However, the mechanical design of a wheel becomes slightly complicated. One drawback of the Swedish wheel is that there is a vertical vibration because of discontinuous contacts during motion. Another drawback is its relatively low durability when compared to conventional tires.

Figure 5: (Left) A mobile robot with three omniwheels. (Right) A mobile robot with four omniwheels [3]

3.1 Kinematic Model of Mecanum-Wheeled Mobile Robot

Mecanum wheels have no vertical axis of rotation, yet are able to move omnidirectionally like the castor wheel. This is possible by adding a degree of freedom to the fixed standard wheel. Swedish wheels consist of a fixed standard wheel with rollers attached to the wheel perimeter with axes that are anti-parallel to the main axis of the fixed wheel component. The exact angle between the roller axes and the wheel plane can vary.

Passive rollers are free to rotate around the axis of rotation, which results in lateral motion of the wheel. As a result, a driving velocity should be controlled, while the lateral velocity is passively determined by the actuation of the other wheels. [1] [2]



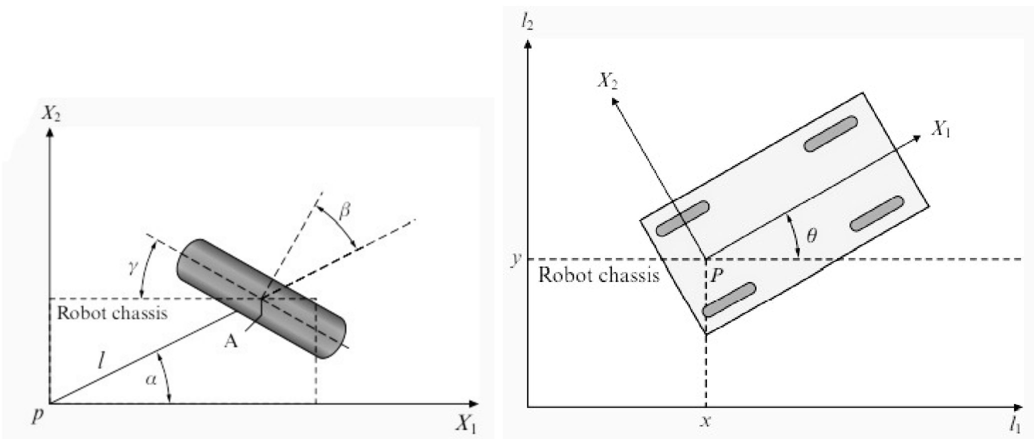
(a) (b)

Figure 6: (a) Mecanum wheeled mobile robot (b) Mecanum wheel [6]

The position of the robot on the plane is described, with respect to an arbitrary inertial frame, by posture vector $\xi = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$, where x and y are the coordinates of reference point P of the robot cart, while θ describes the orientation of mobile robot frame (local reference frame) attached to the robot, with respect to inertial frame (global reference frame).

In other words, in order to specify the position of the robot on the plane, we establish a relationship between the global reference frame of the plane and the local reference frame of the robot. To describe robot motion in terms of component motions, it will be necessary to map motion along the axes of the global reference frame to motion along the axes of the robot's local reference frame.

The position of a Swedish wheel with respect to the cart is described by three constant parameters: α, β and l. An additional parameter is required to characterize the direction, with respect to the wheel plane, of the zero component of the velocity at the contact point of the wheel. This parameter is γ , which is the angle between the axle of the rollers and the wheel plane. The radius of the wheel is denoted by r, and its angle of rotation around its horizontal axle is denoted by $\dot{\varphi}$. [1] [2]



(a) (b)

Figure 7: Visualization of the mentioned parameters [1]

3.1.1 Kinematic Constraints of a Single Mecanum Wheel

Considering the reference point that is part of the robot chassis (local reference frame). As shown in Figure 7, the wheel is located at point A of distance l from the reference point. The translational (absolute) speed of the robot at the reference point is given by $\begin{bmatrix}\dot{x} & \dot{y} & 0\end{bmatrix}^T$ and rotational (relative) speed is given by angular $\dot{\theta}$ velocity around the z axis.

Then the velocity of A in local reference frame is:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ 0\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \dot{\theta}\end{bmatrix} \times \begin{bmatrix}l_{ix}\\ l_{iy}\\ 0\end{bmatrix} = \begin{bmatrix}\dot{x}\\ \dot{y}\\ 0\end{bmatrix} + \begin{vmatrix}i & j & k\\ 0 & 0 & \dot{\theta}\\ l_{ix} & l_{iy} & 0\end{vmatrix} = \begin{bmatrix}\dot{x}\\ \dot{y}\\ 0\end{bmatrix} + \begin{bmatrix}-l_{iy}\dot{\theta}\\ l_{ix}\dot{\theta}\\ 0\end{bmatrix} = \begin{bmatrix}\dot{x} - l_{iy}\dot{\theta}\\ \dot{y} + l_{ix}\dot{\theta}\\ 0\end{bmatrix} \tag{1}$$

$$\begin{bmatrix}\dot{x} - l_{iy}\dot{\theta}\\ \dot{y} + l_{ix}\dot{\theta}\end{bmatrix} \rightarrow \begin{bmatrix}1 & 0 & -l_{iy}\\ 0 & 1 & l_{ix}\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} \tag{2}$$

Next, we must consider point A in terms of the wheel reference frame. Point A is located in the wheel axis. The mecanum wheel performs a rotational movement around the horizontal axis. The resulting speed at point A in terms of the wheel reference frame is given by:

$$\begin{bmatrix}v_i\cos\gamma\\ v_i\sin\gamma\\ 0\end{bmatrix} + \begin{bmatrix}0\\ \dot{\varphi}\\ \dot{\theta}\end{bmatrix} \times \begin{bmatrix}0\\ 0\\ r\end{bmatrix} = \begin{bmatrix}v_i\cos\gamma\\ v_i\sin\gamma\\ 0\end{bmatrix} + \begin{vmatrix}i & j & k\\ 0 & \dot{\varphi} & \dot{\theta}\\ 0 & 0 & r\end{vmatrix} = \begin{bmatrix}v_i\cos\gamma\\ v_i\sin\gamma\\ 0\end{bmatrix} + \begin{bmatrix}r\dot{\varphi}\\ 0\\ 0\end{bmatrix} = \begin{bmatrix}v_i\cos\gamma + r\dot{\varphi}\\ v_i\sin\gamma\\ 0\end{bmatrix} \tag{3}$$

$$\begin{bmatrix}v_i\cos\gamma + r\dot{\varphi}\\ v_i\sin\gamma\end{bmatrix} \rightarrow \begin{bmatrix}r & \cos\gamma\\ 0 & \sin\gamma\end{bmatrix}\begin{bmatrix}\dot{\varphi}\\ v_i\end{bmatrix} \tag{4}$$

Computation the wheel's motion in local wheel reference frame from motion in its wheel reference frame is given by:

$$\begin{bmatrix}\cos(\alpha+\beta) & -\sin(\alpha+\beta)\\ \sin(\alpha+\beta) & \cos(\alpha+\beta)\end{bmatrix}\begin{bmatrix}r & \cos\gamma\\ 0 & \sin\gamma\end{bmatrix}\begin{bmatrix}\dot{\varphi}\\ v_i\end{bmatrix} = \begin{bmatrix}1 & 0 & -l_{iy}\\ 0 & 1 & l_{ix}\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} \tag{5}$$

Then, the inverse kinematics equation of each mecanum wheel is derived as:

$$\begin{bmatrix}\dot{\varphi}\\ v_i\end{bmatrix} = \begin{bmatrix}r & \cos\gamma\\ 0 & \sin\gamma\end{bmatrix}^{-1}\begin{bmatrix}\cos(\alpha+\beta) & -\sin(\alpha+\beta)\\ \sin(\alpha+\beta) & \cos(\alpha+\beta)\end{bmatrix}^{-1}\begin{bmatrix}1 & 0 & -l_{iy}\\ 0 & 1 & l_{ix}\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} \tag{6}$$

The inverse of a 2x2 matrix is given by the formula:

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}^{-1} = \frac{1}{ad-cb}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix} \tag{7}$$

After using formula (7), we obtain equation (6) in the following form:

$$\begin{bmatrix}\dot{\varphi}\\ v_i\end{bmatrix} = \begin{bmatrix}\frac{1}{r} & -\frac{\cos\gamma}{r\sin\gamma}\\ 0 & \frac{1}{\sin\gamma}\end{bmatrix}\begin{bmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta)\\ -\sin(\alpha+\beta) & \cos(\alpha+\beta)\end{bmatrix}\begin{bmatrix}1 & 0 & -l_{iy}\\ 0 & 1 & l_{ix}\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} \tag{8}$$

$$\begin{bmatrix}\dot{\varphi}\\ v_i\end{bmatrix} = \begin{bmatrix}\frac{\sin(\alpha+\beta+\gamma)}{r\sin\gamma} & -\frac{\cos(\alpha+\beta+\gamma)}{r\sin\gamma} & \frac{-l_{ix}\cos(\alpha+\beta+\gamma)-l_{iy}\sin(\alpha+\beta+\gamma)}{r\sin\gamma}\\ -\csc\gamma\sin(\alpha+\beta) & \csc\gamma\cos(\alpha+\beta) & l_{ix}\csc\gamma\cos(\alpha+\beta)+l_{iy}\csc\gamma\sin(\alpha+\beta)\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} \tag{9}$$

3.2 Kinematics of a Four-Wheeled Mecanum Mobile Robot

In terms of control, we are interested in the angular velocities $\dot{\varphi}_i$ of individual wheels, where n is number of wheels. The parameters of the robot: $\alpha + \beta = 90°$, γ corresponds to figure 8.

$$\begin{bmatrix}\dot{\varphi}\\ v_i\end{bmatrix} = \begin{bmatrix}A & B & C\\ a & b & c\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} \rightarrow \begin{bmatrix}\dot{\varphi}_1\\ \dot{\varphi}_2\\ \vdots\\ \dot{\varphi}_n\end{bmatrix} = \begin{bmatrix}A_1 & B_1 & C_1\\ A_2 & B_2 & C_2\\ \vdots & \vdots & \vdots\\ A_n & B_n & C_n\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} \tag{10}$$

$$\begin{bmatrix}\dot{\varphi}_1\\\dot{\varphi}_2\\\dot{\varphi}_3\\\dot{\varphi}_4\end{bmatrix} = \frac{1}{r\sin\gamma}\begin{bmatrix}\sin(\alpha+\beta+\gamma) & -\cos(\alpha+\beta+\gamma) & -l_{ix}\cos(\alpha+\beta+\gamma)-l_{iy}\sin(\alpha+\beta+\gamma)\\\sin(\alpha+\beta+\gamma) & -\cos(\alpha+\beta+\gamma) & -l_{ix}\cos(\alpha+\beta+\gamma)-l_{iy}\sin(\alpha+\beta+\gamma)\\\sin(\alpha+\beta+\gamma) & -\cos(\alpha+\beta+\gamma) & -l_{ix}\cos(\alpha+\beta+\gamma)-l_{iy}\sin(\alpha+\beta+\gamma)\\\sin(\alpha+\beta+\gamma) & -\cos(\alpha+\beta+\gamma) & -l_{ix}\cos(\alpha+\beta+\gamma)-l_{iy}\sin(\alpha+\beta+\gamma)\end{bmatrix}\begin{bmatrix}\dot{x}\\\dot{y}\\\dot{\theta}\end{bmatrix} \tag{11}$$

$$J = \frac{1}{\sin\gamma}\begin{bmatrix}\sin(\alpha+\beta+\gamma) & -\cos(\alpha+\beta+\gamma) & -l_{ix}\cos(\alpha+\beta+\gamma)-l_{iy}\sin(\alpha+\beta+\gamma)\\\sin(\alpha+\beta+\gamma) & -\cos(\alpha+\beta+\gamma) & -l_{ix}\cos(\alpha+\beta+\gamma)-l_{iy}\sin(\alpha+\beta+\gamma)\\\sin(\alpha+\beta+\gamma) & -\cos(\alpha+\beta+\gamma) & -l_{ix}\cos(\alpha+\beta+\gamma)-l_{iy}\sin(\alpha+\beta+\gamma)\\\sin(\alpha+\beta+\gamma) & -\cos(\alpha+\beta+\gamma) & -l_{ix}\cos(\alpha+\beta+\gamma)-l_{iy}\sin(\alpha+\beta+\gamma)\end{bmatrix} \tag{12}$$

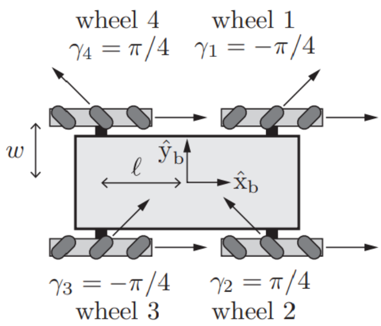


Figure 8: Configuration of robot [3]

By substituting the parameters we get the inverse kinematic Jacobian matrix:

$$J = \begin{bmatrix}-1 & 1 & (L+l)\\1 & 1 & (L+l)\\-1 & 1 & -(L+l)\\1 & 1 & -(L+l)\end{bmatrix} \rightarrow \begin{bmatrix}\dot{\varphi}_1\\\dot{\varphi}_2\\\dot{\varphi}_3\\\dot{\varphi}_4\end{bmatrix} = \frac{1}{r}J\begin{bmatrix}\dot{x}\\\dot{y}\\\dot{\theta}\end{bmatrix} \tag{13}$$

To control the robot in terms of the global reference frame, we must perform remapping using a matrix:

$$R(\theta) = \begin{bmatrix}\cos\theta & \sin\theta & 0\\-\sin\theta & \cos\theta & 0\\0 & 0 & 1\end{bmatrix} \rightarrow \begin{bmatrix}\dot{\varphi}_1\\\dot{\varphi}_2\\\dot{\varphi}_3\\\dot{\varphi}_4\end{bmatrix} = \frac{1}{r}JR(\theta)\begin{bmatrix}\dot{x}\\\dot{y}\\\dot{\theta}\end{bmatrix} \tag{14}$$

4 Conclusion

The introduction of mobile robotics and classification of omnidirectional mobility is followed by a key part dealing with the kinematic interpretation of a mobile robot, which is demonstrated on the example of an omnidirectional mobile robot with four mecanum wheels, where the inverse kinematics equation of a given mobile robot is derived.

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