NAME:- MOHAMMED JALALUDDIN JUNAID
ID:- 1002172658

HANDS-ON 6.

3. Mathematically derive the average runtime complexity of the non-random pivot version of quicksort.

A) For array of size n, recurrence relation for T(n) is given by

$$T(n) = T(n) + T(n-k-1) + \Theta(n)$$

where k = no. of elements in left sub-array.

$\Theta(n)$ = time partition of array.

$$\Rightarrow T(n) = \frac{1}{n}\sum_{k=0}^{n-1} T(n) + T(n-k-1) + \Theta(n)$$

$\Rightarrow$ Since left & right part follow symmetry, we have

$$T(n) = \frac{2}{n}\sum_{k=0}^{n-1} T(n) + \Theta(n)$$

$\Rightarrow$ Let's assume $S(n) = \sum_{k=0}^{n-1} T(n)$

$$\therefore S(n) = \int_0^n n.\log x \, dx = \frac{n^2\log n}{2} - \frac{n^2}{4} = n^2\log n/2$$

$$\therefore T(n) = \frac{\cancel{2}}{n}\cdot\frac{n^2\log n}{\cancel{2}} + \Theta(n) = n\log n + \Theta(n) = O(n\log n)$$

$\Rightarrow$ Therefore average runtime complexity of the non-random pivot version of quicksort is $O(n\log n)$