4-2 4-3

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3. (1) 显然 $\alpha_3 = \alpha_1 - 2\alpha_2$

$\therefore$ $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ 线性相关

(2) 计算 $\begin{vmatrix} 3 & 1 & 4 \\ -1 & 1 & 0 \\ 1 & 2 & 5 \end{vmatrix} = \begin{vmatrix} 3 & 1 & 0 \\ -1 & 1 & 0 \\ 1 & 2 & 2 \end{vmatrix} = 2\times(3+1) = 8 \neq 0$

$\therefore$ $\alpha_1, \alpha_2, \alpha_4$ 线性无关

4. $\det \begin{vmatrix} \vdots & \vdots & \vdots \\ \alpha_1 & \alpha_1+\alpha_2 & \alpha_1+\alpha_2+\alpha_3 \\ \vdots & \vdots & \vdots \end{vmatrix} = \begin{vmatrix} \vdots & \vdots & \vdots \\ \alpha_1 & \alpha_2 & \alpha_3 \\ \vdots & \vdots & \vdots \end{vmatrix} \neq 0$

$\therefore$ $\alpha_1, \alpha_1+\alpha_2, \alpha_1+\alpha_2+\alpha_3$ 线性无关.

5. $\beta_n = -\alpha_n + (\alpha_1+\alpha_2+\cdots+\alpha_s)$

$= -\alpha_n + \frac{1}{s-1}(\beta_1+\beta_2+\cdots+\beta_s)$

$\therefore$ 对 $\forall\, n \in \mathbb{R}$, 且 $1 \leq n \leq s$,

都有 $\alpha_n = \frac{1}{s-1}\beta_1 + \frac{1}{s-1}\beta_2 + \cdots + \frac{2-s}{s-1}\beta_n + \cdots + \frac{1}{s-1}\beta_s$

Q.E.D