1-5 展开定理

2023年10月5日 21:09

12. (1) 左式 $\xlongequal{c_2+c_3} \begin{vmatrix} 2 & 2 & 2\lg 5 \\ 1 & 1 & \sin^2\alpha \\ -1 & -1 & \frac{1}{a^2+1} \end{vmatrix} = 0$

(2) 左式 $\xlongequal{r_1-r_2+r_3} \begin{vmatrix} 2x_1 & 2x_2 & 2x_3 \\ y_1+z_1 & y_2+z_2 & y_3+z_3 \\ z_1+x_1 & z_2+x_2 & z_3+x_3 \end{vmatrix}$

$= 2\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1+z_1 & y_2+z_2 & y_3+z_3 \\ z_1+x_1 & z_2+x_2 & z_3+x \end{vmatrix}$

$\xlongequal{r_2-r_3+r_1} 2\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1+x_1 & z_2+x_2 & z_3+x \end{vmatrix}$

$\xlongequal{r_3-r_1}$ 右式

(3) 左式 $\xlongequal[(2\le i\le n)]{c_i-i\times c_1} \begin{vmatrix} a & 2(1-a) & 3(1-a) & \cdots & n(1-a) \\ 1 & a-1 & 0 & \cdots & 0 \\ 1 & 0 & a-1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 0 & 0 & \cdots & a-1 \end{vmatrix}$

$\xlongequal[(2\le i\le n)]{r_1+i\times r_i} \begin{vmatrix} a+2+3+\cdots+n & 0 & 0 & \cdots & 0 \\ 1 & a-1 & 0 & \cdots & 0 \\ 1 & 0 & a-1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 0 & 0 & \cdots & a-1 \end{vmatrix}$

$\xlongequal{\text{以}r_1\text{展开}} \left[a+\frac{(n-1)(n+2)}{2}\right](a-1)^{n-1} =$ 右式

(4) 设左展为D_n

$D_n \xlongequal{\text{以}r_1\text{展开}} (a+b)D_{n-1} - ab\begin{vmatrix} 1 & ab & \cdots & 0 & 0 \\ 0 & a+b & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & a+b & ab \\ 0 & 0 & \cdots & 1 & a+b \end{vmatrix}$

$$\xlongequal[\text{以}C_1\text{展开}]{} (a+b)D_{n-1} - ab\,D_{n-2}$$

首先 我们有 $D_1 = a+b = \sum_{i=0}^{1} a^i b^{1-i}$

$$D_2 = (a+b)^2 - ab = a^2+ab+b^2 = \sum_{i=0}^{2} a^i b^{2-i}$$

假设 当 $n=k-2$ 和 $k-1$ 时 原命题成立

那么就有 $D_k = (a+b)\sum_{i=0}^{k-1} a^i b^{k-1-i} - ab\sum_{i=0}^{k-2} a^i b^{k-2-i}$

$$= \sum_{i=0}^{k-1} a^{i+1} b^{k-1-i} + \sum_{i=0}^{k-1} a^i b^{k-i} - \sum_{i=0}^{k-2} a^{i+1} b^{k-1-i}$$

$$= \sum_{i=1}^{k} a^i b^{k-i} + \sum_{i=0}^{k-1} a^i b^{k-i} - \sum_{i=1}^{k-1} a^i b^{k-i}$$

$$= \sum_{i=0}^{k} a^i b^{k-i}$$

综上 由数学归纳法可知

$$\begin{vmatrix} a+b & ab & 0 & \cdots & 0 & 0 \\ 1 & a+b & ab & \cdots & 0 & 0 \\ 0 & 1 & a+b & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a+b & ab \\ 0 & 0 & 0 & \cdots & 1 & a+b \end{vmatrix} = \sum_{i=0}^{n} a^i b^{n-i}$$

14. (1) 原式 $\xlongequal[\text{以}r_2\text{展开}]{} (-1)^{2+2} \times 20 \times \begin{vmatrix} 7 & 1 & 1 \\ -3 & -1 & 5 \\ -2 & -3 & 1 \end{vmatrix}$

$$= 20 \times \begin{vmatrix} 9 & 4 & 0 \\ 7 & 14 & 0 \\ -2 & -3 & 1 \end{vmatrix}$$

$$= 20 \times (-1)^{3+3} \times 1 \times \begin{vmatrix} 9 & 4 \\ 7 & 14 \end{vmatrix}$$

$$= 20 \times (9\times14 - 4\times7) = 1960$$

(2) 原式 $\xlongequal{\text{分块}} \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{vmatrix}$

$$= \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \times \begin{vmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{vmatrix}$$

$$= a_{11}a_{22}a_{33}a_{44} - a_{11}a_{22}a_{34}a_{43}$$

$$- a_{12}a_{21}a_{33}a_{44} + a_{12}a_{21}a_{34}a_{43}$$

(3) 原式 $\xlongequal[\text{以}r_1\text{展开}]{} (-1)^{1+1}\cdot x \cdot x^{n-1} + (-1)^{1+n}\cdot y \cdot y^{n-1}$

$= x^n + (-1)^{n+1} y^n$