Problem 230: Sierpinski's Triangle

Difficulty: Easy

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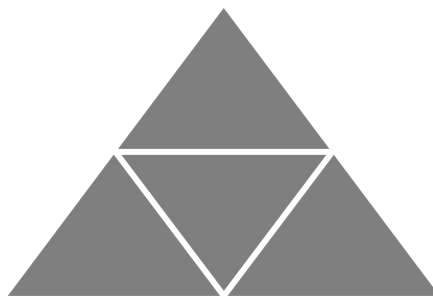
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Problem Background

Named after Polish mathematician Wacław Sierpiński but seen in decorative works for centuries before his birth, a Sierpiński Triangle is a fractal design built out of a series of triangles. The pattern can be generated in a number of ways, but consists of repeatedly removing equilateral-triangle-shaped portions from an existing equilateral triangle, resulting in a lattice pattern. This design has a number of applications in mathematics and chaos theory.

Problem Description

Your team is conducting some research into the properties of Sierpiński Triangles and needs a program that can quickly produce information about these properties. Specifically, after a certain number of iterations, you need to know the total number of triangles remaining in a Sierpiński Triangle and the total area of those triangles.
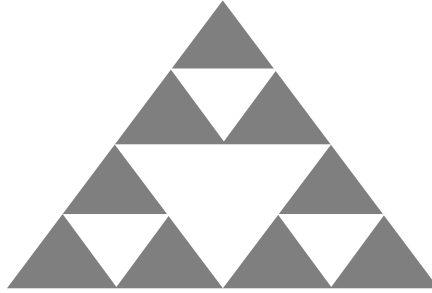
Your Sierpiński Triangles will be generated by starting with a single triangle of a specified size; this single triangle will represent the design following iteration 0. In each successive iteration, you'll divide each triangle remaining in the design into four equal parts; the division for iteration 1 is shown below:

Then, remove the central (upside-down) triangle:

This will leave three triangles for each triangle that was in the design prior to the iteration. After iteration 2, your design will look like this:

Again, given a number of iterations and the width of the original triangle, your program must report the number and total area of triangles remaining in the design. Remember that the area of an equilateral triangle can be calculated using this formula:

$$A = \frac{\sqrt{3}}{4} w^2$$

Sample Input

The first line of your program's input, received from the standard input channel, will contain a positive integer representing the number of test cases. Each test case will include a single line that contains the following values separated by spaces:

- A positive number, W, the width of the original triangle in centimeters
- A non-negative integer, N, the number of iterations to use to generate a Sierpiński Triangle

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3
10 0
5 1
12 2
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Sample Output

For each test case, your program must print a single line containing the following values, separated by spaces:

- An integer indicating the number of triangles remaining in the design after the given number of iterations have been completed, and
- The total area of those triangles in square centimeters, rounded to two decimal places and including any trailing zeroes.

```
1 43.30
3 8.12
9 35.07
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