11.16

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习题 4.1 (A)

5. (1) $x\sin x$ 在 $(0,\pi)$ 上大于 0

$\therefore \int_0^{\pi} x\sin x\,dx > 0$

(2) $x^2\ln x$ 在 $(\frac{1}{2},1)$ 上小于 0

$\therefore \int_{1/2}^{1} x^2\ln x\,dx < 0$

6. (1) $0 < \sin^{10}x < \sin^2 x < 1$, $(0<x<\frac{\pi}{2})$

$\therefore \int_0^{\pi/2} \sin^{10}x\,dx < \int_0^{\pi/2} \sin^2 x\,dx$

(2) $0 < e^{-x} < e^{-x^2}$, $(0<x<1)$

$\therefore \int_0^1 e^{-x}dx < \int_0^1 e^{-x^2}dx$

(3) $0 < \sin x < x$, $(0<x<\frac{\pi}{2})$

$\therefore \int_0^{\pi/2} x\,dx > \int_0^{\pi/2} \sin x\,dx$

(B)

4. 即证: $\left(\int_a^b f(x)\cdot 1\,dx\right)^2 \le (b-a)\int_a^b f^2(x)\,dx$

$\left(\int_a^b f(x)\cdot 1\cdot dx\right)^2 \le \int_a^b 1^2\cdot dx \cdot \int_a^b f^2(x)\,dx$

$= (b-a)\cdot\int_a^b f^2(x)\,dx$

5. $f(0) = 3\int_{2/3}^{1} f(x)\,dx = 3\cdot f(\xi)(1-\frac{2}{3}) = f(\xi)$, $(\frac{2}{3}<\xi<1)$

$\Rightarrow 0 = f(0)-f(\xi) = f'(c)(\xi-0)$, $(0<c<\xi)$

$\Rightarrow f'(c)=0$ Q.E.D