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Elec 2200

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Hw 2

# 2.1.2:

2. a.)  $\exists x : (x + x = 1)$

$$\begin{aligned} x + x &= 1 \\ &= 2, \quad x = \frac{1}{2} \neq 2 \end{aligned}$$

false statement

b.)  $\exists x : (x + 2 = 1)$

$$\begin{aligned} x + 2 &= 1 \\ x &= -1 \end{aligned}$$

true statement

c.)  $\forall x (x^2 - x \neq 1)$

$$\begin{aligned} x &= 0 \\ &= 0^2 - 0 \neq 1 \end{aligned}$$

true statement

d.)  $\forall x (x^2 - x \neq 0)$

$$\begin{aligned} x &= 1 \\ &= 1^2 - 1 = 0 \end{aligned}$$

false statement

e.)  $\forall x (x^2 > 0)$

$$x = 0 \rightarrow x^2 = 0^2 = 0 \not> 0$$

false statement

f.)  $\exists x : (x^2 > 0)$

$$\begin{aligned} x &= -1 \\ (-1)^2 &= 1 > 0 \end{aligned}$$

true statement

# 2.1.4

4. a.)  $\forall x P(x) = \text{true}$ , all inputs are True
- b.)  $\exists x P(x)$  True because there are some inputs for which  $P$  is true
- c.)  $\forall x Q(x) = \text{False}$ , majority of inputs are false
- d.)  $\exists x Q(x) = \text{True}$ , only one input is True
- e.)  $\forall x R(x) = \text{False}$ ,  $R$  is never True
- f.)  $\exists x R(x) = \text{False}$ ,  $R$  is never True

# 2.2.2

a.)  $\exists x : E(x)$

b.)  $\forall x : E(x) \wedge T(x)$

c.)  $\exists x : T(x) \wedge E(x)$

d.)  $\exists x : E(x) \wedge \neg T(x)$

#2.2.4

a.)  $\exists x S(x)$

b.)  $\forall x (\neg S(x) \wedge W(x))$

c.)  $\forall x (S(y) \rightarrow \neg W(x))$

d.)  $\exists x (S(x) \wedge W(x))$

e.)  $\forall x (\neg W(x) \rightarrow S(x))$

f.)  $\forall x (\neg W(x) \rightarrow (S(x) \vee V(x)))$

g.)  $\exists x (\neg W(x) \wedge \neg S(x) \wedge \neg V(x))$

h.)  $\forall x (\neg W(x) \rightarrow (S(x) \vee V(x)))$

#2.3.1

$$\text{a.) } \neg \exists x P(x) \\ = \forall x \neg P(x)$$

$$\text{b.) } \neg \exists x [P(x) \vee Q(x)] \\ = \forall x \neg [P(x) \vee Q(x)] \\ = \forall x [\neg P(x) \wedge \neg Q(x)]$$

$$\text{c.) } \neg \forall x [P(x) \wedge Q(x)] \\ = \exists x \neg [P(x) \wedge Q(x)] \\ = \exists x [\neg P(x) \wedge \neg Q(x)]$$

$$\text{d.) } \forall x [P(x) \wedge [Q(x) \vee R(x)]] \\ = \exists x \neg [P(x) \wedge [Q(x) \vee R(x)]] \\ = \exists x [\neg P(x) \wedge \neg [Q(x) \vee R(x)]] \\ = \exists x [\neg P(x) \wedge [\neg Q(x) \wedge \neg R(x)]]$$

# 2.3.2

a.)  $\exists x (D(x))$

Negation  $\neg \exists x (D(x))$

DeMorgan law  $\forall x (\neg D(x))$

English: No patient given medicine

b.)  $\forall x (D(x) \vee P(x))$

Negation:  $\neg (\forall x (D(x) \vee P(x)))$

$$= \exists x \neg (D(x) \vee P(x))$$

DeMorgan  $= \exists x (\neg D(x) \wedge \neg P(x))$

English: There is a patient who was not given any medicine or placebo

c.)  $\exists x (D(x) \wedge M(x))$

Negation  $\neg \exists x (D(x) \wedge M(x))$

$$\forall x \neg (D(x) \wedge M(x))$$

DeMorgan  $\forall x (\neg D(x) \vee \neg M(x))$

English: Everyone either not given medicine or not given migraines or both

d.)  $\forall x (P(x) \Rightarrow M(x))$

Negation  $\exists x \neg (\neg P(x)) \wedge \neg M(x)$

De Morgan  $\exists x P(x) \wedge \neg M(x)$

English: There is a person who took Placebo and not having migraines

e.)  $\exists x (M(x) \wedge P(x))$

Negation  $\neg \exists x (M(x) \wedge P(x))$

DeMorgan  $\forall x (\neg M(x) \vee \neg P(x))$

Every patient had no migraines or not given Placebo

# 2.4.1

a.)  $M(1,1)$  is a proposition

True

b.)  $\forall y N(x,y)$  is not a proposition as the  
value of  $x$  is not defined

Truth value not obtainable

c.)  $\exists x M(x,3)$  is a proposition

Truth value is true

d.)  $\exists x \exists y M(x,y)$  is a proposition

True

e.)  $\exists x \forall y M(x,y)$  is a proposition

False

f.)  $M(x,2)$  is not a proposition

Truth value is not obtainable

g.)  $\exists y \forall x M(x,y)$  is a proposition

True

#2.4.3

- a.) False
- b.) True
- c.) True
- d.) False
- e.) True
- f.) True
- g.) False
- h.) True
- i.) True
- j.) False
- k.) False

# 2.5.1

a.)  $\forall x \forall y M(x, y) = \text{False}$

As for  $x=2$  and  $y=2$  ( $\text{False}$ )  
 $x=3$  and  $y=3$  ( $\text{False}$ )

b.)  $\forall x \forall y ((x \neq y) \rightarrow M(x, y)) = \text{True}$   
 $x \neq y$  ( $\text{True}$ )

c.)  $\exists x \exists y \neg M(x, y) = \text{True}$

exist some  $x$  and some  $y$ ,  $M(x, y)$  is not true

d.)  $\exists x \exists y ((x \neq y) \wedge \neg M(x, y)) = \text{False}$

There doesn't exist any  $x \neq y$ ,  $M(x, y)$  is not true

e.)  $\forall x \exists y \neg M(x, y) = \text{False}$

$x=1$ , there exist no  $y$ , not true

$x=2$ ,  $y=2$  exist  $M(x, y)$  not true

$x=3$  and  $y=3$ ,  $M(x, y)$  is not true

f.)  $\exists x \forall y M(x, y) = \text{True}$

$y=1$ , there exist

$y=2$ , also exist

$y=3$ , also exist