Problem 128: Be Rational

Difficulty: Hard

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Problem Background

Pi (π) is a very important number. It defines the ratio between a circle's circumference and its diameter, and has a number of interesting mathematical properties. One of these properties is the fact that it is irrational. Rational numbers are those that can be accurately defined with a proper fraction, be it as simple as $^{1}/_{2}$ or as complicated as $^{123456789}/_{987654321}$. Irrational numbers, like π, can't be written this way. You may have seen π written before as $^{22}/_{7}$ or $^{333}/_{106}$, but these are just approximations; they don't accurately represent the full value of π.

To represent π accurately as a fraction, you have to use an infinitely-long series of nested fractions, known as a "continued fraction":

$$\pi = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \ddots}}}}$$

You might be wondering where those numbers are coming from; they are calculated using this process. Take the number you wish to represent as a continued fraction and discard the portion to the left of the decimal point (in the case of π, you would subtract 3, leaving you with 0.14159...). Divide 1 by this number to determine the denominator for the next level. Continue repeating until you reach the desired level:

$$\pi = 3.1415926535\ldots$$
$$\pi = 3 + 0.1415926535\ldots$$
$$\pi = 3 + \cfrac{1}{7.0625133104\ldots}$$
$$\pi = 3 + \cfrac{1}{7 + \cfrac{1}{15.9965932606\ldots}}$$
$$\pi = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1.0034183849\ldots}}}$$

These integer values that appear along the left side of the fraction are known as the "terms" of the continued fraction. Since π is an irrational number, it has an infinitely long list of these terms.

However, you don't have to have an irrational number to create a continued fraction; any number can be represented this way! The main difference is that with a rational number, you will eventually get an exact integer, and be forced to stop. For example, if we use only two decimal places for π, we only get the terms 3, 7, and 7 (see below).

$$3.14 = 3 + 0.14 = 3 + \frac{14}{100}$$
$$3.14 = 3 + \frac{1}{\frac{100}{14}}$$
$$3.14 = 3 + \frac{1}{7 + \frac{2}{14}}$$
$$3.14 = 3 + \frac{1}{7 + \frac{1}{\frac{14}{2}}}$$
$$3.14 = 3 + \frac{1}{7 + \frac{1}{7}}$$

You'll notice that this time, we converted the decimal portion of the number into a fraction. Because computers store numbers in a binary format, they're notoriously bad about working with decimal numbers. Attempting to calculate continued fractions with decimal numbers will usually fail; in this case, the initial decimal value of 0.14 may end up being recorded as something like 0.1400000000012 - those extra decimal places will throw off your calculations and prevent the algorithm from terminating when expected. Converting the decimal to a fraction first allows us to work with integers, which can be recorded and calculated accurately.

Problem Description

In this problem, you will need to calculate the first ten terms of the continued fractions of a list of decimal numbers. Since these numbers will all be rational numbers (since it's impossible to print an irrational number in full), not all of these numbers will actually *have* ten terms. If this happens, simply print as many terms that do exist.

As mentioned in the last paragraph of the background section, we strongly recommend converting any decimals to fractions before attempting to do your calculations. Attempting to calculate a continued fraction with decimal values will likely result in incorrect answers.

Sample Input

The first line of your program's input, received from the standard input channel, will contain a positive integer representing the number of test cases. Each test case will include a single line, containing a positive decimal number.

```
4
3.14
3.14159
3.1415926535
1.6180339887
```

Sample Output

For each test case, your program must print the first ten integer terms of the continued fraction representation of the given number, on a single line, with each term separated by a space. If a continued fraction has fewer than ten terms, print all of them.

```
3 7 7
3 7 15 1 25 1 7 4
3 7 15 1 292 1 1 6 2 13
1 1 1 1 1 1 1 1 1 1
```