10.26

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习题 3.3

(A) 9. (1) $\frac{dy}{dx} = \frac{\tan\theta - \cot 3\theta}{1+\tan\theta\cot 3\theta} = -\cot 3\theta$

$\rho\sin\theta - |a|\sqrt{2\cos 2\theta_0}\sin\theta_0 = -\cot 3\theta_0\left(\rho\cos\theta - |a|\sqrt{2\cos 2\theta_0}\cos\theta_0\right)$

$(\sin\theta + \cos\theta\cot 3\theta)\rho = |a|\sqrt{2\cos 2\theta_0}\left(\sin\theta_0 + \cot 3\theta_0\cos\theta_0\right)$

$\therefore \rho = \frac{|a|\sqrt{2\cos 2\theta_0}\left(\sin\theta_0 + \cot 3\theta_0\cos\theta_0\right)}{(\sin\theta + \cot 3\theta_0\cos\theta)}$

(2) $\rho\sin\theta - ae^{m\theta_0}\sin\theta_0 = \tan(\theta_0 + \arctan\frac{1}{m})\cdot\left(\rho\cos\theta - ae^{m\theta_0}\cos\theta_0\right)$

$\therefore \rho = \frac{ae^{m\theta_0}\left[\sin\theta_0 - \tan(\theta_0 + \arctan\frac{1}{m})\cos\theta_0\right]}{\sin\theta - \tan(\theta_0 + \arctan\frac{1}{m})\cos\theta}$

(B) 1. (1) $y' = e^x + e^{e^x}\cdot e^x = e^x(1+e^{e^x})$

(2) $y' = a^{x^a}\cdot\ln(x^a)\cdot ax^{a-1} + a^{a^x}\cdot\ln(a^x)\cdot a^x\ln a$

$= a^2x^{a-1}\cdot\ln x\cdot a^{x^a} + xa^{a^x}\cdot\ln^2 a\cdot a^{a^x}$

(3) $y' = 2^{\tan\frac{1}{x}}\cdot\ln(\tan\frac{1}{x})\cdot\sec^2(\frac{1}{x})\cdot -\frac{1}{x^2}$

(4) $\ln y = x\ln\frac{a}{b} + a\ln\frac{b}{x} + b\ln\frac{x}{a}$

$= x\ln\frac{a}{b} + a\ln b - a\ln x + b\ln x - b\ln a$

$\frac{y'}{y} = \ln\frac{a}{b} - \frac{a}{x} + \frac{b}{x}$

$y' = \left(\frac{a}{b}\right)^x\left(\frac{b}{x}\right)^a\left(\frac{x}{a}\right)^b\left(\ln\frac{a}{b} + \frac{b-a}{x}\right)$

(5) $y' = e^x\left(1 + \cot\frac{x}{2} - \frac{1}{2}\csc^2\frac{x}{2}\right)$

(6) $y' = \ln 3\cdot 3^x\cdot\ln x + \frac{3^x}{x}$

习题 3.4

(A) 2. (1) $e^x(x^2+2x)\,dx$

(2) $x\sin x\,dx$

(3) $-\frac{2}{x^3}dx$

(4) $\frac{x}{\sqrt{a^2+x^2}}dx$

(5) $\frac{2x}{x^2-1}dx$

(6) $\frac{1-\frac{1}{2}\ln x}{x\sqrt{x}}dx$

4. (1) $dy=\ln x\,dx$

(2) $dy=-\frac{x}{|x|\sqrt{1-x^2}}dx=-\frac{\operatorname{sgn}x}{\sqrt{1-x^2}}dx$

(3) $dy=(2x\cos 2x-2x^2\sin 2x)dx$

(4) $dy=5^x\ln 5\,dx$

6. (1) $\sin 29^\circ=\sin(30^\circ-1^\circ)=\sin(\frac{\pi}{6}-\frac{\pi}{180})$

$\therefore \sin 29^\circ\approx \sin\frac{\pi}{6}+\cos\frac{\pi}{6}\times(-\frac{\pi}{180})$

$=\frac{1}{2}-\frac{\sqrt{3}}{2}\times\frac{\pi}{180}\approx 0.4849$

(2) $\sqrt[3]{1.02}=\sqrt[3]{1+0.02}$

$\therefore \sqrt[3]{1.02}\approx \sqrt[3]{1}+\frac{1}{3}\times 1^{-\frac{2}{3}}\times 0.02$

$=1+\frac{1}{3}\times 0.02\approx 1.0067$

(B) 1. (1) $d^2y=(1+x^2)^{-\frac{3}{2}}dx^2$

(2) $d^2y=\frac{2\ln x-3}{x^2}dx^2$

2. $V(r)=\frac{4}{3}\pi r^3$

$V(D)=\frac{1}{6}\pi D^3$

$\frac{V'(D_0)}{V(D_0)}\delta_D=\frac{\frac{1}{2}\pi D_0^2}{\frac{1}{6}\pi D_0^3}\delta_D=3\frac{\delta_D}{D_0}\le 2\%$

$\therefore \frac{\delta_D}{D_0}\le 0.67\%$