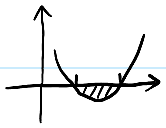
12-7

2023年12月9日 9:54

习题4.8 (A)

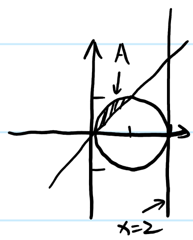
3. 

$$V=\int_1^2 2\pi x\left|(x-1)(x-2)\right|dx$$
$$=2\pi\int_1^2\left|x^3-3x^2+2x\right|dx$$
$$=-2\pi\left(\frac{x^4}{4}-x^3+x^2\right)\Big|_1^2$$
$$=\frac{\pi}{2}$$

5. $$S=\int_0^{1/2}\sqrt{dx^2+dy^2}=\int_0^{1/2}\sqrt{1+\left(\frac{-2x}{1-x^2}\right)^2}\,dx=\int_0^{1/2}\frac{1+x^2}{1-x^2}dx$$
$$=\int_0^{1/2}-1+\frac{2}{1-x^2}dx=-x\Big|_0^{1/2}+\int_0^{1/2}\frac{1}{1-x}+\frac{1}{1+x}dx$$
$$=-\frac{1}{2}+\ln\left|\frac{x+1}{x-1}\right|\Bigg|_0^{1/2}=-\frac{1}{2}+\ln 3.$$

7. $$\frac{dy}{dt}=at\sin t\qquad\frac{dx}{dt}=at\cos t$$
$$\Rightarrow\frac{dy}{dx}=\tan t$$
$$S=\int_0^{2\pi}\sqrt{1+\tan^2 t}\;at\cos t\,dt$$
$$=a\int_0^{2\pi}t\,dt=2\pi^2 a$$

(B) 3. $$A:\begin{cases}x^2+y^2\le 2x\\ y\ge x\end{cases}\Rightarrow\begin{cases}(x-1)^2+y^2\le 1\\ y\ge x\end{cases}\Rightarrow$$



$$V_1=\int_0^1\pi\left(1+\sqrt{1-y^2}\right)^2dy$$
$$=\pi\int_0^1 2-y^2+2\sqrt{1-y^2}\,dy$$
$$=\pi\left(2y-\frac{y^3}{3}\Big|_0^1+2\int_0^1\sqrt{1-y^2}\,dy\right)$$
$$=\pi\left(\frac{5}{3}-\frac{\pi}{2}\right)=\frac{5\pi}{3}+\frac{\pi^2}{2}$$
$$V_2=\int_0^1\pi(2-y)^2dy=\pi\int_0^1 y^2-4y+4\,dy$$
$$=\pi\left(\frac{y^3}{3}-2y^2+4y\right)\Big|_0^1=\frac{7\pi}{3}$$
$$V=V_1-V_2=\frac{\pi^2}{2}-\frac{2\pi}{3}$$

5. $y' = 2ax+b \quad y'' = 2a$

$$K = \frac{|y''|}{(1+y'^2)^{3/2}} = \frac{|2a|}{[1+(2ax+b)^2]^{3/2}}$$

$\therefore$ 当 $x = -\frac{b}{2a}$ 时 K 最大.

6. (1) $S = 2\pi \int_0^{\pi} \sin x \sqrt{1+\cos^2 x}\, dx$

$$= 2\pi \int_{-1}^{1} \sqrt{1+t^2}\, dt$$

$$= 4\pi \int_0^1 \sqrt{1+t^2}\, dt$$

$$= \left(\sqrt{2} - \int_0^1 \sqrt{1+t^2}\, dt + \int_0^1 \frac{1}{\sqrt{1+t^2}}\, dt\right) 4\pi$$

$$= 2\pi\left[\sqrt{2} + \ln(1+\sqrt{2})\right]$$

(2) $S = 2\pi \int_0^{2\pi} (2a-y)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dt$

$$= 2\pi a^2 \int_0^{2\pi} (1+\cos t)\sqrt{2(1-\cos t)}\, dt$$

$$= 8\pi a^2 \int \cos^2\frac{t}{2} \sin\frac{t}{2}\, dt$$

$$= -16\pi a^2 \int_2^{\pi} \cos^2 u\, d(\cos u)$$

$$= \frac{32\pi a^2}{3}$$

(3) $S = 2\pi \int_0^{\pi} r\sin\theta \sqrt{r^2 + r'^2}\, d\theta$

$$= 2\sqrt{2}\pi a^2 \int_0^{\pi} \sin\theta\, (1+\cos\theta)^{3/2}\, d\theta$$

$$= \frac{32\pi a^2}{5}.$$