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COMP 4200

Homework 1

EXERCISES

0.1 Examine the following formal descriptions of sets so that you understand which members they contain. Write a short informal English description of each set.

a. $\{1, 3, 5, 7, \ldots\}$
b. $\{\ldots, -4, -2, 0, 2, 4, \ldots\}$
c. $\{n \mid n = 2m \text{ for some } m \text{ in } \mathcal{N}\}$
d. $\{n \mid n = 2m \text{ for some } m \text{ in } \mathcal{N}, \text{ and } n = 3k \text{ for some } k \text{ in } \mathcal{N}\}$
e. $\{w \mid w \text{ is a string of 0s and 1s and } w \text{ equals the reverse of } w\}$
f. $\{n \mid n \text{ is an integer and } n = n + 1\}$

F.) This set is an empty set. E.g., n=3 => 3 != 3+1

E.) This set holds the string *w* of 0s and 1s while having the reverse values of *w*.

0.6 Let X be the set $\{1, 2, 3, 4, 5\}$ and Y be the set $\{6, 7, 8, 9, 10\}$. The unary function $f: X \longrightarrow Y$ and the binary function $g: X \times Y \longrightarrow Y$ are described in the following tables.

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

g	6	7	8	9	10
1	10	10	10	10	10
2	7	8	9	10	6
3	7	7	8	8	9
4	9	8	7	6	10
5	6	6	6	6	6

a. What is the value of $f(2)$?
b. What are the range and domain of f?
c. What is the value of $g(2, 10)$?
d. What are the range and domain of g?
e. What is the value of $g(4, f(4))$?

D.)

The range of function g is {6,7,8,9,10}

The domain of function g will contain the values of X and Y.

X ~ {1,2,3,4,5}

Y ~ {6,7,8,9,10}

Domain is equal to X * Y =

{(1, 6) (1, 7) (1, 8) (1, 9) (1, 10) (2, 6) (2, 7) (2, 8) (2, 9) (2, 10) (3, 6) (3, 7) (3, 8) (3, 9) (3, 10) (4, 6) (4, 7) (4, 8) (4, 9) (4,10) (5, 6) (5, 7) (5, 8) (5, 9) (5, 10)}

E.) The value of g(4, f(4)) is g(4, 7) which is the number **8**.

Problem 2

Total: 40 points (20 points each)
Prove the following by mathematical induction. For each solution, please specify your (1) base case; (2) induction hypothesis; and (3) inductive step.

1. For all $n \in \mathbb{N}$:
$$5^n + 5 < 5^{n+1}$$

2. For all $n \in \mathbb{N}$:
$$\sum_{i=1}^{n} (-1)^i i^2 = (-1)^n \frac{n(n+1)}{2}$$

1.

Basis Step: $n=1$

$5^{(1)} + 5 < 5^{(1)+1}$

$10 < 25$

✓

Induction Hypothesis.

Assume the formula is true for $n=k$

$k = 5^{(k)} + 5 < 5^{(k)+1}$

Inductive Step:

$n = k+1$

$k+1 = 5^{(k+1)} + 5 < 5^{(\ \ )+1}$ (k+1)

- The formula remains true with *n* being either 1,2, or k+1. The induction hypothesis was correct.

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2. Basis Step
$n=1$

LHS

$$\sum_{i=1}^{n} (-1)^1 (1)^2 = -1$$

RHS

$$\frac{(-1)^1 \cdot 1(1+1)}{2} = -1$$

Induction Hypothesis:

Assume the formula is true for $n=k$

Inductive Step: $n=k+1$

$$\sum_{i=1}^{k+1} (-1)^i i^2 = \sum_{i=1}^{k} (-1)^i i^2 + (-1)^{k+1} (k+1)^2$$

$$= \frac{(-1)^k k(k+1)}{2} + (-1)^{k+1} (k+1)^2$$

$$= \frac{(-1)^k (k+1)}{2} (k - 2(k+1))$$

$$= \frac{(-1)^k (k+1)}{2} (-k-2)$$

$$= \boxed{\frac{(-1)^{k+1} (k+2)}{2}}$$

- The formula remains true with *n* being either 1,2, or k+1. The induction hypothesis was correct.