Jalen Powell 10/3/2021
Comp 3240

Hw 6

6.1.1

a.) No. (a,b) is not in G^2; it takes either 1 or >2 length to reach b from a

b. Yes. (b,e) is in G^3; because there is a path of length 3 ($b \to c \to f \to e$) to reach e from b.

c. No. (g,g) is not in G^3; it takes either 2 or >3 length to reach g from g.

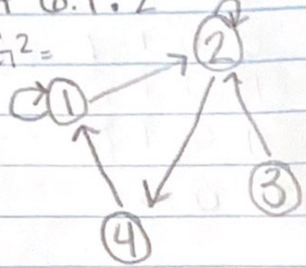
d. Yes. (g,g) is in G^4; there is a path of length 4 to reach g from g.

e. Yes. (b,b) is in G^3, there is a path of length 3 to reach b from b.

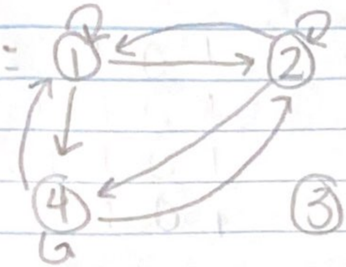
f. No, (b,d) is not in G^5, there isn't a path of length 5 to reach d from b; it is either 2,3,4, but not 5

6.1.2

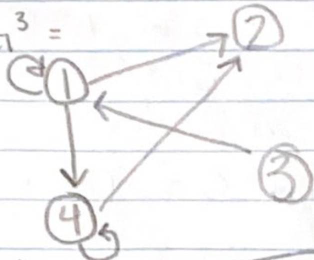
a. $G^2 =$



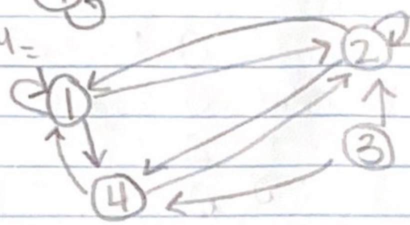
$G^+ =$



$G^3 =$



$G^4 =$



6.1.3

a)

$$\begin{matrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{matrix}$$

b)

$$\begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{matrix}$$

c)

$$\begin{matrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{matrix}$$

6.2.1

a.)

$$\begin{array}{c|c} & \begin{matrix}1 & 2 & 3 & 4\end{matrix} \\ \begin{matrix}1\\2\\3\\4\end{matrix} & \begin{bmatrix}0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\\0 & 1 & 0 & 0\end{bmatrix}\end{array}$$

$$g^2 = \begin{array}{c|c} & \begin{matrix}1 & 2 & 3 & 4\end{matrix} \\ \begin{matrix}1\\2\\3\\4\end{matrix} & \begin{bmatrix}0 & 0 & 1 & 0\\0 & 0 & 0 & 1\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\end{bmatrix}\end{array}$$

$$g^3 = \begin{array}{c|c} & \begin{matrix}1 & 2 & 3 & 4\end{matrix} \\ \begin{matrix}1\\2\\3\\4\end{matrix} & \begin{bmatrix}0 & 0 & 0 & 1\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}\end{array}$$

$$g^4 = \begin{array}{c|c} & \begin{matrix}1 & 2 & 3 & 4\end{matrix} \\ \begin{matrix}1\\2\\3\\4\end{matrix} & \begin{bmatrix}0 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}\end{array}$$

$$g^+ = \begin{array}{c|c} & \begin{matrix}1 & 2 & 3 & 4\end{matrix} \\ \begin{matrix}1\\2\\3\\4\end{matrix} & \begin{bmatrix}0 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}\end{array}$$

6.2.2

a.)

$$A = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 1\\1 & 0 & 0 & 0 & 0 & 0\\0 & 1 & 0 & 1 & 0 & 0\\0 & 0 & 1 & 0 & 1 & 0\\0 & 0 & 0 & 0 & 1 & 1\\0 & 1 & 0 & 0 & 0 & 0\end{bmatrix}$$

b.) $A^2 = A \cdot A$

$$\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 1\\1 & 0 & 0 & 0 & 0 & 0\\0 & 1 & 0 & 1 & 0 & 0\\0 & 0 & 1 & 0 & 1 & 0\\0 & 0 & 0 & 0 & 1 & 1\\0 & 1 & 0 & 0 & 0 & 0\end{bmatrix} \cdot \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 1\\1 & 0 & 0 & 0 & 0 & 0\\0 & 1 & 0 & 1 & 0 & 0\\0 & 0 & 1 & 0 & 1 & 0\\0 & 0 & 0 & 0 & 1 & 1\\0 & 1 & 0 & 0 & 0 & 0\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0 & 0 & 0\\0 & 0 & 0 & 0 & 0 & 1\\1 & 0 & 1 & 0 & 1 & 0\\0 & 1 & 0 & 1 & 1 & 1\\0 & 1 & 0 & 0 & 1 & 1\\0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

c) The 4 vertices are (V_2, V_4, V_5, V_6) can be reached from vertex V_4 by length 2

d) The vertices V_1, V_4, and V_5 can reach vertex V_2 by walk of length 2

6.2.3

a) The vertices that reach 2 with walk of 3 are V_2, V_4, V_5.

b) The out-degree of 4 in closure is 5

c) There is no walk of 4 from $4 \to 5$

d) Entry is 1, so (2,2) is there in closure

e) No, the entry is 0 at $A^3(5,3)$

f) Yes, there are entries '1' for $A^3(1,1)$, $A^3(2,2)$, $A^3(3,3)$. There are 3 circuits with length 3.

6.3.1

a) The minimal elements of the partial order are {J, I, A, F}

b.) The maximal elements of the partial order are {J, H, D, G}

c.) The comparable pairs are (A, D), (G, F), (D, B), (H, I) only.

6.4.1

a) Not necessarily an equivalence relation. The Statement do not state that the relation P is transitive.

b.) Equivalence relation. Reflexive, the person has the same mother as herself or himself as one individual. Symmetric and transitive.

c.) Not an equivalence relation. The relation S is not reflexive because a person cannot be married to himself / herself

d.) Equivalence relation. Reflexive, Symmetric, and transitive

e.)

#6.4.2

a) xDy is Reflexive, Symmetric, and transitive.

Partition of [2] = { 2, 34 } = equivalence class of [34]

Equivalence class of [7] = { 7, 99, 31 } = E. class of [31][99]

E. class of [13] = { 13, 17 } = E. class of [17]

E. class of [44] = { 44, 56, 4 } = E. class of [56], [4]

Partitions of D = {2, 34}, {7, 99, 31}, {13, 17}, {44, 56, 4}

#6.4.3

a) Let $(x, z) \in P^+ \setminus P$

there exist an element $y \in A$ such that

$(x, y) \in P$ and $(y, z) \in P$

Since p is Symmetric relation

$(y, x) \in P$ and $(z, y) \in P$

$= (z, x) \in P^+$ $\because$ P^+ is transitive relation

P^+ is Symmetric relation

b.) P is reflexive and symmetric relation on set A

P^+ is reflexive relation

P^+ is equivalence relation

c.) Yes, $R = \{ (1,2), (2,1), (2,3), (3,2) \}$ an

$A = \{ 1, 2, 3, 4 \}$

Them $R^+ = \{ (1,1), (2,2), (3,3), (1,2), (2,1), (2,3),$

$(3,2), (1,3), (3,1) \}$

not equivalence

R^+ is not reflexive