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Eigenmodes in nearly integrable quantum chains

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Index number: 250193

Master thesis
under supervision of
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Wrocław
University
of Science
and Technology

Wrocław 2023

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I would like to express my sincere gratitude to prof. dr hab. Marcin Mierzejewski for being my

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Abstract

Apparent incompatibility of classical irreversible thermodynamics with

Keywords: *integrals of motion, ETH, integrability breaking, XXZ model*

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Contents

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1 Introduction

The results concerning spin transport in the long range XXZ have already been published in Mierzejewski et al. [1].

2

Lanczős based numerical methods for quantum many-body systems

One of the two purposes of this thesis is to develop and test a set of numerical tools based on the Lanczős algorithm, which is an iterative method for finding extremal eigenvalues of a large, sparse matrices. Therefore, this chapter serves as a pedagogical introduction to the core ideas of these methods, however without going too deep into details of algorithm analysis. In the exposition we follow the excellent treatments found in Sandvik [2] and PhD thesis by Crivelli [3]. The interested and mathematically inclined reader is referred to the classic textbook of numerical linear algebra by Trefethen and Bau [4].

In the first part of this chapter, the Lanczős algorithm is developed in its simplest form which in itself is useful for efficient calculation of a ground state eigenvalue and eigenvector of sparse Hamiltonians, and thus for determining their ground state properties. However, in this work we are mainly interested in infinite temperature correlation functions, which in principle require sampling of the whole spectrum. To this end, in subsequent sections we develop a scheme for time evolution of arbitrary state, called the Krylov propagator [5], and combine it with the idea of Dynamical Quantum Typicality (DQT), which states that a single pure state can have the same properties as an ensemble density matrix [6, 7, 8]. We finish this chapter with a proposal of employing this method to the identification of local integrals of motion (cite my bachelors)

2.1 Lanczos method for ground state calculation

3 Summary

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