Jalen Powell 10/3/2021
Comp 3240

Hw 6

6.1.1

a.) No. (a, b) is not in G^2; it takes either 1 or >2 length to reach b from a

b. Yes. (b, e) is in G^3; because there is a path of length 3 ($b \to c \to f \to e$) to reach e from b.

c. No. (g, g) is not in G^3; it takes either 2 or >3 length to reach g from g.

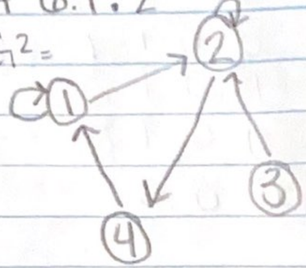
d. Yes. (g, g) is in G^4; there is a path of length 4 to reach g from g.

e. Yes. (b, b) is in G^3, there is a path of length 3 to reach b from b.

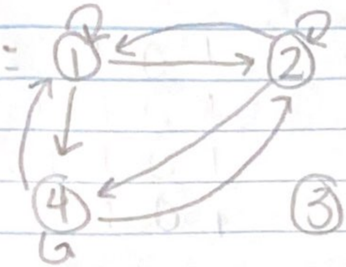
f. No, (b, d) is not in G^5, there isn't a path of length 5 to reach d from b; it is either 2, 3, 4, but not 5

6.1.2

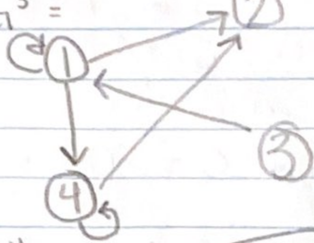
a. $G^2 =$



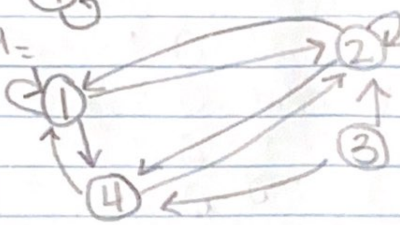
$G^+ =$



$G^3 =$



$G^4 =$



6.1.3

a)

1	1	0	0
1	1	0	0
1	1	1	1
1	1	0	1

b)

1	1	1	1	1
0	1	1	1	1
0	0	1	0	0
0	0	1	1	0
0	0	1	1	1

c)

1	1	0	0	1	0
0	1	0	0	1	0
0	0	1	1	0	1
0	0	1	1	0	1
0	0	0	0	1	0
0	0	1	1	0	1

#6.2.1

a) $\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 1 & 0 & 0 \end{array}$

$g^2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$

$g^3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 & 1 \end{array}$

$g^4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \end{array}$

$g^+ = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \end{array}$

#6.2.2

a.) $A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

b.) $A^2 = A \cdot A$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

c) The 4 vertices are (V_2, V_4, V_5, V_6) can be reached from vertex V_4 by length 2

d) The vertices V_1, V_4, and V_5 can reach vertex V_2 by walk of length 2

6.2.3

a) The vertices that reach 2 with walk of 3 are V_2, V_4, V_5.

b) The out-degree of 4 in closure is 5

c) There is no walk of 4 from $4 \to 5$

d) Entry is 1, so (2,2) is there in closure

e) No, the entry is 0 at $A^3(5,3)$

f) Yes, there are entries '1' for $A^3(1,1)$, $A^3(2,2)$, $A^3(3,3)$. There are 3 circuits with length 3.

6.3.1

a) The minimal elements of the partial order are {J, I, A, F}

b.) The maximal elements of the partial order are {J, H, D, G}

c.) The comparable pairs are (A,D), (G,F), (D,B), (H, I) only.

6.4.1

a) Not necessarily an equivalence relation. The Statement do not state that the relation P is transitive.

b.) Equivalence relation. Reflexive, the person has the same mother as herself or himself as one individual. Symmetric and transitive.

c.) Not an equivalence relation. The relation S is not reflexive because a person cannot be married to himself / herself

d.) Equivalence relation. Reflexive, Symmetric, and transitive

e.)

#6.4.2

a.) xDy is Reflexive, Symmetric, and transitive.
Partition of $[2] = \{2, 34\}$ = equivalence class of $[34]$
Equivalence class of $[7] = \{7, 99, 31\}$ = E. class of $[31][99]$
E. class of $[13] = \{13, 17\}$ = E. class of $[17]$
E. class of $[44] = \{44, 56, 4\}$ = E. class of $[56], [4]$
Partitions of $D = \{2,34\}, \{7, 99, 31\}, \{13, 17\}, \{44, 56, 4\}$

#6.4.3

a.) Let $(x, z) \in P^+ \setminus P$
there exist an element $y \in A$ such that
$(x, y) \in P$ and $(y, z) \in P$
Since p is symmetric relation
$(y, x) \in P$ and $(z, y) \in P$
$= (z, x) \in P^+$ $\because P^+$ is transitive relation
P^+ is symmetric relation

b.) P is reflexive and symmetric relation on set A
P^+ is reflexive relation
P^+ is equivalence relation

c.) Yes, $R = \{(1,2), (2,1), (2,3), (3,2)\}$ an
$A = \{1, 2, 3, 4\}$
Them $R^+ = \{(1,1), (2,2), (3,3), (1,2), (2,1), (2,3),$
$(3,2), (1,3), (3,1)\}$
not equivalence
R^+ is not reflexive

#6.4.2

a) xDy is Reflexive, Symmetric, and transitive.

Partition of $[2] = \{2, 34\}$ = equivalence class of $[34]$

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E. class of $[13] = \{13, 17\}$ = E. class of $[17]$

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Partitions of $D = \{2, 34\}, \{7, 99, 31\}, \{13, 17\}, \{44, 56, 4\}$

#6.4.5

a.) $x - x = 0$. $x - x = 3 \cdot 0$ {relation R is reflexive}

$x - y = 3m$

$y - x = -3m$

$y - x = 3(-m)$ {relation R is symmetric}

$x - y = 3m$ and $y - z = 3n$

$x - y + y - z = 3m + 3n$

$x - z = 3(m+n)$ {R is transitive}

R is equivalence relation

b.) $x + x = 2x$ $x + x = 2x$ {relation R is reflexive}

$x + y = 3m$

$y + x = 3m$ {relation R is symmetric}

$x + y = 3m$ and $y + z = 3n$

$x + y - (y + z) = 3m - 3n$

$x + y - y - z = 3(m - n)$

$x - z = 3(m - n)$

$x - z = 3(m - n)$ is not the form of $x + y = 3m$

{R is not transitive}

R is not equivalence relation