10.13

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总习题(2)

16. $\lim\limits_{x\to0^-} f(x)=\lim\limits_{x\to0^-}\frac{ax}{\sqrt{\frac{1}{2}x^2}}=\sqrt{2}a$

$f(0)=b$

$\lim\limits_{x\to0^+} f(x)=\lim\limits_{x\to0^+}\frac{\ln\frac{1}{x+1}}{x}=\lim\limits_{x\to0^+}\left[-\frac{\ln(x+1)}{x}\right]$

$=-1$

$\therefore\begin{cases}a=-\frac{\sqrt{2}}{2}\\ b=-1\end{cases}$

21. 令$y=0$ 得 $f(x)=f(x)+f(0)$ $\therefore f(0)=0$

$\because f(x)$在$x=0$处连续

$\therefore \forall\varepsilon>0,\exists\delta>0,\ni\forall|x-0|<\delta$，都有$|f(x)-f(0)|<\varepsilon$

即 $\lim\limits_{x\to0} f(x)=0$

$|f(x_1)-f(x_2)|=|f(x_2+\Delta x)-f(x_2)|$

$=|f(\Delta x)|$

$\because \lim\limits_{\Delta x\to0}|f(\Delta x)|=0$

$\therefore f(x)$在R上连续

23. $\because x_1$和x_2是相邻的两根.

$\therefore (x_1,x_2)$之间没有根.

又$\because \exists c\in(x_1,x_2),\ni f(c)>0$

$\therefore \exists a\in(-\infty,x_1),\ni f(a)\cdot f(c)<0$ 即$f(a)<0$

同理$\exists b\in(x_2,+\infty)$ 使$f(b)<0$

$\therefore$ 不能存在一个$\xi\in(x_1,x_2)$，使$f(\xi)<0$

即 $\forall x\in(x_1,x_2)$，$\ni f(x)>0$

24. 原方程 $\Longleftrightarrow 28x^2-101x+83=0$

记 $f(x)=28x^2-101x+83$

$\because f(1)f(2)<0$

$\therefore$ 在 $(1,2)$ 上有一根

同理 在 $(2,3)$ 上有一根

25. (1) $\lim\limits_{x\to 0}\dfrac{\ln\cos ax}{\ln\cos bx}=\lim\limits_{x\to 0}\dfrac{\ln[1+(\cos ax-1)]}{\ln[1+(\cos bx-1)]}$

$=\lim\limits_{x\to 0}\dfrac{\cos ax-1}{\cos bx-1}$

$=\lim\limits_{x\to 0}\dfrac{(\frac{ax}{2})^2}{(\frac{bx}{2})^2}=\dfrac{a^2}{b^2}$

(2) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x\sin x}-1}{e^{x^2}-1}=\lim\limits_{x\to 0}\dfrac{\frac{1}{2}x\sin x}{e^{x^2}-1}$

$=\dfrac{1}{2}\lim\limits_{x\to 0}\dfrac{\frac{\sin x}{x}}{\frac{e^{x^2}-1}{x^2}}=\dfrac{1}{2}$

(3) 原式 $=\lim\limits_{x\to 0}\dfrac{\sqrt[n]{1+\alpha x}-1}{x}-\lim\limits_{x\to 0}\dfrac{\sqrt[m]{1+\beta x}-1}{x}$

$=\lim\limits_{x\to 0}\dfrac{\frac{1}{n}\alpha x}{x}-\lim\limits_{x\to 0}\dfrac{\frac{1}{m}\beta x}{x}$

$=\dfrac{\alpha}{n}-\dfrac{\beta}{m}$

(4) 考虑极限 $\lim\limits_{x\to 0}\dfrac{1}{x}\ln\left(\dfrac{a^x+b^x+c^x}{3}\right)$

$\lim\limits_{x\to 0}\dfrac{1}{x}\ln\left(\dfrac{a^x+b^x+c^x}{3}\right)$

$=\lim\limits_{x\to 0}\dfrac{1}{x}\ln\left(1+\dfrac{a^x-1+b^x-1+c^x-1}{3}\right)$

$=\lim\limits_{x\to 0}\dfrac{a^x-1+b^x-1+c^x-1}{3x}$

$=\dfrac{1}{3}\left(\lim\limits_{x\to 0}\dfrac{a^x-1}{x}+\lim\limits_{x\to 0}\dfrac{b^x-1}{x}+\lim\limits_{x\to 0}\dfrac{c^x-1}{x}\right)$

$=\dfrac{1}{3}\left(\lim\limits_{x\to 0}\dfrac{e^{x\ln a}-1}{x}+\lim\limits_{x\to 0}\dfrac{e^{x\ln b}-1}{x}+\lim\limits_{x\to 0}\dfrac{e^{x\ln c}-1}{x}\right)$

$=\dfrac{1}{3}(\ln a+\ln b+\ln c)=\dfrac{\ln(abc)}{3}$

(5) 原式 $=\lim\limits_{x\to 0}\dfrac{x\cdot 1\cdot(2x)^2\cdot x^2}{\frac{1}{4}x^4\cdot 8x}$

$=\lim\limits_{x\to 0}\dfrac{4x^5}{2x^5}=2$