Resonance in a RLC circuit

Vibration are defined as processes in which certain quantities alternately decrease and increase over time. They can be divided into free, damped and forced. The phenomenon of **resonance** can be observed for forced vibrations. It consists in amplifying the amplitude of vibrations, which is caused by the influence of the external force, variable in time, on the vibrating system, characterized by the resonance frequency (or close to it). The simulation will work for series and parallel RLC circuit, i.e. one in which the resistor, inductor and capacitor are connected in series or in parallel. Each of these elements has its own impedance:

- resistor impedance $Z_R = R$; R - resistance in ohms [Ω]
- inductor impedance $Z_L = i\omega L$; L - inductor inductance in henry [H]
- capacitor impedance $Z_C = \frac{1}{i\omega C}$; C - capacitor capacity in farad [F]

In general, each impedance has two components: real, i.e. resistance R, and imaginary, i.e. reactance X.

$$Z(\omega) = R(\omega) + iX(\omega)$$

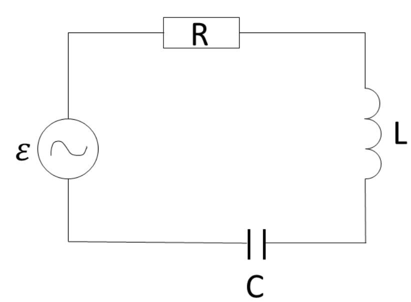
For both of them $\varepsilon(t) = Asin(\omega t)$

1 Series circuit

In this case, the elements are connected in series, so the equivalent impedance is the sum of the impedances of the individual elements.

$$Z = Z_R + Z_L + Z_C$$

$$Z = R + i\left(\omega L - \frac{1}{\omega C}\right)$$



The reactance for this circuit is $X(\omega) = \omega L - \frac{1}{\omega C}$. Resonance occurs when the reactance is equal to 0. The resonance will occur when $\omega L = \frac{1}{\omega C}$. Thus, the resonant frequency is $\omega_0 = \frac{1}{\sqrt{LC}}$, hence the resonant frequency, due to the fact that $\omega = 2\pi f$ will be $f_0 = \frac{1}{2\pi\sqrt{LC}}$
The Kirchhoff's law for this circuit is:

$$\omega Acos(\omega t) = \frac{d\varepsilon(t)}{dt} = \frac{di(t)}{R} + L\frac{d^2i(t)}{dt^2} + \frac{i(t)}{C}$$

It is known that impedance in general is a complex quantity and current intensity is a real quantity. The impedance modulus is expressed by the formula

$$|Z| = \sqrt{R^2 + X^2}$$

Solving the differential equation with respect to i(t) gives the solution:

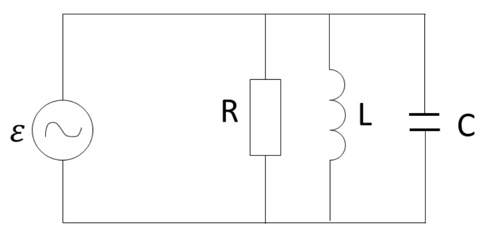
$$i(\omega) = \frac{A}{|Z|} sin(\omega t - \varphi_s)$$

$$\varphi_s = -arctg \frac{\omega L - \frac{1}{\omega C}}{R}$$

φ_s this is the phase shift between current i(t) and $\varepsilon(t)$, constituting the exciting force of the circuit. The amplitude of the current flowing in the circuit is given by the formula

$$\mathbf{I_{max}(\omega) = \frac{A}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}}$$

2 Parallel circuit



In a parallel circuit, the reciprocal of the impedance is the sum of the reciprocal of the impedance of the individual circuit components. This quantity is called admittance Y.

$$Y = \frac{1}{Z} = \frac{1}{R} + i\omega C - i\frac{1}{\omega L}$$

The admittance modulus is equal to $|Y| = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$
The dependence of the amplitude of the current flowing in the circuit on the frequency is

$$I_{max}(\omega) = A\sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$\mathbf{I_{max}(f) = A\sqrt{\left(\frac{1}{R}\right)^2 + \left(2\pi f C - \frac{1}{2\pi f L}\right)^2}}$$

For this circuit, resonance will occur when the current flowing in the circuit reaches its minimum value. Thus, the resonant frequency is derived from the formula $\omega L = \frac{1}{\omega C}$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\mathbf{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

The phase shift between current and voltage is

$$\varphi_d = -arctg\left(R\left(\omega C - \frac{1}{\omega L}\right)\right)$$