ELEC 4700 Assignment 2 Jalil (Rohana) Aalab #100995788 Feb/25/2018

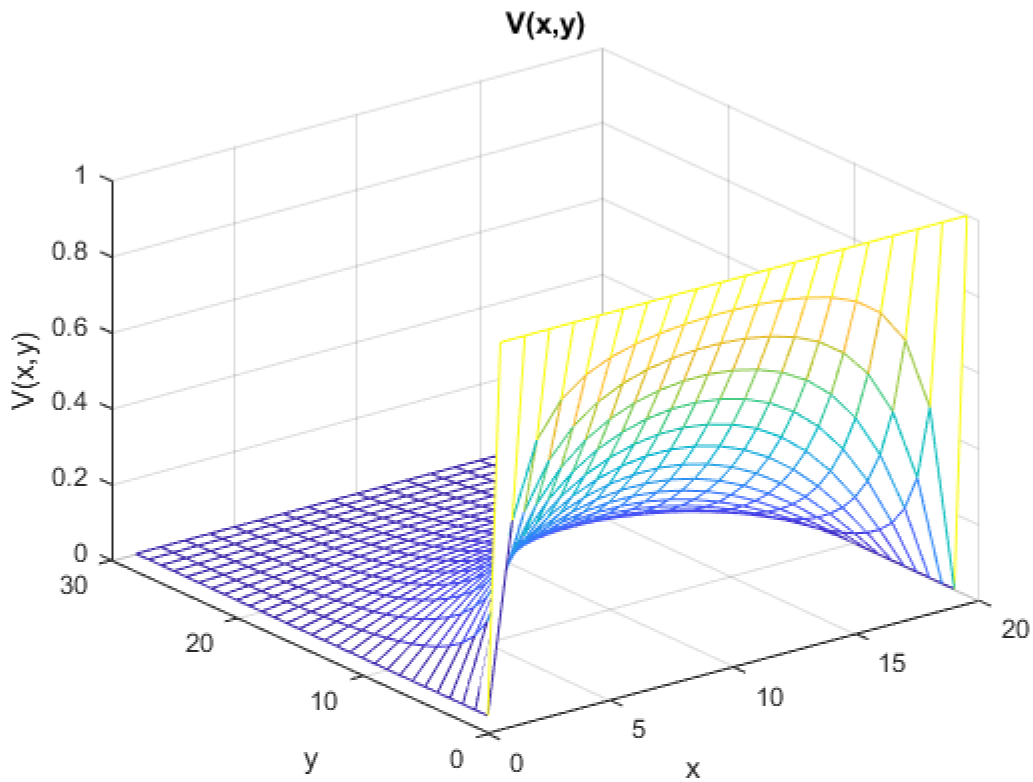
Table of Contents

Part 1 A

This code is used to provide a visual understanding of the electrostatic potential that is present within a given 2D rectangular region. This intial section will set up the grid and basic variables prior to evaluating the potentials. The first case will assume that the potential does not change with respect to the y-coordinates, and therefore will be treated as a 1D situation.

```
% Inputs
L = 30;
W = 20;

%Grid
x = linspace(0,L);
y = linspace(0,W);
dx = x(2) - x(1);
dy = y(2) - y(1);

% Make matrices
N = L*W;
M = zeros(N,N);
B = zeros(N,1);

% Interior Points
for i = 2:L-1
    for j = 2:W-1
        n = i + (j-1)*L;
        M(n,n) = -4;
        M(n,n-1) = 1;
        M(n,n+1) = 1;
        M(n,n-L) = 1;
        M(n,n+L) = 1;
        B(n,1) = 0;
    end
end
```

```
% Left BC point
i = 1;
for j = 1:W
    n = i + (j-1)*L;
    M(n,n) = 1;
     B(n,1) = 1;
end

% Right BC
i = L;
for j = 1:W
    n = i + (j-1)*L;
    M(n,n) = 1;
    B(n,1) = 0;
end

% Bottom BC
j = 1;
for i = 1:L
    n = i + (j-1)*L;
    M(n,n) = 1;
end

% Top BC
j = W;
for i = 1:L
    n = i + (j-1)*L;
    M(n,n) = 1;
end

% Solve for potential
phi_vec = M\B;

% Convert vector to 2D array
for i = 1 : L
    for j = 1 : W
        n = i + (j-1)*L;
        phi(i,j) = phi_vec(n);
    end
end

% Plot
figure(1);
mesh(phi);
xlabel('x');
ylabel('y');
zlabel('V(x,y)');
title('V(x,y)');
```



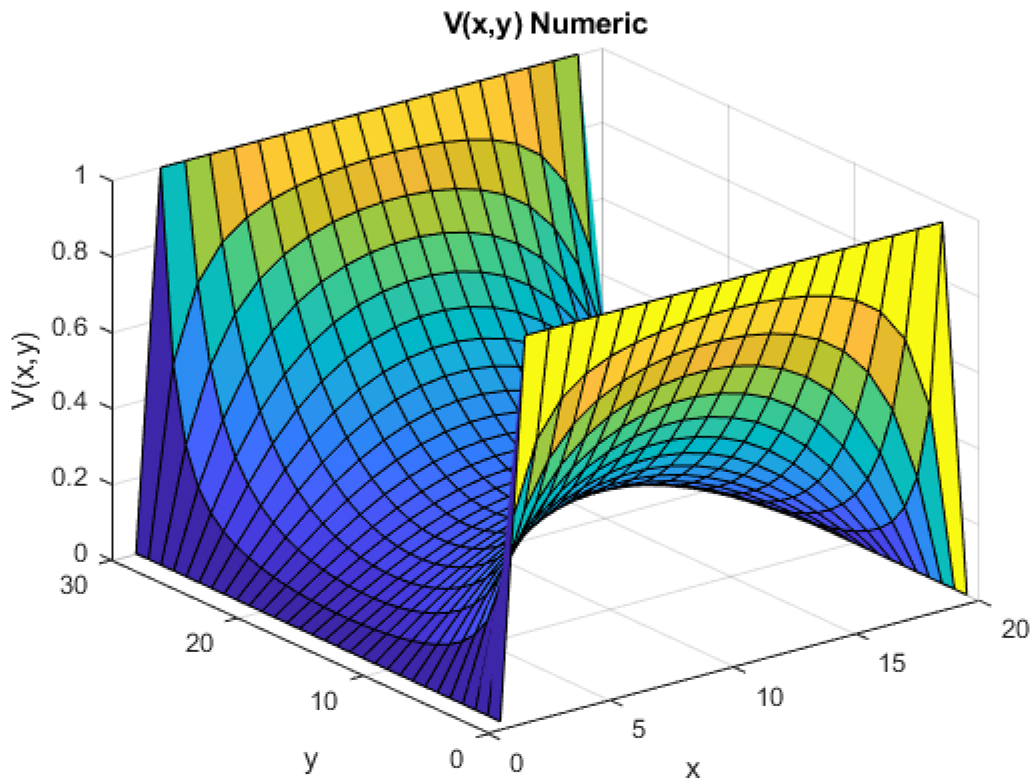
The graph shows that the potential (V) is 1 on one end of the region, and on the other end it is 0. Between these sides, the potential slowly drops as it is distributed from the V = 1 side to the zero potential side. This produces the curve that lowers down to a flat line, and this represents that the raised potential spreads out to the other side but the value of the potential drops as it is moved further away from the V = 1 side.

Part 1 B

This section provides a visual representation of the variance of electric potential over a 2D rectangular region.

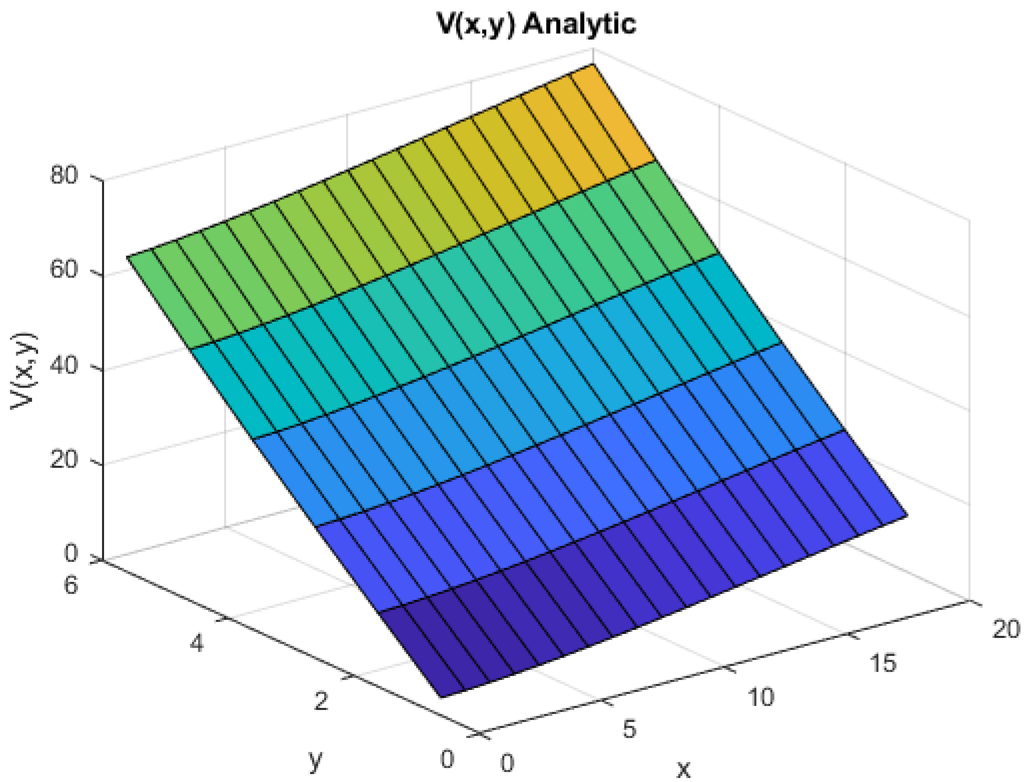
```
% Interior Points
for i = 2:L-1
    for j = 2:W-1
        n = i + (j-1)*L;
        M(n,n) = -4;
        M(n,n-1) = 1;
        M(n,n+1) = 1;
        M(n,n-L) = 1;
        M(n,n+L) = 1;
        B(n,1) = 0;
    end
end

% Left BC point
i = 1;
for j = 1:W
```

```
        n = i + (j-1)*L;
        M(n,n) = 1;
        B(n,1) = 1;
    end

    % Right BC
    i = L;
    for j = 1:W
        n = i + (j-1)*L;
        M(n,n) = 1;
        B(n,1) = 1;
    end

    % Bottom BC
    j = 1;
    for i = 1:L
        n = i + (j-1)*L;
        M(n,n) = 1;
        B(n,1) = 0;
    end

    % Top BC
    j = W;
    for i = 1:L
        n = i + (j-1)*L;
        M(n,n) = 1;
        B(n,1) = 0;
    end

    % Solve for potential
    phi_vec = M\B;

    % Convert vector to 2D array
    for i = 1 : 30
        for j = 1 : 20
            n = i + (j-1)*L;
            phi(i,j) = phi_vec(n);
        end
    end

    % Plot
    figure(2);
    surf(phi);
    xlabel('x');
    ylabel('y');
    zlabel('V(x,y)');
    title('V(x,y) Numeric');
```



As opposed to the previous figure, this region has boundary conditions such that both ends have a raised potential at V = 1. The areas of the region closest to either end are influenced by the raised potential and the potential of those areas rises as well. However, the potential drops in proportion to the distance away from either end. Therefore, the middle of the region is furtherst away from either end and has the lowest potential because it is far from the influence of the raised voltage.

```
% Part 1 b analytical
phi2 = phi;
a = 30;
b = 10;
added = 0;
for i = 1:L
    for j = 1:W
        for n = 1:2:1003
            added = added + ((1/n)*(cosh(n*pi*i/a))*(sin(n*pi*j/
a))*(1/(cosh(n*pi*b/a))));
        end
    phi2(i,j) = (4/pi) * added;
    end
end
figure(3);
surf(phi2);
xlabel('x');
ylabel('y');
zlabel('V(x,y)');
title('V(x,y) Analytic');
```



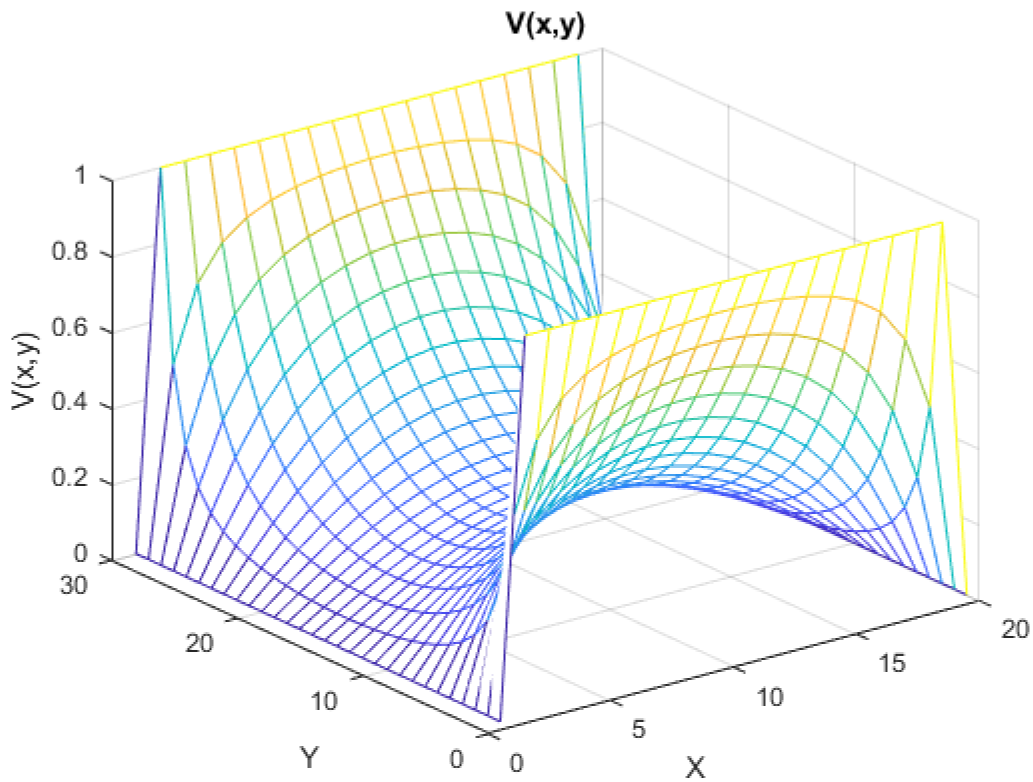
This graph is the analytic solution for the electric potential. It is derived from using the analytic series solution, which is described above by the variable 'added'. It can be seen that this analytic solution is not as accurate as the previous numeric solution. This may be due to the fact that the variable n is supposed to be all the odd numbers from 1 to infinity, however this code only had a maximum n of 1003. Therefore, the graph does not properly show the raised voltage of 1 V on both ends of the region. For this example, the superior solution is therefore the numeric solution.

Part 2 A

This section provides a visual understanding of the current flow in a given 2D rectangular region. The distribution of the conductivity (sigma(x,y)), current density(J(x,y)), horizontal electric field (Ex), vertical electric field (Ey), and electric potential (V(x,y)) are displayed. The region has a 'bottleneck' area which is highly resistive, and the effect of this on other variables is explored.

```
% Interior Points
for i = 2:L-1
    for j = 2:W-1
        n = i + (j-1)*L;
        M(n,n) = -4;
        M(n,n-1) = 1;
        M(n,n+1) = 1;
        M(n,n-L) = 1;
        M(n,n+L) = 1;
        B(n,1) = 0;
    end
end
```

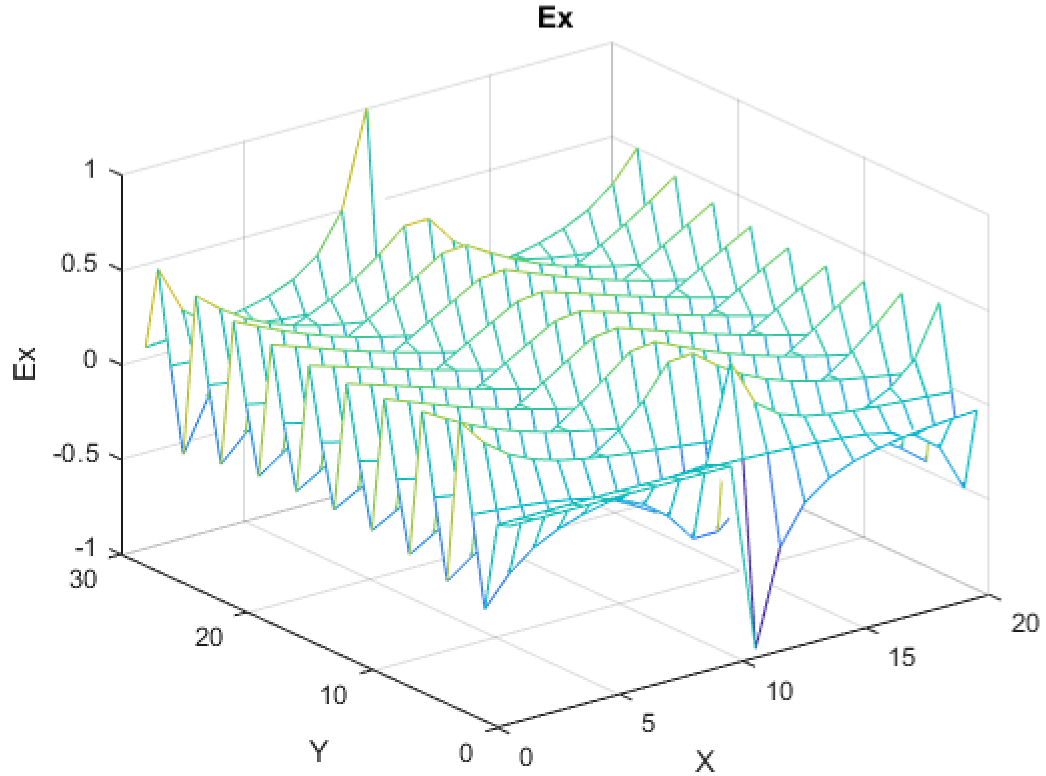
```
% Left BC point
i = 1;
for j = 1:W
    n = i + (j-1)*L;
    M(n,n) = 1;
    B(n,1) = 1;
end

% Right BC
i = L;
for j = 1:W
    n = i + (j-1)*L;
    M(n,n) = 1;
    B(n,1) = 1;
end

% Bottom BC
j = 1;
for i = 1:L
    n = i + (j-1)*L;
    M(n,n) = 1;
    B(n,1) = 0;
end

% Top BC
j = W;
for i = 1:L
    n = i + (j-1)*L;
    M(n,n) = 1;
    B(n,1) = 0;
end

% Solve for potential
phi_vec = M\B;

% Convert vector to 2D array
for i = 1 : 30
    for j = 1 : 20
        n = i + (j-1)*L;
        phi(i,j) = phi_vec(n);
    end
end

% Plot
figure(4);
mesh(phi);
xlabel('X');
ylabel('Y');
zlabel('V(x,y)');
title('V(x,y)');
```



This figure is the same as the previous figure with identical potentials and boundary conditions. Other variables and factors are changed in order to compare their effects, therefore this figure is a 'control' variable to relate the differences of various physical effects.

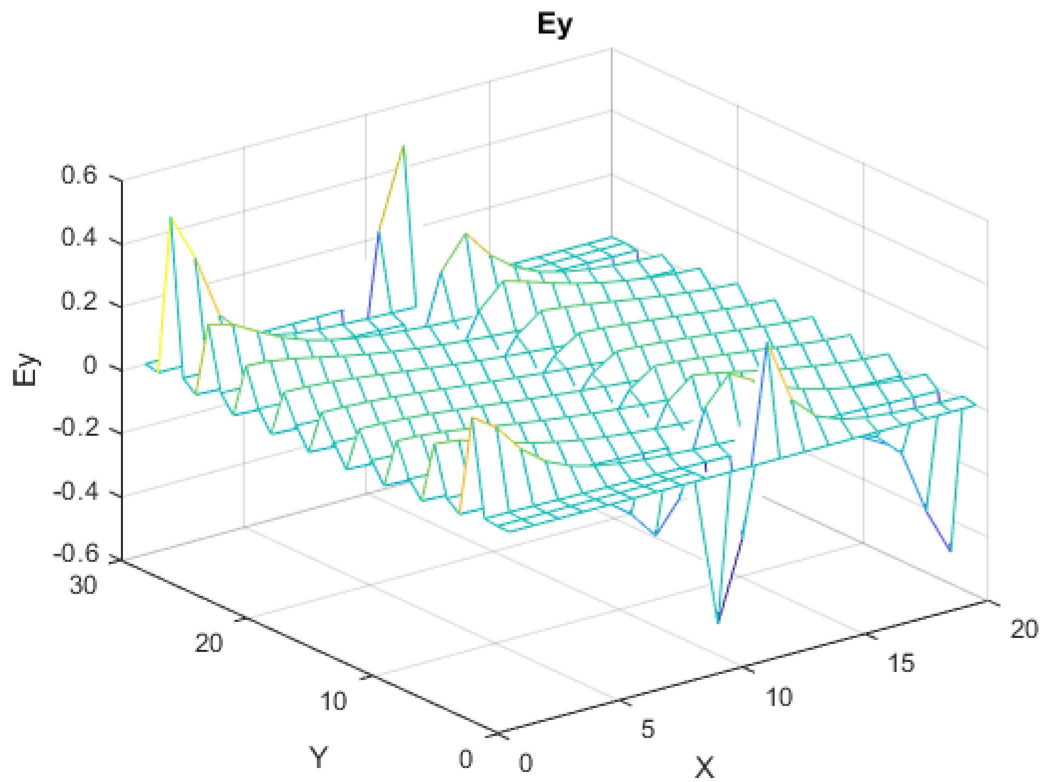
```
% Electric Field
Vmap = zeros(L,W);
for i = 1:L
    for j = 1:W
        n = j + (i-1)*W;
        Vmap(i,j) = phi_vec(n);
    end
end

for i = 1:L
    for j = 1:W
        if i == 1
            Ex(i,j) = (Vmap(i+1,j) - Vmap(i,j));
        elseif i == L
            Ex(i,j) = (Vmap(i,j) - Vmap(i-1,j));
        else
            Ex(i,j) = (Vmap(i+1,j) - Vmap(i-1,j))*0.5;
        end
        if j == 1
            Ey(i,j) = (Vmap(i,j+1) - Vmap(i,j));
        elseif j == W
            Ey(i,j) = (Vmap(i,j) - Vmap(i,j-1));
```

```
        else
            Ey(i,j) = (Vmap(i,j+1) - Vmap(i,j-1))*0.5;
        end
    end
end

Ex = -Ex;
Ey = -Ey;

figure(5);
mesh(Ex);
xlabel('X');
ylabel('Y');
zlabel('Ex');
title('Ex');
```



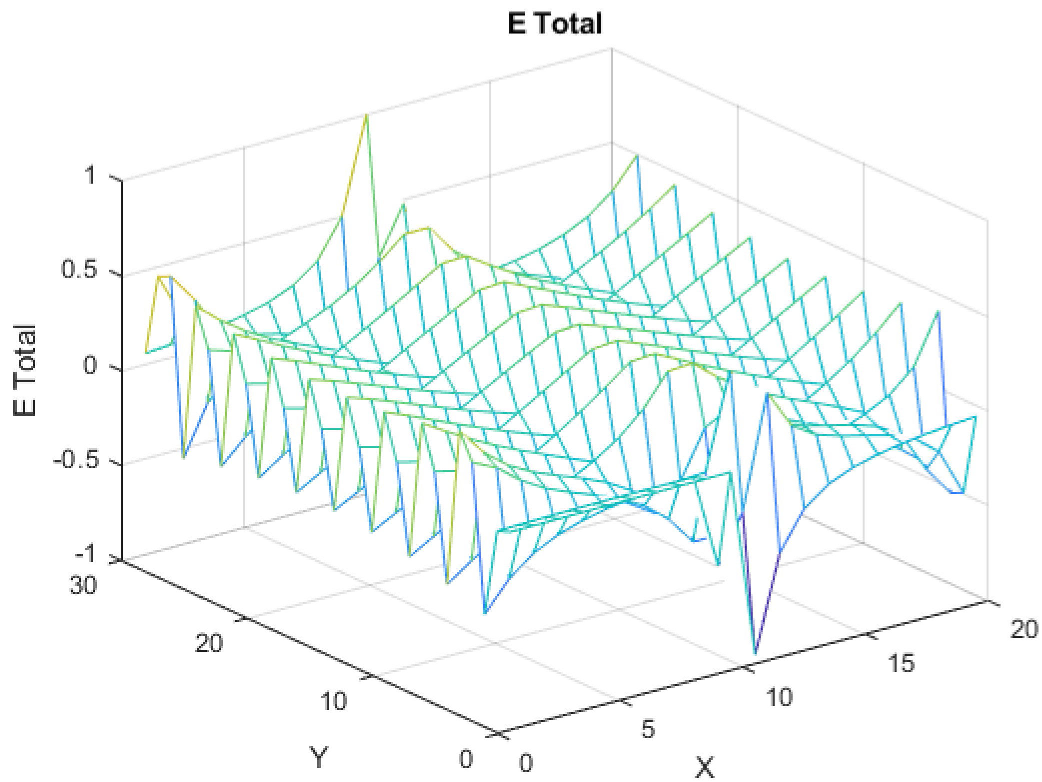
This is the graph of the horizontal electric field, or the electric field that is produced in the 'x' direction. The electric field is the gradient of electric potential (Voltage V), and Ex is the gradient in the 'x' component.

```
figure(6);
mesh(Ey);
xlabel('X');
ylabel('Y');
zlabel('Ey');
title('Ey');
```



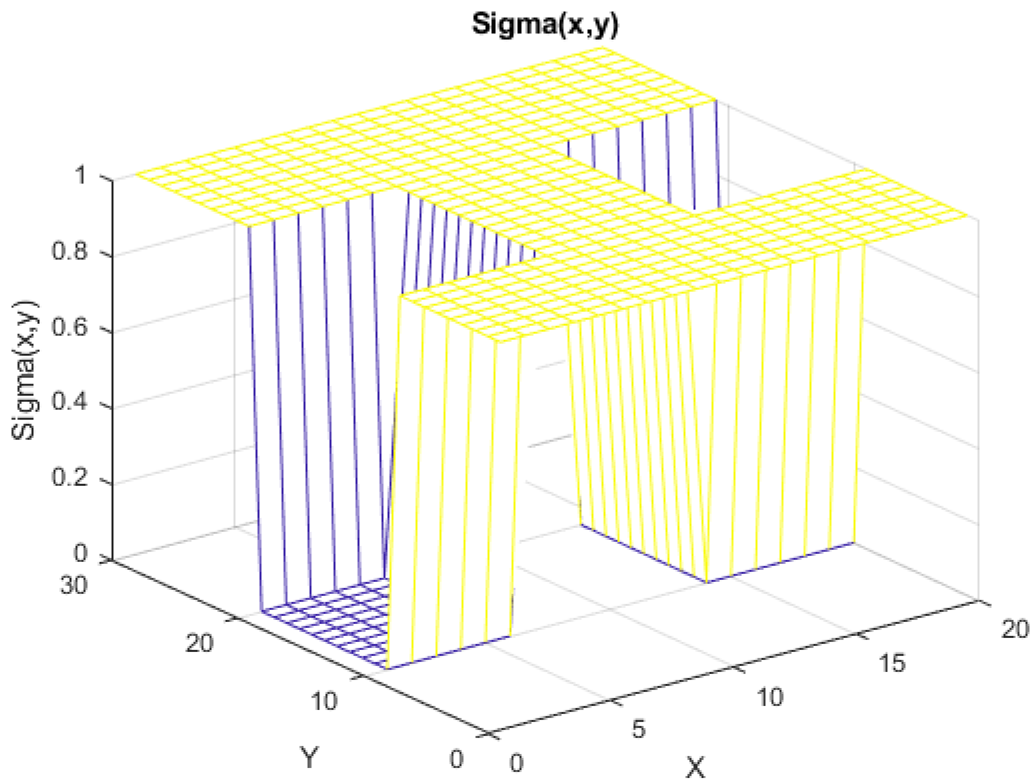
This is the graph of the vertical electric field, or the electric field that is produced in the 'y' direction. The electric field is the gradient of electric potential (Voltage V), and Ey is the gradient in the 'y' component.

```
Et = Ex + Ey;
figure(7);
mesh(Et);
xlabel('X');
ylabel('Y');
zlabel('E Total');
title('E Total');
```



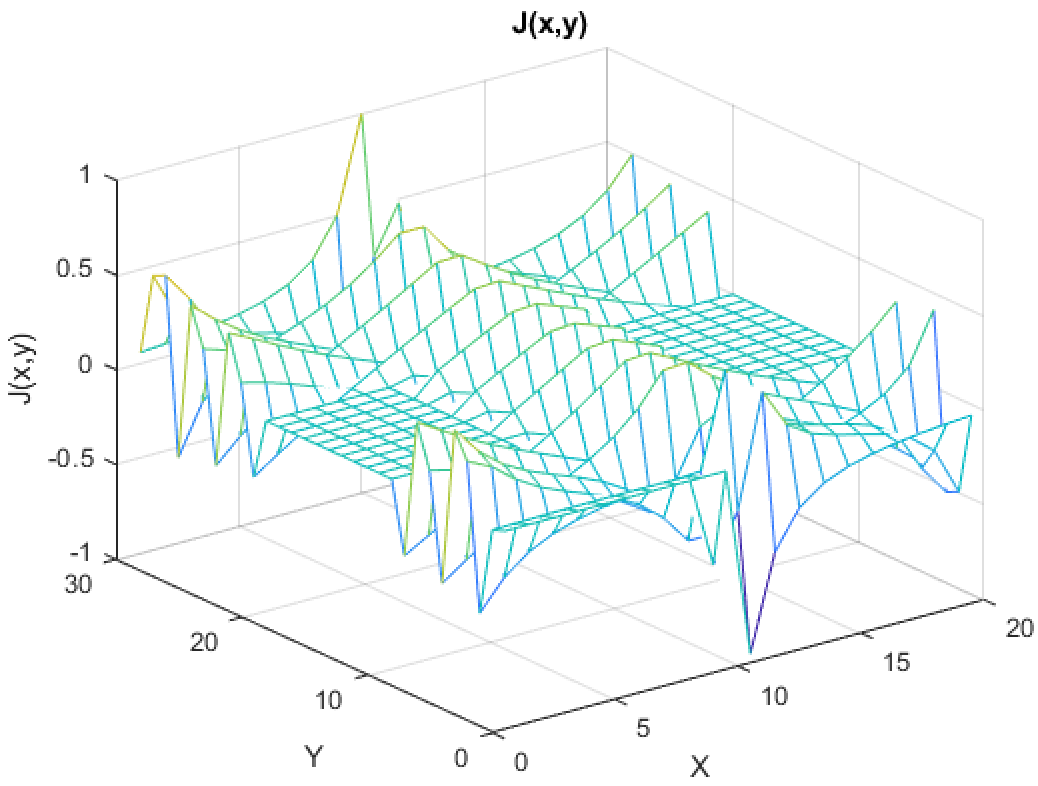
This is the graph of the total electric field, which is produced by adding components of the electric field in both the 'x' and 'y' directions. Therefore, this graph shows the electric field as the complete gradient of the 2D electric potential (Voltage V).

```
% Sigma
sigma = ones(L,W);
for i = 1:L
    for j = 1:W
        if j <= (W/3) || j >= (W*2/3)
            if i >= (L/3) && i <= (L*2/3)
                sigma(i,j) = 10^-12;
            end

        end
    end
end

figure(8);
mesh(sigma);
xlabel('X');
ylabel('Y');
zlabel('Sigma(x,y)');
title('Sigma(x,y)');
```



This graph depicts the distribution of the variable sigma over the region. The variable sigma is the conductivity of different areas of the region, and is the inverse of the resistivity of an area. The graph shows the 'bottle-neck' area, where there are 2 'boxes' that contain a conductivity that is significantly lower than that of the surrounding area. In a physical sense, these 2 boxes represent areas of the region that are highly resistive, and which will resist the flow of electric current.

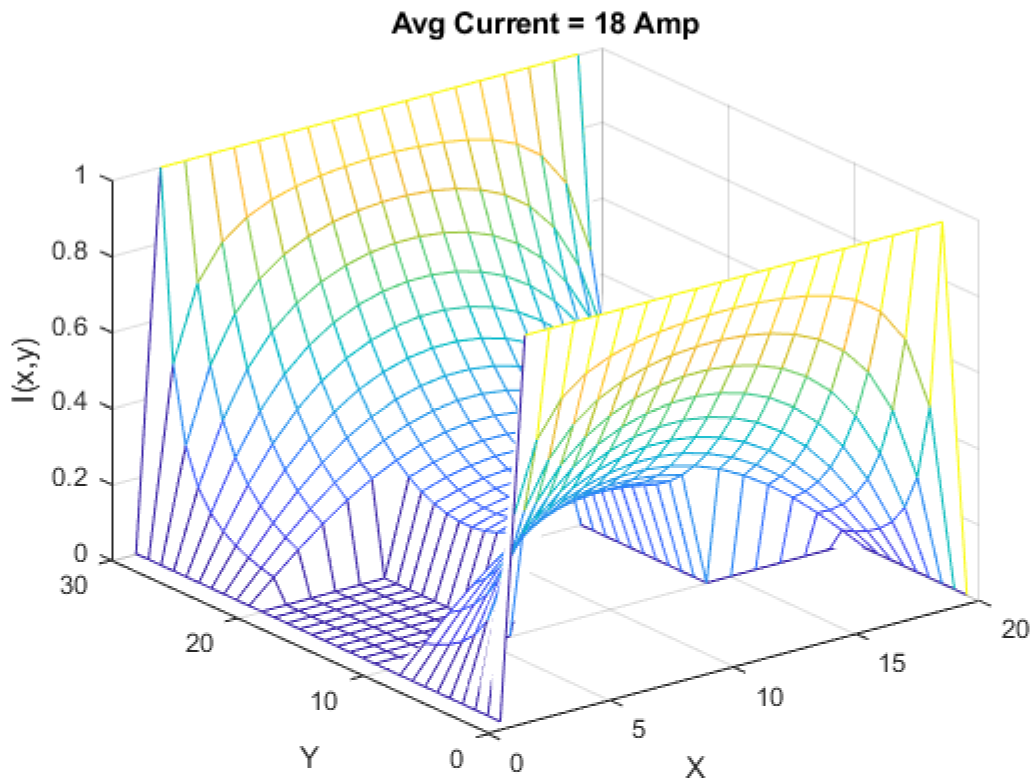
```
% Current Density
J = sigma .* Et;
figure(9);
mesh(J);
xlabel('X');
ylabel('Y');
zlabel('J(x,y)');
title('J(x,y)');
```



The graph shown depicts the distribution of current density (J) over the region. From physics, Ohm's Law is J = sigma * E. This formula states that the current density is equal to the product of the electric field and conductivity. This graph is similar to the previous figure of the total electric field (Et), however this graph has 2 'boxes' near the middle where there is a near 0 current density. These 2 areas exist because the conductivity of those areas is extremely low (meaning that the resistance in those areas is extremely high), and this decreases the ability of current to flow through these areas. Therefore, the lack of current in those areas produces a near 0 current density in those areas.

```
% Current Flow I = V/R
R = sigma;
for i = 1:L
    for j = 1:W
        if sigma(i,j) == (10^-12)
            R(i,j) = 1 / (10^-12);
        end
    end
end

Current = phi ./ R;
C0 = sum(Current(1,:));
CL = sum(Current(L,:));
c = (C0 + CL) / 2;
figure(10);
mesh(Current);
xlabel('X');
ylabel('Y');
```

```
zlabel('I(x,y)');
title(['Avg Current = ', num2str(c), ' Amp']);
```



The graph represents the flow of current throughout the region. Another form of Ohm's Law is I = V/R, which states that the current is equal to the voltage divided by the resistance. This graph is similar the previous graph of the distribution of electric potential (V(x,y)), however there are 2 'boxes' in the region where the current is 0. These 2 areas are the areas with high resistivity. From the formula, a high resistance with respect to voltage produces a low (or 0) current. Therefore, these 2 boxes act as a 'bottle-neck' region which narrows the path of current. The average current is stated and is calculated by averaging the current at both ends of the region.

Part 2 B

This section explores the relationship between the mesh size and the electric current.

```
% Inputs
L = 30;
W = 20;

%Grid
x = linspace(0,L);
y = linspace(0,W);
dx = x(2) - x(1);
dy = y(2) - y(1);

% Make matrices
```

```
N = L*W;
M = zeros(N,N);
B = zeros(N,1);
```

Part 2 C

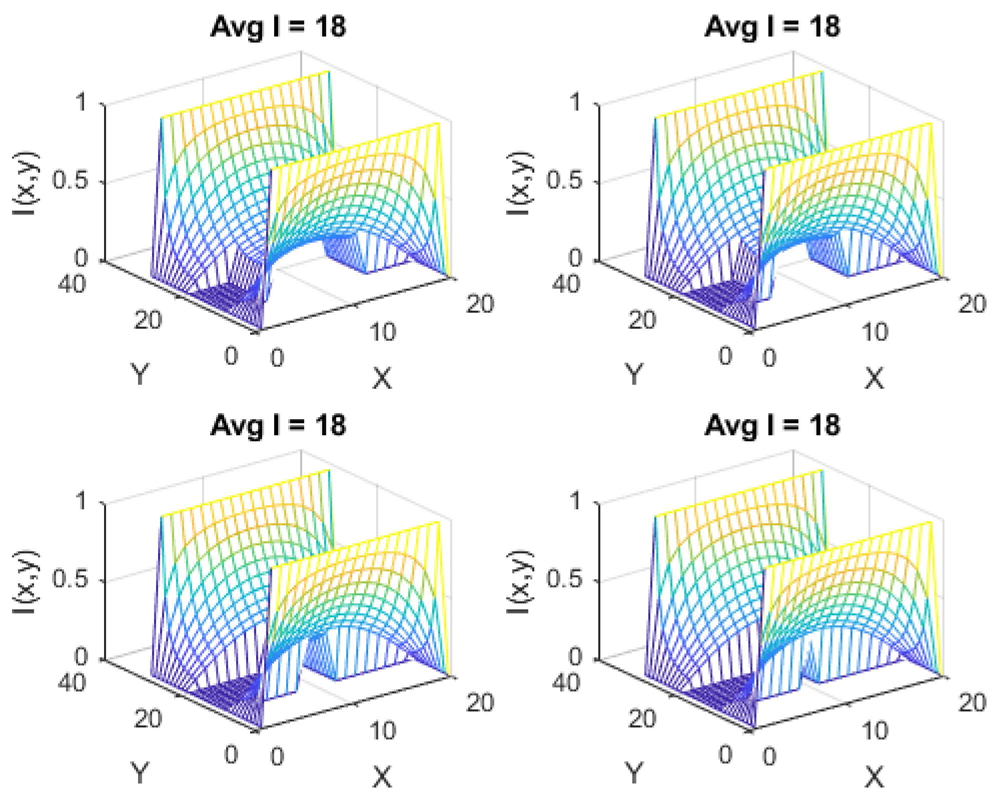
This section investigates the change of the current with respect to the change in the width of the bottle-neck region.

```
% Interior Points
for i = 2:L-1
    for j = 2:W-1
        n = i + (j-1)*L;
        M(n,n) = -4;
        M(n,n-1) = 1;
        M(n,n+1) = 1;
        M(n,n-L) = 1;
        M(n,n+L) = 1;
        B(n,1) = 0;
    end
end

% Left BC point
i = 1;
for j = 1:W
    n = i + (j-1)*L;
    M(n,n) = 1;
    B(n,1) = 1;
end

% Right BC
i = L;
for j = 1:W
    n = i + (j-1)*L;
    M(n,n) = 1;
    B(n,1) = 1;
end

% Bottom BC
j = 1;
for i = 1:L
    n = i + (j-1)*L;
    M(n,n) = 1;
    B(n,1) = 0;
end

% Top BC
j = W;
for i = 1:L
    n = i + (j-1)*L;
    M(n,n) = 1;
    B(n,1) = 0;
end
```

```
% Solve for potential
phi_vec = M\B;

% Convert vector to 2D array
for i = 1 : 30
    for j = 1 : 20
        n = i + (j-1)*L;
        phi(i,j) = phi_vec(n);
    end
end

% Sigma
sigma1 = ones(L,W);
sigma2 = ones(L,W);
sigma3 = ones(L,W);
sigma4 = ones(L,W);
for i = 1:L
    for j = 1:W
        if j <= (W/4) || j >= (W*3/4)
            if i >= (L/3) && i <= (L*2/3)
                sigma1(i,j) = 10^-12;
            end
        end
        if j <= (W/3.1) || j >= (W - (W/3.1))
            if i >= (L/3) && i <= (L*2/3)
                sigma2(i,j) = 10^-12;
            end
        end
        if j <= (W/2.5) || j >= (W - (W/2.5))
            if i >= (L/3) && i <= (L*2/3)
                sigma3(i,j) = 10^-12;
            end
        end
        if j <= (W/2.1) || j >= (W - (W/2.1))
            if i >= (L/3) && i <= (L*2/3)
                sigma4(i,j) = 10^-12;
            end
        end
    end
end

% Current Flow I = V/R
R1 = sigma1;
R2 = sigma2;
R3 = sigma3;
R4 = sigma4;
for i = 1:L
    for j = 1:W
        if sigma1(i,j) == (10^-12)
            R1(i,j) = 1 / (10^-12);
        end
    end
end
```

```
for i = 1:L
    for j = 1:W
        if sigma2(i,j) == (10^-12)
            R2(i,j) = 1 / (10^-12);
        end
    end
end
for i = 1:L
    for j = 1:W
        if sigma3(i,j) == (10^-12)
            R3(i,j) = 1 / (10^-12);
        end
    end
end
for i = 1:L
    for j = 1:W
        if sigma4(i,j) == (10^-12)
            R4(i,j) = 1 / (10^-12);
        end
    end
end

Current1 = phi ./ R1;
C01 = sum(Current1(1,:));
CL1 = sum(Current1(L,:));
c1 = (C01 + CL1) / 2;
figure(11);
subplot(2,2,1);
mesh(Current1);
xlabel('X');
ylabel('Y');
zlabel('I(x,y)');
title(['Avg I = ', num2str(c1)]);

Current2 = phi ./ R2;
C02 = sum(Current2(1,:));
CL2 = sum(Current2(L,:));
c2 = (C02 + CL2) / 2;
subplot(2,2,2);
mesh(Current2);
xlabel('X');
ylabel('Y');
zlabel('I(x,y)');
title(['Avg I = ', num2str(c2)]);

Current3 = phi ./ R3;
C03 = sum(Current3(1,:));
CL3 = sum(Current3(L,:));
c3 = (C03 + CL3) / 2;
subplot(2,2,3);
mesh(Current3);
xlabel('X');
ylabel('Y');
zlabel('I(x,y)');
```

```
title(['Avg I = ', num2str(c3)]);

Current4 = phi ./ R4;
C04 = sum(Current4(1,:));
CL4 = sum(Current4(L,:));
c4 = (C04 + CL4) / 2;
subplot(2,2,4);
mesh(Current4);
xlabel('X');
ylabel('Y');
zlabel('I(x,y)');
title(['Avg I = ', num2str(c4)]);
```



The figure shows 4 different graphs which all show the effect of different conductivity regions on the current. The different graphs show 'bottle-neck' regions where the path for current to flow through the middle region becomes increasingly narrow. However, each graph produces the same average current, meaning that both ends of each graph are at the same current. This observation would suggest that the narrowing of the 'bottle-neck' region has no effect on the current flowing from one end to the other. This conclusion can be supported by the fact that the average current is calculated from the average of the current at both ends of the region, and current is defined as I = V/R. The voltage distribution and resistivity of both ends has not changed by the narrowing of the 'bottle-neck' in the middle of the region, therefore the currents would be identical.

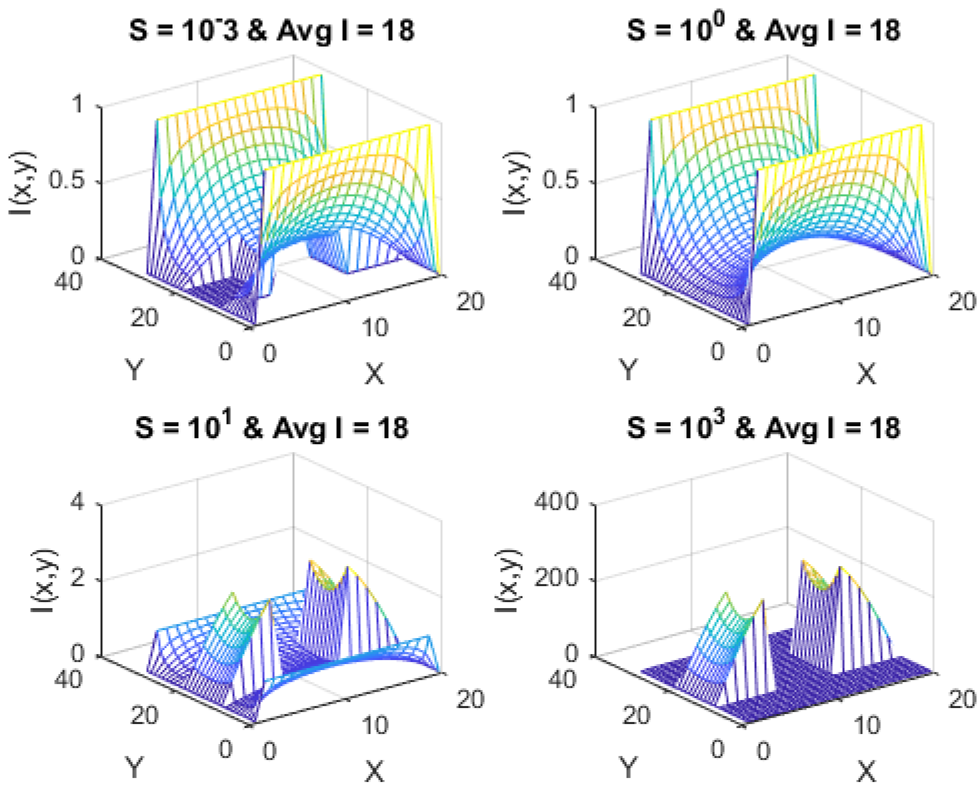
Part 2 D

This section analyzes the effects of various conductivities (sigma(x,y)) on the current.

```
% Interior Points
for i = 2:L-1
    for j = 2:W-1
        n = i + (j-1)*L;
        M(n,n) = -4;
        M(n,n-1) = 1;
        M(n,n+1) = 1;
        M(n,n-L) = 1;
        M(n,n+L) = 1;
        B(n,1) = 0;
    end
end

% Left BC point
i = 1;
for j = 1:W
    n = i + (j-1)*L;
    M(n,n) = 1;
    B(n,1) = 1;
end

% Right BC
i = L;
for j = 1:W
    n = i + (j-1)*L;
    M(n,n) = 1;
    B(n,1) = 1;
end

% Bottom BC
j = 1;
for i = 1:L
    n = i + (j-1)*L;
    M(n,n) = 1;
    B(n,1) = 0;
end

% Top BC
j = W;
for i = 1:L
    n = i + (j-1)*L;
    M(n,n) = 1;
    B(n,1) = 0;
end

% Solve for potential
phi_vec = M\B;

% Convert vector to 2D array
for i = 1 : 30
    for j = 1 : 20
        n = i + (j-1)*L;
        phi(i,j) = phi_vec(n);
    end
```

```
end

% Sigma
sigma = ones(L,W);
for i = 1:L
    for j = 1:W
        if j <= (W/3) || j >= (W*2/3)
            if i >= (L/3) && i <= (L*2/3)
                sigma(i,j) = 10^-12;
            end

        end
    end
end

% Current Flow I = V/R
R1 = sigma;
R2 = sigma;
R3 = sigma;
R4 = sigma;

for i = 1:L
    for j = 1:W
        if sigma(i,j) == (10^-12)
            R1(i,j) = 1 / (10^-3);
            R2(i,j) = 1 / (10^0);
            R3(i,j) = 1 / (10^1);
            R4(i,j) = 1 / (10^3);
        end
    end
end

Current = phi ./ R1;
C0 = sum(Current(1,:));
CL = sum(Current(L,:));
c = (C0 + CL) / 2;
figure(12);
subplot(2,2,1);
mesh(Current);
xlabel('X');
ylabel('Y');
zlabel('I(x,y)');
title(['S = 10^-3 & Avg I = ', num2str(c)]);

Current = phi ./ R2;
C0 = sum(Current(1,:));
CL = sum(Current(L,:));
c = (C0 + CL) / 2;
subplot(2,2,2);
mesh(Current);
xlabel('X');
ylabel('Y');
zlabel('I(x,y)');
title(['S = 10^0 & Avg I = ', num2str(c)]);
```

```
Current = phi ./ R3;
C0 = sum(Current(1,:));
CL = sum(Current(L,:));
c = (C0 + CL) / 2;
subplot(2,2,3);
mesh(Current);
xlabel('X');
ylabel('Y');
zlabel('I(x,y)');
title(['S = 10^1 & Avg I = ', num2str(c)]);

Current = phi ./ R4;
C0 = sum(Current(1,:));
CL = sum(Current(L,:));
c = (C0 + CL) / 2;
subplot(2,2,4);
mesh(Current);
xlabel('X');
ylabel('Y');
zlabel('I(x,y)');
title(['S = 10^3 & Avg I = ', num2str(c)]);
```



The figure shows 4 different graphs that depict the effect of various magnitudes of conductivity on the current. The different graphs show that increasing the conductivity of the 2 'boxes' in the 'bottle-neck' region proportionally increases the current in those 2 areas. However, all the graphs produce the same average current. This observation suggests that manipulation of the conductivity of the 'bottle-neck' areas (and therefore the resistivity) has no effect on the average current flowing from one end to the other. This conclusion can be supported by the fact that the average current is calculated as the average of the current on both ends of the region. Current is defined as I = V/R, and the change in the magnitude of conductivity in the 'bottle-neck' region does not affect the voltage distribution or resistivity of either end of the region; therefore the currents should be equal.

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