Problem 179: This Is Rocket Science

Difficulty: Hard

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Problem Background

Lockheed Martin and the UK Space Agency are working together to develop a new system for deploying CubeSats - small satellites often used for scientific research. The Small Launch Orbital Manoeuvering Vehicle - or SL-OMV - can hold up to 6 CubeSats at a time and launch them at the optimal times and positions for their respective missions. This makes launching CubeSats considerably easier, particularly since CubeSats are usually too small to have any sort of propulsion systems of their own. That said, launching the satellites is still not an easy task.

Problem Description

Your team is working with the UK Space Agency to create a program to determine how much fuel must be loaded onto the SL-OMV for each mission. The navigational systems onboard the vehicle allow it to move as though on a two-dimensional grid. On each mission, the vehicle will have up to six CubeSats which must each be launched at a specific location on that grid while the vehicle is at a complete stop relative to the grid. This ensures that each CubeSat is placed into the appropriate orbit for its mission. However, the vehicle must also launch each CubeSat at a specific time to ensure that the CubeSat's orbit won't interfere with that of another satellite. Collisions in space are a very serious concern, and can cause catastrophic damage not just to the colliding objects, but also to other satellites in nearby orbits.

The SL-OMV has four thrusters, each allowing it to accelerate along either the X or Y axis. Each thruster provides a constant acceleration of 1 m/s^2 while it is being fired. Keep in mind that the vehicle is operating in space - with no air resistance or other friction to slow it down, the vehicle will continue to move at a constant rate of speed once its thrusters are shut off. This means you must fire the opposite thruster(s) to slow the vehicle down or cause it to reverse direction.

For example, let's say that the SL-OMV must launch a CubeSat at coordinates (3,3) within 5 seconds. It starts at coordinates (0,0) moving at 0 m/s (note that all speeds shown here are relative to the grid; in reality, orbital speeds are measured in *kilometers* per second). Getting to the launch coordinates is easy; a short burst on both the positive X and positive Y thrusters will cause the vehicle to start moving in the right direction; once it approaches the launch coordinates, bursts on the negative thrusters of equal duration will cause it to stop on target. However, a short 0.1 second burst on each thruster wouldn't move the vehicle very quickly; there's no way it would arrive in time to be able to launch the CubeSat without interfering with other satellites.

We'll need to get the vehicle moving a bit faster in order to launch in time. The CubeSat must be launched after 5 seconds. Once we get the launch vehicle up to speed, we'll have to expend the same amount of time and fuel to slow it back down again. This means we can focus simply on the first half of the trip; the speeding up. Once we determine how much fuel is needed to reach halfway, we can simply double our calculations to get the total amount of fuel required to speed up *and* slow down.

Let's begin our calculations. If we're only concerned about getting halfway, we need to be at coordinates (1.5,1.5) after 2.5 seconds. Part of this time (t_1 seconds) will be spent firing our thrusters; once we get to the necessary speed, we can stop firing, and simply coast for the remaining time (t_2 seconds). Since our total journey will take 2.5 seconds, $t_2 = 2.5 - t_1$.

The distance travelled by a moving object can be calculated using this formula:

$$x = \frac{1}{2}at^2 + ut + x_0$$

Here, x represents distance, a represents acceleration, t represents time, and u represents the object's initial velocity (before it started accelerating). x_0 represents the starting position of the object. While we're accelerating, a is equal to 1 m/s², and u is 0. Once we stop accelerating, a becomes 0, and v becomes however fast we're moving (note that we've switched to v, as now it's our *final* velocity). This means we'll need to use two equations to accurately represent the distance we've travelled:

$$x_1 = \frac{1}{2}a{t_1}^2$$

$$x_2 = vt_2$$

$$x = x_1 + x_2$$

$$t = t_1 + t_2$$

How fast will we be moving once we shut off the thrusters, however? As it turns out, we don't really need to know; velocity can be calculated by multiplying your acceleration by the amount of time you spend accelerating. In other words, we can replace v with at_1 and not have to worry about its actual value.

With all of this knowledge, we can finally combine all of our knowledge into a single equation that we can solve to determine how long the thrusters need to be fired.

$$x = x_1 + x_2$$

$$x = \frac{1}{2}a{t_1}^2 + vt_2$$

$$x = \frac{1}{2}a{t_1}^2 + at_1(2.5 - t_1)$$

$$x = \frac{1}{2}a{t_1}^2 + 2.5at_1 - a{t_1}^2$$

$$a = 1; x = 1.5$$

$$1.5 = 2.5t_1 - 0.5{t_1}^2$$

$$0 = -0.5t^2 + 2.5t - 1.5$$

We've managed to remove all of the other variables, but this equation isn't something we can solve on its own. By rearranging it into the format shown above, the equation is now something called a quadratic equation; an equation of the form $0 = ax^2 + bx + c$. The equation below can be used to solve this type of equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-2.5 \pm \sqrt{2.5^2 - (4 * -0.5 * -1.5)}}{2 * -0.5}$$

$$t = \frac{-2.5 \pm \sqrt{6.25 - 3}}{-1}$$

$$t = 2.5 \pm \sqrt{3.25}$$

$$t = 0.6972 \; or \; 4.3027$$

This equation always gives two answers, but the interesting thing about it is you can always pick whichever answer makes sense for your situation. In this case, 4.3 seconds would be far too much time to fire the thrusters; we'd overshoot the launch coordinates and ruin the mission. On the other hand, firing the thrusters for 0.6972 seconds fits very nicely into our timeframe, and should allow us to reach the launch coordinates on time.

Keep in mind, though, this only covers half the trip; we also have to fire the opposite thrusters for 0.6972 seconds in order to slow down again. We've also only been focused on movement along the X axis during these calculations; the calculations would need to be repeated for movement along the Y axis. As it turns out, we're moving the same distance along both axes, so the numbers end up being the same. This results in a final thruster burn time of 2.7888 seconds (0.6972 seconds in each direction) to go from our starting position of (0,0) to our launch coordinates of (3,3) in exactly five seconds.

Again, your team will need to calculate how much fuel (measured in seconds of burn time) will be required in order to bring the SL-OMV launch vehicle to each of the provided launch coordinates in the specified amount of time, coming to a complete stop each time.

Sample Input

The first line of your program's input, received from the standard input channel, will contain a positive integer representing the number of test cases. Each test case will include:

- A line containing information about the initial state of the launch vehicle. This information includes the following values, separated by spaces:
  - A positive integer less than or equal to 6, S, representing the number of CubeSats on board
  - An integer representing the starting X coordinate (in meters)
  - An integer representing the starting Y coordinate (in meters)
- S lines containing information about the launch parameters for each CubeSat on board the launch vehicle, listed in the required launch order. This information includes the following values, separated by spaces:
  - An integer representing the launch X coordinate (in meters)
  - An integer representing the launch Y coordinate (in meters)
  - A non-negative decimal value representing the number of seconds the vehicle must take to travel from its previous location to these coordinates

```
2
2 0 0
3 3 5.0
5 5 3.0
2 0 0
1 7 10.0
5 3 6.0
```

Sample Output

For each test case, your program must print the minimum amount of fuel required to launch the CubeSats in the order listed with the given launch parameters. Fuel amounts should be measured in seconds of burn time, rounded to two decimal places, and include any trailing zeroes.

```
6.79
4.77
```