1-5 展开定理2

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15. 原式 $= \prod_{1\le j\le i\le n}(a_i-a_j)$

$= 2\times(-1)\times4\times(1-\frac{\sqrt2}{2})\times(\frac{\sqrt2}{2}+1)$

$\times(\frac{\sqrt2}{2}-2)\times(\frac{\sqrt2}{2}+3)\times(-2)\times(-5)$

$\times 3$

$= 2\times4\times\frac12\times(2-\frac{\sqrt2}{2})\times(3+\frac{\sqrt2}{2})\times10\times3$

$= 120\times(\frac{11}{2}-\frac{\sqrt2}{2}) = 60(11-\sqrt2)$

16. 原式 $\xlongequal[c_2-c_1]{c_4-c_3 \\ c_3-c_2} \begin{vmatrix} a^2 & 2a+1 & 2a+3 & 2a+5 \\ b^2 & 2b+1 & 2b+3 & 2b+5 \\ c^2 & 2c+1 & 2c+3 & 2c+5 \\ d^2 & 2d+1 & 2d+3 & 2d+5 \end{vmatrix}$

$\xlongequal[c_3-c_2]{c_4-c_3} \begin{vmatrix} a^2 & 2a+1 & 2 & 2 \\ b^2 & 2b+1 & 2 & 2 \\ c^2 & 2c+1 & 2 & 2 \\ d^2 & 2d+1 & 2 & 2 \end{vmatrix} = 0$

17. 原式 $= \begin{vmatrix} x^n & (x+1)^n & (x+2)^n & \cdots & (x+n-1)^n & (x+n)^n \\ x^{n-1} & (x+1)^{n-1} & (x+2)^{n-1} & \cdots & (x+n-1)^{n-1} & (x+n)^{n-1} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ x & x+1 & x+2 & \cdots & x+n-1 & x+n \\ 1 & 1 & 1 & \cdots & 1 & 1 \end{vmatrix}$

$= (-1)^{n+(n-1)+\cdots+1} \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ x & x+1 & x+2 & \cdots & x+n-1 & x+n \\ \vdots & \vdots & \vdots & & \vdots & \vdots \end{vmatrix}$

$$= (-1)^{n+(n-1)+\cdots+1} \begin{vmatrix} x & x+1 & x+2 & \cdots & x+n-1 & x+n \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ x^{n-1} & (x+1)^{n-1} & (x+2)^{n-1} & \cdots & (x+n-1)^{n-1} & (x+n)^{n-1} \\ x^n & (x+1)^n & (x+2)^n & \cdots & (x+n-1)^n & (x+n)^n \end{vmatrix}$$

$$= (-1)^{\frac{n(n+1)}{2}} \prod_{i=1}^{n} i!$$

18. (2) $\begin{bmatrix} 3 & 1 & 2 & 0 \\ 0 & 5 & 1 & 4 \\ 4 & 0 & 2 & 1 \\ 2 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{13} & -\frac{4}{39} & -\frac{1}{39} & \frac{17}{39} \\ 0 & 1 & 0 & 0 & \frac{6}{13} & \frac{2}{39} & -\frac{19}{39} & \frac{11}{39} \\ 0 & 0 & 1 & 0 & \frac{2}{13} & \frac{5}{39} & \frac{11}{39} & -\frac{31}{39} \\ 0 & 0 & 0 & 1 & -\frac{8}{13} & \frac{2}{13} & \frac{7}{13} & -\frac{2}{13} \end{array}\right]$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{13} & -\frac{4}{39} & -\frac{1}{39} & \frac{17}{39} \\ \frac{6}{13} & \frac{2}{39} & -\frac{19}{39} & \frac{11}{39} \\ \frac{2}{13} & \frac{5}{39} & \frac{11}{39} & -\frac{31}{39} \\ -\frac{8}{13} & \frac{2}{13} & \frac{7}{13} & -\frac{2}{13} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ -\frac{2}{3} \\ 0 \end{bmatrix}$$

综上 $\begin{cases} x_1 = x_2 = \frac{1}{3} \\ x_3 = -\frac{2}{3} \\ x_4 = 0 \end{cases}$

20. 方程组有非零解 $\Leftrightarrow D=0$

$$D=\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 2 \\ \lambda^2 & 1 & \lambda \end{vmatrix} = \begin{vmatrix} 0 & 1-\lambda^2 & 1-2\lambda \\ 1 & \lambda & 2 \\ 0 & 1-\lambda^3 & \lambda-2\lambda^2 \end{vmatrix}$$

$$=(-1)^{2+1}\begin{vmatrix} 1-\lambda^2 & 1-2\lambda \\ 1-\lambda^3 & \lambda-2\lambda^2 \end{vmatrix}$$

$$=(1-\lambda^3)(1-2\lambda)-(1-\lambda^2)(\lambda-2\lambda^2)$$

$$=2\lambda^2-3\lambda+1=(\lambda-1)(2\lambda-1)$$

综上 $\lambda=1$ 或 $\frac{1}{2}$

21. 已知有
$$\begin{cases} C_{n-1}a_1^{n-1}+C_{n-2}a_1^{n-2}+\cdots+C_1a_1+C_0=b_1 \\ C_{n-1}a_2^{n-1}+C_{n-2}a_2^{n-2}+\cdots+C_1a_2+C_0=b_2 \\ \vdots \\ C_{n-1}a_n^{n-1}+C_{n-2}a_n^{n-2}+\cdots+C_1a_n+C_0=b_n \end{cases}$$

也就是说 $C_i\ (0\le i\le n-1)$ 是该方程组的根

那么命题成立