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Comp 3240

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Hw 13

13.1.1

a) ancestors of vertex n are m, h, l, p

b) descendants of vertex i are f, g, c, d, a, b, c

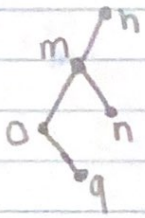
c) leaves in tree are j, q, n, k, e, a, b, c, g

d) level of vertex d is 4

e) height of tree is 6

f) level 4 vertices are m, e, d

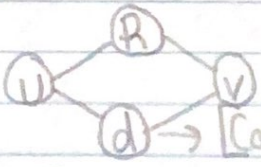
g)



h) siblings of vertex i are l, k

13.1.3

a)



Depth first Traversal is R → U → d → V

or

R → V → d → U

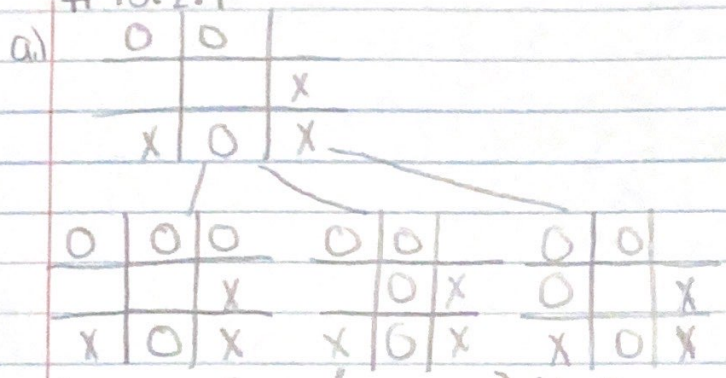
#13.1.3

In the first case, you can see that v is a descendant of vertex u, since discovery time of u < d. time of v.

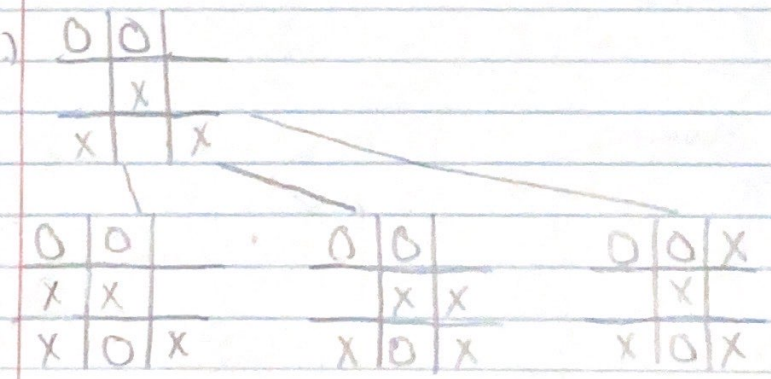
In the second case, you can see that u is a descendant of vertex v, since discovery time of v < d. time of u.

#13.2.1

a)



b)



#13.2.2

a.) d = 1110
a = 0
y = 1111
= day is 111001111

b.) c = 1100
a = 0
n = 1101
d = 1110
y = 1111
= candy is 110001101110 1111

c.) 1110,10,1101

1110 is d
10 is e
1101 is n
the word is den

d. 1110,0,1101,1100,10

1110 is d
0 is a
1101 is n
1100 is c
10 is e
the word is dance

13.2.3

a.) # of bits used per character

5 a → 2 (00)
5 b → 2 (01)
10 c → 3 (110)
15 d → 3 (101)
25 e → 3 (110)
40 f → 3 (111)

$$\frac{[5(2)+5(2)+10(3)+15(3)+25(3)+40(3)]}{100} = \frac{290}{100} = 2.9 \text{ bits/char}$$

b.) # of bits used as per Huffman code

5 a → (5) (11100)
5 b → (5) (11101)
10 c → (4) (1111)
15 d → (3) (110)
25 e → (2) (10)
40 f → (1) (0)

$$\frac{[5(5)+5(5)+10(4)+15(3)+25(2)+40(1)]}{100} = \frac{225}{100} = 2.25 \text{ bits/char}$$

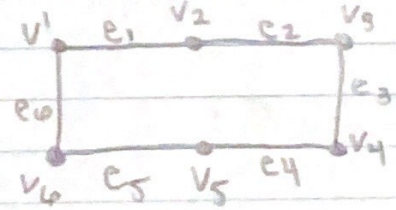
13.3.1

a.) G is a tree 7 vertices, number of edges in G is 6

Total degree $= \sum_{v \in V} \deg(v)$

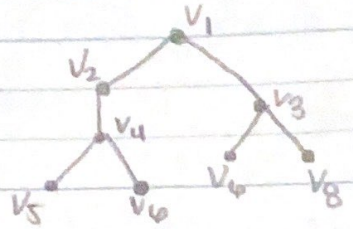
$= 2 \times 6 = 12 \neq 14$ There is no graph

b.)



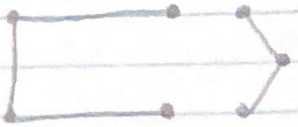
yes, There is a graph with 6 vertices and 6 edges

c.



Yes there is a tree.

d.



In fact it is a disconnected graph and has two connected components

e. $|E(G)| = n-1 \rightarrow \sum_{i=1}^{n} \deg(v_i) = 2|E|$

$= 2n = 2(n-1)$ which is not true

Hence there is no tree

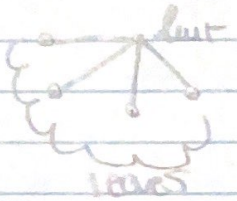
13.3.2

a) For $n=1$ T is •

only 1 out no leave

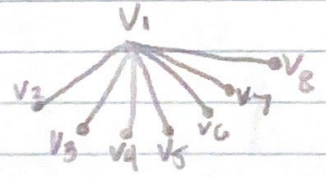
For $n=2$ T is |

one out and one leave



Similarly for any n we can draw a T with and root and $n-1$ leaves

b.)



Have $n=8$

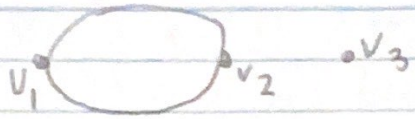
V_1 is root and $\{v_2, v_3, \ldots v_8\}$ is set of leaves it had.

#13.3.3

a) No, the converse is not true.

Counter-example -

V_8) and 2 edges as below-



It is not tree since vertex V_3 is not connected to any other vertex

#13.3.5

a) $n(n-1)/2 - k(n-k)$

$\downarrow$

$n^2 - n - 2kn + 2k^2 \geq n^2 - 3n + 2$

$\downarrow$

$n - k(n-k) - 1 \geq 0$

$\downarrow$

$-(n-2)^2/4 = n - n^2/4 - 1 \geq 0$

which is not possible

#13.4.1

IDK