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COMP 4200

Assignment 6

Problem 1

Total: 15 points
Problem 2.30 (textbook, page 157)
Use Pumping Lemma for CFLs to prove the following language is not context-free.

$$A = \{0^n 1 0^{2n} 1 0^{3n} \mid n \geq 0\}$$

Ans:

We can prove by Contradiction that the language above is not context-free. We must first show the language as context-free so our pumping lemma will prove it. Using the terms w = uv^ixy^iz, we can assign 'uvxy' to the leftmost 0's in the language and 'z' to the rest. 0^n cannot be pumped up or down as it will not satisfy with the other grammar in the language. 0^{2n} and 0^{3n} depends on the leftmost 0's.

Problem 2

Total: 15 points
Is the following language B context-free? If yes, show a context-free grammar (CFG) that generates B. If no, please prove it using Pumping Lemma for CFLs.

$$B = \{0^n 0^{2n} 1 0^{3n} \mid n \geq 0\}$$

Note: B is the same as A in Problem 1, except that there is one less 1 in each string of B compared to the strings in A.

Ans:

In the language above, it can be simplified to B = $\{0^{3n}\#0^{3n} \mid n>= 0\}$. This shows that the # of 0's will be equal on both sides of the '1'. Using this, we can construct a context-free grammar:

S -> ASA | 1
A -> 000

This allows recursion after giving the three 0's alongside the 1 in the language. B is context-free.

Problem 3

Total: 15 points
Problem 2.31 (textbook, page 157)
Let C be the language of all palindromes over $\{0,1\}$ containing equal numbers of 0s and 1s. Show that C is not context free.

Ans:

For the language C = {w -> {0,1} | #w(0) = #w(1)}, the number of 0's and 1's must be equal and be a palindrome. Some acceptable strings for this language would be ({}, 1001, 0110, 10011001, 01100110,etc). Looking at the sample string of 1001, we can look at like $1^p\,0^{2p}\,1^p$. In the first case, we can assign $uv^ixy^i = 1^p$ and $z = 0^{2p}1^p$. Pumping up or down will not keep the string in the language as the rest of the string depends on the first set of 1's. This proves that the language is not context-free.

Problem 4

Total: 15 points
Problem 2.31 (textbook, page 157)
Let $\Sigma = \{1,2,3,4\}$ and $D = \{w \in \Sigma^* \mid \text{in } w$, the number of 1s equals the number of 2s, and the number of 3s equal the number of 4s$\}$. Show that D is not context free.

Ans:

While looking at this language, I thought of 2 cases that that will prove through pumping lemma that it is not context-free. Case 1 would look like the following,

D = $(1,2)^n(3,4)^m$ and uv^ixy^i = 1 and z = 2, 3, and 4

This will lead the string not belonging to the language as the number of 1's will not equal the number of 2s. The next case is,

Uv^ixy^i = 1,2,and 3, and x = 4

This will also take the string out of the language as the number of 1s and 2s equal, but the 3s will not equal the 4s.