

Lec 14: Unsolved Problems

Problem 1. We are given a coin for which the probability of heads is p ($0 < p < 1$) and the probability of tails is $1 - p$. Consider a sequence of independent flips of the coin.

- (a) Let Y be the number of flips up to and including the flip on which the first head occurs. Write down the PMF of Y .
- (b) Let X be the number of heads that occur on any particular flip. Write down $\mathbf{E}[X]$ and $\text{var}(X)$.
- (c) Let K be the number of heads that occur on the first n flips of the coin. Determine the PMF, mean, and variance of K .
- (d) Given that a total of exactly six heads resulted from the first nine flips, what is the conditional probability that both the first and seventh flips were tails?
- (e) Let H be the number of heads that occur on the first twenty flips. Let C be the event that a total of exactly ten heads resulted from the first eighteen flips. Find $\mathbf{E}[H | C]$ and the conditional variance $\text{var}(H | C)$.

Problem 2. At each trial of a game, Don and Greg flip biased coins, simultaneously but independently. For each trial, the probability of heads is p_D and p_G for Don and Greg, respectively.

- (a) Given that the flips on a particular trial resulted in 2 heads, find the PMF of the number of additional trials up to and including the next trial on which 2 heads result.
- (b) Given that the flips on a particular trial resulted in at least one head, find the probability that Don flipped a head on that trial.
- (c) Starting from a trial on which no heads result, find the probability that Don's next flip of a head will occur before Greg's next flip of a head.