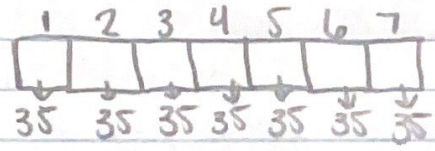
Jalen Powell 10/17/2021
Comp 3240

Hw 8

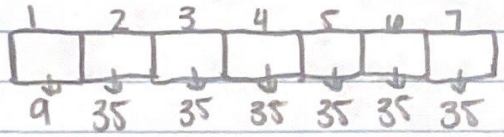
8.1.1

a.) No constraints



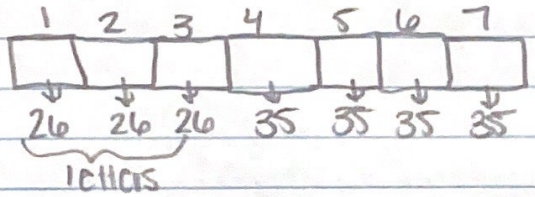
Each character can have 35 value
$\therefore 35^7$

b.) Liscence plate starts with a digit



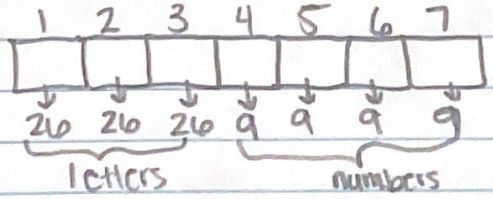
First plate can have 9 values while the rest have 35. $= 9 * 35^6$

c.) First three are letters



First three have 26 values and the rest have $35 = 26^3 * 35^4$

d.) First three are letters / last 4 are #s



$26^3 * 9^4$

8.1.2

a) String of length 6 is formed with each place filling 40 ways. $(40)^6$

b) String of length 7 is $(40)^7$
String of length 8 is $(40)^8$
String of length 9 is $(40)^9$
digits or letters = $(40)^7 + (40)^8 + (40)^9$

c) Total strings of length 7, 8, or 9 where first characters can not be letters = $14(40^6 + 40^7 + 40^8)$

8.2.2

a) $f(y_1) = f(y_2)$ $\quad y_1 = x_1 x_1^R$ and
$x_1^R = x_2^R$ $\quad y_2 = x_2 x_2^R$
$x_1 x_1^R = x_2 x_2^R$
$y_1 = y_2$ = F is one to one

$(x x^R)^R = (x^R)^R x^R = x x^R$
$f(x x^R) = x$
F is onto
f is a bijection between P_6 and B^3

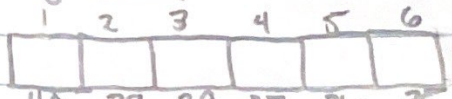
b) B^3 $\quad |P_6| = |B^3| = |B|^3 = 2^3 = 8$
$|P_6| = 8$

c) $x_1 x_2 x_3 x_4 = x_1' x_2' x_3' x_4'$
$x_1 = x_1'$ / $x_2 = x_2'$, $x_3 = x_3'$ and $x_4 = x_4'$
$y_1 = y_2$ g is one to one

$x_1 y_2 x_3 x_4 y_3 x_2 y_1 \in P_7$ $\quad g(y) = x_1 x_2 x_3 y_4 = x$
g is onto
$|P_7| = |B^4| = |B|^4 = 2^4 = 16$
$|P_7| = 16$

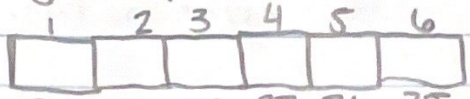
8.3.1

a) Length of password is 6.



The number of passwords is 2763633600

b) Length of password is 6. First position = not special



The number of passwords is 2487270240

8.3.2

a) First letter: 3 ways $= 3+2+2+2+2+2+2+2+2+2$
Second letter: 2 ways $= 3 + 2^9$
3rd, 4th, ... 10th: 2 ways $= 1536$

8.4.1

a) $f: \{0,1\}^7 \to \{0,1\}^7$
$2^7 = 128$

b.) First element = f(0000000), $2^7 - 1$
Second element = f(0000001)
$2^7 (2^7-1)(2^7-2)\ldots$
$(2^7)! = (128)$

c.)

d) $f\{0,1\}^5 \to \{0,1\}^7$
Domain = $\{0,1\}^5$ = 2^5 elements
Domain = $\{0,1\}^7$ = 2^7 elements
$(2^5 < 2^7)$
$\{0,1\}^5 \to \{0,1\}^7 = (2^5)!$
$= (32)!$

#8.5.2

a.) $S = (a,b,c,d,e,F,g)$, ${}^6C_4$

$S = 15$

$S' = \{a,b,c,d\}$

b.) 15 subsets of S has 4 elements

$S^1 = \{a,b,c,d\}$

$S^2 = \{a,b,c,e\}$

$S^3 = \{a,b,c,f\}$

c.) ${}^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$

4 elements = 15

d) subsets of S having 3 elements are ${}^6C_3$

${}^6C_3 + {}^6C_4 = 20 + 15 = 35$

#8.6.1

a) 4 - students out of 37

${}^{37}C_4 = 66,045$

b.) A, B, C, D = selected 4 students

A → pick up homework

B → hand out PS

C → staple worksheets

D → organize the CL

4-students out of 37

$${}^{37}P_4 = \frac{37!}{(37-4)!} = 1,585,080$$

#8.7.2

a.) Standard deck of cards has 13 clubs and 39 other suits.

$= {}^{39}C_5$

5 hand card $= {}^{52}C_5$

5 hand card that has atleast one club

${}^{52}C_5 - {}^{39}C_5$

$= 2598960 - 575757 - 2023203$

b.) 5 card hands with all ranks different

$= {}^{13}C_5 \cdot 4^5$ (4 choices)

$= {}^{52}C_5 - {}^{13}C_5 \cdot 4^5$

$= 1281072$

#8.7.3

a.) 8-bit $= 2^8 = 256$

Only two possible, they are 10101010 (c6)

01010101

$2^8 - 2 = \underline{254}$

b.) 8-bit $= 2^5$ (000)

$= 2^8 - 2^5 = 224$