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Comp 3420

Hw 7

7.1.2

a) $a_{n+1} - a_n = (n+1)^2 - 2(n+1) - n^2 + 2n$
$= a_{n+1} - a_n = 2n - 1 > 0$ for all $n \geq 1$
This sequence is increasing.

b) $a_{n+1} - a_n = (n+1)^2 - 3(n+1) - n^2 + 3n$
$= 2n - 2 \geq 0$ for all $n \geq 1$.
This sequence is non-decreasing.

c.) $a_{n+1} - a_n = (n+1)^2 - 4(n+1) - n^2 + 4n$
$= 2n - 3$
This sequence is non of the mentioned properties.

d.) $a_{n+1} - a_n = 2^{n+1} - (n+1)! - 2^n + n!$
$= 2^n - n \times n$
This sequence is non of the mentioned properties.

e.) $a_{n+1} - a_n = 2^{n+1} - 3^{n+1} - 2^n + 3^n$
$= 2^n - 2 \times 3^n < 0$
This sequence is decreasing.

#7.1.3

a.) first value, a = 2, common ratio = 3

$t_1 = a$

$t_2 = t_1 \times r$

$\boxed{t_{n+1} = t_n \times r = ar^n}$

Six terms: $\underset{1}{2}$ $\underset{2}{6}$ $\underset{3}{18}$ $\underset{4}{54}$ $\underset{5}{162}$ $\underset{6}{486}$

b.) $t_1 = a = 2$

$t_2 = a + r$

$t_3 = a + 2r$

$\boxed{t_n = a + (n-1)r}$

Six terms: $\underset{1}{2}$ $\underset{2}{5}$ $\underset{3}{8}$ $\underset{4}{11}$ $\underset{5}{14}$ $\underset{6}{17}$

c.) $a = 27$

$r = 1/3$

Six terms: $\underset{1}{27}$ $\underset{2}{9}$ $\underset{3}{3}$ $\underset{4}{1}$ $\underset{5}{1/3}$ $\underset{6}{1/9}$

d.) $a = 3$

$d = -1/2$

Six terms: $\underset{1}{3}$ $\underset{2}{5/2}$ $\underset{3}{2}$ $\underset{4}{3/2}$ $\underset{5}{1}$ $\underset{6}{1/2}$

7.2.1

a.) 1, 2, 3, 5, 8, 13
1, 2, 3, 4, 5, 6

b.) 1, 5, 13, 41, 121, 365
1, 2, 3, 4, 5, 6

c.) 2, 1, 5, 21, 110, 681
1, 2, 3, 4, 5, 6

d.) 4, 5, 20, 100, 2000, 200000
1, 2, 3, 4, 5, 6

e.) 1, 3, -4, -25, 3, 178
1, 2, 3, 4, 5, 6

f.) 1, 1, 2, 5, 27, 734
1, 2, 3, 4, 5, 6

g.) 0, 2, 10, 46, 210, 958
1, 2, 3, 4, 5, 6

7.3.1

a.) $\sum^{4} k^2 \quad k=-1$

$= (-1)^2 + (0)^2 + (1)^2 + (2)^2 + (3)^2 + (4)^2$

$= 1 + 0 + 1 + 4 + 9 + 16$

$= \underline{31}$

b.) $\sum^{4} k=0 \quad 2^k$

$= 2^0 + 2^1 + 2^2 + 2^3 + 2^4$

$= 1 + 2 + 4 + 8 + 16$

$= \underline{31}$

c.) $\sum^{2} k=-3 \quad k^3$

$= (-3)^3 + (-2)^3 + (-1)^3 + (0)^3 + 1^3 + 2^3$

$= -27 - 8 - 1 + 0 + 1 + 8$

$= \underline{-27}$

d.) $\sum^{3} k=0 \quad 3^k$

$= 3^0 + 3^1 + 3^2 + 3^3$

$= 1 + 3 + 9 + 27$

$= \underline{40}$

e.) $\sum^{200} k=0 \ (2+3k)$

$= \underline{60702}$

f.) $\sum^{200} k=0 \quad 2(1.01)^k$

$= \underline{1277.84}$

g.) $\sum^{100} k=0 \ (3+5k)$

$= \underline{25,553}$

h.) $\sum^{100} k=0 \quad 3(1.1)^k$

$= \underline{454730,208}$

#7.3.2

a.) $(-2)^5 + (-1)^5 + \dots + 7^5$

Summation Notation

$\sum^{7} i=2 \ (i)^5$

b.) $(-2)+(-1)+0+1+2+3+4+5$

Summation Notation

$\sum^{5} i=-2 \ i$

c.) $2^2+2^3+2^4+2^5+2^6+2^7+2^8$

Summation Notation

$\sum^{8} i=2 \ (2)^i$

d. $0^3+1^3+2^3+3^3+4^3+\dots+17^3$

Summation Notation

$\sum^{17} i=0 \ (i)^3$

e. Sum of cubes = 15 positive integers.

Summation Notation

$\sum^{15} i=1 \ (i)^3$ ($\because$ positive integers start from 1)

f. sum of squares = odd integers between 0 and 100

Summation Notation

$\sum^{50} i=1 \ (2i-1)^2$

#7.4.1

a.) $p(3)$ = true

$1+4+9 = 3 \cdot 4 \cdot 7/6$

$14=14$ = $p(3)$ = true

b.) $p(k) = 1^2+2^2+3^2+\dots+(k-1)^2+k^2 = (k(k+1)(2 \cdot k+1))/6$

c.) $p(k+1)$

$1^2+2^2+3^2+\ldots+(k-1)^2+k^2+(k+1)^2=((k+1)(k+1+1)\rightarrow$
$(2(k+1)+1))/6$

d. $n=1$ / $1=1 \rightarrow$ true

e.) True for $n=1 \rightarrow$ true for $n=k$
true for $n=k \rightarrow$ prove for $n=k+1$
Show true for $n=k+1$ its a inductive step

f.) $n=k \rightarrow$ induction hypothesis = true for $n=k+1$

g.) true $= n=1 \rightarrow$ true for $n=k$
Sum of first k terms $= S_k$
$S_k = k(k+1)(2k+1)/6$
Show statement is true for $n=k+1$
$= 1+4+9+\ldots+n^2 = n(n+1)(2n+1)/6$ for a positive Integers

#7.4.3

a.) $3^n > 2^n+n^2$
true for $n=2$
$\boxed{3^{n+n} > 2^{n+1}+(n+1)2}$
$= \boxed{3^n > 2^n+n^2 \quad \forall n \geq 2}$

b.) $n! \geq 2^n$ for $n=4 \rightarrow 4! = 4\times3\times2\times1 = 24$
$2^4=16 \rightarrow 4! > 2^4$
true for $n=4$
$\boxed{n! \geq 2^n \text{ true for } n \geq 4} \rightarrow$ method of induction

c.) $\sum_{j=1}^{n} 1/j^2 \leq 2 - 1/n$

for $n=1$ L.H.S

$\boxed{\sum_{j=1}^{k+1} 1/j^2 \leq 2 + 1/(k+1)^2}$

$\rightarrow$ true for $n = k+1$ / $n \geq 4$

d.) $3^n \geq n^3$ for $n=3$

$3 = 3 \rightarrow$ true for $n=3$

$3^{k+1} \geq (k+1)^3 \rightarrow$ true for $n = k+1$

$\boxed{3^n \geq n^3}$

7.4.2

a.) $\sum_{j=1}^{1} j^3 = 1^3 = \boxed{1}$

$= \left(\frac{k(k+1)}{2}\right)^2$

$= \boxed{\left(\frac{(k+1)((k+1)+1)}{2}\right)^2}$

b.) $\sum_{j=1}^{n} j \cdot 2^j = (n-1)2^{n+1} + 2$

$= (k+1-1)2^{k+1+1} + 2$

R.H.S for $n = k+1$

#7.5.1

a.) $n=1$ $3^2-1=9-1=8$

for $n=1$ $3^{2n}-1$ is divisible by 4.

$n=2$ $3^4-1=81-1=80$

4 divides $3^{2n}-1$

b. $n=1$ $7^r-1=6$

6 divides 7^n-1, for $n=1$

$n=2$ $7^2-1=49-1=48$

6 divides $7^n-1 \rightarrow 7^{k+1}-1$

for any positive integer 6 divide (evenly) 7^n-1

c. $n=1$ $11^n-7^n=11-7=4$

$4/11-7 \rightarrow 4/4$

$4/11^k-7^k \rightarrow 4/11^{k+1}-7^{k+1}$

for any positive integer n, 4 evenly divides 11^n-7^n

d.) $n=1$ $9^1-2^1=7$ true

$9^k-2^k=7m$

$=7(9^k+2m)$ 7 evenly divides 9^n-2^n for $n>0$

e.) $n=1$ $1^2-5.1+2$

$=1-5+2$

$=-2$ true equality hold for $n=1$

$k^2-k+2=2m$

$=2(m+k-2)$ true for $n=k+1$

2 evenly divides n^2-5n+2, $n>0$

f.) $n=1$ $n^3-4n+r=(1)^3-4(1)+r=3$

$=3/3$ true for $n=1$

$3/k^3-4k+6 \rightarrow 3/(k+1)^3-4(k+1)+6$

true for all positive integers

3 evenly divides n^3-4n+6.

#7.5.3

a.) $C_0 - 5,\ C_k = (C_{k-1})^2$ for $k \geq 1$

$C_0 = 5$ and $5^{2^0} = 5^1 = 5$

$C_0 = 5^{2^0}$

$k > 0 \rightarrow C_k = (c_{k-1})^2 = 5^{2^k} = 5^{2^k}$

Result hold for $n = k+1 \rightarrow C_{k+1} = 5^{2^{k+1}}$

$C_n = 5^{2^n}$, for all $n \geq 0$

b. $b_0 = 1,\ b_k = 2\,b_{k-1} + 1,$ for $k \geq 1$