10.24

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习题3.3

3. (1) $e^x - e^y \cdot y' + y + xy' = 0$

$\therefore \; y' = \frac{e^x + y}{e^y - x}$

(4) $\frac{1}{1+(\frac{y}{x})^2} \cdot (\frac{y}{x})' = \frac{1}{2} \cdot \frac{2x+2yy'}{x^2+y^2}$

$\frac{x^2}{x^2+y^2} \cdot \frac{y'x - y}{x^2} = \frac{x+yy'}{x^2+y^2}$

$y'x - y = x + yy' \qquad \therefore \; y' = \frac{x+y}{x-y}$

(7) $y^2 + 2xyy' + e^y \cdot y' = -\sin(x+y^2) \cdot (1+2yy')$

$y' = \frac{y^2 + \sin(x+y^2)}{e^y + 2xy + 2y\sin(x+y^2)}$

4. (1) $\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{\sqrt{t} \cdot \sqrt[3]{(1-\sqrt{t})^2}}{\sqrt[3]{t^2} \cdot \sqrt{1-\sqrt[3]{t}}}$

(3) $\frac{dy}{dx} = \frac{2te^{2t}}{3t^2} = \frac{2e^{2t}}{3t^2}$

$\because \; x=2 \quad \therefore \; t=1$

$\therefore \; \frac{dy}{dx}\Big|_{x=2} = \frac{2e^2}{3}$

(5) $\frac{dy}{dx} = \frac{e^{-t}(-\cos t - \sin t)}{e^t(\sin t + \cos t)} = -\frac{1}{e^{2t}}$

5. (1) $\ln y = \sin x \cdot \ln x$

$\frac{y'}{y} = \cos x \cdot \ln x + \frac{\sin x}{x}$

$\therefore \; y' = x^{\sin x}(\cos x \ln x + \frac{\sin x}{x})$

(3) $\ln y = \sin x \cdot \ln a$

$\frac{y'}{y} = \cos x \cdot \ln a$

$\therefore \; y' = a^{\sin x} \cdot \cos x \cdot \ln a$

(5) $\ln y = 2\ln(x+5) + \frac{1}{3}\ln(x-4) - 5\ln(x+2) - \frac{1}{2}\ln(x+4)$

$\frac{y'}{y} = \frac{2}{x+5} + \frac{1}{3(x-4)} - \frac{5}{x+2} - \frac{1}{2(x+4)}$

$\therefore \; y' = \frac{(x+5)^2 (x-4)^{\frac{1}{3}}}{(x+2)^5 (x+4)^{\frac{1}{2}}} \cdot \left[\frac{2}{x+5} + \frac{1}{3(x-4)} - \frac{5}{x+2} - \frac{1}{2(x+4)}\right]$

$$\therefore\ y' = \frac{(x+5)^2 (x-4)^{\frac{1}{3}}}{(x+2)^5 (x+4)^{\frac{1}{2}}} \cdot \left[\frac{2}{x+5} + \frac{1}{3(x-4)} - \frac{5}{x+2} - \frac{1}{2(x+4)}\right]$$

6. (2) $\frac{dy}{dx} = \frac{3-3t^2}{2-2t} = \frac{3}{2}(1+t)$

$\frac{d^2y}{dx^2} = \frac{\frac{3}{2}}{2-2t} = \frac{3}{4(1-t)}$

(4) $\frac{dy}{dx} = 2t \Big/ \frac{1+\frac{t}{\sqrt{t^2+1}}}{t+\sqrt{t^2+1}} = 2t\sqrt{t^2+1}$

$\frac{d^2y}{dx^2} = \left(2\sqrt{t^2+1} + \frac{2t^2}{\sqrt{t^2+1}}\right) \Big/ \frac{1+\frac{t}{\sqrt{t^2+1}}}{t+\sqrt{t^2+1}} = 2(t^2+1) + 2t^2$

$= 4t^2+2$

7. (1) $3x^2 + 3y^2y' - 3a(y+xy') = 0$

$x^2 + y^2y' - ay - axy' = 0 \Rightarrow y' = \frac{ay-x^2}{y^2-ax}$

$\Downarrow$

$2x + 2yy'^2 + y^2y'' - ay' - a(y' + xy'') = 0$

$\Rightarrow y'' = \frac{2}{y^2-ax}\left[\frac{a(ay-x^2)}{y^2-ax} - y\left(\frac{ay-x^2}{y^2-ax}\right)^2 - x\right]$

(3) $\ln x + \ln y = x+y$

$\frac{1}{x} + \frac{y'}{y} = 1+y' \Rightarrow y' = \frac{\frac{1}{x}-1}{1-\frac{1}{y}} = \frac{y-xy}{xy-x}$

$\Downarrow$

$-\frac{1}{x^2} + \frac{y''y - y'^2}{y^2} = y''$

$\Rightarrow y'' = \frac{y}{x-xy} + \frac{(x+y-2)(xy-y)}{(x-xy)^2} + \frac{x(xy-y^2)}{(x-xy)^3}$

9. (1) $r = |a|\sqrt{2\cos 2\theta}$

$\frac{r}{r'} = |a|\sqrt{2\cos 2\theta} \Big/ |a| \frac{-4\sin 2\theta}{2\sqrt{2\cos 2\theta}} = \frac{2\cos 2\theta}{-2\sin 2\theta} = -\cot 2\theta$

$\frac{dy}{dx} = \frac{\tan\theta - \cot 2\theta}{1+\tan\theta\cot 2\theta}$

$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{6}} = \frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+\sqrt{3}\cdot\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}}{3}$

(2) $\frac{r}{r'} = ae^{m\theta} \Big/ ame^{m\theta} = \frac{1}{m}$

$\frac{dy}{dx} = \frac{\tan\theta + \frac{1}{m}}{1-\frac{\tan\theta}{m}} = \frac{m\tan\theta+1}{m-\tan\theta}$