

HANDS-ON 11

1. Given a dynamic table that doubles in size when it needs more space. Find the amortized runtime for inserting n elements.

a) Using the aggregate method:-

\Rightarrow For the amortized runtime, analysis of inserting n elements into a dynamic array that doubles in size.

1) Insertion cost:- Each insertion typically costs $O(1)$.

\Rightarrow When the array is full, resizing occurs, costing $O(n)$ to copy existing elements.

2) Total resizing cost:- In the form of geometric series

$$1 + 2 + 4 + 8 + \dots + n/2 = n - 1$$

The total cost $T(n)$ for n insertion is 2

$$T(n) = O(n) + O(n) = O(n)$$

3. Amortized cost per insertion is:- 01

$$\text{Amortized cost} = \frac{T(n)}{n} = \frac{O(n)}{n} = O(1)$$

b) Using the accounting method:-

The amortized runtime of inserting n elements into dynamic array using the accounting method:

1) Cost setup:- i) Assign an amortized cost of 3 units of each insertion.

ii) Each insertion uses 1 unit for insertion itself & stores 1 unit as 'credit' for future resizing.

2) Cost Breakdown:-

Regular Insertions:- Costs 1 unit, leaving 2 units in credit.
Insertion with Resizing - Array doubles in size, we have enough saved credit to cover the resizing cost.

3) Amortized cost per insertion:- For 3 units = $O(1)$, means each insertion has a constant time cost on average, even with occasional resizing.

b) Using the accounting method:-

The amortized runtime of inserting n elements into dynamic array using the accounting method:-

1) Cost setup:- i) Assign an amortized cost of 3 units of each insertion.

ii) Each insertion uses 1 unit for insertion itself & stores 1 unit as 'credit' for future resizing.

2) Cost Breakdown:-

Regular Insertion:- Costs 1 unit, leaving 2 units in credit.

Insertion with Resizing - Array doubles in size, we have enough saved credit to cover the resizing cost.

3) Amortized cost per insertion:- For 3 units = $O(1)$, means each insertion has a constant time cost on average, even with occasional resizing.