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习题 4.3 (A)

2\. (2) $\frac{5x^2}{2}+C$

(4) $\frac{t^3}{3}+\frac{t^2}{2}+C$

(6) $\frac{2z^{\frac{3}{2}}}{3}+C$

(8) $-\frac{1}{t}+C$

(10) e^x+C

(12) $\frac{5t^3}{3}+C$

(14) $\frac{y^5}{5}+\ln y+C$

(16) $\frac{t^2}{2}+\ln|t|+C$

(18) $e^t+e^{5t}+C$

(20) $\frac{e^{5x}}{5}+C$

(22) $\sin t+\tan t+C$

(24) $\sin(t^3+7)+C$

(26) $e^y+\frac{2^y}{\ln 2}+C$

(28) $\arcsin x+\arctan x+C$

(B)

1\. (2) 原式 $=\int \frac{1}{1+x^2}+\frac{1}{x}\,dx = \arctan x+\ln|x|+C$

(4) 原式 $=\int x^{\frac{3}{4}}dx = \frac{4x^{\frac{7}{4}}}{7}+C$

(6) 原式 $=2\int x^{\frac{3}{2}}dx = 2\,\frac{2x^{\frac{5}{2}}}{5}+C$

$= \frac{4x^{\frac{5}{2}}}{5}+C$

(8) 原式 $=\int \sqrt{x}+\frac{1}{\sqrt{x}}\,dx = \frac{2x^{\frac{3}{2}}}{3}+2\sqrt{x}+C$

(10) 原式 $=\frac{2^x}{\ln 2}+\frac{3^x}{\ln 3}+C$

(12) 原式 $=\int\left(\frac{1}{x^4}-\frac{1}{x^2}+\frac{1}{1+x^2}\right)dx$

$= -\frac{1}{3x^3}+\frac{1}{x}+\arctan x+C$

2\. (2) 原式 $=-2\cos\theta-3\sin\theta+C$

(4) 原式 $=\int \frac{\sin^2\frac{t}{2}+\cos^2\frac{t}{2}}{\sin^2\frac{t}{2}\cos^2\frac{t}{2}}\,dt$

$=\int\left(\sec^2\frac{t}{2}+\csc^2\frac{t}{2}\right)dt$

$=2\int\left(\sec^2\frac{t}{2}+\csc^2\frac{t}{2}\right)d\left(\frac{t}{2}\right)$

$=2\left(\tan\frac{t}{2}-\cot\frac{t}{2}\right)+C$

(6) 原式 $=\int \frac{4\sin^2 x+3\cos^2 x}{\cos^2 x}\,dx$

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$= \int 4\tan^2x+3\,dx$

$= \int 4\sec^2x-1\,dx$

$= 4\tan x - x + C$

(8) 原式 $= \int \frac{dx}{\cos^2x} = \tan x + C$

(10) 原式 $= \int \frac{\cos^2x-\sin^2x}{\cos x-\sin x}dx$

$= \int \cos x+\sin x\,dx$

$= \sin x - \cos x + C$

(12) 原式 $= \int 2-5\left(\frac{2}{3}\right)^x dx$

$= 2x - \frac{5}{\ln2-\ln3}\left(\frac{2}{3}\right)^x + C$

习题 4.4 (A)

2. (2) 原式 $= \frac{1}{2}\int (x^2-4)^{7/2}d(x^2-4) = \frac{(x^2-4)^{9/2}}{9} + C$

(4) 原式 $= -\frac{1}{5}\int (1-5x)^{\frac{1}{3}}d(1-5x) = -\frac{3(1-5x)^{4/3}}{20} + C$

(6) 原式 $= \frac{1}{2}\int \frac{dx}{1+\frac{3}{2}x^2} = \frac{1}{2}\cdot\frac{2}{\sqrt6}\int \frac{1}{1+\frac{3}{2}x^2}d\left(\frac{\sqrt6}{2}x\right)$

$= \frac{\sqrt6}{6}\arctan\left(\frac{\sqrt6}{2}x\right) + C$

(8) 原式 $= \frac{1}{\sqrt2}\cdot\frac{2}{\sqrt6}\int \frac{1}{\sqrt{1-\frac{3}{2}x^2}}d\left(\frac{\sqrt6}{2}x\right)$

$= \frac{\sqrt3}{3}\arcsin\left(\frac{\sqrt6}{2}x\right) + C$

(10) 原式 $= \int \frac{d(\ln x)}{\ln x\ln(\ln x)} = \int \frac{d[\ln(\ln x)]}{\ln(\ln x)}$

$= \ln[\ln(\ln x)] + C$

4. (2) 设 $x = a\sin t$，$t \in [0, \frac{\pi}{2}]$

原式 $= \int_0^{\pi/4} \frac{a\cos t\,dt}{(a^2\cos^2t)^{3/2}} = \int_0^{\pi/4}\frac{dt}{a^2\cos^2t}$

$= \frac{1}{a^2}\tan t\Big|_0^{\pi/4} = \frac{1}{a^2}$

(4) 设 $x = t^2$，则：

原式 $= \int \frac{2t\,dt}{1+t} = 2\int 1-\frac{1}{1+t}dt$

$= 2(t-\ln|1+t|) + C$

$= 2(\sqrt x - \ln|1+\sqrt x|) + C$

(6) 设 $x = \sin t$，则：

原式 $= \int \frac{\cos t\,dt}{1+\cos t} = \int 1-\frac{1}{1+\cos t}dt$

$= t - \int \frac{dt}{2\cos^2\frac{t}{2}}$

$= t - \tan\frac{t}{2} + C$

$= \arcsin x - \tan\left(\frac{\arcsin x}{2}\right) + C$

(8) 设 $x = a\sin t$，则

(8) 设 $x=a\sin t$，则

$$原式 = \int_0^{\pi/2} a^2\sin^2 t\cdot a\cos t\cdot a\cos t\,dt$$

$$= a^4\int_0^{\pi/2}\sin^2 t\cos^2 t\,dt$$

$$= \frac{a^4}{4}\int_0^{\pi/2}\sin^2 2t\,dt$$

$$= \frac{a^4}{8}\int_0^{\pi/2} 1-\cos 4t\,dt$$

$$= \frac{a^4}{32}\left(4t-\sin 4t\Big|_0^{\pi/2}\right)$$

$$= \frac{a^4}{32}\cdot 2\pi = \frac{\pi a^4}{16}$$

(10) 先计算 $d(\arctan\sqrt{x}) = \frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}dx$

所以 $原式 = 2\int \arctan\sqrt{x}\,d(\arctan\sqrt{x})$

$$= \arctan^2\sqrt{x}+C$$

(B)

2. (2) 设 $x=\sin t$，则:

$$原式 = \int\frac{\cos t\,dt}{\sin t+\cos t} = \frac{1}{2}\int\frac{\sin t+\cos t+\cos t-\sin t}{\sin t+\cos t}dt$$

$$= \frac{1}{2}\int 1+\frac{\cos t-\sin t}{\sin t+\cos t}dt$$

$$= \frac{1}{2}\left[t+\int\frac{1}{\sin t+\cos t}d(\sin t+\cos t)\right]$$

$$= \frac{1}{2}\left(t+\ln|\sin t+\cos t|\right)+c$$

$$= \frac{1}{2}\left(\arcsin x+\ln|x+\cos\arcsin x|\right)+C$$

(4) 设 $x=\tan t$，则:

$$原式 = \int\frac{\sec^2 t\,dt}{(\sec^2 t)^{3/2}} = \int\cos t\,dt = \sin t+C = \sin\arctan x+C$$

(6) $原式 = -\int-\frac{\ln x+1}{(x\ln x)^2}dx = -\int d\left(\frac{1}{x\ln x}\right) = -\frac{1}{x\ln x}+C$

(8) 设 $x=a\sin t$，则:

$$原式 = \int_0^{\pi/2}\sqrt{\frac{1-\sin t}{1+\sin t}}\,a\cos t\,dt = a\int_0^{\pi/2}\left(\cos\tfrac{t}{2}-\sin\tfrac{t}{2}\right)\frac{\cos^2\frac{t}{2}-\sin^2\frac{t}{2}}{\sin\frac{t}{2}+\cos\frac{t}{2}}dt$$

$$= a\int_0^{\pi/2}\left(\sin\tfrac{t}{2}-\cos\tfrac{t}{2}\right)^2dt = a\int_0^{\pi/2}1-\sin t\,dt$$

$$= a\left(t+\cos t\Big|_0^{\pi/2}\right) = a\left(\frac{\pi}{2}-1\right)$$