11.17

2023年11月10日 11:37

习题 4.2 (A)

5. (1) $2x\sqrt{1+x^2}$

(2) $(b-a)(2x+a+b+2)$

7. 原式 $= \lim\limits_{x\to+\infty}\dfrac{(\arctan x)^2}{\frac{x}{\sqrt{x^2+1}}} = \lim\limits_{x\to+\infty}\arctan^2 x\sqrt{1+\frac{1}{x^2}}$

$= \lim\limits_{x\to+\infty}\arctan^2 x\cdot\lim\limits_{x\to+\infty}\sqrt{1+\frac{1}{x^2}} = \dfrac{\pi^2}{4}$

(B)

1. 原式 $= \lim\limits_{x\to0}\dfrac{\frac{x^2}{\sqrt{a+x}}}{b-\cos x} = 1$

$\because$ 分子极限为0 且 极限存在

$\therefore$ 分母极限也为0 $\therefore b=1$

原式 $= \lim\limits_{x\to0}\dfrac{\frac{x^2}{\sqrt{a-x^2}}}{1-\cos x} = \lim\limits_{x\to0}\dfrac{2}{\sqrt{a-x^2}} = \dfrac{2}{\sqrt{a}} = 1$

$\therefore a=4$ 综上 $\begin{cases} a=4 \\ b=1 \end{cases}$

2. 考虑 $\lim\limits_{x\to+\infty}\dfrac{\int_0^x e^{t^2}dt}{\frac{1}{2x}e^{x^2}} = 2\cdot\lim\limits_{x\to+\infty}\dfrac{e^{x^2}}{\frac{2x^2e^{x^2}-e^{x^2}}{x^2}}$

$=2\cdot\lim\limits_{x\to+\infty}\dfrac{x^2}{2x^2-1} = 1$ $\therefore$ 当 $x\to+\infty$ 时，$\int_0^x e^{t^2}dt \sim \frac{1}{2x}e^{x^2}$

3. $F'(x) = f(\ln x)\frac{1}{x} - f(\frac{1}{x})(-\frac{1}{x^2}) = \dfrac{f(\ln x)}{x} + \dfrac{f(\frac{1}{x})}{x^2}$

4. $\left(\int_0^1 f'(x)dx\right)^2 = \int_0^1 f'(x)\cdot1\cdot dx \le \int_0^1[f'(x)]^2dx\cdot\int_0^1 1^2dx$

$= \int_0^1[f'(x)]^2dx\cdot(1-0)$

即 $\int_0^1[f'(x)]^2dx \ge \left(\int_0^1 f'(x)dx\right)^2 = [f(1)-f(0)]^2 = 1$ Q.E.D