5-1

2023年11月26日 16:03

2.(1) $\det(A-\lambda E)=\begin{vmatrix} -\lambda & 1 & 1 & \cdots & 1 \\ 1 & -\lambda & 1 & \cdots & 1 \\ 1 & 1 & -\lambda & \cdots & 1 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 1 & 1 & \cdots & -\lambda \end{vmatrix}$

$$=\begin{vmatrix} -\lambda & 1 & 1 & \cdots & 1 & 1 & 1 \\ 1+\lambda & -1-\lambda & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1+\lambda & -1-\lambda & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1-\lambda & 0 & 0 \\ 0 & 0 & 0 & \cdots & 1+\lambda & -1-\lambda & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1+\lambda & -1-\lambda \end{vmatrix}$$

$$=\begin{vmatrix} -\lambda+n-1 & n-1 & \cdots & 2 & 1 \\ 0 & -1-\lambda & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & -1-\lambda & 0 \\ 0 & 0 & \cdots & 0 & -1-\lambda \end{vmatrix} = (-\lambda+n-1)(-1-\lambda)^{n-1}$$

$\therefore \lambda = n-1$ 或 -1

1° 当 $\lambda = n-1$ 时 对 $(A-\lambda E)x=0$ 求解

$$A-\lambda E=\begin{bmatrix} 1-n & 1 & \cdots & 1 \\ 1 & 1-n & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 1-n \end{bmatrix} \to \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1-n & 1 & 1 & \cdots & 1 & 1 \\ 0 & n & -n & 0 & \cdots & 0 & 0 \\ 0 & 0 & n & -n & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & & n & -n \end{bmatrix}$$

$$\to \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1-n & 1 & 1 & \cdots & 1 & 1 \\ 0 & 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix}$$

$$\to \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix}$$

$\therefore x=\begin{bmatrix} 1 \\ 1 \\ \vdots \end{bmatrix}$

$$\therefore x = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

2° 当 $\lambda = -1$ 时 对 $(A-\lambda E)x=0$ 来说:

$$A-\lambda E = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\therefore x = \begin{bmatrix} -1 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 1 \end{bmatrix} \text{或} \begin{bmatrix} -1 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \end{bmatrix} \cdots\cdots$$

(2) 记(1)中的 A 为 A' 则(2)中的 $A = A'+E$

$\therefore \lambda - 1 = \lambda' = n-1$ 或 -1

$\therefore \lambda = n$ 或 0

特征向量与(1)中的两种情况分别相同

3. 想要证: $A^*\alpha = \frac{|A|}{\lambda}\alpha \iff A^{-1}\alpha = \frac{1}{\lambda}\alpha$

$$\iff \alpha = \frac{A}{\lambda}\alpha$$

$$\iff A\alpha = \lambda\alpha \quad \text{Q.E.D}$$