10.10

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习题 2.6 (B)

2. (1) $\lim\limits_{x\to0}\frac{\tan 5x}{2x}=\frac{5}{2}$

(2) $\lim\limits_{x\to0}\frac{\sin(x^m)}{(\sin x)^n}=\lim\limits_{x\to0}x^{m-n}=\begin{cases}1, & m=n\\ \infty, & m<n\\ 0, & m>n\end{cases}$

(3) $\lim\limits_{x\to0}\frac{\tan x-\sin x}{\sin^3 x}=\lim\limits_{x\to0}\frac{\frac{1}{2}x^3}{x^3}=\frac{1}{2}$

(4) $\lim\limits_{x\to0}\frac{\sqrt{1+x^2}-1}{1-\cos x}=\lim\limits_{x\to0}\frac{\frac{1}{2}x^2}{\frac{1}{2}x^2}=1$

总习题 (2)

4. (7) ∵ 当 $x\to0$ 时 $\tan x\to0$

∴ 原式 $=\lim\limits_{t\to0}(1+3t^2)^{\frac{1}{t^2}}$

$=\lim\limits_{t\to0}\left[(1+3t^2)^{\frac{1}{3t^2}}\right]^3=e^3$

(8) $\frac{\sin\alpha}{1-(\frac{\alpha}{\pi})^2}=\pi^2\frac{\sin\alpha}{\pi^2-\alpha^2}$

$=\frac{\pi^2}{\pi+\alpha}\frac{\sin(\pi-\alpha)}{\pi-\alpha}$

∴ 原式 $=\lim\limits_{\alpha\to\pi}\frac{\pi^2}{\pi+\alpha}\cdot\lim\limits_{\alpha\to\pi}\frac{\sin(\pi-\alpha)}{\pi-\alpha}$

$=\frac{\pi}{2}$

(9) 法一: $\lim\limits_{x\to\infty}\left(\frac{x^2}{x^2-1}\right)^x=\lim\limits_{x\to\infty}\left[\left(1+\frac{1}{x^2-1}\right)^{x^2-1}\right]^{\frac{x}{x^2-1}}$

$=e^{\lim\limits_{x\to\infty}\frac{1}{x-\frac{1}{x}}}$

$=e^0=1$

法二: 先证 $\lim\limits_{x\to0}\frac{\ln(1+x)}{x}=1$:

有不等式 $1-\frac{1}{x}\le\ln x\le x-1\quad(x>0)$

将 $x\to x+1$ 得 $\frac{x}{x+1}\le\ln(x+1)\le x\quad(x>-1)$

即 $\begin{cases}\frac{1}{x+1}<\frac{\ln(x+1)}{x}<1, & x>0\\ \frac{1}{x+1}>\frac{\ln(x+1)}{x}>1, & -1<x<0\end{cases}$

$$\left\{ \frac{1}{x+1} > \frac{\ln(x+1)}{x} > 1, \quad -1 < x < 0 \right.$$

$\because \lim_{x\to 0} \frac{1}{x+1} = 1$

$\therefore$ 由夹逼定理可知 $\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$

$\because \left(\frac{x^2}{x^2-1}\right)^x = e^{x\ln\left(\frac{x^2}{x^2-1}\right)}$

$\therefore$ 考虑求 $\lim_{x\to\infty} x\ln\frac{x^2}{x^2-1}$

$$\lim_{x\to\infty} x\ln\frac{x^2}{x^2-1} = \lim_{x\to\infty} \frac{\ln\left(\frac{1}{1-\frac{1}{x^2}}\right)}{\frac{1}{x}}$$

$$\underline{\underline{\text{记}\ \tfrac{1}{x}=t}}\ \lim_{t\to 0} \frac{-\ln(1-t^2)}{t}$$

$$= -\frac{\ln(1+t)}{t} + \frac{\ln(1-t)}{-t}$$

$$= -1+1 = 0$$

$\therefore$ 原式 $= e^0 = 1$

9. (1) $\lim_{x\to 0} \frac{\sqrt{1+\tan x}-\sqrt{1-\sin x}}{x} = \lim_{x\to 0}\left(\frac{\sqrt{1+\tan x}-1}{x} - \frac{\sqrt{1-\sin x}-1}{x}\right)$

$$= \lim_{x\to 0}\frac{\sqrt{1+\tan x}-1}{x} - \lim_{x\to 0}\frac{\sqrt{1-\sin x}-1}{x}$$

$$= \lim_{x\to 0}\frac{\frac{1}{2}\tan x}{x} - \lim_{x\to 0}\frac{-\frac{1}{2}\sin x}{x}$$

$$= \frac{1}{2} - \left(-\frac{1}{2}\right) = 1$$

$\therefore \sqrt{1+\tan x}-\sqrt{1-\sin x}$ 对 x 是 1 阶

(2) $\lim_{x\to 0} \frac{\sqrt{x^2+\sqrt[3]{x^4}}}{x^{\frac{2}{3}}} = \lim_{x\to 0}\sqrt{1+x^{\frac{2}{3}}} = 1$

$\therefore \sqrt{x^2+\sqrt[3]{x^4}}$ 对 x 是 $\frac{2}{3}$ 阶.

(3) $\lim_{x\to 0} \frac{\frac{1}{1-x}-1-x-x^2}{x^3} = \lim_{x\to 0}\frac{\frac{x^3}{1-x}}{x^3} = \lim_{x\to 0}\frac{1}{1-x} = 1$

$\therefore \frac{1}{1-x}-1-x-x^2$ 对 x 是 3 阶.

(4) $\lim_{x\to 0} \frac{2\sin x-\sin 2x}{x^3} = \lim_{x\to 0}\frac{2\sin x(1-\cos x)}{x^3}$

$$= \lim_{x\to 0}\frac{2x\cdot\frac{1}{2}x^2}{x^3} = 1$$

$$= \lim_{x\to 0} \frac{2x\cdot\frac{1}{2}x^2}{x^3} = 1$$

$\therefore$ $2\sin x - \sin 2x$ 对x是3阶.

习题 2.7 (A)

2. $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} (1-x)^{\frac{1}{x}} = \lim\limits_{x\to 0^+}\left[(1-x)^{-\frac{1}{x}}\right]^{-1} = \frac{1}{e}$

$\because f(0)=B \quad \therefore B=\frac{1}{e}$

$\because \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-}\left(x\sin\frac{1}{x}+A\right) = A$

$\therefore A=\frac{1}{e}$ 综上 $A=B=\frac{1}{e}$

3. $a=b$