

Stat753_HW11_JaleesaHoule

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[4]: import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
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0.1 Take a standard Brownian motion W . Find the probability that its maximum over $[0,9]$ is greater than 5, and $W(9) > 1$.

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[5]: def brownian_motion_sim( dt, T, mu=0, sigma=1, x0=None):
    N = int(T/dt)
    BM = np.append(np.zeros(1),np.cumsum(np.random.normal(mu*dt, sigma*np.
    ↪sqrt(dt), N)))
    if x0 is not None:
        BM= np.cumsum(np.append(x0, np.random.normal(mu*dt, sigma*np.sqrt(dt),
    ↪N)))
    time = np.linspace(0,T, N+1)

    return BM, np.round(time,2)

def find_value_at_T(values,jumptimes, T):
    idx = np.where(jumptimes == T)[-1][-1]
    return values[idx]
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[6]: dt = 0.01
N = 10000
T=9
sims=[]
np.random.seed(1000)
for i in range(N):
    a,b = brownian_motion_sim(dt, T)
    sims.append(a)
    time = b
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[9]: countw_9 = 0

sims_idx=[]
for i in range(N):
    w_9 = find_value_at_T(sims[i], time, 9)
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if (np.max(sims[i][:900])>5) and (w_9>1):  
    countw_9+=1
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[10]: print('P(M(9)>5, W(9)>1)=', countw_9/N)
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P(M(9)>5, W(9)>1)= 0.0912

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[ ]:
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