Jalen Powell 09/12/21
Comp 3240

Hw 3

3.1.1

a)

p	q	$p \vee q$	$(p \vee q) \wedge p$	$(p \vee q) \wedge p \Rightarrow q$
T	T	T	T	T
T	F	T	T	F
F	T	T	F	T
F	F	F	F	T

– Arguement is invalid, last column's values not T.

b.)

p	q	$p \leftrightarrow q$	$(p \vee q) \wedge (p \leftrightarrow q)$	$(p \vee q) \wedge (p \leftrightarrow q) \Rightarrow p$
T	T	T	T	T
T	F	T	F	T
F	T	T	F	T
F	F	F	F	T

– Arguement is valid, last column is all true.

c.)

p	q	$p \leftrightarrow q$	$(p \wedge q) \Rightarrow (p \leftrightarrow q)$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	T	T

– Arguement is valid, the last column is all T.

3.1.1

d.)

p	q	¬q	p v q	p <-> q	(p v q) ^ ¬q	(p v q) ^ ¬ => p <-> q
T	T	F	T	T	F	T
T	F	T	T	F	T	F
F	T	F	T	F	F	T
F	F	T	F	T	F	T

- Arguement is invalid, the last column is not all T.

e.)

p	q	r	(p v q)	(p v q) -> r	(p ^ q)	(p ^ q) -> R
T	T	T	T	T	T	T
T	T	F	T	F	T	F
T	F	T	T	T	F	T
T	F	F	T	F	F	T
F	T	T	T	T	F	T
F	T	F	T	F	F	T
F	F	T	F	T	F	T
F	F	F	F	T	F	T

- The arguement is valid

f.)

p	q	r	(p ^ q)	(p ^ q) -> r	(p v q)	(p v q) -> r
T	T	T	T	T	T	T
T	T	F	T	F	T	F
T	F	T	F	T	T	T
T	F	F	F	T	T	F
F	T	T	F	T	T	T
F	T	F	F	T	T	F
F	F	T	F	T	F	T
F	F	F	F	T	F	T

- The arguement is invalid.

3.1.1

g.)

p	q	$q \to p$	$\neg q$	p
T	T	T	F	T
T	F	T	T	T
F	T	F	F	F
F	F	T	T	F

- Arguement is invalid

h.)

p	q	$p \to q$	$\neg(p \to q)$	$q \to p$	$\neg q$
T	T	T	F	T	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	T

- The arguement is valid

i.)

p	q	$p \to q$	$\neg(p \to q)$	$q \to p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	F	T

- Arguement is valid.

j.

p	q	$q \to p$	$p \to q$	$\neg(p \to q)$
T	T	T	T	F
T	F	T	F	T
F	T	F	T	F
F	F	T	T	F

3.2.1

a) $p \vee q$
$\underline{q}$
$\neg p$ → This statement is invalid

b.) $q \Rightarrow p$
$\underline{\neg q}$
$\neg q$ → This statement is invalid

c) $\underline{p}$
$\therefore p \vee q$ → This statement is valid

d) $q \rightarrow p$
$\underline{\neg p}$ → This statement is valid
$\therefore \neg q$

#3.3.2

a.) S: Student of my class

$A(x)$: Getting an A

b: John

$\exists s, A(s)$ The statement is invalid

$b \neq s$

b.) G: girl scout

C: Selling 50 boxes of cookies

P: Getting a prize

s: Suzy

$\forall x \in G, C(x) \rightarrow P(x)$

$s \in G, P(s)$ This statement is invalid

3.4.5

a) neither

b) neither

c.) prime

d.) prime

e.) composite = $21 = 3 \times 7$, $1 < 3 < 21$ and $1 < 7 < 21$

f.) composite = $1 < 2 < 56328$

g.) prime

3.5.2

a.) $n=0$, then $(n+1)^2 = (0+1)^2 = 1^2 = 1$
and $n^3 = 0^3 = 0$
$\therefore 1 > 0 = (n+1)^2 > n^3$ for $n=0$

$n=1$, then $(n+1)^2 = (1+1)^2 = 2^2 = 4$
and $n^3 = 1^3 = 1$
$\therefore 4 > 1 = (n+1)^2 > n^3$ for $n=1$

$n=2$, then $(n+1)^2 = (2+1)^2 = 3^2 = 9$
and $n^3 = 2^3 = 8$
$\therefore 9 > 8 = (n+1)^2 > n^3$ for $n=2$
Therefore, n is an integer

b. $n=0$, then $2^{(n+2)} = 2^2 = 4$, $3^n = 3^0 = 1$
$\therefore 4 > 1 = 2^{n+2} > 3^n$ for $n=0$

$n=1$, then $2^{n+2} = 2^3 = 8$, $3^n = 3^1 = 3$
$\therefore 8 > 3 = 2^{n+2} > 3^n$ for $n=1$

$n=2$, then $2^{n+2} = 2^4 = 16$, $3^n = 3^2 = 9$
$\therefore 16 > 9 = 2^{n+2} > 3^n$ for $n=2$

$n=3$, then $2^{n+2} = 2^5 = 32$, $3^n = 3^3 = 27$
$\therefore 32 > 27 = 2^{n+2} > 3^n$ for $n=3$
Therefore n is an integer

c.) $n=0$, then $(n+1)^3 = 1^3 = 1$, $3^n = 3^0 = 1$
$\therefore 1 = 1 = (n+1)^3 \geq 3^n$ for $n=0$

$n=1$, then $(n+1)^3 = 2^3 = 8$, $3^n = 3^1 = 3$
$\therefore 8 > 3 = (n+1)^3 \geq 3^n$ for $n=1$

$n=2$, then $(n+1)^3 = 3^3 = 27$, $3^n = 3^2 = 9$

$\therefore 27 > 9 = (n+1)^3 \geq 3^n$ for $n=2$

$n=3$, then $(n+1)^3 = 4^3 = 64$, $3^n = 3^3 = 27$

$\therefore 64 \geq 27 = (n+1)^3 \geq 3^n$ for $n=3$

$n=4$, then $(n+1)^3 = 5^3 = 125$, $3^n = 3^4 = 81$

$\therefore 125 \geq 81 = (n+1)^3 \geq 3^n$ for $n=4$

Therefore, n is all positive integers $n \leq 4$

#3.6.2

a.) invalid statement:

w divides x $\therefore$ $x = kw$

y divides z $\therefore$ $z = ky$

k might not be the same for both.

b.) Missing a step where it says that k, j are integers & their product is also another integer, m.

c.) It misses a step that says why xz is some integer times wy.

d.) Its our assumption that w divides x but the result is invalid.

3.7.1

a) even number: $2k$

odd number: $2k+1$

even + odd: $2k + 2k+1$

$= \underline{4k+1} \rightarrow$ is an odd number

b.) odd $\rightarrow 2k+1$

odd + odd $= 2k+1 + 2k+1$

$= 4k+2$

$= 2(2k+1) =$ Even

odd + odd = even

c.) odd $\rightarrow 2k+1$

$\text{odd}^2 = (2k+1)^2$

$= 4k^2 + 4k + 1$

$= 4k^2$ is even as 4 is even

$= 4k+1$

$\text{odd}^2 =$ even + odd

$\text{odd}^2 =$ odd

d.) odd $\rightarrow 2k+1$

odd x odd $\rightarrow (2k+1)(2k+1)$

$= \underbrace{4k^2}_{\text{even}} + \underbrace{4k+1}_{\text{odd}}$

odd x odd = odd

e.) x is an even $\rightarrow 2k$

y is an odd $\rightarrow 2k+1$

$x^2 + y^2$ is odd

$(2k)^2 + (2k+1)^2 \rightarrow 4k^2 + 4k^2 + 4k+1$

$= \underbrace{8k^2}_{\text{even}} + \underbrace{4k+1}_{\text{odd}}$

$x^2+y^2 =$ odd

f.) $x = \text{even} \rightarrow 2k$

$y = \text{odd} \rightarrow 2k+1$ even + even = even

$3x + 2y = \text{even}$

$3(2k) + 2(2k+1)$

$\underbrace{6k}_{\text{even}} + \underbrace{2y}_{\text{even}} = y$

g.) $x = \text{even} \rightarrow 2k$

$y = \text{odd} \rightarrow 2k+1$

$2x + 3y = y \text{ odd}$

$2(2k) + 3(2k+1) = 4k + 6k + 3$

$\underbrace{10k}_{\text{even}} + \underbrace{3}_{\text{odd}}$

even + odd → odd

h.) $y - \text{odd} = y \; 2k+1$

$-y = -(2k+1)$

$= -(\text{odd}) = -y = \text{odd only}$

i.) $x \rightarrow \text{even}$ $(-1)^x = 1$

$(-1)^{2k} = ((-1)^2)^k = ((-1)^2)^k$

$= (1)^k \therefore (-1)^2 = 1$

So $(-1)^x = 1$

j.) $x = \text{odd} = 2k+1$

$(-1)^x = (-1)^{2k+1} = (-1)^{2k} \times (-1)^1$

$= 1 \times -1$

$= -1$

$(-1)^{2k} = 1$

$(-1)^1 = -1$

$(-1)^x = -1$ when x is odd

3.8.1

a.) Contrapositive: If n is not odd, then n^2 is not odd
Then, $n^2 = 4m^2$ is even
n is not odd, then n^2 is not odd

b.) Contrapositive: If n is not even, then n^3 is not even
if n is not even → $n = 2m+1$, m(-2)
n^3 is not even

c.) Contrapositive: If n is not odd, then $5n+3$ is not even
$5n+3 = 5(2m)+3 = 10m+3 = 2(5m+1)+1$
$= 5n+3$ is not even

d.) Contrapositive: if n is not odd, then n^2-2n+1 is not even
$n^2-2n+1 = 4m^2-4m+1 = 2(2m^2-2m+3)+1$
n^2-2n+1 is not even

e.) Contrapositive: If n is not odd, then n^2 is divisible by 4
$n^2 = 4m^2$ is divisible by 4

f.) Contrapositive: If both x and y are odd, then xy is not even
$x = 2m+1$ $y = 2n+1$ $xy = (2m+1)(2n+1)$
$= 4mn + 2m + 2n + 1$
xy = is not even

g.) Contrapositive: If both x and y are even, then $x-y$ is not odd
$x = 2m$, $y = 2n$ $x-y = 2(m-n)$ is not odd.

h.) <u>Contrapositive:</u> If n is not odd, then 2^n-1 is not prime

$$2^n-1 = 2^{2m}-1 = (2^m)^2-1$$
$$= (2^m+1)(2^m-1)$$

$2^m-1 \neq 1 \qquad m \geq 2$

2^m-1 is not prime

#3.9.1

a) $\frac{\sqrt{2}}{2} = \frac{p}{q} = 2p = q\sqrt{2}$

$= \sqrt{2} = \frac{2p}{p} \quad q \neq 0$

$\frac{2p}{q}$ is rational number

$\sqrt{2}$ is rational number

$\frac{\sqrt{2}}{2}$ must be irrationals number

b.) $2 - \sqrt{2} = \frac{x}{y}$

$-\sqrt{2} = \frac{x}{y} - 2$

$-\sqrt{2} = \frac{x-2}{y}$

$-\sqrt{2}$ is rational

$\frac{x-2}{y}$ is rational

$-2 - \sqrt{2}$ is irrational

c.) $3 + \sqrt{2}$ and $3 - \sqrt{2}$ are irrational

$3 + \sqrt{2} + 3 - \sqrt{2} = 6$

This Statement is false

d) $\sqrt{8}$ and $\sqrt{2}$ are irrational

$\sqrt{8} \cdot \sqrt{2} = \sqrt{16} = 4$ is rational

This statement is false

3. 10. 2