Tutorial: Exploring Random Effects What Do Participants and Items Tell us Beyond the Fixed Effects?

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Outline

- Fixed and random effects?
- Random Intercepts and Slopes?
- ▶ Why are they important in linguistics' research? And beyond?

All material available here: https://shorturl.at/M6deV

- In linguistics (and other disciplines), we rarely use data coming from one participant and/or from one item/utterance (or corpora)
- Having multiple participants and/or items/utterances allows to reduce Type I error, controls for Type II error, Type S error and increases power.

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- ▶ But choosing the right model is an *art* and depends heavily on the dataset you are working on.
- 1. $y = x\beta + \varepsilon$
 - ▶ $y \rightarrow \text{outcome (DV)} \Rightarrow \text{known}$
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 - ightharpoonup Z
 ightharpoonuprandom effects term \Rightarrow known
 - $ightharpoonup u
 ightharpoonup random effects coefficients <math>\Rightarrow$ unknown

Types of Errors?

- Type I (or "false positive") ⇒ falsely concluding there is an effect when none exists. (generally *rightarrow* inaccurate modelling strategies)
- Type II (or "false negative") ⇒ falsely concluding there is no effect when one in fact exists (generally *rightarrow* inaccurate modelling strategies)

		Statistical analysis result (sample)	
		Reject H_0	Don't reject H_0
Reality (population)	H_0 is true H_0 is false	Type I error (α) Correct decision (significant)	Correct decision (null result) Type II error (β)

- 3. Type S \Rightarrow Inaccurate sign (generally due to *hidden* multicollinearity and low power = 1- β)
- 4. Type M \Rightarrow Inaccurate magnitude (generally due to *hidden* multicollinearity and low power = 1- β)

Sonderegger, M. (2023). Regression Modeling for Linguistic Data. The MIT Press.

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- 1. Multiple speakers
- 2. Multiple Items (words)
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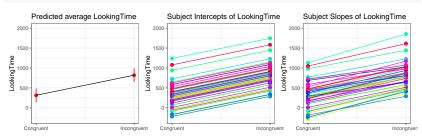
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- Random effects
 - Are random selections of the **population** you have and you want to generalise over them.
 - ► These are ⇒ subjects; Items; Utterances; corpora; etc.
 - You are not using all the **population** of subjects, listeners, items, or utterances in your data!

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Random Intercepts and Random Slopes?

- ▶ Random Intercepts ⇒ averages of your population; and these are used in your statistical model to estimate the population-specific error term
- Random Slopes ⇒ adjustments to your populations' observations as a function of your fixed effects (within-subject or within-item)



A concrete example

We use a simulated dataset with \Rightarrow 40 subjects responded to a task involving 40 items in a fully crossed design, with two IVs: Condition with congruent and incongruent (within-subject and within-item) and Age with young and old (between-subject and within-item). The DV is LookingTime (in msec)

```
set.seed(42)
# define parameters
Subj_n = 40 # number of subjects
Item n = 40 # number of items
b0 = 100 # intercept
b1 = 2.5 * b0 # fixed effect of condition
u0s_sd = 300 # random intercept SD for subjects
u0i_sd = 200  # random intercept SD for items
uls sd = 100 # random b1 slope SD for subjects
u1i_sd = 50 # random b1 slope SD for items
r01s = -0.3 # correlation between random effects 0 and 1 for subjects
r01i = 0.2 # correlation between random effects 0 and 1 for items
sigma sd = 150 # error SD
# set up data structure
dataCong <- add random(Subj = Subj_n, Item = Item_n) %>%
  # add within and then between categorical variable for subject
 add_within("Subj", Cond = c("Congruent", "Incongruent")) %>%
 add recode("Cond", "Cond.Incongruent", Congruent = 0, Incongruent = 1) %>%
 add_between("Subj", Age = c("Young", "Old")) %>%
 add_recode("Age", "Age.Old", Young = 0, Old = 1) %>%
  # add random effects
 add ranef("Subj", u0s = u0s_sd, u1s = u1s_sd, .cors = r01s) %>%
  add_ranef("Item", u0i = u0i_sd, u1i = u1i_sd, .cors = r01i) %>%
 add ranef(sigma = sigma_sd) %>%
  # calculate DV
 mutate(LookingTime = b0 + b1 + u0s + u0i + #u0si + u1si +
           (((b1 + u1s) + 0.5) * Cond.Incongruent) + (((b1 + u1s) + 0.9) * Age.Old) +
           (((b1 + u1i) - 0.3) * Cond.Incongruent) + (((b1 + u1i) - 0.25) * Age.Old) + sigma)
```

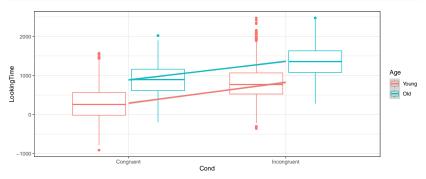
RQ + Hypotheses

Our research question is as follows \Rightarrow Age of subject will impact the Looking Time in the two conditions.

Our hypothesis is \Rightarrow The older a subject is, the more the looking time it is to the incongruent condition

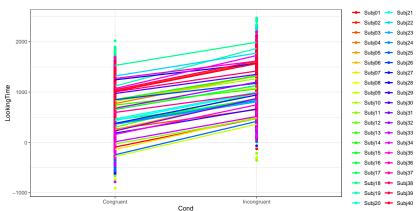
Visualisation I

An increase in LookingTime in the incongruent condition and overall, older participants show an increase in LookingTime. BUT there is no clear interaction



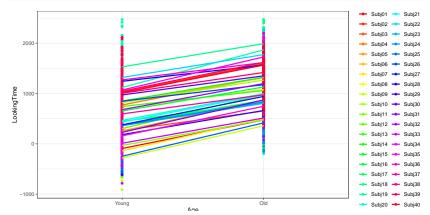
Visualisation II

This figure shows that subjects are variable in how they responded to this task



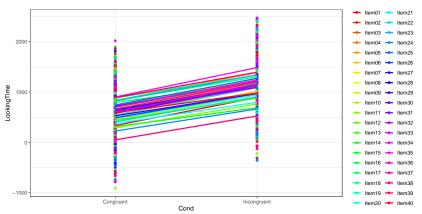
Visualisation III

This figure shows that subjects had an impact on the LookingTime in both age groups, simply due to their variable responses to the different items



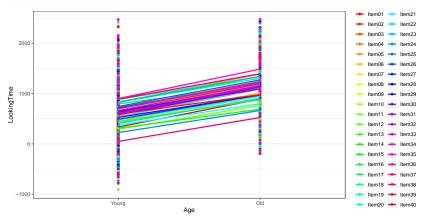
Visualisation IV

This figure shows that items had an impact on the LookingTime in both conditions



Visualisation V

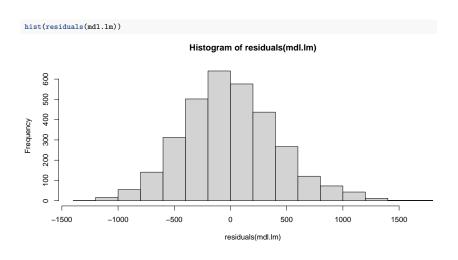
This figure shows that items had an impact on the LookingTime in both age groups



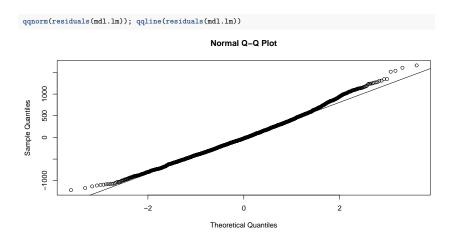
Simple Linear Model - Summary

```
mdl.lm <- dataCong %>%
 lm(LookingTime ~ Cond + Age, data = .)
summarv(mdl.lm)
##
## Call:
## lm(formula = LookingTime ~ Cond + Age, data = .)
##
## Residuals:
               1Q Median
##
       Min
                                 3Q
                                         Max
## -1215.59 -290.17 -21.78 264.89 1661.41
##
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                 304.95
                             12.91 23.61 <2e-16 ***
## CondIncongruent 504.35 14.91 33.82 <2e-16 ***
## AgeOld
                566.44 14.91 37.98 <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 421.8 on 3197 degrees of freedom
## Multiple R-squared: 0.4472, Adjusted R-squared: 0.4469
## F-statistic: 1293 on 2 and 3197 DF, p-value: < 2.2e-16
```

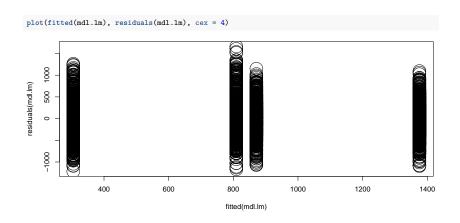
Simple Linear Model - Model Criticism I



Simple Linear Model - Model Criticism II



Simple Linear Model - Model Criticism III



Modelling strategy I

Due to the variation observed in the data, one needs to model both random intercepts and random slopes.

```
## Crossed random intercepts
xmdl.rand.Interc <- dataCong %>%
 lmer(LookingTime ~ Cond + Age +
         (1 | Subj) +
         (1 | Item), data = ., REML = FALSE,
       control = lmerControl(optimizer = "bobyga", optCtrl = list(maxfun = 1e5)))
## Crossed random intercepts + By-speaker random slopes
xmdl.rand.Slope1 <- dataCong %>%
 lmer(LookingTime ~ Cond + Age +
         (1 + Cond | Subj) +
         (1 | Item), data = ., REML = FALSE,
       control = lmerControl(optimizer = "bobyga", optCtrl = list(maxfun = 1e5)))
## Crossed random intercepts + By-speaker and by-item random slopes
xmdl.rand.Slope2 <- dataCong %>%
 lmer(LookingTime ~ Cond + Age +
         (1 + Cond | Subi) +
         (1 + Cond | Item), data = ., REML = FALSE,
       control = lmerControl(optimizer = "bobyga", optCtrl = list(maxfun = 1e5)))
## Crossed random intercepts + By-speaker and by-item random slopes
xmdl.rand.Slope3 <- dataCong %>%
 lmer(LookingTime ~ Cond + Age +
         (1 + Cond | Subi) +
         (1 + Cond + Age | Item), data = ., REML = FALSE,
       control = lmerControl(optimizer = "bobyga", optCtrl = list(maxfun = 1e5)))
```

Modelling strategy II

We test with interactions

```
## Crossed random intercepts + Interaction
xmdl.rand.Interc.Int <- dataCong %>%
 lmer(LookingTime ~ Cond * Age +
         (1 | Subj) +
         (1 | Item), data = ., REML = FALSE,
       control = lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e5)))## Crossed random interce
xmdl.rand.Slope1.Int <- dataCong %>%
 lmer(LookingTime ~ Cond * Age +
         (1 + Cond | Subi) +
         (1 | Item), data = ., REML = FALSE,
       control = lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e5)))
## Crossed random intercepts + By-speaker and by-item random slopes + Interaction
xmdl.rand.Slope2.Int <- dataCong %>%
 lmer(LookingTime ~ Cond * Age +
         (1 + Cond | Subj) +
         (1 + Cond | Item), data = .. REML = FALSE,
       control = lmerControl(optimizer = "bobyga", optCtrl = list(maxfun = 1e5)))
## Crossed random intercepts + By-speaker and by-item random slopes
xmdl.rand.Slope3.Int <- dataCong %>%
 lmer(LookingTime ~ Cond * Age +
         (1 + Cond | Subj) +
         (1 + Cond * Age | Item), data = .. REML = FALSE.
       control = lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e5)))
```

Model Comparison

We use a formal model comparison via a Maximum Likelihood χ^2 Test. Model xmdl.rand.Slope3 is the optimal model as it improved the model fit over a simpler one

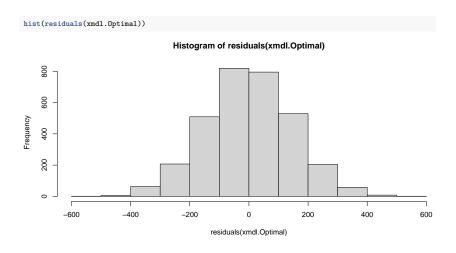
anova(xmdl.rand.Interc, xmdl.rand.Slope1, xmdl.rand.Slope2, xmdl.rand.Slope3, xmdl.rand.Interc.Int, xmdl.

```
## Data: .
## Models:
## xmdl.rand.Interc: LookingTime ~ Cond + Age + (1 | Subj) + (1 | Item)
## xmdl.rand.Interc.Int: LookingTime ~ Cond * Age + (1 | Subj) + (1 | Item)
## xmdl.rand.Slope1: LookingTime ~ Cond + Age + (1 + Cond | Subj) + (1 | Item)
## xmdl.rand.Slope1.Int: LookingTime ~ Cond * Age + (1 + Cond | Subj) + (1 | Item)
## xmdl.rand.Slope2: LookingTime ~ Cond + Age + (1 + Cond | Subj) + (1 + Cond | Item)
## xmdl.rand.Slope2.Int: LookingTime ~ Cond * Age + (1 + Cond | Subj) + (1 + Cond | Item)
## xmdl.rand.Slope3: LookingTime ~ Cond + Age + (1 + Cond | Subj) + (1 + Cond + Age | Item)
## xmdl.rand.Slope3.Int: LookingTime ~ Cond * Age + (1 + Cond | Subj) + (1 + Cond * Age | Item)
##
                       npar AIC
                                  BIC logLik deviance
                                                          Chisa Df Pr(>Chisa)
## xmdl.rand.Interc
                          6 42074 42110 -21031
                                                 42062
## xmdl.rand.Interc.Int 7 42050 42093 -21018
                                               42036
                                                        25.8359 1 3.717e-07
## xmdl.rand.Slope1
                       8 41834 41883 -20909 41818 217.8699 1 < 2.2e-16
## xmdl.rand.Slope1.Int 9 41833 41888 -20908
                                                 41815
                                                         3.0247 1
                                                                       0.0820
## xmdl.rand.Slope2
                                                 41788 27.3253 1 1.719e-07
                     10 41808 41869 -20894
## xmdl.rand.Slope2.Int 11 41807 41874 -20892
                                                        3.0149 1
                                                 41785
                                                                       0.0825
## xmdl.rand.Slope3
                                                 41754 31.2599 2 1.629e-07
                      13 41780 41858 -20877
## xmdl.rand.Slope3.Int 18 41786 41895 -20875
                                                        3.3401 5
                                                 41750
                                                                       0.6477
##
## ymdl rand Interc
## xmdl.rand.Interc.Int ***
## xmdl.rand.Slope1
                       ***
## xmdl.rand.Slope1.Int .
## xmdl.rand.Slope2
                       ***
## xmdl.rand.Slope2.Int .
## xmdl.rand.Slope3
                       ***
## xmdl.rand.Slope3.Int
```

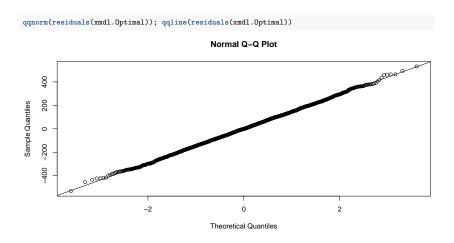
Optimal model

We run the model via a REstricted Maximum Likelihood

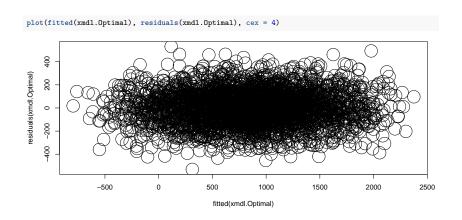
Model criticism I



Model criticism II



Model criticism III



Summary(xmdl.Optimal)

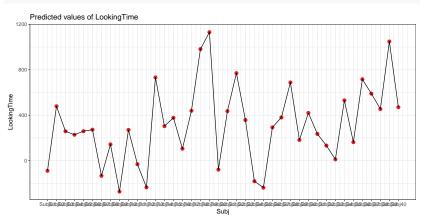
```
## Linear mixed model fit by REML ['lmerMod']
## Formula: LookingTime ~ Cond + Age + (1 + Cond | Subj) + (1 + Cond + Age |
##
      Item)
##
     Data:
## Control: lmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e+05))
##
## REML criterion at convergence: 41724.6
##
## Scaled residuals:
##
      Min
              10 Median
                              30
                                    Max
## -3.5337 -0.6485 -0.0054 0.6647 3.5358
##
## Random effects:
## Groups
                          Variance Std.Dev. Corr
            Name
           (Intercept)
                          123480
                                  351.40
##
   Subj
##
            CondIncongruent 10746 103.66
                                          -0.26
## Item
           (Intercept) 38781 196.93
##
            CondIncongruent 1872
                                  43.27
                                          0.31
                                  43.03
                                          -0.13 0.69
##
            AgeOld
                            1851
## Residual
                            22613 150.38
## Number of obs: 3200, groups: Subj, 40; Item, 40
##
## Fixed effects:
##
                 Estimate Std. Error t value
## (Intercept)
                   315.04
                              83.42 3.777
## CondIncongruent 504.35
                             18.54 27.204
## AgeOld
                   546.28 107.70 5.072
##
## Correlation of Fixed Effects:
##
             (Intr) CndInc
## CndIncngrnt -0.120
## AgeOld
              -0.646 0.016
```

ANOVA

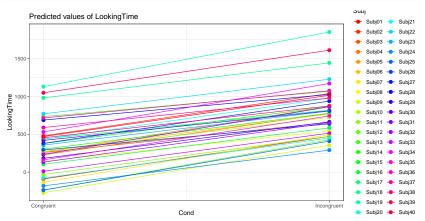
Anova(xmdl.Optimal)

```
## Analysis of Deviance Table (Type II Wald chisquare tests)
##
## Response: LookingTime
## Chisq Df Pr(>Chisq)
## Cond 740.067 1 < 2.2e-16 ***
## Age 25.729 1 3.928e-07 ***
## ---
## ---
```

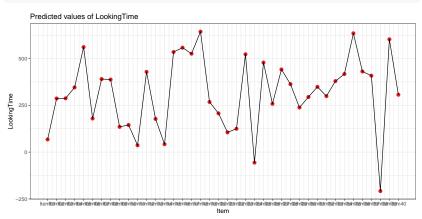
Subject specific-variation



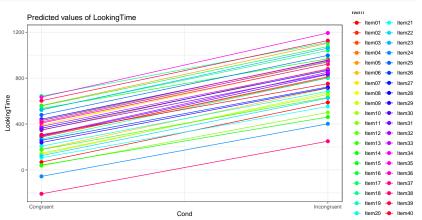
Subject specific-variation by Condition



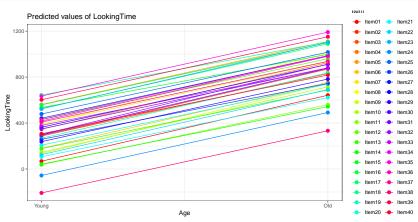
Item specific-variation



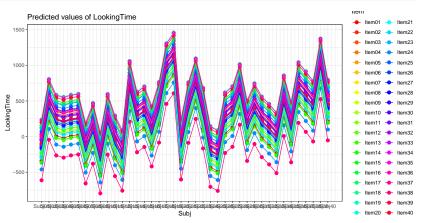
Item specific-variation by Condition



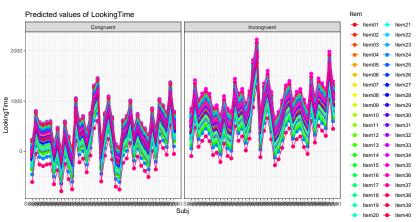
Item specific-variation by Age



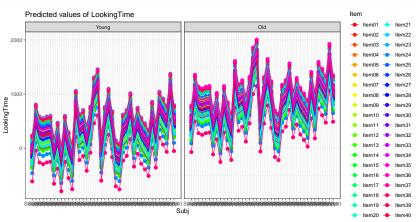
Item specific-variation by Subj



Item specific-variation by Subj by Cond



Item specific-variation by Subj by Age



Conclusion

- This tutorial showed how one can explore random effects and formally assess the need for Random slopes
- As a rule of thumb ⇒ Any within-subject (or within-item) should be tested for a potential inclusion as a random slope
- Fixed effects provides averages over all observations, even when using mixed effects regressions; we need to explore what random effects (intercepts and slopes) tell us.
- ▶ In this example, we see that many subjects vary beyond the fixed effect; Standard Errors are not enough to quantify this type of variation. The same is true for items that are not explored routinely!

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- ▶ In this example, we see that many subjects vary beyond the fixed effect; Standard Errors are not enough to quantify this type of variation. The same is true for items that are not explored routinely!

I hope this tutorial helped you to uncover the role of participants and items and what they can tell us beyond the fixed effect!

Questions?