Problem 219: Maxwell's Demon

Difficulty: Hard

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Problem Background

Maxwell's demon is a thought experiment posed by physicist James Clerk Maxwell which seemingly violates the second law of thermodynamics. According to that law, energy will only naturally flow from hotter (more energetic) areas to colder (less energetic) areas. Maxwell's demon is proposed as an entity who controls a "door" between two chambers containing gases of different temperatures, allowing energy to instead flow in the opposite direction.

Imagine placing a piston between the two chambers; the hotter gas has more energy and will push the piston towards the chamber with less energy. This results in a change in volume that will eventually equalize the temperatures, as the compressed gas will become hotter while the decompressed gas will become cooler. Once this equilibrium is reached, the piston will stop. Now imagine that below the cylinder there is a door which Maxwell's demon controls. As the temperature changes, the demon selectively admits molecules between the chambers to reverse the situation. The gas on the compressed side is now hotter than the gas on the decompressed side and the piston is pushed back the other way. If there is enough gas and the demon is fast enough, a reciprocating engine can be built without any outside driver to the system, granting free work, and a perpetual motion machine for the demon's nefarious purposes.

Problem Description

Your challenge will be to develop a demon that will partition a one-meter-long, one-dimensional container of gas into two chambers. The chamber contains a number of particles; you will be given their masses and initial velocities. You must model the movement of these particles as they collide with each other and the ends of the container. After a certain period of time, your demon will close his door at the halfway point within the container (0.5 meters). This will trap the particles on one side or the other, creating two chambers of (most likely) unequal energy levels. You must then calculate the energy level on each side of the demon's door.

Positions within the container will range from 0.0 meters to 1.0 meters. Note that because we're in one dimension, particles will constantly be colliding with their neighbors and the walls at 0.0 and 1.0 meters. All collisions between particles will be completely elastic; this means that when two particles collide, the combined momentum of those particles prior to the collision will be equal to their combined momentum after the collision. When a particle collides with a wall, it simply reverses direction; its velocity retains the same magnitude, but switches from positive to negative (or vice versa).

You might find these equations helpful as you solve this problem:

Terms and Units	Equations
$x = position\ (meters)$	$x_f = vt + x_0$
$t = time\ (seconds)$	$E = \frac{1}{2}mv^2$
$m = mass\ (grams)$	$p = mv$
$v = velocity\ \left(\frac{meters}{second}\right)$	$p_{a0} + p_{b0} = p_{af} + p_{bf}$
$E = energy\ \left(millijoules = \frac{grams \times meters^2}{seconds^2}\right)$	$v_{af} = \left(\frac{m_a - m_b}{m_a + m_b}\right)v_{a0} + \left(\frac{2m_b}{m_a + m_b}\right)v_{b0}$
$p = momentum\ \left(\frac{grams \times meters}{seconds}\right)$	$v_{bf} = \left(\frac{m_b - m_a}{m_a + m_b}\right)v_{b0} + \left(\frac{2m_a}{m_a + m_b}\right)v_{a0}$
$(something)_0 = initial\ something$	
$(something)_f = final\ something$	

Again, using these equations, model the interactions between all particles in the container until the stated time has elapsed. At that point, calculate the total energy of all particles on each side of the door located at 0.5 meters. If no particles exist on one side of the door, the energy on that side is 0.

One last important note: Normally we recommend that you do not attempt to round intermediate values when performing calculations, and that you only round the final results. For this problem, we strongly recommend that you **round each particle's position and velocity to no fewer than six decimal places following each recalculation**. Computers struggle to accurately store decimal numbers, and this will reduce the risk of errors resulting from this difficulty.

Sample Input

The first line of your program's input, **received from the standard input channel**, will contain a positive integer representing the number of test cases. Each test case will include:

- A line containing the following values, separated by spaces:
  - A positive integer, **N**, representing the number of particles in the chamber.
  - A positive decimal value, **T**, representing the time (in seconds) at which the "demon" will separate the chamber into two halves.

- N lines containing information about the particles. Each line contains the following values, separated by spaces:
  - A positive decimal number representing the particle's mass in grams
  - A decimal number between 0.0 and 1.0 inclusive representing the particle's initial position within the chamber, in meters
  - A decimal number representing the particle's initial velocity in meters per second.

```
2
2 4.0
1.8 0.1 0.3
2.6 0.9 -0.1
4 4.0
1.1 0.3 0.4
4.3 0.1 -0.2
0.8 0.4 1.3
2.2 0.7 -0.1
```

Sample Output

For each test case, your program must print a single line, containing the word "LEFT" if the total kinetic energy of all particles on the left side of the door (from 0.0 meters inclusive to 0.5 meters exclusive) is greater than that on the right side of the door (from 0.5 meters exclusive to 1.0 meters inclusive) after T seconds have elapsed, or the word "RIGHT" otherwise. No particles will be located at exactly 0.5 meters when the stated time has elapsed.

```
RIGHT
RIGHT
```