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Beta Binomiale Conjugaison

Posterior: $$P(p|x) = \frac{P(x|p)\;\; P(p)}{P(x)}$$

- $P(x|p) = \begin{bmatrix} n \\ x \end{bmatrix} p^{x} (1-p)^{n-x}$
- $P(p) = \dfrac{1}{B(\alpha,\beta)} p^{\alpha-1} (1-p)^{\beta-1}$

So: $$P(p|x) = \frac{\begin{bmatrix} n \\ x \end{bmatrix} p^{x} (1-p)^{n-x} \frac{1}{B(\alpha,\beta)} p^{\alpha-1}(1-p)^{\beta-1}}{\int_0^1 \begin{bmatrix} n \\ x \end{bmatrix} p^{x} (1-p)^{n-x} \frac{1}{B(\alpha,\beta)} p^{\alpha-1} (1-p)^{n-x} \, dp}$$

$$= \frac{p^{x+\alpha-1} (1-p)^{n-x+\beta-1}}{\underbrace{\int_0^1 p^{x+\alpha-1} (1-p)^{n-x+\beta-1} \, dp}}$$

We Know $\displaystyle\int_0^1 \beta(p \mid x+\alpha, n-x+\beta) = 1$

$$\Leftrightarrow \frac{1}{B(x+\alpha, n-x+\beta)} \int_0^1 p^{x+\alpha-1} (1-p)^{n-x+\beta-1} \, dp = 1$$

$$\Leftrightarrow \int_0^1 p^{x+\alpha-1} (1-p)^{n-x+\beta-1} \, dp = B(x+\alpha, n-x+\beta)$$

Finally:

$$P(p|x) = \frac{1}{B(x+\alpha, n-x+\beta)} p^{x+\alpha-1} (1-p)^{n-x+\beta-1}$$

$$\boxed{P(p|x) \sim \text{Beta}(x+\alpha,\; n-x+\beta)}$$

Gamma Poisson distribution

Posterior $\quad P(\lambda|D) = \frac{P(D|\lambda)\, P(\lambda)}{P(D)}$

$D = \{D_0, \text{---} D_N\}$, D_i are iid

$$\begin{cases} \lambda \sim \text{Gamma}(\alpha, \beta) \Rightarrow P(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda} \\ D_i \sim P_0(\lambda) \Rightarrow P(D_i | \lambda) = \frac{\lambda^{D_i} e^{-\lambda}}{D_i!} \end{cases}$$

$$\Rightarrow P(D|\lambda) = \prod_i p(D_i|\lambda) = \frac{\lambda^{\sum_i D_i} e^{-N\lambda}}{\prod_i D_i!}$$

So $\quad P(\lambda|D) = \frac{P(D|\lambda)\, P(\lambda)}{P(D)} \propto P(D|\lambda)\, P(\lambda)$

$$\propto \frac{\lambda^{\sum_i D_i} e^{-N\lambda}}{\prod_i D_i!} \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}$$

$$\propto \lambda^{\sum_i D_i} e^{-N\lambda} \lambda^{\alpha-1} e^{-\beta\lambda}$$

$$\propto \lambda^{\sum_i D_i + \alpha - 1} e^{-(N+\beta)\lambda}$$

Thus $\quad P(\lambda|D) \sim \text{Gamma}\left(\sum_i D_i + \alpha,\ N+\beta\right)$