Stat753_HW12_JaleesaHoule

April 30, 2024

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

1 Question 1

Consider the switch SDE: $dX(t) = -c \cdot \mathrm{sgn}(X(t) - m)dt + \sigma dW(t)$. Here, $c, \sigma > 0$, and sgn is the signum function:

$$\mathrm{sgn}(y) := \begin{cases} +1, y > 0; \\ 0, y = 0; \\ -1, y < 0. \end{cases} \tag{1}$$

Show that the Laplace distribution with mean m and variance v is a stationary distribution, and find the variance v.

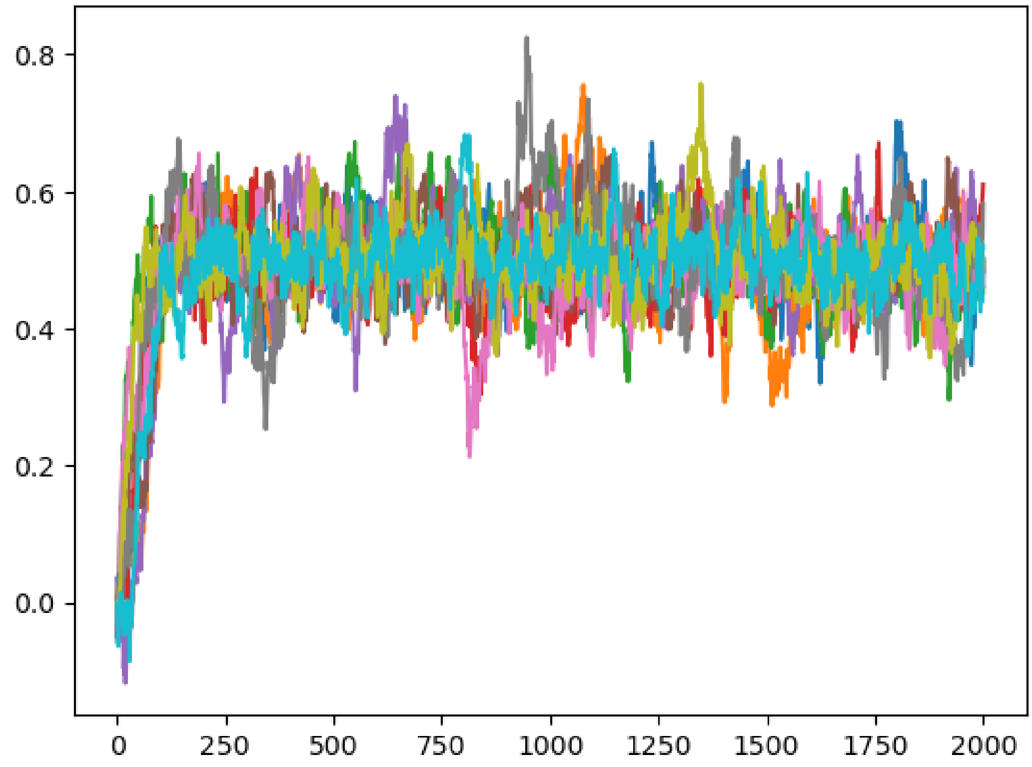
[2]:
```
def SDE1(mu, sigma,c, dt, T):
    simX=[0]
    N=int(T/dt)
    noise = np.random.normal(0,np.sqrt(dt),N)

    for i in range(N):
        old = simX[-1]
        if (old-mu)>0:
            new= -c*dt + sigma*noise[i] + old
        elif (old-mu)<0:
            new= c*dt + sigma*noise[i] + old
        elif (old-mu)==0:
            sigma*noise[i] + old

        simX.append(new)
    return simX
```

[3]:
```
sigma=.2
m=.5
c=.5
b= sigma**2/(2*c)
dt=0.01
```

```
T=20
```

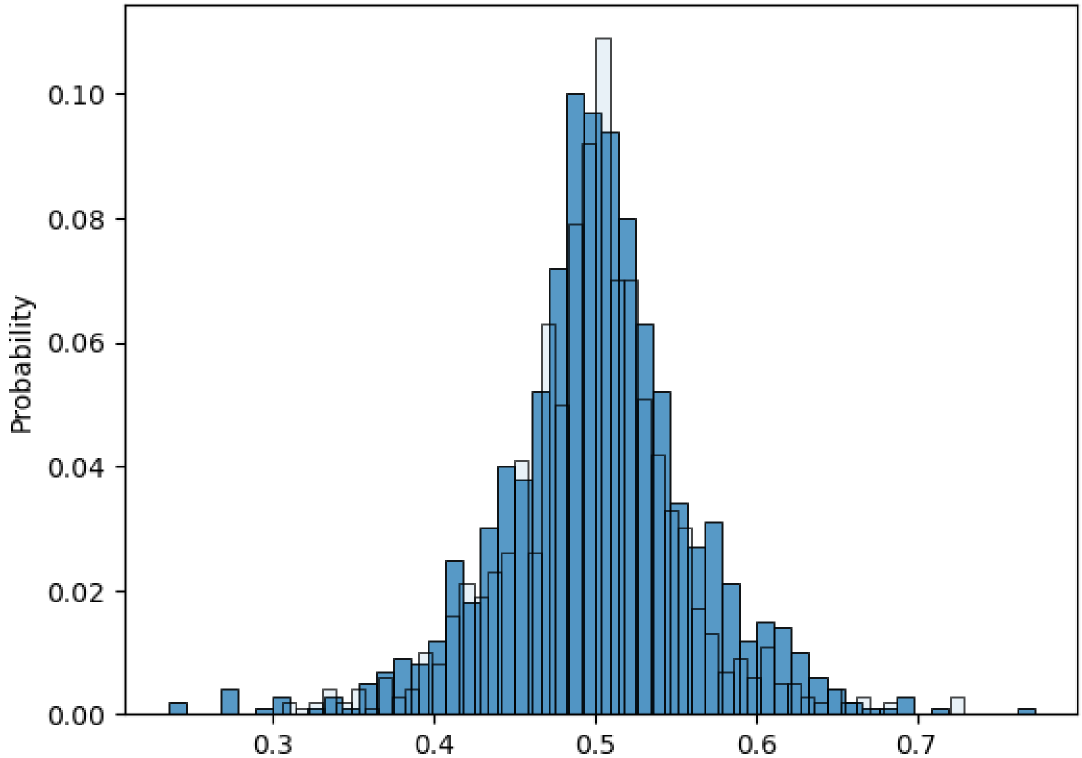
[4]:
```
sims=[]
np.random.seed(1)
for i in range(1000):
    sims.append(SDE1(m,sigma,c, dt, T))
```

[5]:
```
for i in range(10):
    plt.plot(sims[i])
```



[6]:
```
X = np.random.laplace(loc=m, scale=b , size=1000)
sns.histplot(np.array(sims)[:,1000],stat='probability', bins=50)
sns.histplot(X, bins=50,stat='probability', alpha=.1)
```

[6]: <AxesSubplot:ylabel='Probability'>



[7]:
```
print('Empirical mean:', np.mean(np.array(sims)[:,500]))
print('Empirical variance:', np.var(np.array(sims)[:,500]))

print('Theoretical mean:', m)
print('Theoretical variance:', 2*b**2)
```

```
Empirical mean: 0.5038519384972718
Empirical variance: 0.0035413485192730544
Theoretical mean: 0.5
Theoretical variance: 0.0032000000000000015
```

2 Question 2

Consider the Ornstein-Uhlenbeck process in the log scale modeling the Volatility Index VIX:

$$d\ln V(t) = (-0.1188\ln V(t) + 0.3482)dt + 0.1589dW(t), V(0) = 16. \tag{2}$$

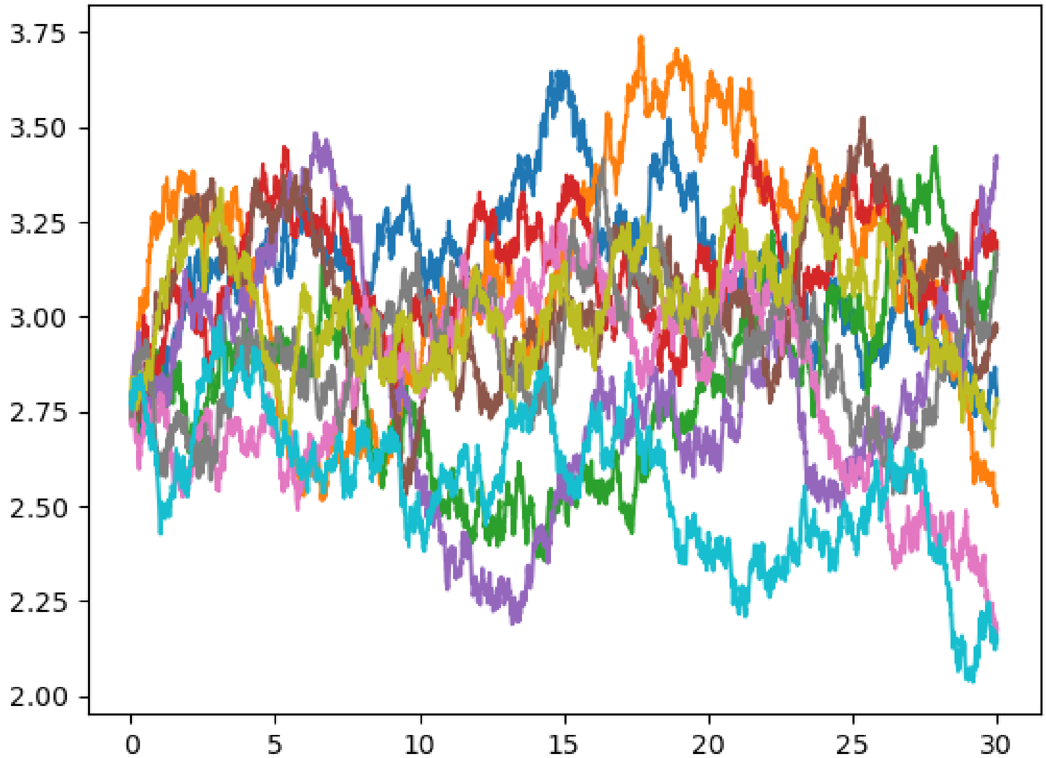
Find the mean and variance of V in its stationary distribution.

[8]:
```
def SDE2(m,c, sigma, dt, T, X0):
    simX=[X0]
    N= int(T/dt)
```

```
    noise = np.random.normal(0,np.sqrt(dt),N)

    for i in range(N):
        old = simX[-1]
        new= (np.exp(c*m*dt)) * (old**(-c*dt)) * np.exp(sigma*noise[i]) * old
        simX.append(new)
    return simX
```

[9]:
```
c = .1188
m = 0.3482/c
sigma=0.1589
dt= .01
T=30
X0=16

p2 = sigma*2 / 2*c

np.random.seed(1)
sims=[]
for i in range(1000):
    sims.append(SDE2(m,c,sigma, dt, T, X0))
```

[10]:
```
time = np.linspace(0,T,int(T/dt)+1)
for i in range(10):
    plt.plot(time, np.log(sims[i]))
```

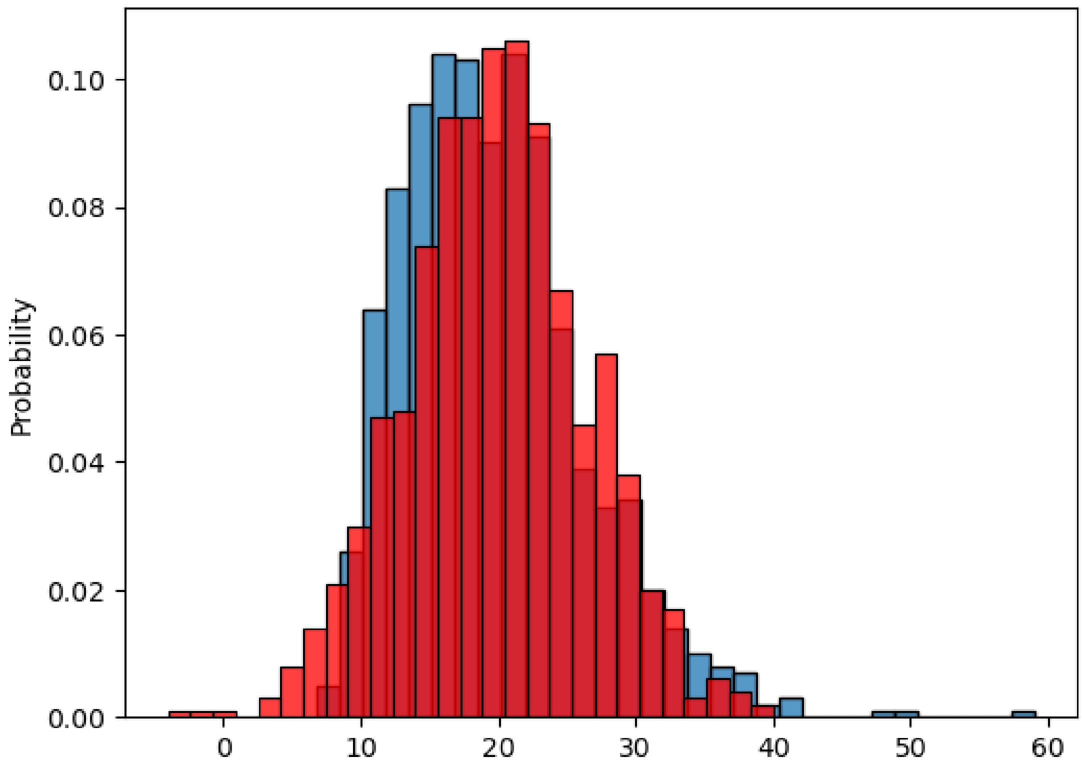


[11]:
```
p2 = sigma**2 / (2*c)
print('Empirical mean:', np.mean(np.array(sims)[:,1000:]))
print('Empirical variance:', np.var(np.array(sims)[:,1000:]))

print('Theoretical mean:', np.exp(m + (p2/2)))
print('Theoretical variance:', (np.exp(2*m + 4*p2/2) -  np.exp(m + p2/2)**2))
```

```
Empirical mean: 19.335080974581142
Empirical variance: 43.44341712167216
Theoretical mean: 19.76890582446917
Theoretical variance: 43.81740930293188
```

[17]:
```
X = np.random.normal(19.76890582446917, np.sqrt(43.81740930293188), size=1000)
sns.histplot(np.array(sims)[:,2500], stat='probability')
sns.histplot(X, stat='probability', color='red')
```

[17]: <AxesSubplot:ylabel='Probability'>


Probability
0.00
0.02
0.04
0.06
0.08
0.10
0
10
20
30
40
50
60

```
[ ]:
```