Jalen Powell 10/24/2021
Comp 3240

Hw 9

9.1.1

a.) Number
- number of ways to permute the letter. $6! = 720$

b.) Discrete
- number of ways to permute the letter. $8!/2!$
('E' repeated twice)

c.) Subsets
- $7!/3! = 840$

9.1.3

a.) $\binom{52}{13} \times \binom{52-13}{13} \times \binom{52-13-13}{13} \times \binom{52-13-13-13}{13}$

1st player 2nd player 3rd player

$= \binom{52}{13} \times \binom{39}{13} \times \binom{26}{13} \times \binom{13}{13}$

$= \binom{52}{13} \times \binom{39}{13} \times \binom{26}{13} \times 1$

$= \binom{52}{13}\binom{39}{13}\binom{26}{13}$

b.) $\binom{52}{7}\binom{52-7}{7}\binom{52-7-7}{7}\binom{52-7-7-7}{7} = \binom{52}{7}\binom{45}{7}\binom{38}{7}\binom{31}{7}$

#9.2.2

a.) $\binom{15+6-1}{15}$ ways $= \binom{20}{15}$ ways $=$ 15504 ways

b.) $\binom{15-k+5-1}{15-k} = \binom{19-k}{15-k} = \binom{19-k}{4}$ ways

$$= \sum_{k=3}^{15} \binom{19-k}{4}$$

c.)

k	$\binom{19-k}{4}$
0	3876
1	3060
2	2380
Total	9316

$= 9316$ ways

#9.2.4

a.) $\binom{25+4-1}{4-1} = \binom{28}{3} = \frac{28!}{3!(28-3)!} = \frac{28 \times 27 \times 26 \times 25!}{3 \times 2 \times 25!}$

$= 14 \times 9 \times 26$

$= 3276$

b.) $\binom{25+4-1-(5\times1)}{4-1} = \binom{29-6}{3} = \binom{23}{3} = \frac{23!}{3!(23-3)!}$

$= \frac{23 \times 22 \times 21 \times 20!}{3 \times 2 \times 20!} = 23 \times 11 \times 7 = 1771$

c.) $\binom{25+4-1}{4-1} - \binom{25+3-1-10}{4-1} = \binom{28}{3} - \binom{17}{3} = \frac{28!}{3!(28-3)!} - \frac{17!}{3!(17-3)!}$

$= \frac{28 \times 27 \times 26 \times 25!}{3 \times 2 \times 25!} - \frac{17 \times 16 \times 15 \times 14!}{3 \times 2 \times 14!} = 3276 - 680$

$= 2596$

#9.3.1

a.) 3 TA is: $\binom{60+3-1}{3-1} = \binom{62}{2} = 1891$

b.) number of ways = 3^{60}

c.) $\frac{60!}{25!\,20!\,15!}$

#9.3.5

a.) number of ways = $^{25}C_{10}$

$= \frac{25!}{15!\,10!}$

$= 3268760$

b.) $25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19 \times 18 \times 17 \times 16$

first lunch box has 25 possibilities

2nd lunch box has 24 possibilities

#9.5.1

a.) $2 \times 3^7 \times 2 = 8748$ → $19683 - 8748 = 10935$

b.) The number of strings of length 8 consecutive a's is 5.

c.) Those strings of length 8 consecutive characters which all are same is 15.

d.) Case 1: $3^7 = 6561$

Case 2: $2 \times 3^7 \times 1 = 4374$

Case 3: $6561 + 4374 + 4374 = 15309$

#9.5.1

e.) $4+4+4=12$

Consecutive a's = $\underline{5}$

$12+5=\underline{17}$

f.) Total # = $(3\times3)+(3\times3)+(3\times3)$

Total # = 27

g.) $2^7 = \frac{9C_2}{21} = 18 \rightarrow 18\times2^7$

$2^6 = \frac{9C_3}{3!} = 14 \rightarrow 14\times2^6$

$18\times2^7 + 14\times2^6 = 2304+1792$

$= 4096$

h.) Total # = $(18\times2^7)+(18\times2^7)+(18\times2^7)$

$= 3\times2304$

$= 6912$

#9.5.2

a.) 0000000000 = $2^{10}-1$

b.) 1111111111 / 0000000000 = $2^{10}-2$

c.) $\binom{9}{5} = |A\cap B|$

$= \binom{10}{5} + (2^9) - \binom{9}{5}$

$= 252 + 512 - 126$

$= 764 - 126$

$= \underline{(638)}$

#9.6.1

a.) $\binom{7}{3}(-3)^3(4)^4 = -241920$

b.) $\binom{9}{2}(5)^2(-1)^7 = -900$

c.) $\binom{8}{5}(3)^5(-4)^3 = -870912$

d.) $\binom{7}{6}(-2)^6(-5)^1 = -2240$

#9.6.2

$(x+y)^n = \sum_{k=0}^{n} x^k y^{n-k}$

a.) $x=3, y=-1$

$(3+(-1))^n = 2^n$

b.) $x=2, y=1$

$(2+1)^n = 3^n$