

TTK4225 2024 - Assignment 01

September 3, 2025

1 TTK4225 Assignment 1

1.0.1 Note

Some of the points below may be either solved by pen and paper, or – for who feels brave but also learn something that will be very useful in your theses – via symbolic computing (e.g., sympy: <https://docs.sympy.org/latest/tutorials/intro-tutorial/solvers.html>).

The most important thing in this assignment is to understand the logics behind each procedural step you take. Thus for the teachers it does not matter which way you choose, the important is that you understand what you do.

1.0.2 Introduction

Consider the following three systems,

- [Mass-spring](#),
- [Lotka Volterra](#), and
- [Van der Pol oscillator](#),

defined through their ODEs as in the links above. In the remainder of the assignment we will analyse them analytically, so to gain intuitions about their properties.

1.0.3 Question 1

Compute the equilibria of each of these systems as functions of the parameters of their models.

- If you are solving this point via pen and paper, then write the procedure you followed to compute the solution in the cell below, using it as a `markdown` cell, and the solution itself.
- If you are solving this point via symbolic computing, then use the cell below as a `code` cell to write the code that computes the solution.

1.0.4 Solution to question 1

Equilibria of the mass spring system: The general ODE for a mass spring system is described by the following,

$$\sum F = -kx - c\dot{x} + F_{external} = m\ddot{x}$$

Since we are solving the system around equilibria, $\dot{x} = 0$ and $\ddot{x} = 0$, we get,

$$\begin{aligned} -kx + F &= 0 \\ \downarrow \\ x &= \frac{F}{k} \end{aligned}$$

Equilibria of the Lotka-Volterra system: The general ODE for a Lotka-Volterra system is described by the following,

$$\begin{aligned} \dot{x} &= \alpha x - \beta xy \\ \dot{y} &= -\gamma y + \zeta xy \end{aligned}$$

Since we are solving the system around equilibria, $\dot{x} = 0$, we get,

$$\begin{aligned} 0 &= \alpha x - \beta xy \\ 0 &= -\gamma y + \zeta xy \\ \downarrow \\ \beta xy &= \alpha x \\ \gamma y &= \zeta xy \\ \downarrow \\ y &= \frac{\alpha}{\beta} \\ x &= \frac{\gamma}{\zeta} \end{aligned}$$

Equilibria of the Van der Pol system: The general ODE for a mass spring system is described by the following,

$$\ddot{x} - \mu(1 - \dot{x}) + x = 0$$

Since we are solving the system around equilibria, $\dot{x} = 0$ and $\ddot{x} = 0$, we get,

$$x = 0$$

1.0.5 Question 2

Using the cell below as a `markdown` cell, describe which of these equilibria are:

- simply stable (i.e., for which there are no trajectories that start in a neighborhood of the equilibrium that ‘escape’ from the equilibrium. Note: simply stable is a synonym for marginally stable),

- asymptotically stable (i.e., not just simply stable but also for which the trajectories that start closeby the equilibrium ‘end’ in the equilibrium),
- or unstable (i.e., for which you can find at least one trajectory that seems starting from the equilibrium and ‘moving away’ from it).

Say also why you believe so. Note that the fastest way to solve this question is likely googling “phase portrait of system XXX”, taking a look at the plots you get, thinking at what these plots mean and imply, and then describe such behaviors.

1.0.6 Solution to question 2

Description of the stability properties of the equilibria of the mass spring system:

The system is asymptotically stable. This is due to the fact that any perturbations will eventually result in the system returning to equilibria, due to friction.

Description of the stability properties of the equilibria of the Lotka-Volterra system:

The system is marginally stable. This is because there are no trajectories that escape from the equilibria.

Description of the stability properties of the equilibria of the Van der Pol system:

The system is unstable. While a stable orbit can be observed, this does not equate a stable equilibria, and there are no stable equilibria.

1.0.7 Question 3

As for the asymptotically stable equilibria, describe their basin of attraction (i.e., which zone of the phase portrait is so that starting from a generic initial condition in that zone makes the system follow a trajectory that converges as time goes to infinity to that equilibrium).

Write your answer using the cell below as a `markdown` cell.

The fastest way to solve this is likely googling “phase portrait of system XXX”, taking a look at the plots you get, thinking at what these plots mean and imply, and then describe such basins.

1.0.8 Solution to question 3

The basin of attraction will converge with any initial condition, regardless of the zone.

1.0.9 Question 4

Think at which physical system has both an **asymptotically** stable equilibrium and an unstable one. Describe such a system, and explain the intuitions you have that make you say that the equilibria have that stability properties.

Write your answer using the cell below as a `markdown` cell.

1.0.10 Solution to question 4

A system with both a asymptotically stable equilibrium and an unstable one could be a pendulum with friction. When a pendulum is at the top of the swing curve, with no velocity, it can balance on that point. However, any infinitely small perturbations will cause it fall. At the bottom,

any perturbation will make the pendulum oscillate with slowly reducing oscillations around the equilibrium point.

1.0.11 Question 5

How would you modify the system in Question 4 so to make the asymptotically stable equilibrium only simply stable? (Note: simply stable is a synonym for marginally stable).

Write your answer using the cell below as a `markdown` cell.

1.0.12 Solution to question 5

Removing friction would make the system a marginally stable equilibria.

1.0.13 Question 6

Use the cell below as a `markdown` cell to describe how model predictive control works from intuitive perspectives.

1.0.14 Solution to question 6

Model Predictive Control (MPC) works by using a known model(states) of a system to predict future states of a system. Then using this knowledge it can apply control outputs that best fits the desired behaviour of said system. For each applied control output, it reasses the future predicted states and corrects the applied control output, this makes it more adaptable to dynamic environments in comparison to LQR. The accuracy and sensitivity of an MPC can be tuned based on how much noise is allowed through.

1.0.15 Question 7

Use the cell below as a `markdown` cell to describe what you think the goal of this course is.

1.0.16 Solution to question 7

The goal of this course is to give us students the ability to intuitively understand, describe and control a systems behaviour and properties.