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ELEC 4700 Assignment 1 Feb/4/2018

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Part 1 : Electron Modelling

This first section covers defining variables. The physical constants that will be used later are defined here.

```
Mo = 9.109 * (10^-31);  
% electron rest mass in kilograms  
Mn = 0.26 * Mo;  
% effective electron mass in kilograms  
L = 200 * (10^-9);  
% length of region in metres  
W = 100 * (10^-9);  
% width of region in metres  
T = 300;  
% Temperature in Kelvin  
Tmn = 0.2 * (10^-12);  
% Mean time of collisions between particles in sec  
dt = 5 * (10^-15);  
% time step in seconds  
TStop = 1000 * dt;  
% stop simulation after 1000 time steps  
B = 1.38 * (10^-23);  
% Boltzmann constant  
old_t = 0;  
% This will be the time before the timestep  
old_temp_test = Temp_test;  
% This will be the old calculated system temp  
  
Temp_test = 300;  
% This tests to see if the temperature remains constant  
% and is set to 300K to check if the system remains at  
% this temperature over time.
```

```
% P is the position matrix of 1000 particles. The first row contains
the
% x-coordinates of the points and the second row has the y-
coordinates. The
% coordinates are scaled from a random generator to be within the
limits of
% the region
p = [zeros(1,1000); zeros(1,1000)];
for i = 1 : 1 : 1000
    p(1,i) = rand * 100 * (10^-9);
    p(2,i) = rand * 100 * (10^-9);
end

% Old P will contain the old position coordinates for the points prior
% to them being updated at every timestep.
old_p = [zeros(1,1000); zeros(1,1000)];
old_p = p;

% Polar coordinates are being used, and theta is the angle that
determines
% the direction of the path of the electron. The values are chosen
from a
% uniformly distributed random generator and are scaled between 0 and
2 pi
% radians.
theta = [zeros(1,1000)];
for a = 1 : 1 : 1000
    theta(a) = rand * (2*pi);
end

% The elements of the bounce vector correspond to each particle; the
value
% of each element is 1 (TRUE) or 0 (FALSE). The value changes when it
is
% detected that the particle needs to bounce.
bounce = [ones(1,1000)];

% The cross and cross1 vectors' elements correspond to each particle
and
% will be used to determine if the right and left boundaries have been
% crossed. The values are each 1 or 0.
cross = [zeros(1,1000)];
cross1 = [zeros(1,1000)];

Undefined function or variable 'Temp_test'.

Error in assignment1 (line 26)
old_temp_test = Temp_test;
```

Thermal Velocity

From the kinetic theory of gases, the kinetic energy is related to temperature and both mass and velocity through the formula below, where m is mass, v is velocity, k is Boltzmann's constant and T is temperature.

$(1/2)m(v^2) = 1/2 * kT$ (per degree of freedom) This simulation modelled electron movement in two dimensions, namely in the x and y directions, and therefore had two degrees of freedom. This changed the formula to: $(1/2)m(v^2) = (1/2*2) kT = kT$ The thermal velocity is calculated by rearranging the above formula: $V_{th} = \sqrt{2mv/m}$ The thermal velocity calculation used the above formula while assuming $T = 300$ K, and an effective electron mass of 2.36834×10^{-31} kilograms. Therefore, $V_{th} = \sqrt{(2k(300))/((2.36834 \times 10^{-31}))} = 1.8698 \times 10^5$ m/s

```
Vth = sqrt((B * T * 2) / Mn);  
% Thermal Velocity in metres per second
```

Mean Free Path

The mean free path is defined as the average distance a particle can travel undisturbed prior to a collision with another particle. It is calculated as the product of the thermal velocity and the mean time between collisions. Therefore, $MFP = V_{th} * T_{mn}$. The velocities of all the particles were fixed at the thermal velocity determined above, and the mean collision time was given as $T_{mn} = 0.2 \times 10^{-12}$ s Therefore, $MFP = V_{th} \times T_{mn} = (1.8698 \times 10^5 \text{ m/s}) \times (0.2 \times 10^{-12} \text{ s}) = 2.6443 \times 10^{-8} \text{ m}$

```
Mean_path = Tmn * Vth;  
% Mean Free Path in metres  
rho = Vth * dt;  
% Polar coordinates are used to determine the magnitude of  
% distance and its angular direction. Rho is the magnitude  
% of the distance in metres.
```

2-D Plot

This section covers setting up the plot. The plot has two graphs on one figure; the electron model movement and the average temperature.

```
figure(1);  
subplot(2,1,1)  
axis([0 L 0 W]);  
hold on;  
title('Model Electron Movement');  
  
subplot(2,1,2)  
hold on;  
title('Temperature over Time');  
xlabel('Time (sec)');  
ylabel('Temperature (Kelvin)');  
  
for t = 0 : dt : (TStop)  
    % goes up to max time steps  
    for k = 1 : 1 : 7  
        % Sets up the different coloured lines  
        if k == 1  
            c = 'y';  
        else  
            if k == 2  
                c = 'm';  
            else  
                if k == 3
```

```
        c = 'c';
    else
        if k == 4
            c = 'r';
        else
            if k == 5
                c = 'g';
            else
                if k == 6
                    c = 'b';
                else
                    if k == 7
                        c = 'k';
                    end
                end
            end
        end
    end
end
end
end
end

subplot(2,1,1)
% Now we fix the case that the line crosses the left/right
% boundaries and we don't want there to be a big line across
the
% graph from the previous and new points. This situation has
been
% called 'cross' if the right is crossed, and 'crossl' if the
% left is crossed. They are both vectors that are initially 0,
and
% if the situation changes they are called TRUE and the old
point
% is moved from the previous side to the new side so that when
a
% line is placed between the old and new points it doesn't
cross
% the whole page.
if cross(k) == true
    old_p(1,k) = 0.004 * (10^-7);
end
if crossl(k) == true
    old_p(1,k) = 1.996 * (10^-7);
end
% The line is made from the old to the new positions
line([old_p(1,k) p(1,k)], [old_p(2,k) p(2,k)], 'Color', c);
end

old_p = p;
% The current positions become the old positions

% This loop updates the positions of the x and y coordinates
for e = 1 : 1 : 1000
    p(1,e) = p(1,e) + (rho * cos(theta(e)));
    % position gets updated
```

```
    if bounce(e) == 1
        p(2,e) = p(2,e) + (rho * sin(theta(e)));
    else
        p(2,e) = p(2,e) - (rho * sin(theta(e)));
        % opposite vertical way
    end
    pause(dt);
end

% This loop determines if the right and left boundaries are
crossed
for d = 1 : 1 : 1000
    % if the line goes past the right boundary
    if p(1,d) > (1.995 * 10^-7)
        % is the x-coordinate past the right?
        p(1,d) = 0.005 * (10^-7);
        % bring it to the left side
        cross(d) = true;
    else
        cross(d) = false;
    end

    % if the line goes past the left boundary
    if p(1,d) < (0.005 * 10^-7)
        % is the x-coordinate past the left?
        p(1,d) = 1.995 * (10^-7);
        % bring it to the right side
        cross1(d) = true;
    else
        cross1(d) = false;
    end
end

for b = 1 : 1 : 1000
    if p(2,b) < (0.005 * (10^-7))
        % if line hits bottom
        bounce(b) = bounce(b) * (-1);
        % make it go the other way
    end
end

for m = 1 : 1 : 1000
    % if line hits top
    if (1 * (10^-7)) - (p(2,m)) < (0.005 * (10^-7))
        bounce(m) = bounce(m) * (-1);
        % make it go the other way
    end
end

% Checks if temperature is constant
Temp_test = ((Mn * ((Vth*Vth))) / (B * 2));
subplot(2,1,2)
line([old_t t], [old_temp_test Temp_test], 'Color', 'r');
old_t = t;
old_temp_test = Temp_test;
```

end

Part 2: Collisions with Mean Free Path (MFP)

The physical constants are defined here.

```
Mo = 9.109 * (10^-31);  
% electron rest mass in kilograms  
Mn = 0.26 * Mo;  
% effective electron mass in kilograms  
L = 200 * (10^-9);  
% length of region in metres  
W = 100 * (10^-9);  
% width of region in metres  
T = 300;  
% Temperature in Kelvin  
Tmn = 0.2 * (10^-12);  
% Mean time of collisions between particles in sec  
dt = 5 * (10^-15);  
% time step in seconds  
TStop = 1000 * dt;  
% stop simulation after 1000 time steps  
B = 1.38 * (10^-23);  
% Boltzmann constant  
old_t = 0;  
% This will be the time before the timestep  
old_temp_test = Temp_test;  
% This will be the old calculated system temp  
Temp_test = 300;  
  
Pscat = 1 - exp((dt * (-1)) / Tmn);  
% Probability of scattering  
  
Max_Bolt = (4*pi)*((Mn/(2*pi*B*T))^(1.5))*((Vth*Vth)*(exp(-Mn*Vth*Vth/  
(2*B*T)))));  
% Assigning random velocities  
rand_v = [zeros(1,1000)];  
for h = 1 : 1 : 1000  
    rand_v(h) = (4*pi)*((Mn/(2*pi*B*T))^(1.5))*((h*h)*(exp(-Mn*h*h/  
(2*B*T)))) * Vth;  
end  
  
rand_rho = [zeros(1,1000)];  
for h1 = 1 : 1 : 1000  
    rand_rho(h1) = rand_v(h1) * dt;  
end  
  
cross = [zeros(1,1000)];  
cross1 = [zeros(1,1000)];  
  
old_p = [zeros(1,1000); zeros(1,1000)];  
  
p = [zeros(1,1000); zeros(1,1000)];
```

```
for i = 1 : 1: 1000
    p(1,i) = rand * 100 * (10^-9);
    p(2,i) = rand * 100 * (10^-9);
end

old_p = p;

theta = [zeros(1,1000)];
% Polar coordinates are used; theta is the direction
for a = 1 : 1 : 1000
    theta(a) = rand * (2*pi);
    % Sets up a random angle of direction for each particle
end

scatter = [zeros(1,1000)];

old_avg_v = mean(rand_rho);

bounce = [ones(1,1000)];
% Setting up the plot
figure(2);
subplot(3,1,1)
axis([0 L 0 W]);
hold on;
title('Model Electron Movement');

subplot(3,1,2)
hold on;
title('Avg Temp');
xlabel('Time (sec)');
ylabel('Temperature (Kelvin)');
```

Histogram

This produces a histogram of the initial velocities.

```
subplot(3,1,3)
hist(rand_rho,100);
title('Velocity Histogram');
xlabel('Velocity');
ylabel('#Particles');

old_t = 0;
```

2-D Plot

```
for t = 0 : dt : (TStop)
    % goes up to max time steps
    for k = 1 : 1 : 7
        % Sets up the different coloured lines
        if k == 1
            c = 'y';
```

```
else
    if k == 2
        c = 'm';
    else
        if k == 3
            c = 'c';
        else
            if k == 4
                c = 'r';
            else
                if k == 5
                    c = 'g';
                else
                    if k == 6
                        c = 'b';
                    else
                        if k == 7
                            c = 'k';
                        end
                    end
                end
            end
        end
    end
end
end
end
end
subplot(3,1,1)
if cross(k) == true
    old_p(1,k) = 0.004 * (10^-7);
end
if cross1(k) == true
    old_p(1,k) = 1.996 * (10^-7);
end
line([old_p(1,k) p(1,k)], [old_p(2,k) p(2,k)], 'Color', c);
end

old_p = p;

% Scatter Check
for sc = 1 : 1 : 1000
    if scatter(sc) == true
        rand_rho(sc) = rand * 100 * (10^4) * dt;
        theta(sc) = rand * (2*pi);
    end
end

for e = 1 : 1 : 1000
    p(1,e) = p(1,e) + (rand_rho(e) * cos(theta(e)));
    % position gets updated
    if bounce(e) == 1
        p(2,e) = p(2,e) + (rand_rho(e) * sin(theta(e)));
    else
        p(2,e) = p(2,e) - (rand_rho(e) * sin(theta(e)));
        % opposite vertical way
    end
end
```



```
    pause(dt);
end

for e1 = 1 : 1 : 1000
    if Pscat > rand
        scatter(e1) = true;
    else
        scatter(e1) = false;
    end
end

for d = 1 : 1 : 1000
    % if the line goes past the right boundary
    if p(1,d) > (1.995 * 10^-7)
        % is the x-coordinate past the right?
        p(1,d) = 0.005 * (10^-7);
        % bring it to the left side
        cross(d) = true;
    else
        cross(d) = false;
    end

    % if the line goes past the left boundary
    if p(1,d) < (0.005 * 10^-7)
        % is the x-coordinate past the left?
        p(1,d) = 1.995 * (10^-7);
        % bring it to the right side
        cross1(d) = true;
    else
        cross1(d) = false;
    end
end

for b = 1 : 1 : 1000
    if p(2,b) < (0.005 * (10^-7))
        % if line hits bottom
        bounce(b) = bounce(b) * (-1);
        % make it go the other way
    end
end

for m = 1 : 1 : 1000
    if (1 * (10^-7)) - (p(2,m)) < (0.005 * (10^-7))
        % if line hits top
        bounce(m) = bounce(m) * (-1);
        % make it go the other way
    end
end

avg_v = mean(rand_rho)/dt;
Temp_test = (Mn * Vth * Vth) / (B * 2);
% Checks if temperature is constant
subplot(3,1,2)
line([old_t t], [old_avg_v avg_v], 'Color', 'r');
```

```
    old_avg_v = avg_v;  
    old_t = t;  
end
```

Part 3: Enhancements

2-D Plot

```
Mo = 9.109 * (10^-31);  
% electron rest mass in kilograms  
Mn = 0.26 * Mo;  
% effective electron mass in kilograms  
L = 200 * (10^-9);  
% length of region in metres  
W = 100 * (10^-9);  
% width of region in metres  
T = 300;  
% Temperature in Kelvin  
Tmn = 0.2 * (10^-12);  
% Mean time of collisions between particles in sec  
dt = 5 * (10^-15);  
% time step in seconds  
TStop = 1000 * dt;  
% stop simulation after 1000 time steps  
B = 1.38 * (10^-23);  
% Boltzmann constant  
Vth = sqrt(B * T / Mn);  
% Thermal Velocity in metres per second  
Mean_path = Tmn * Vth;  
% Mean Free Path in metres  
rho = Vth * dt;  
% Gives the distance between each timestep of new point  
Temp_test_old = 300;  
% This tests to see if the temperature remains constant  
t_old = 0;  
% This is the old time prior to the new timestep  
  
x1 = (0.8 * (10^-7));  
x2 = (1.2 * (10^-7));  
y1 = (0.4 * (10^-7));  
y2 = (0.6 * (10^-7));  
y3 = (1 * (10^-7));  
% These are coordinates of the bottle neck boundaries  
  
Pscat = 1 - exp((dt * (-1)) / Tmn);  
% Probability of scattering  
Max_Bolt = (sqrt(Mn / (2 * pi * B * T))) * (exp((-Mn * Vth * Vth) / (2  
    * B * T)));  
  
% Assigning random velocities  
rand_v = [zeros(1,1000)];  
for h = 1 : 1 : 7
```

```
    rand_v(h) = (4*pi)*((Mn/(2*pi*B*T))^(1.5))*((h*h)*(exp(-Mn*h*h/
(2*B*T)))) * Vth;
end

rand_rho = [zeros(1,1000)];
for h1 = 1 : 1 : 7
    rand_rho(h1) = rand_v(h1) * dt;
end

cross = [zeros(1,1000)];
cross1 = [zeros(1,1000)];

p = [zeros(1,1000); zeros(1,1000)];
for i = 1 : 1: 1000
    p(1,i) = rand * 100 * (10^-9);
    while (p(1,i) >= x1) && (p(1,i) <= x2)
        p(1,i) = rand * 100 * (10^-9);
    end
    p(2,i) = rand * 100 * (10^-9);
end

old_p = [zeros(1,1000); zeros(1,1000)];
old_p = p;

% Polar coordinates are used; theta is the direction
theta = [zeros(1,1000)];
for a = 1 : 1 : 7
    theta(a) = rand * (2*pi);
    % Sets up a random angle of direction for each particle
end

% The scatter vector has values either 0 or 1 to determine
% if the electron path should scatter
scatter = [zeros(1,1000)];

% Setting up the plot
bounce = [ones(1,1000)];
% The xbounce vector has values corresponding to the particles. It is
% either 1 or 0 if the particle bounces off the bottleneck areas or
not
xbounce = [ones(1,1000)];
figure(3);
subplot(3,1,1)
axis([0 L 0 W]);
% Setting up boxes in plot
line([x1 x2], [y1 y1]);
line([x1 x2], [y2 y2]);
line([x1 x1], [0 y1]);
line([x2 x2], [0 y1]);
line([x1 x1], [y2 y3]);
line([x2 x2], [y2 y3]);
hold on;
title('Model Electron Movement');
```

```
for t = 0 : dt : (TStop)
    % goes up to max time steps
    for k = 1 : 1 : 7
        % Sets up the different coloured lines
        if k == 1
            c = 'y';
        else
            if k == 2
                c = 'm';
            else
                if k == 3
                    c = 'c';
                else
                    if k == 4
                        c = 'r';
                    else
                        if k == 5
                            c = 'g';
                        else
                            if k == 6
                                c = 'b';
                            else
                                if k == 7
                                    c = 'k';
                                end
                            end
                        end
                    end
                end
            end
        end
    end

    subplot(2,1,1)
    if cross(k) == true
        old_p(1,k) = 0.004 * (10^-7);
    end
    if cross1(k) == true
        old_p(1,k) = 1.996 * (10^-7);
    end
    line([old_p(1,k) p(1,k)], [old_p(2,k) p(2,k)], 'Color', c);
end

old_p = p;

for sc = 1 : 1 : 1000
    % Scatter Check
    if scatter(sc) == true
        rand_rho(sc) = rand * 100 * (10^4) * dt;
        theta(sc) = randi([1 179]);
    end
end

for n = 1 : 1 : 1000
```

```
    if xbounce(n) == 1
        p(1,n) = p(1,n) + (rand_rho(n) * cos(theta(n)));
        % position gets updated
    else
        p(1,n) = p(1,n) - (rand_rho(n) * cos(theta(n)));
        % position gets updated
    end
end

for e = 1 : 1 : 1000
    if bounce(e) == 1
        p(2,e) = p(2,e) + (rand_rho(e) * sin(theta(e)));
    else
        p(2,e) = p(2,e) - (rand_rho(e) * sin(theta(e)));
        % opposite vertical way
    end
    pause(dt);
end

for e1 = 1 : 1 : 1000
    if Pscat > rand
        scatter(e1) = true;
    else
        scatter(e1) = false;
    end
end

for d = 1 : 1 : 1000
    % if the line goes past the right boundary
    if p(1,d) > (1.995 * 10^-7)
        % opposite vertical way
        p(1,d) = 0.005 * (10^-7);
        % bring it to the left side
        cross(d) = true;
    else
        cross(d) = false;
    end

    % if the line goes past the left boundary
    if p(1,d) < (0.005 * 10^-7)
        % is the x-coordinate past the left?
        p(1,d) = 1.995 * (10^-7);
        % bring it to the right side
        cross1(d) = true;
    else
        cross1(d) = false;
    end
end

for q1 = 1 : 1 : 1000
    if (x1 - p(1,q1)) < (0.005 * (10^-7)) && (p(2,q1) < y1)
        xbounce(q1) = xbounce(q1) * (-1);
    end
end
```

```
for q2 = 1 : 1 : 1000
    if (x1 - p(1,q2)) < (0.005 * (10^-7)) && (p(2,q2) > y2)
        xbounce(q2) = xbounce(q2) * (-1);
    end
end

for q3 = 1 : 1 : 1000
    if (p(1,q3) - x2) > (0.005 * (10^-7)) && (p(2,q3) < y1)
        xbounce(q3) = xbounce(q3) * (-1);
    end
end

for q4 = 1 : 1 : 1000
    if (p(1,q4) - x2) > (0.005 * (10^-7)) && (p(2,q4) > y2)
        xbounce(q4) = xbounce(q4) * (-1);
    end
end

for b = 1 : 1 : 1000
    if p(2,b) < (0.005 * (10^-7))
        % if line hits bottom
        bounce(b) = bounce(b) * (-1);
        % make it go the other way
    end
end

    for f = 1 : 1 : 1000
        if ((p(2,f)) - (0.4 * (10^-7))) < (0.005 * (10^-7)) &&
(p(1,f) >= x1) && (p(1,f) <= x2)
            bounce(f) = bounce(f) * (-1);
            % make it go the other way
        end
    end

    for m = 1 : 1 : 1000
        if (1 * (10^-7)) - (p(2,m)) < (0.005 * (10^-7))
            % if line hits top
            bounce(m) = bounce(m) * (-1);
            % make it go the other way
        end
    end

    for g = 1 : 1 : 1000
        if ((0.6 * (10^-7)) - (p(2,g)) < (0.005 * (10^-7))) &&
(p(1,g) >= x1) && (p(1,g) <= x2)
            bounce(g) = bounce(g) * (-1);
            % make it go the other way
        end
    end

t_new = t;
Temp_test_new = (Mn * Vth * Vth) / (B * 2);
```

```
% This plots the temperature over time, updating time and
temperature
subplot(2,1,2)
line([t_old t_new], [Temp_test_old
Temp_test_new], 'Color', 'r');
Temp_test_old = Temp_test_new;
t_old = t_new;

% This is the new idea for the temperature map
% h = histogram2(x,y,'Display Style','tile','ShowEmptyBins','on');

end
```

Final Electron Position Density Map

This is the electron density map of the figure.

```
subplot(3,1,2);
h = histogram2(p(1,:), p(2,:), [20 10]);
```

Final Temperature Plot

This is the temperature map of the figure.

```
subplot(3,1,3);
h = histogram2(rand_v(1,:),[200 100], 'FaceColor', 'flat');
colorbar
```

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