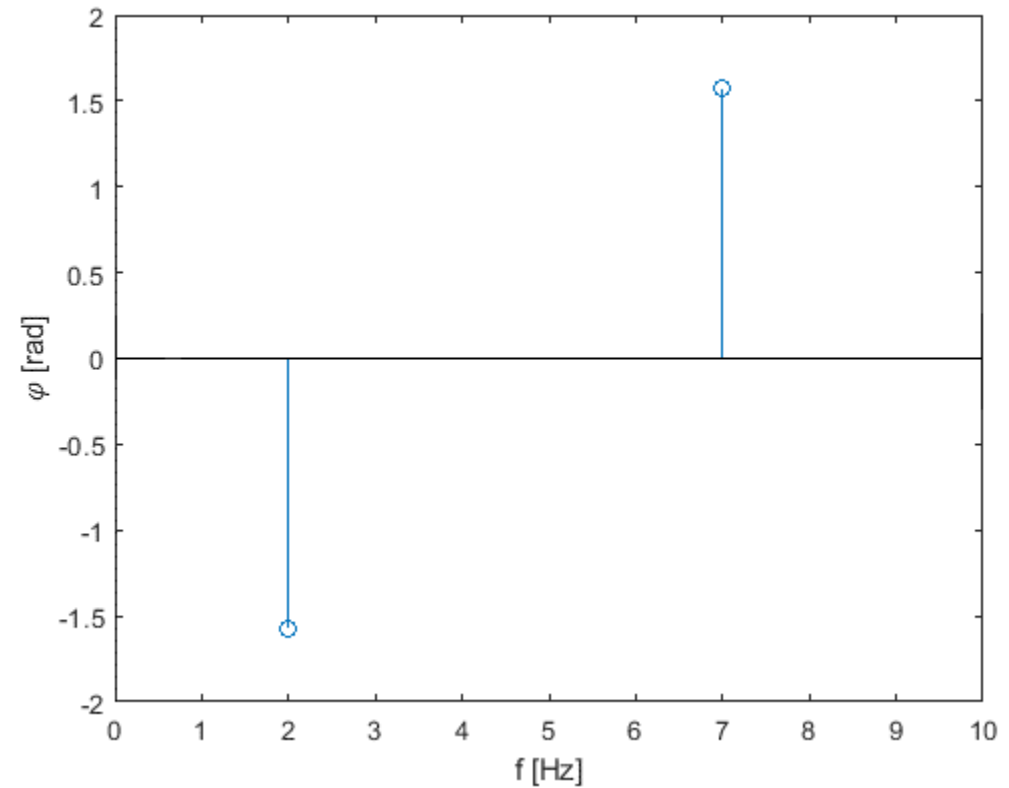
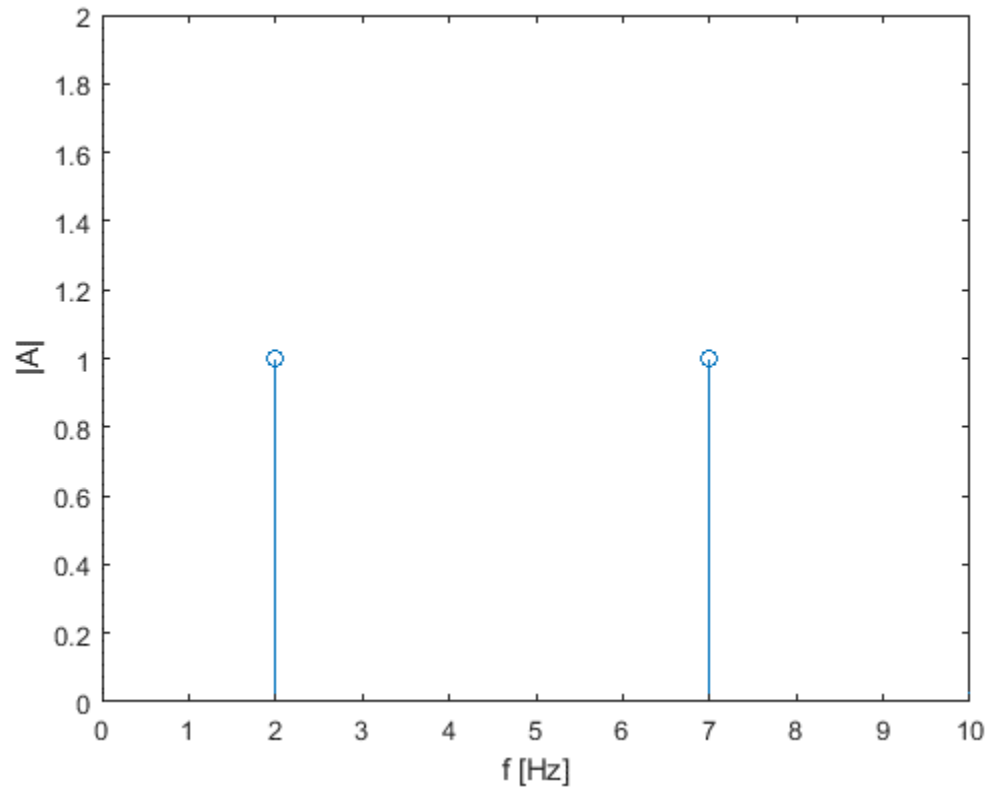


Signály a informace

7. cvičení

Spektrum - příklad

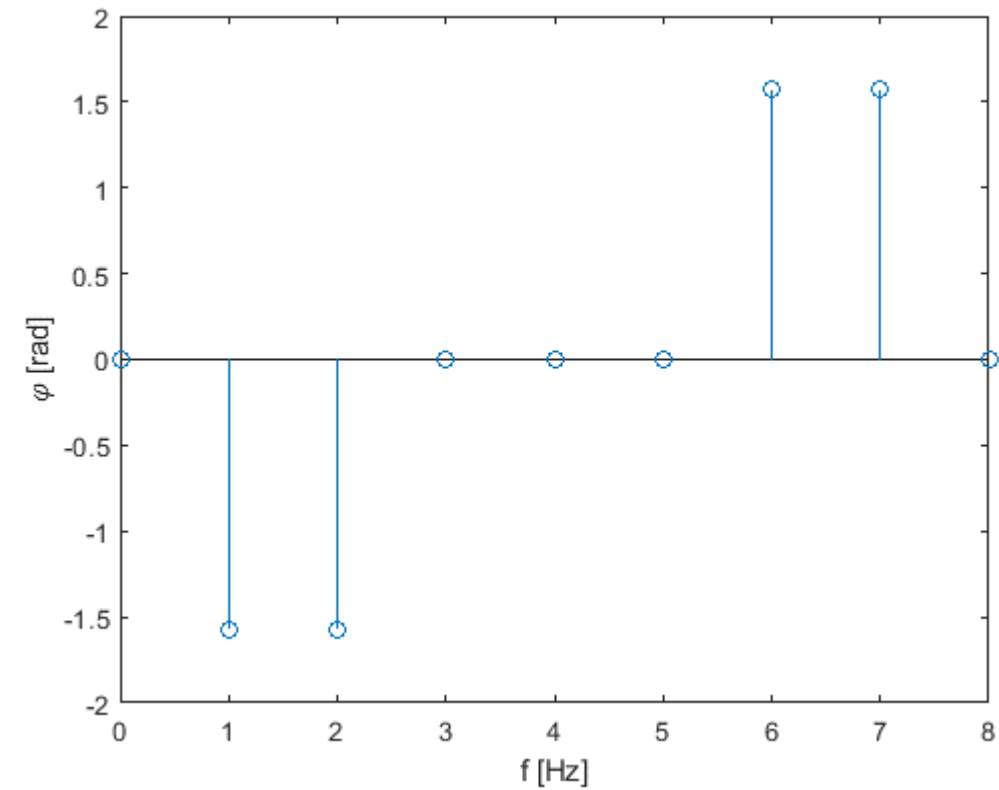
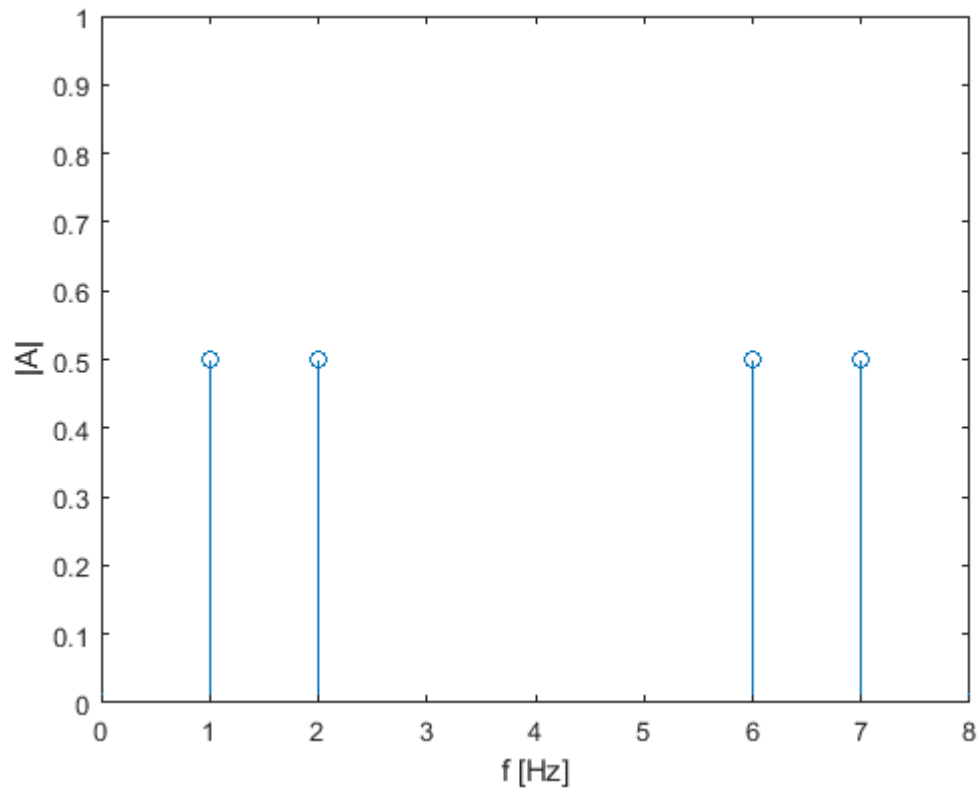
$$x(t) = \sin(2\pi 2t) + \cos\left(14\pi t + \frac{\pi}{2}\right) = \cos\left(2\pi 2t - \frac{\pi}{2}\right) + \cos\left(2\pi 7t + \frac{\pi}{2}\right)$$



Spektrum - příklad

$$x(t) = \cos\left(2\pi 2t - \frac{\pi}{2}\right) + \cos\left(2\pi 7t + \frac{\pi}{2}\right)$$

$$F_s = 8 \text{ Hz}$$



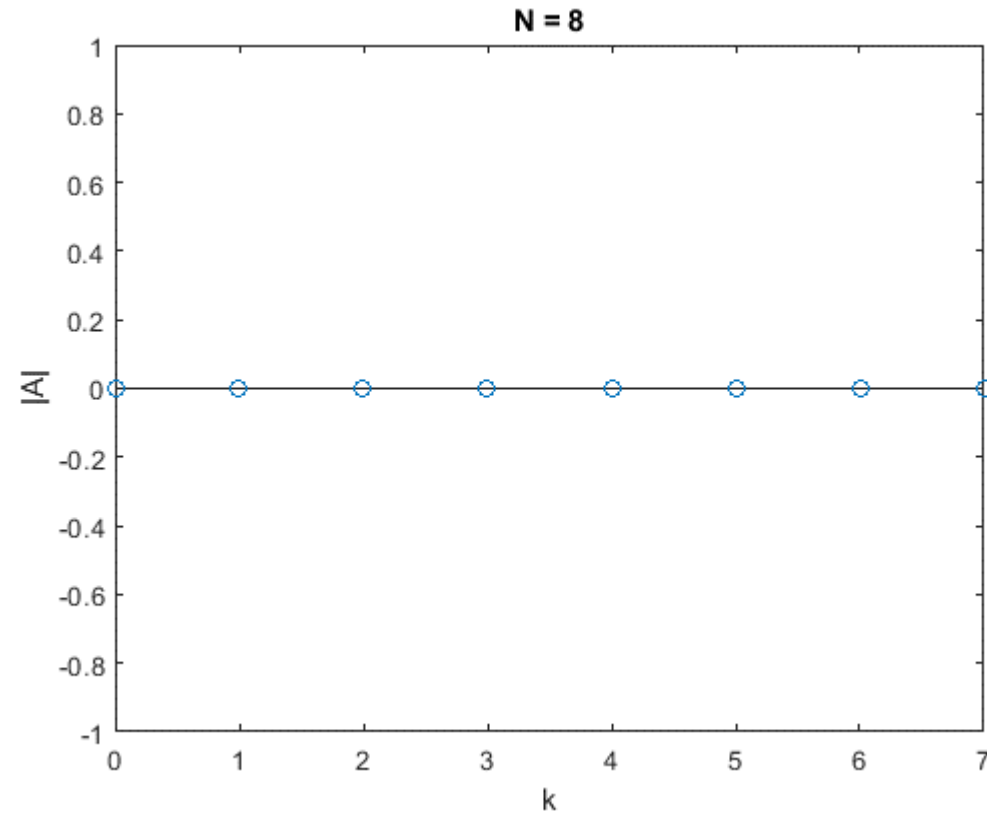
DFT – Diskrétní Fourierova transformace

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-\frac{i2\pi nk}{N}}$$

$$k = 0 \dots N - 1$$

N – bodová DFT

$$\Delta f = \frac{F_s}{N}$$



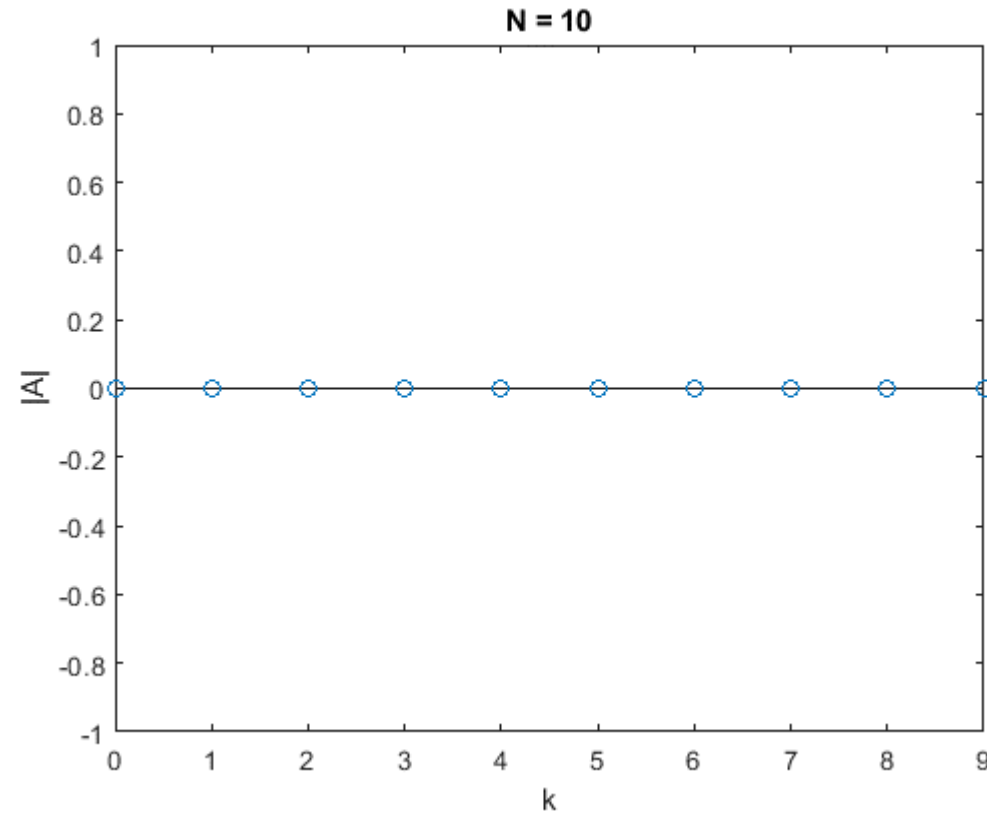
DFT – Diskrétní Fourierova transformace

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-\frac{i2\pi nk}{N}}$$

$$k = 0 \dots N - 1$$

N – bodová DFT

$$\Delta f = \frac{F_s}{N}$$



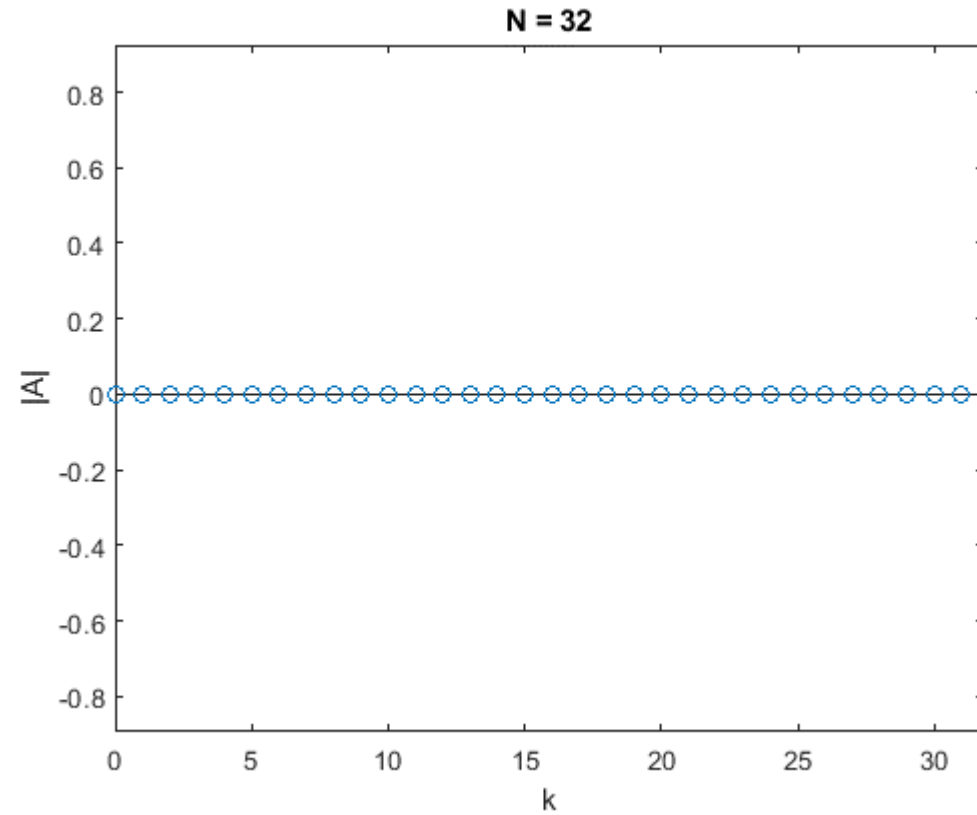
DFT – Diskrétní Fourierova transformace

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-\frac{i2\pi nk}{N}}$$

$$k = 0 \dots N - 1$$

N – bodová DFT

$$\Delta f = \frac{F_s}{N}$$



DFT – Diskrétní Fourierova transformace

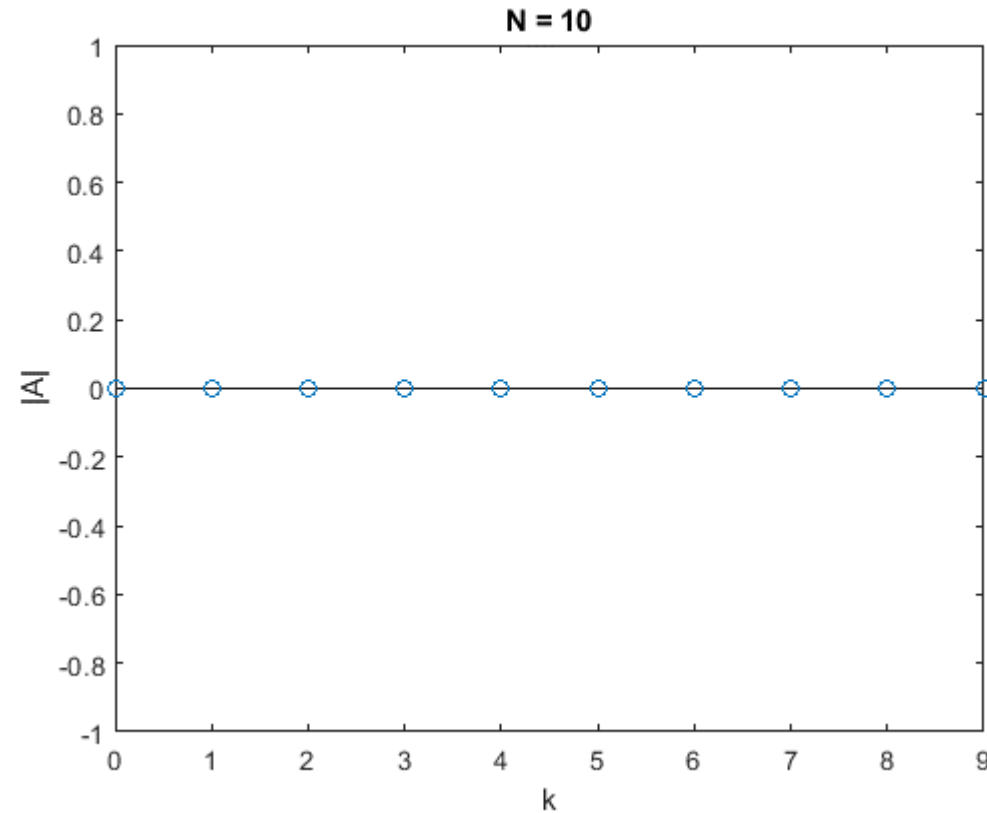
$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-\frac{i2\pi nk}{N}}$$

$$k = 0 \dots N - 1$$

N – bodová DFT

$$\Delta f = \frac{F_s}{N}$$

$$f = k \cdot \frac{F_s}{N}$$



DFT – Diskrétní Fourierova transformace

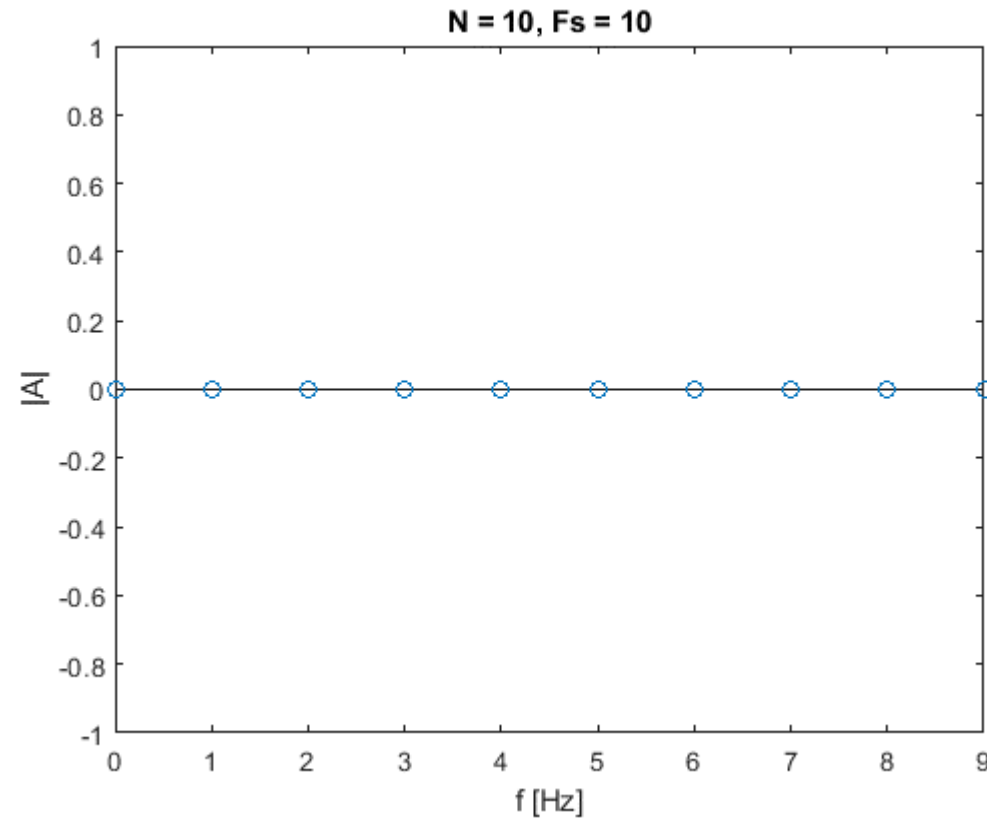
$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-\frac{i2\pi nk}{N}}$$

$$k = 0 \dots N - 1$$

N – bodová DFT

$$\Delta f = \frac{F_s}{N}$$

$$f = k \cdot \frac{F_s}{N}$$



DFT – Diskrétní Fourierova transformace

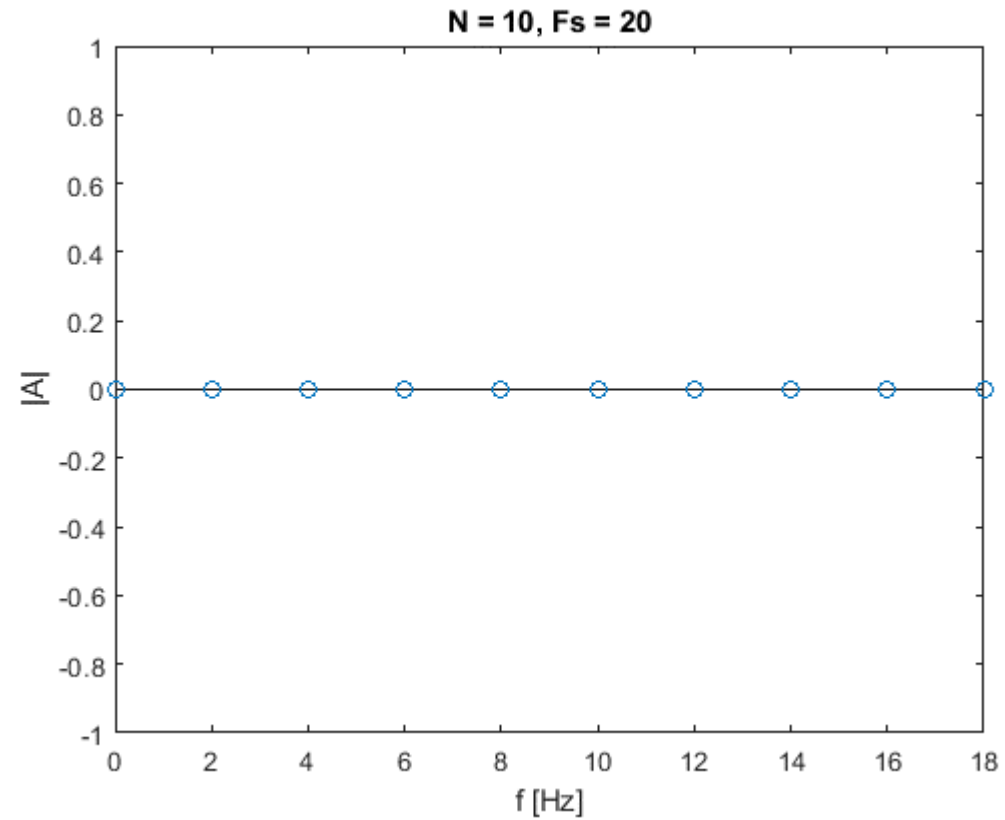
$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-\frac{i2\pi nk}{N}}$$

$$k = 0 \dots N - 1$$

N – bodová DFT

$$\Delta f = \frac{F_s}{N}$$

$$f = k \cdot \frac{F_s}{N}$$



DFT – Diskrétní Fourierova transformace

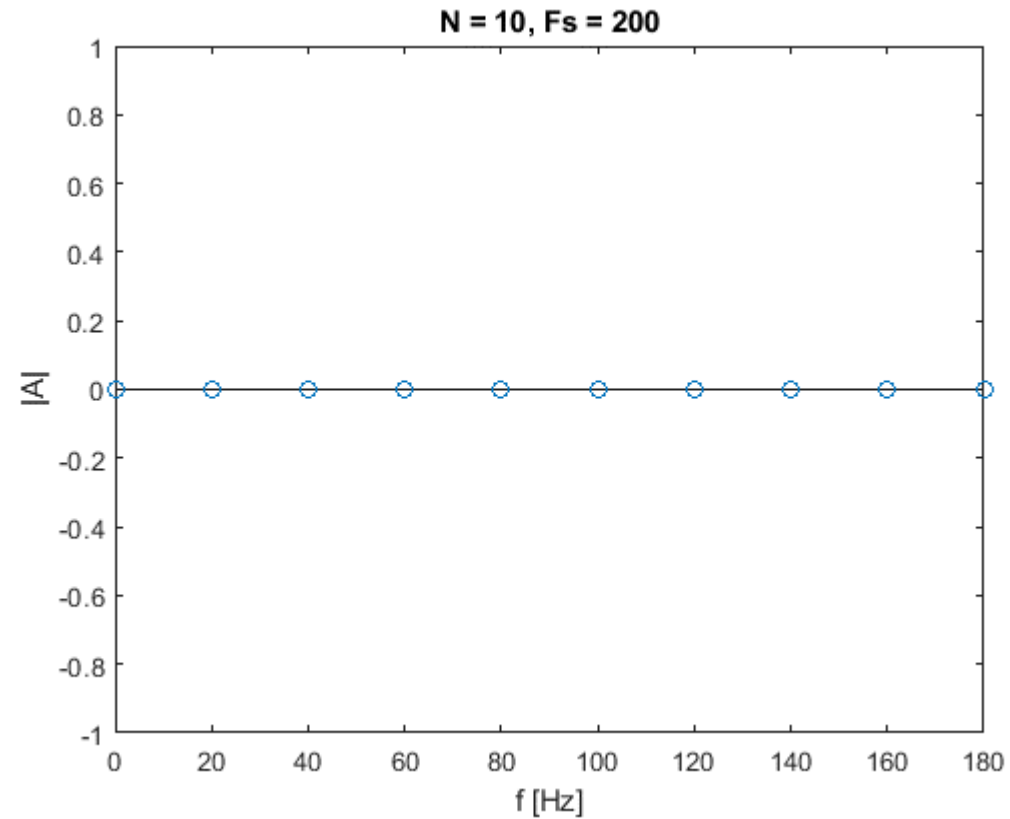
$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-\frac{i2\pi nk}{N}}$$

$$k = 0 \dots N - 1$$

N – bodová DFT

$$\Delta f = \frac{F_s}{N}$$

$$f = k \cdot \frac{F_s}{N}$$



DFT – Příklad

$$x(t) = \sin(2\pi 2t)$$

$$F_s = 8 \text{ Hz} \quad N = 4$$

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-\frac{i2\pi nk}{N}}$$

$$t = [0, \frac{1}{8}, \frac{2}{8}, \frac{3}{8}]$$

$$x(0) = \sin(2\pi 2 \cdot 0) = 0$$

$$x\left(\frac{1}{8}\right) = \sin\left(2\pi 2 \cdot \frac{1}{8}\right) = 1$$

$$x[n] = [0, 1, 0, -1]$$

$$x\left(\frac{2}{8}\right) = 0$$

$$x\left(\frac{3}{8}\right) = -1$$

DFT – Příklad

$$x(t) = \sin(2\pi 2t)$$

$$F_s = 8 \text{ Hz} \quad N = 4$$

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-\frac{i2\pi nk}{N}}$$

$$x[n] = [0, 1, 0, -1]$$

$$X[k] = [0, -\frac{i}{2}, 0, \frac{i}{2}]$$

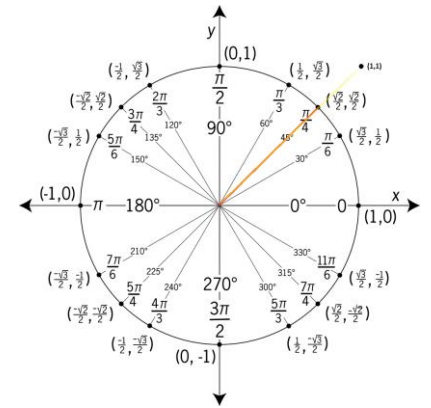
$$f = [0, 2, 4, 6]$$

$$X[0] = \frac{1}{4} \left(0e^{\frac{-i2\pi \cdot 0 \cdot 0}{4}} + 1e^{\frac{-i2\pi \cdot 1 \cdot 0}{4}} + 0e^{\frac{-i2\pi \cdot 2 \cdot 0}{4}} - 1e^{\frac{-i2\pi \cdot 3 \cdot 0}{4}} \right) = \frac{1}{4} (0 + 1 + 0 - 1) = 0$$

$$X[1] = \frac{1}{4} \left(0e^{\frac{-i2\pi \cdot 0 \cdot 1}{4}} + 1e^{\frac{-i2\pi \cdot 1 \cdot 1}{4}} + 0e^{\frac{-i2\pi \cdot 2 \cdot 1}{4}} - 1e^{\frac{-i2\pi \cdot 3 \cdot 1}{4}} \right) = \frac{1}{4} (0 - i + 0 - i) = -\frac{i}{2}$$

$$X[2] = \frac{1}{4} \left(0e^{\frac{-i2\pi \cdot 0 \cdot 2}{4}} + 1e^{\frac{-i2\pi \cdot 1 \cdot 2}{4}} + 0e^{\frac{-i2\pi \cdot 2 \cdot 2}{4}} - 1e^{\frac{-i2\pi \cdot 3 \cdot 2}{4}} \right) = \frac{1}{4} (0 - 1 + 0 + 1) = 0$$

$$X[3] = \frac{1}{4} \left(0e^{\frac{-i2\pi \cdot 0 \cdot 3}{4}} + 1e^{\frac{-i2\pi \cdot 1 \cdot 3}{4}} + 0e^{\frac{-i2\pi \cdot 2 \cdot 3}{4}} - 1e^{\frac{-i2\pi \cdot 3 \cdot 3}{4}} \right) = \frac{1}{4} (0 + i + 0 + i) = \frac{i}{2}$$



DFT – Příklad

$$x(t) = \sin(2\pi 2t)$$

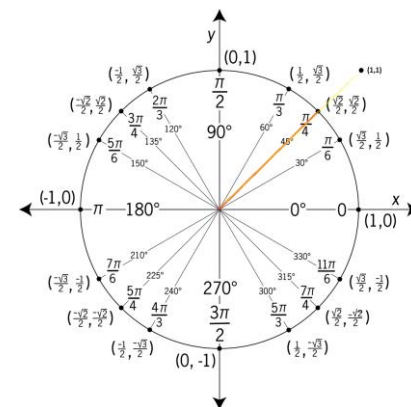
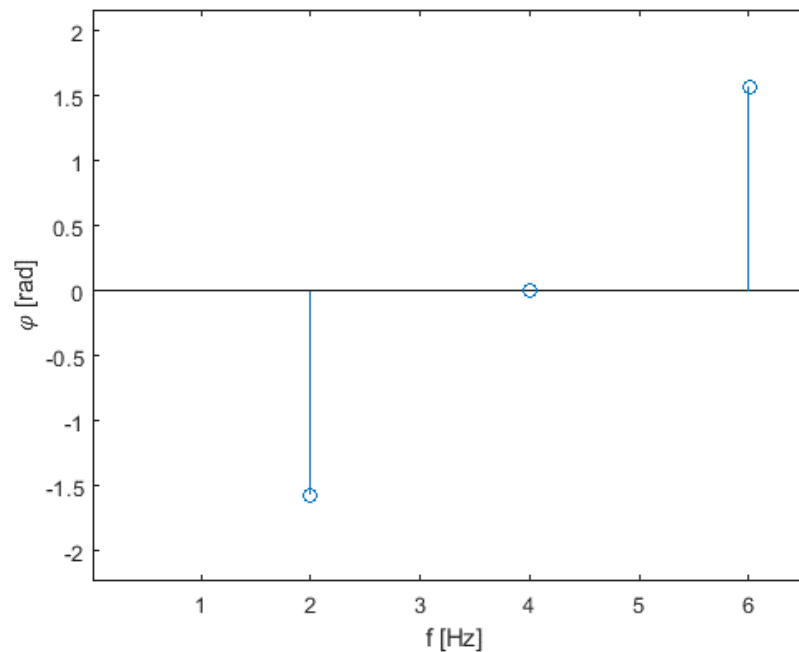
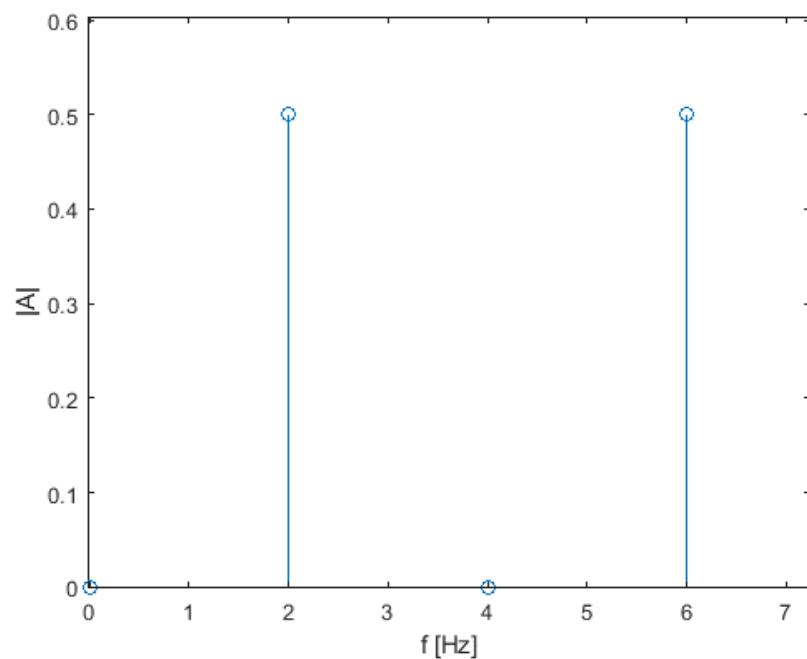
$$F_s = 8 \text{ Hz} \quad N = 4$$

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-\frac{i2\pi nk}{N}}$$

$$x[n] = [0, 1, 0, -1]$$

$$X[k] = [0, -\frac{i}{2}, 0, \frac{i}{2}]$$

$$f = [0, 2, 4, 6]$$



DFT – Příklad 2

$$x(t) = 2 + 4\sin(2\pi 2t - \frac{\pi}{2})$$

$$F_s = 8 \text{ Hz}$$

$$N = 4$$

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-\frac{i2\pi nk}{N}}$$

$$x(0) = 2 + 4\sin\left(2\pi 2 \cdot 0 - \frac{\pi}{2}\right) = -2$$

$$x\left(\frac{1}{8}\right) = 2$$

$$x\left(\frac{2}{8}\right) = 6$$

$$x\left(\frac{3}{8}\right) = 2$$

$$x[n] = [-2, 2, 6, 2]$$

DFT – Příklad 2

$$x(t) = 2 + 4\sin(2\pi 2t - \frac{\pi}{2})$$

$$F_s = 8 \text{ Hz}$$

$$N = 4$$

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-\frac{i2\pi nk}{N}}$$

$$x[n] = [-2, 2, 6, 2]$$

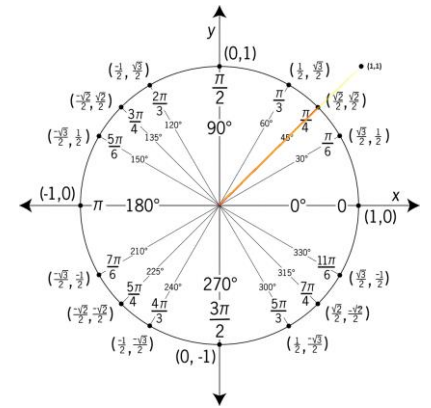
$$X[k] = [2, -2, 0, -2]$$

$$X[0] = \frac{1}{4} \left(-2e^{\frac{-i2\pi \cdot 0 \cdot 0}{4}} + 2e^{\frac{-i2\pi \cdot 1 \cdot 0}{4}} + 6e^{\frac{-i2\pi \cdot 2 \cdot 0}{4}} + 2e^{\frac{-i2\pi \cdot 3 \cdot 0}{4}} \right) = \frac{1}{4} (-2 + 2 + 6 + 2) = 2$$

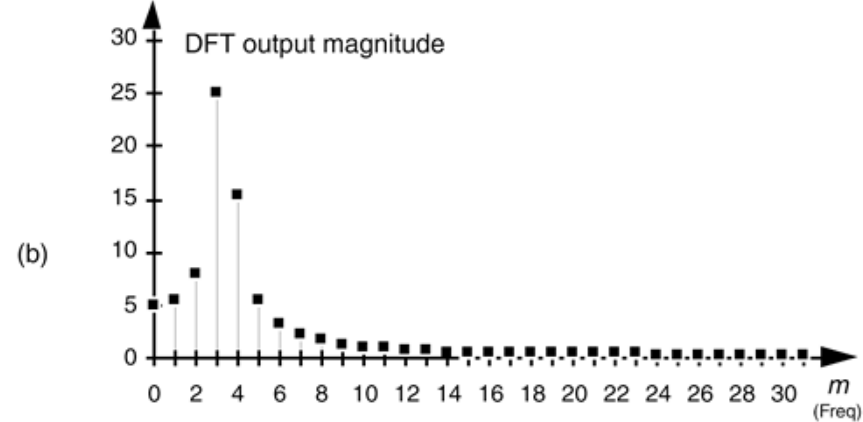
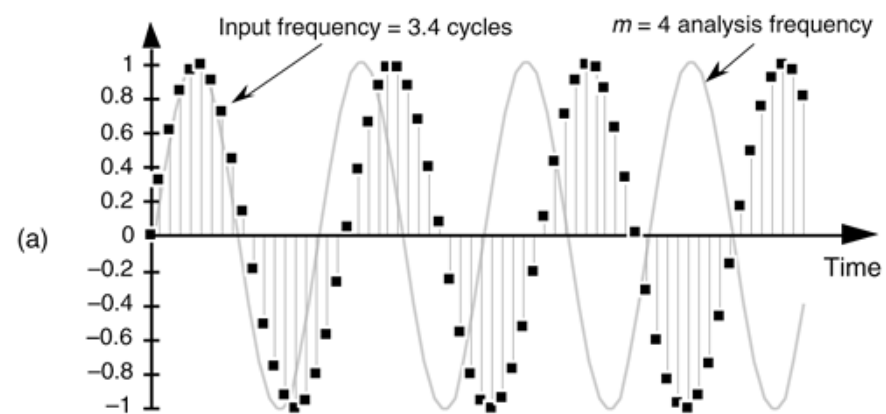
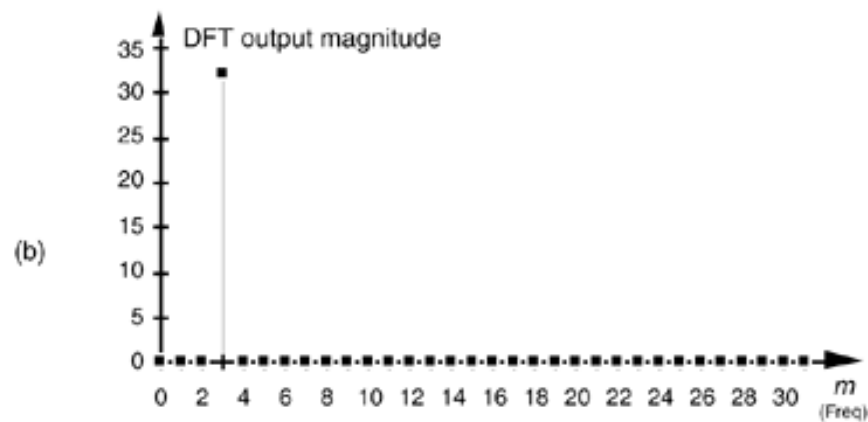
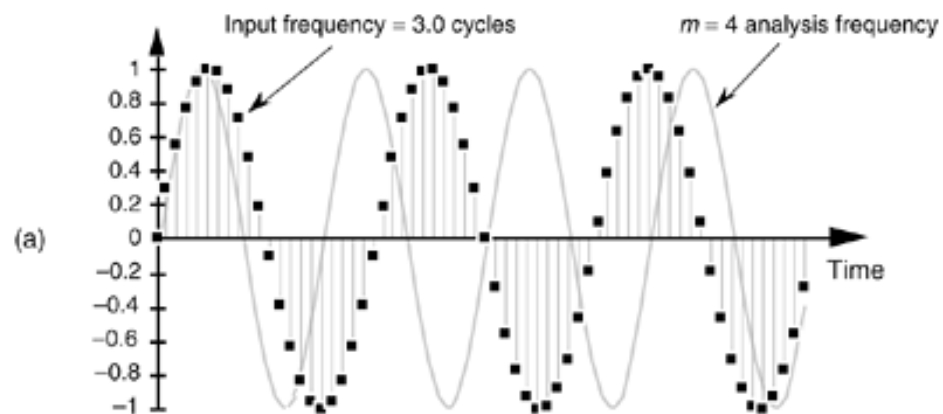
$$X[1] = \frac{1}{4} \left(-2e^{\frac{-i2\pi \cdot 0 \cdot 1}{4}} + 2e^{\frac{-i2\pi \cdot 1 \cdot 1}{4}} + 6e^{\frac{-i2\pi \cdot 2 \cdot 1}{4}} + 2e^{\frac{-i2\pi \cdot 3 \cdot 1}{4}} \right) = \frac{1}{4} (-2 - 2i - 6 + 2i) = -2$$

$$X[2] = \frac{1}{4} \left(-2e^{\frac{-i2\pi \cdot 0 \cdot 2}{4}} + 2e^{\frac{-i2\pi \cdot 1 \cdot 2}{4}} + 6e^{\frac{-i2\pi \cdot 2 \cdot 2}{4}} + 2e^{\frac{-i2\pi \cdot 3 \cdot 2}{4}} \right) = \frac{1}{4} (-2 - 2 + 6 - 2) = 0$$

$$X[3] = \frac{1}{4} \left(-2e^{\frac{-i2\pi \cdot 0 \cdot 3}{4}} + 2e^{\frac{-i2\pi \cdot 1 \cdot 3}{4}} + 6e^{\frac{-i2\pi \cdot 2 \cdot 3}{4}} + 2e^{\frac{-i2\pi \cdot 3 \cdot 3}{4}} \right) = \frac{1}{4} (-2 + 2i - 6 - 2i) = -2$$



DFT - Rozmazání spektra



DFT - Rozmazání spektra

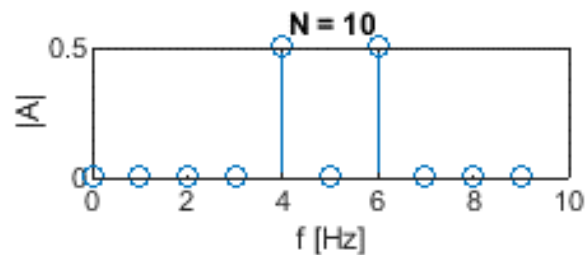
$$x(t) = \cos(2\pi f t)$$

$$F_s = 10 \text{ Hz}$$

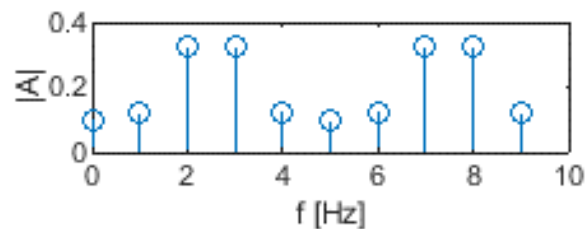
$$N = 10 \rightarrow \Delta f = 1 \text{ Hz}$$

$$N = 20 \rightarrow \Delta f = 0,5 \text{ Hz}$$

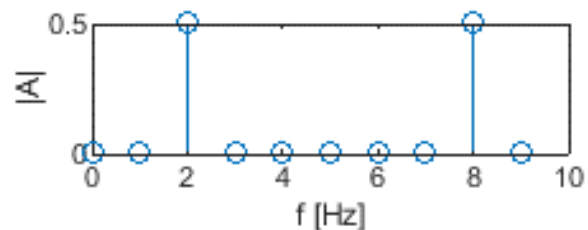
$$f_1 = 4 \text{ Hz}$$



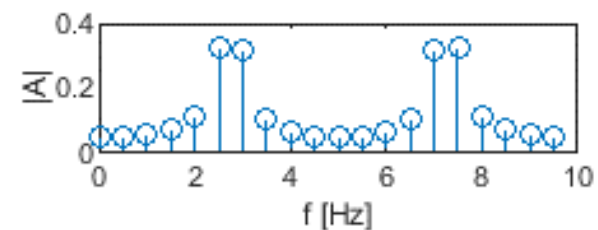
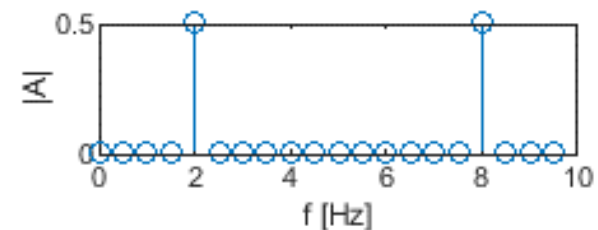
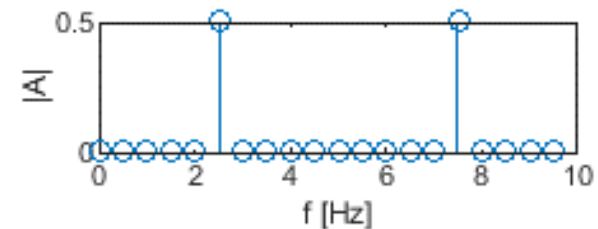
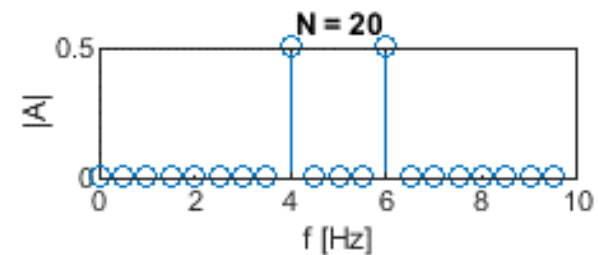
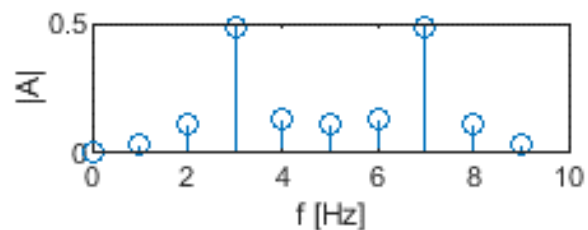
$$f_2 = 2,5 \text{ Hz}$$



$$f_3 = 12 \text{ Hz}$$



$$f_4 = 7,25 \text{ Hz}$$



Úloha k odevzdání

Naprogramujte funkci DFT dle vztahu $X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-\frac{i2\pi nk}{N}}$

```
function X = DFT(x, N)
```

Pomocí vytvořené funkce zobrazte magnitudová spektra čtyř kosinových signálů o frekvencích:

$$f_1 = 4 \text{ Hz}, f_2 = 2,5 \text{ Hz}, f_3 = 12 \text{ Hz}, f_4 = 7,25 \text{ Hz}$$

Vzorkovací frekvenci zvolte 10 Hz, trvání 2 s a hodnotu N zvolte nejprve 10 a poté 20.

Pozor na indexování!!!

