Ćwiczenia

Jakub Kowal

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1 Lista 1

1.5

33-bitowe słowo $x = sm2^c$
cecha 8 bitów ze znakiem
mantysa 24 bity $\in [\frac{1}{2}, 1)$

1.5.a)

$x_{max} = (0.11...1)_2 * 2^{127} = (1 - 2^{-24}) * 2^{127} = 1,7 * 10^{38}$
$x_{min} = (0.10...0)_2 * 2^{-127} = 2^{-1} * 2^{-127} = 2^{-128}$
$[-x_{max}, -x_{min}] \cup [x_{min}, x_{max}]$

1.5.b)

$(-x_{min}, x_{min})$

1.5.c)

$\epsilon = \frac{1}{2} * \beta^{1-t} = 2^{-1} * 2^{-23} = 2^{-24}$

Single

1.5.a)

$x_{max} = (2 * 2^{-23}) * 2^{127} \approx 3,4 * 10^{38}$
$x_{min} = 1 * 2{-}126 = 2^{-126}$
$x_{minsub} = (0.00.....01)_2 * 2^{-126} = 2^{-23} * 2^{-126} = 2^{-149}$
$[-x_{max}, -x_{min}] \cup \{0\} \cup [x_{min}, x_{max}]$

1.5.b)

$(-x_{minsub}, x_{minsub})$

1.5.c)

$\epsilon = \frac{1}{2}\beta^{1-t} = 2^{-1} * 2^{-23} = 2^{-24}$

1.6

1.6.a)

x $= 1 + 2^{-24}$
$\epsilon = 2^{-23}$
$x^- = 1$
$x^+ = 1 + 2^{-23}$

1.6.b)

$x \bigoplus 1 = 1$
$x \in (0, 2^{-24} + 2^{-25}]$

1.6.c)

$x \bigoplus = x$
$x < \frac{x}{2^{23}}$

1.7

$A(a_1, ..., a_n) = (...(a_1 \oplus a_2) \oplus a_3) \oplus ...) \oplus a_n = (...(a_1 + a_2)(1+\delta_1) + a_3)(1+\delta_2) + ...) + a_n)(1+\delta_{n-1}) = a_1 + a_1 * E_1 + a_2 + a_2 * E_1 + a_3 + a_3 * E_2 ... + a_n + a_n * E_{n-1} = (a_1 + a_2 + ... + a_n) + (a_1 * E_1 + a_2 * E_1 + ... + a_n * E_{n-1}) = S + (a_1 * E_1 + a_2 * E_1 + ... + a_n * E_{n-1})$
$|\delta| \leq \epsilon$
$1 + E_k = \prod_{i=1}^{k}(1 + \delta_k) \leftarrow$ *wprowadzamy*
$(a_1 * E_1 + a_2 * E_1 + ... + a_n * E_{n-1}) = E_{max} = \prod_{i=1}^{n-1}(1 + |\delta_i|) - 1 \leq \prod_{i=1}^{n-1}(1+\epsilon) - 1$
$a_1 * E_1 + a_2 * E_1 + ... + a_n * E_{n-1} \leq (a_1 + a_2 + ... + a_n) * E_{max} \leq ((a_1 + a_2 + ... + a_n) * \prod_{i=1}^{n-1}(1 + \epsilon)) - 1 = S(1 + \epsilon)^{n-1} - 1$
$\frac{|\tilde{S} - S|}{|S|} \leq \frac{S + S*(1+s)}{|S|} \leftarrow$ Zmazał za szybko
$\tilde{S} \leq S + S * (1 + \epsilon)^{n-1} - 1$

2 Lista 2

2.1

$y = \sqrt{x^2+1} - 1$
TW z wykładu:
Jeśli x,y - dodatnie liczby w dwójkowej arytmetyce float, takie że:
$x > y, 2^{-q} \leq 1 - \frac{y}{x}$
to przy odejmowaniu tracimy najwyżej q bitów q = 2
$2^{-2} \leq 1 - \frac{1}{\sqrt{x^2+1}}$
$\frac{1}{4} = 1 - \frac{1}{\sqrt{x^2+1}}$
$\frac{1}{\sqrt{x^2+1}} \leq \frac{3}{4}$
$\sqrt{x^2+1} >= \frac{3}{4}$

2.4

$f(x) = \sqrt{x+2} - \sqrt{x}$
Problem dla dużych x : $\sqrt{x} \approx \sqrt{x+2}$
$\sqrt{x+2} - \sqrt{x} / * \frac{\sqrt{x+2}+\sqrt{x}}{\sqrt{x+2}+\sqrt{x}}$
$\frac{x+2-x}{\sqrt{x+2}+\sqrt{x}} = \frac{2}{\sqrt{x+2}+\sqrt{x}}$
$a^2 - b^2 = (a-b)(a+b)$