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Comp 3240

10/31/2021

Hw 10

10.1.1

a.) $A = \{HHHH, HHTH, HTHH, HTTH\}$

$N(A) = 4$

$$P(A) = \frac{N(A)}{N(S)} = \frac{4}{16} = \boxed{.25} \text{ probability}$$

b.) $N(B) = 8$

$$P(B) = \frac{N(B)}{N(S)} = \frac{8}{16} = \boxed{.5} \text{ probability}$$

c.) $N(C) = 2$

$$P(C) = \frac{N(C)}{N(S)} = \frac{2}{16} = \boxed{.125} \text{ probability}$$

10.1.2

a.) $$\text{probability} = \frac{N(E)}{N(S)} = \frac{(n-1)!}{n!} = \frac{(n-1)!}{n(n-1)!} = \boxed{\frac{1}{n}}$$

b.) $$\text{probability} = \frac{N(E)}{N(S)} = \frac{(n-2)!}{n!} = \frac{(n-2)!}{n(n-1)(n-2)!} = \boxed{\frac{1}{n(n-1)}}$$

c.) $$\text{probability} = \frac{N(E)}{N(S)} = \frac{2(n-1)!}{n!} = \frac{2(n-1)!}{n(n-1)!} = \boxed{\frac{2}{n}}$$

#10.1.4

a.) $N(S) = {}^{10}C_5 = 252$

The size of the Sample space is $\boxed{252}$

b.) $N(A) = 2 \times {}^{8}C_3$
$= 2 \cdot 56$
$= 112$

$P(A) = \frac{n(A)}{N(S)} = \frac{112}{252} = \boxed{\frac{4}{9}}$

c.) Required Probability $= \frac{2}{N(S)} = \frac{2}{252} = \boxed{\frac{1}{126}}$

#10.2.1

a.) $\left(\frac{1}{2}\right)^n + n\left(\frac{1}{2}\right)^n = (n+1)\left(\frac{1}{2}\right)^n$

(probability of n-1 flips)

b.) $2\left(\frac{1}{2}\right)^n \rightarrow 1 - 2\left(\frac{1}{2}\right)^n = 1 - \frac{1}{2^{n-1}}$

(probability of atleast two consecutive flips)

c. $n = 2s$ Heads = P

Tails = q

$P(P \neq q) \rightarrow 1 - p(P = q)$

$= 1 - \begin{bmatrix} n \\ n/2 \end{bmatrix}\left(\frac{1}{2}\right)^n$

$= 1 - \begin{bmatrix} 2s \\ s \end{bmatrix}\left(\frac{1}{2}\right)^{2s}$

$= 1 - \frac{(2s)!}{s!\,s!}\left[\frac{1}{2}\right]^n$

$= \boxed{1 - \frac{n!}{(n/2)!\,(n/2)!} \cdot \left(\frac{1}{2}\right)^n}$

10.2.2

a.) probability $= \frac{(n-1)!}{n!} = \frac{(n-1)!}{n(n-1)!}$ $\boxed{p = \frac{1}{n}}$

$n! = n(n-1)(n-2)\ldots\ldots$

b.) probability $= \frac{(n-2)!}{n!} = \frac{\cancel{(n-2)!}}{n(n-1)\cancel{(n-2)!}}$

$\boxed{p = \frac{1}{n(n-1)}}$

c.) $p = \frac{2(n-1)!}{n!} = \frac{2 \cdot (n-1)!}{n(n-1)!} = \frac{2}{n}$

$\boxed{p = \frac{2}{n}}$

d.

e.

10.2.5

a.) $(26+26+10)^{10} = (62)^{10}$

characters $= 26 \cdot 26 \cdot 10 \cdot (62)^7$

$= \frac{26 \cdot 26 \cdot 10 \cdot (62)^7}{(62)^{10}} = 0.02836$

10.3.1

a.) $P(A) = 1/2$, $P(B) = 1/6$, $P(C) = 1/6$

b.) $P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{3}{6} = \frac{1}{2}$

c. $P(B/C) = \frac{P(B \cap C)}{P(C)} = \frac{2/36}{6/36} = \frac{1}{3}$

d.) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{3/36}{6/36} = \frac{1}{2}$

e.) $A, B = \boxed{4, 6} \rightarrow B, C = \boxed{5, 5} \rightarrow A, C = \boxed{5, 1}$

#10.3.3

a.) $7! \cdot 2! =$ total arrangements

probability $= \frac{7! \cdot 2!}{8!} = .25$

b.) $1 \cdot 7! = 7! =$ total arrangements

prob $= \frac{7!}{8!} = \frac{1}{8}$

c.) Yes both are Independent since no relation b/w them

#10.4.1

a.) $P(F) = P(S) = 0.5$

$P(H/F) = \binom{10}{7} (0.5)^7 (1-0.5)^{10-7} = 0.1172$

$P(H|S) = \binom{10}{7} (.75)^7 (1-0.75)^{10-7} = 0.2503$

$P(H) = P(H|S)P(S) + P(H|F)P(F) = 0.2503 \cdot 0.5 \cdot 0.1172 \cdot 0.5$
$= 0.18375$

$P(S|H) = \frac{P(H|S)P(S)}{P(H)} = \frac{0.2503 \cdot 0.5}{0.18375}$
$= \boxed{0.6811}$

#10.4.4

a.) $P(D)$ = probability of having HIV

$P(T)$ = probability of +ve test

$P(D|T) = \frac{P(T|D) \cdot P(D)}{P(T|D) \cdot P(D) + P(T|ND)P(ND)}$

$P(ND)$ = probability of not having HIV

$P(D) = 0.0001$

$P(ND) = 0.9999$

$P(T|D) = 1$

$P(T|ND) = 0.025$

$P(D|T) = \frac{1(0.0001)}{1 \cdot (0.0001) + (0.025) \cdot (0.9999)} = 0.0039$