MATHEMATICS IN THE MODERN WORLD
MODULE 1

I. TOPIC: ***PATTERN, PATTERNS IN NATURE & TRANSFORMATION***

II. OBJECTIVE(S):

1. **Familiarization of patterns;**
2. **Discuss the patterns in nature;**
3. **Identify different type of patterns;**
4. **Enumerate & discuss the different types of transformation.**

III. INTRODUCTION:

Patterns exist in different variety of forms. The petals of a flower, arrangement of leaves reveals a sequential pattern. Natures are bounded by different colors and shapes – the rainbow mosaic of a butterfly's wings, the undulating ripples of a desert dune. But these miraculous creations not only delight the imagination, they also challenge our understanding. How do these patterns develop? What sorts of rules and guidelines, shape the patterns in the world around us?

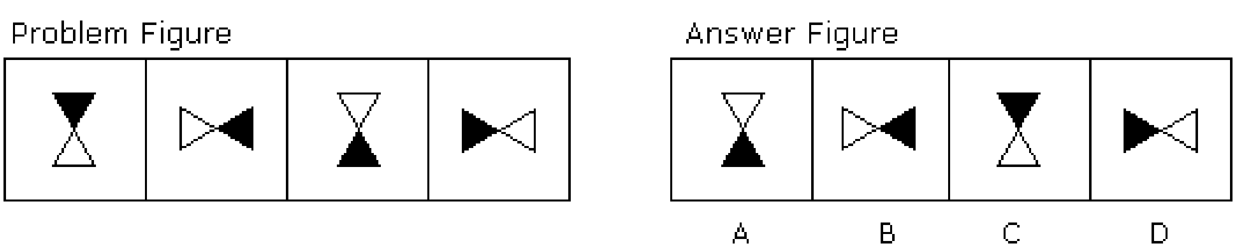
Some patterns are molded with a strict regularity. At least superficially, the origin of regular patterns often seems easy to explain. Thousands of times over, the cells of a honeycomb repeat their hexagonal symmetry. The honeybee is a skilled and tireless artisan with an innate ability to measure the width and to gauge the thickness of the honeycomb it builds. Although the workings of an insect's mind may baffle biologists, the regularity of the honeycomb attests to the honey bee's remarkable architectural abilities.

IV. **DISCUSSION:**

1. A pattern is something which helps us anticipate what we might see or expect to happen next. It may also help us know what may have come before or what we are seeing currently. There are four types of patterns; (1) logic patterns, (2) number patterns, (3) geometric patterns and (4) word patterns.

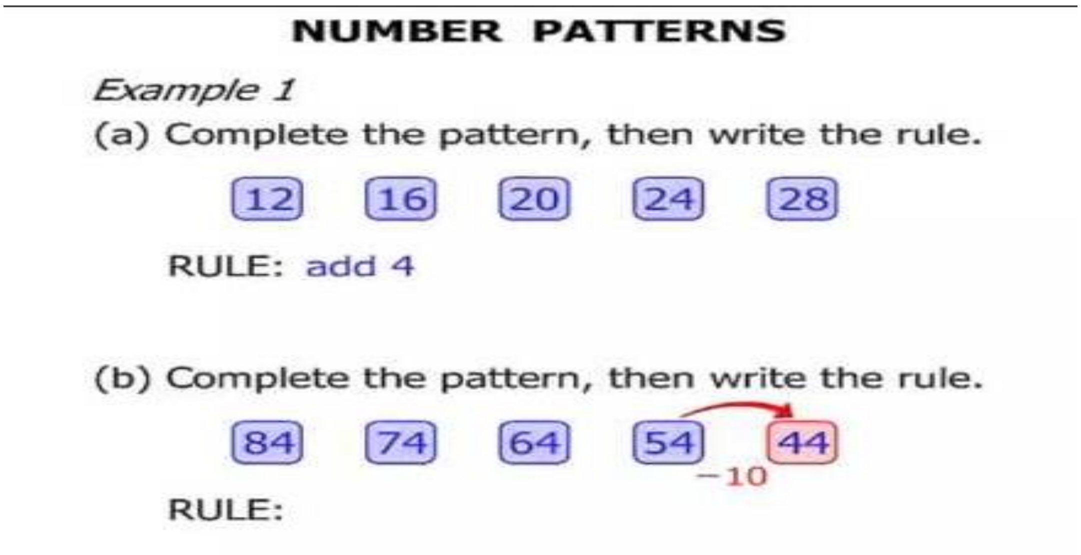
A. **Logic pattern** is the ability to discover meaningful patterns in strange and unpredictable situations. When you enter a strange space, like a new job, you spontaneously search for patterns to influence how you think and act and speak in the new space.

Example of logical patterns:



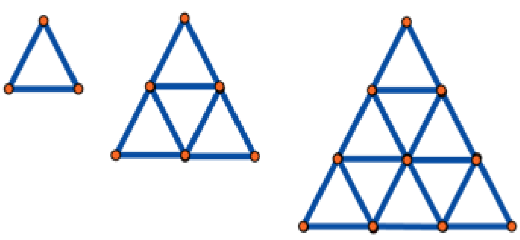
B. **Number pattern** is a sequence of number that are formed in accordance with a definite rule. We can often describe number patterns in more than one way. To illustrate this, consider the following sequence of numbers {1, 3, 5, 7, 9, …}. Clearly, the first term of this number pattern is 1; and the terms after the first term are obtained by adding 2 to the previous term. We can also describe this number pattern as a set of odd numbers.

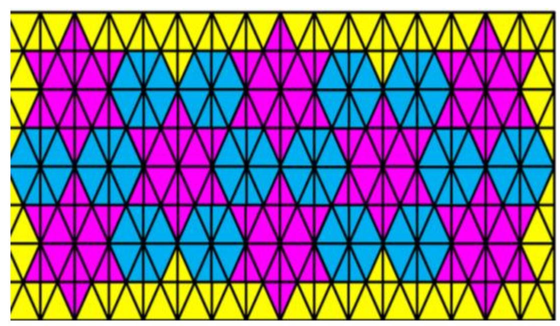
Example of number patterns:



C. Geometric pattern is a pattern that represented by geometrical figures such as polygons and isometric shapes.

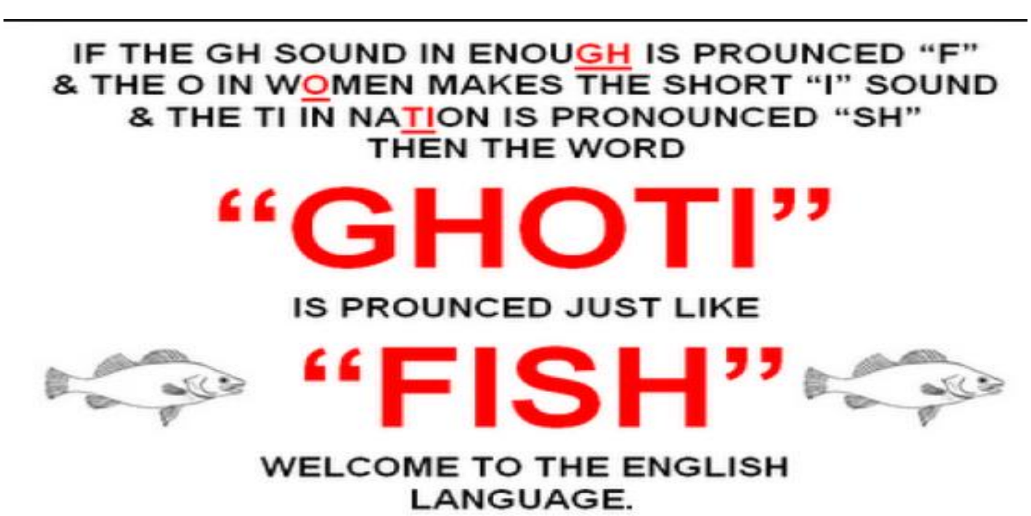
Examples of geometric patterns:





D. Word pattern are represented by jumbled words and analyzed the hidden logic in it.

Example of word pattern:

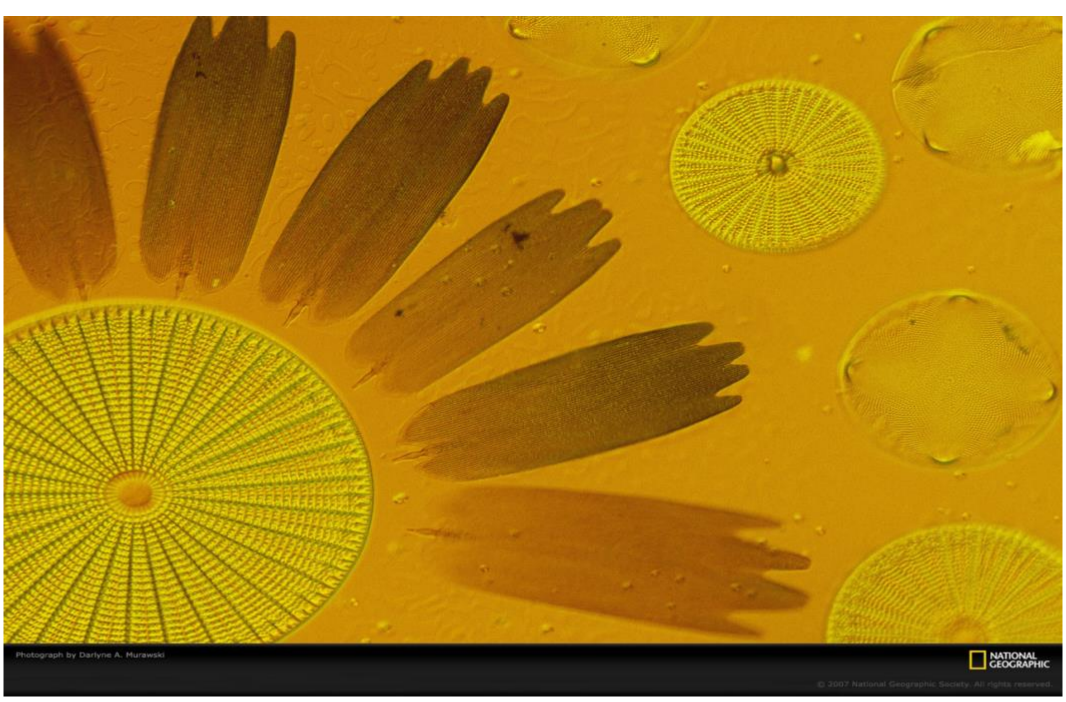


Patterns are part of our everyday life and are visible in shapes, color, number and object repetition. Below are some of the patterns in nature;

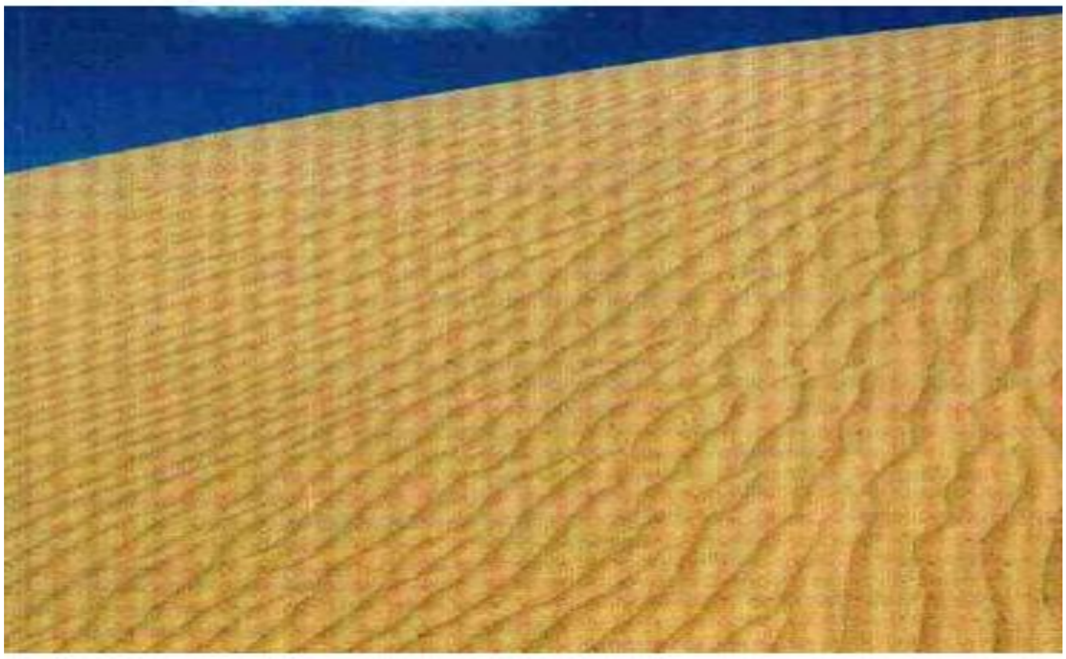
1. Arrangement of trees covered by snow

2. A
still lake reflects sky and trees

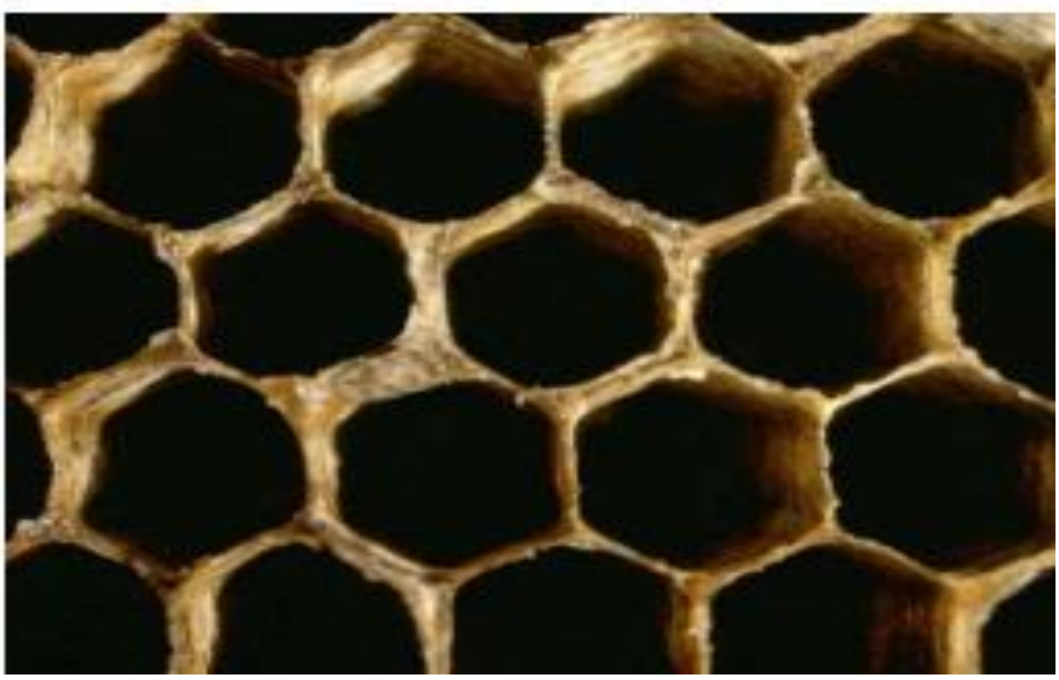
3. Scales from butterfly wings radiate from a glass-shelled diatom.



4. Rippled pattern observed on the desert sand.

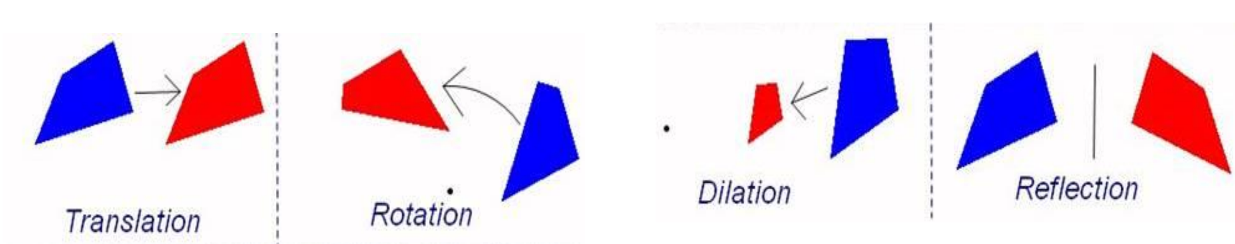


5. Honeycomb structure

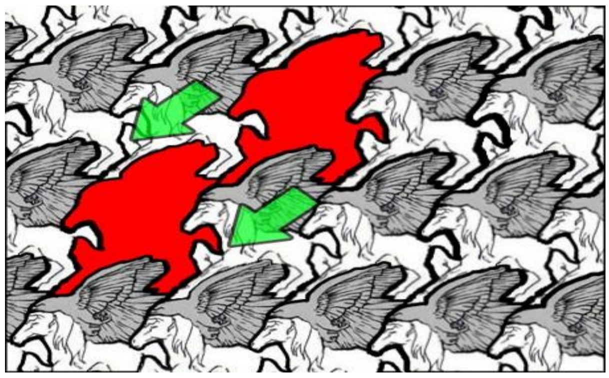


A pattern has symmetry. Isometry of the plane that preserves the pattern. It is a way of transforming the plane that preserves geometrical properties such as length. There are four types of isometries according to Euclidian isometry of plane transformation (1) Translation (2) Reflection (3) Rotation (4) Dilation. Moreover, we have to consider sometimes the combination of Reflection, translation and rotations makes another isometry called rigid transformation which leave the dimensions of the object and its image unchanged.

TRANSFORMATION
is a process which shifts points of the plane to possibly new locations on the plane.

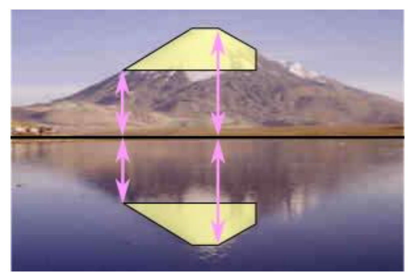


1. Translation or slide moves a shape in a given direction by sliding it up, down, sideways or diagonally.

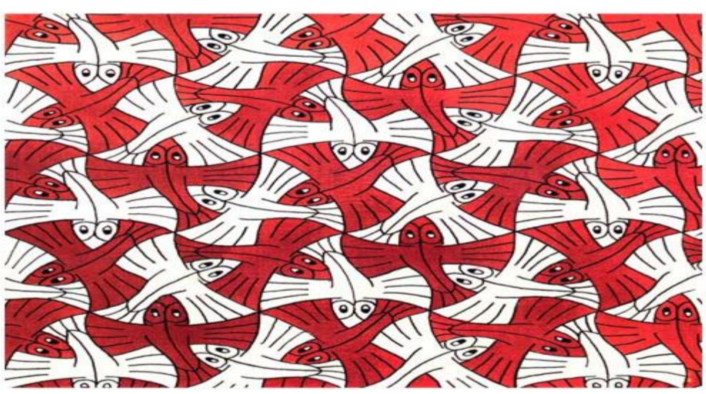


2. A reflection (or a flip) can be thought of a getting a mirror image. It has a line of reflection or mirror line where the distance between the image and the mirror line is the same as that between the original figure and the mirror line.

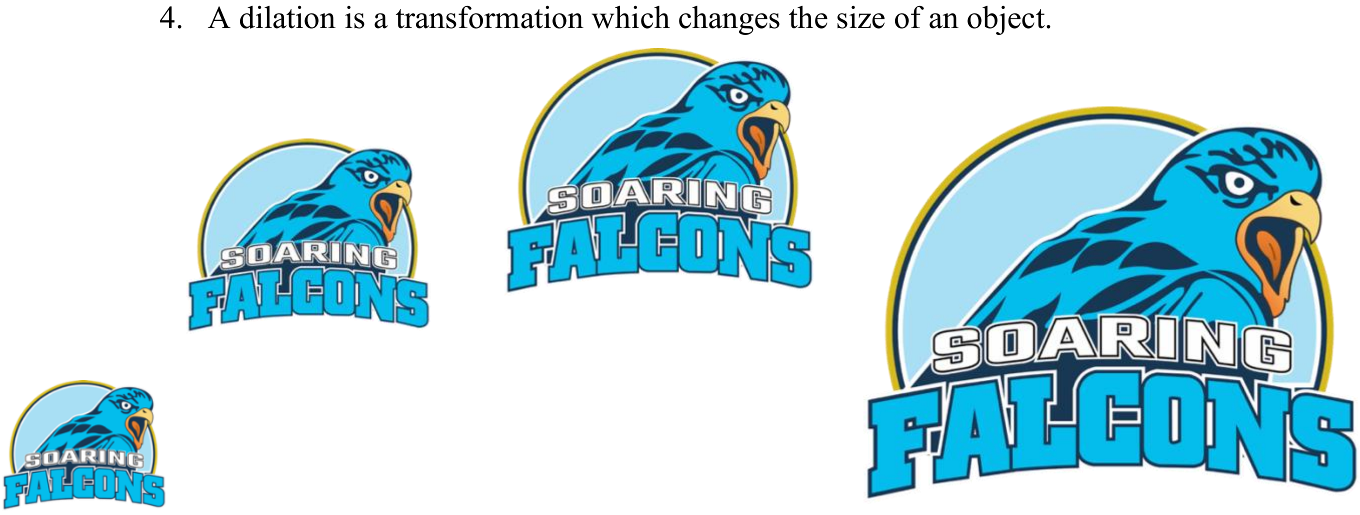




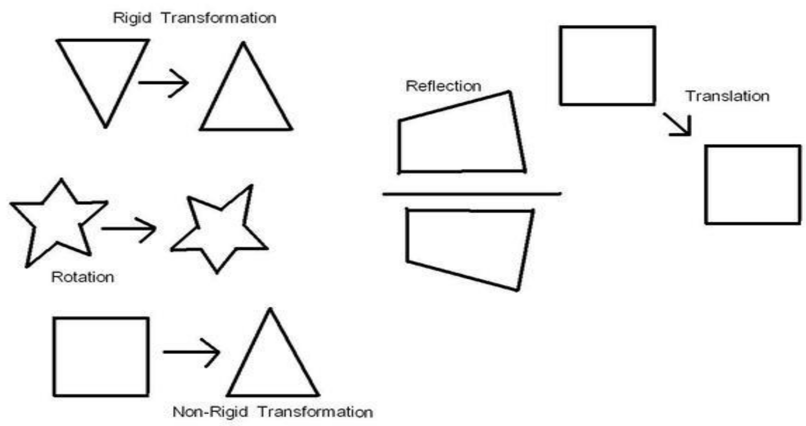
3. Rotation (or a turn) has a point about which the rotation is made and an angle that says how far to rotate.



4. A dilation is a transformation which changes the size of an object.



(5). Rigid Transformation



Note: An isometry of the plane is a mapping that preserves distance (and therefore shape):

$$d\,(f\,(x)\,,f\,(y)\,) = d\,(x,y)$$

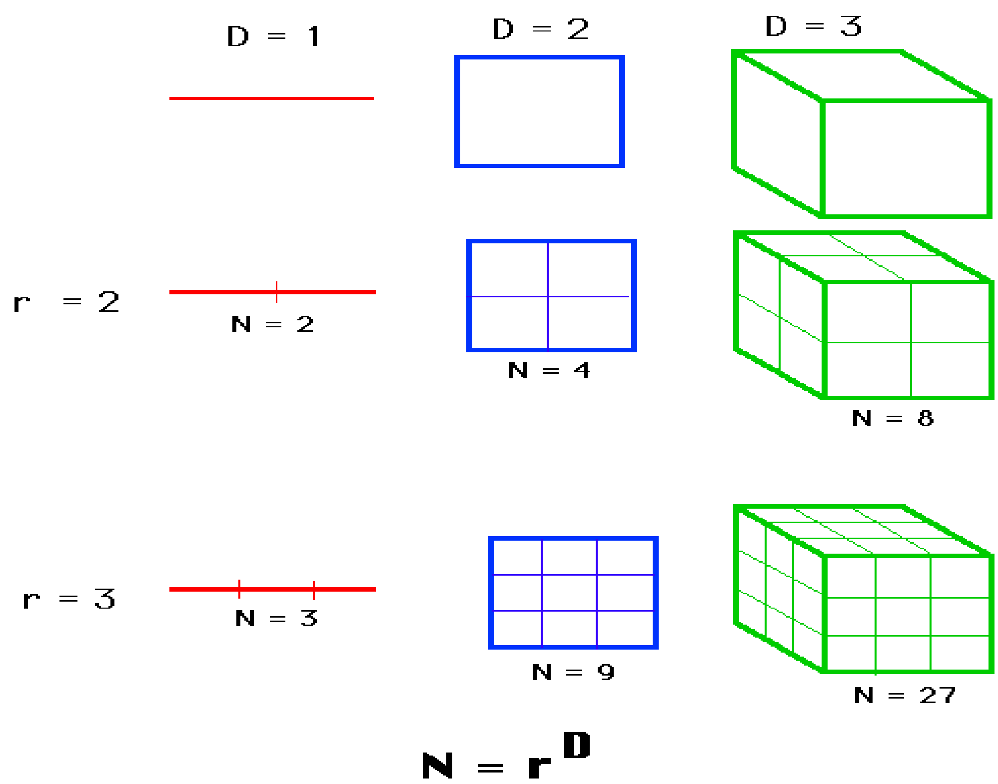
by studying patterns in mathematics, humans become aware of patterns in our world. Pattern observation makes an individual develop their ability to predict future behavior of natural organism (biology) and phenomena. Civil engineers can use their observations of traffic patterns to construct safer cities. Meteorologist use patterns to predict thunderstorms, tornadoes and even hurricanes. Seismologist use patterns to forecast earthquakes and landslides. Mathematical patterns are useful in all areas of science.

FRACTALS

A *fractal* is an object or quantity that displays self-similarity, in a somewhat technical sense, on all scales. Fractals need not exhibit *exactly* the same structure at all scales, but the same "type" of structures must appear on all scales.

What do we mean by **dimension**? Consider what happens when you divide a line segment in two on a figure. How many smaller versions do you get?

Consider a line segment, a square and a cube.



SELF-SIMILARITY

An object is **self-similar** if it can be formed from smaller versions of itself (with no gaps or overlap). A square is self-similar, a circle is not. Many objects in nature have self-similarity.

SELF-SIMILAR FRACTALS

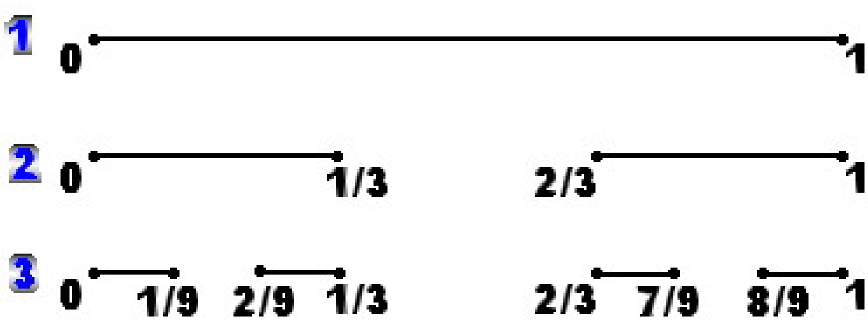
Start with some basic geometrical object like a line segment or triangle and perform some operation. Then repeat the process indefinitely (this is called iterating). Each iteration produces a more complicated object.

The fractal dimension D can be found by considering the scaling at each iteration, where r is the scaling amount and N is the number of smaller pieces.

$$r^D = N \text{ so } D = \ln N/\ln r$$

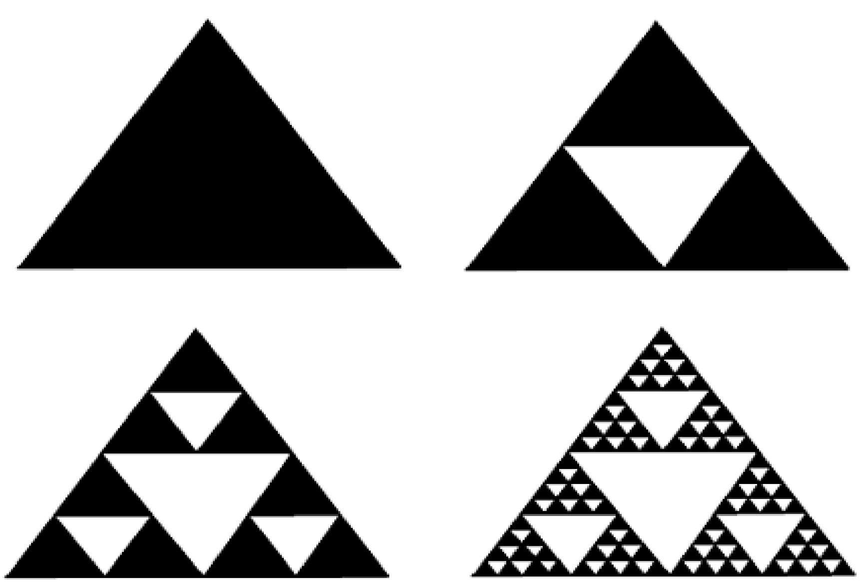
CANTOR SET

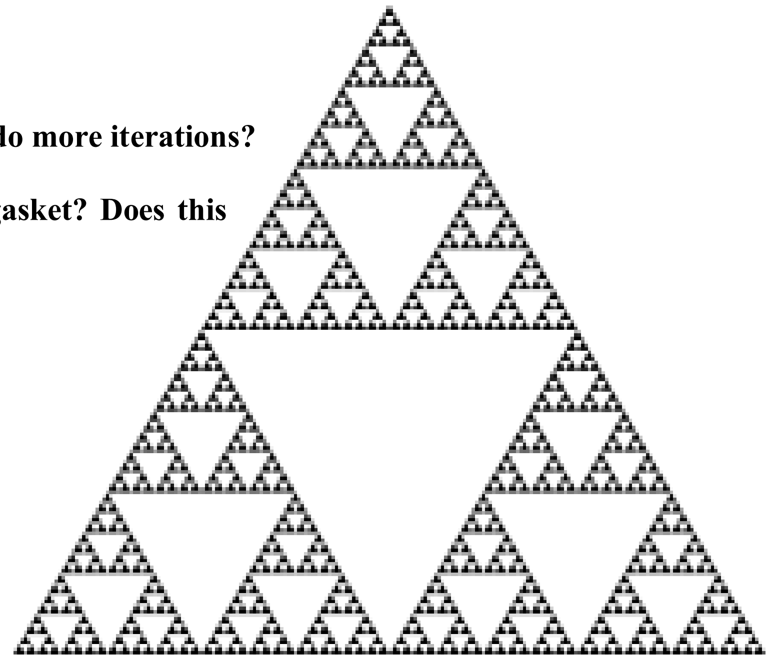
Start with the line segment of length 1 between 0 and 1. Remove the middle third segment. Repeat this process to the remaining two line segments. At each iteration you scale down by 3 to get 2 new pieces. What is the fractal dimension?



SIERPINSKI GASKET

Start with an equilateral triangle. Divide each side in half and remove the middle triangle. Repeat this process indefinitely.

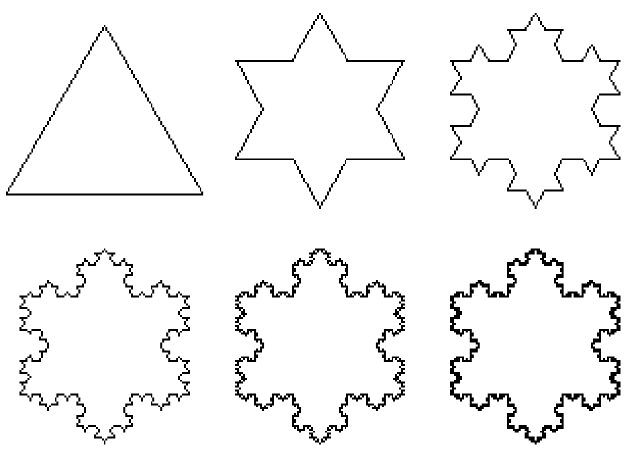


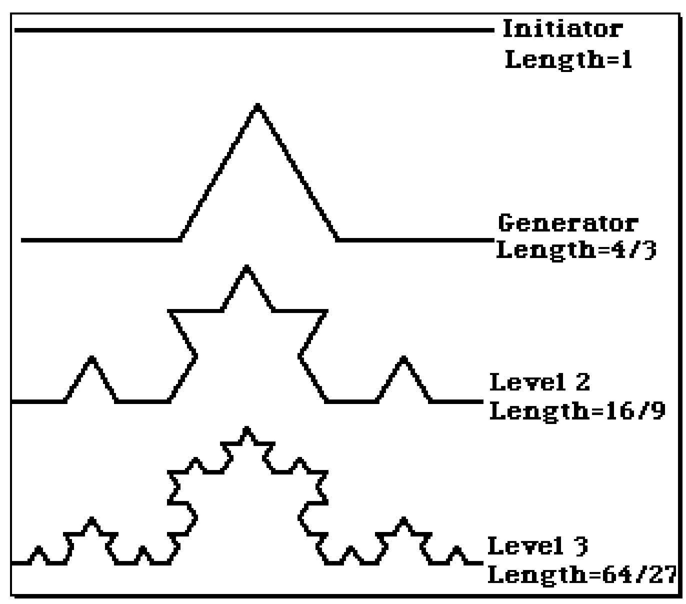


1. What happens to the perimeter as you do more iterations?
2. What about area?
3. What is the fractal dimension of the gasket? Does this make sense?

KOCH SNOWFLAKE

Start with equilateral triangle. Iteration rule:





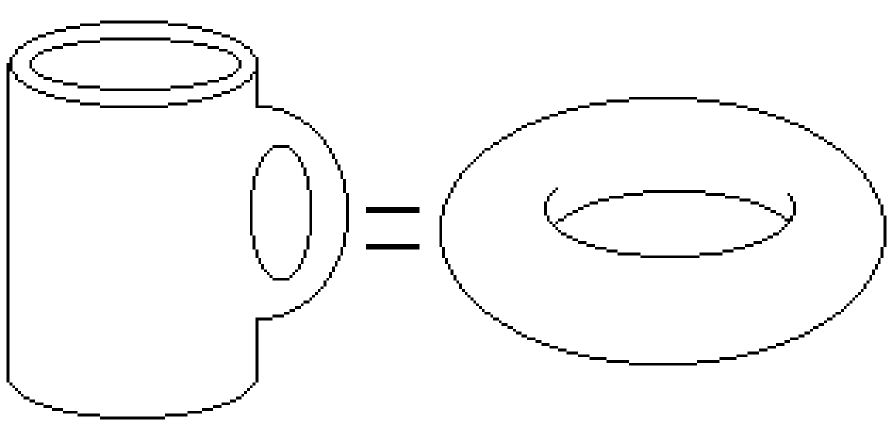
1. What happens to the perimeter? Area?
2. What is the fractal dimension?

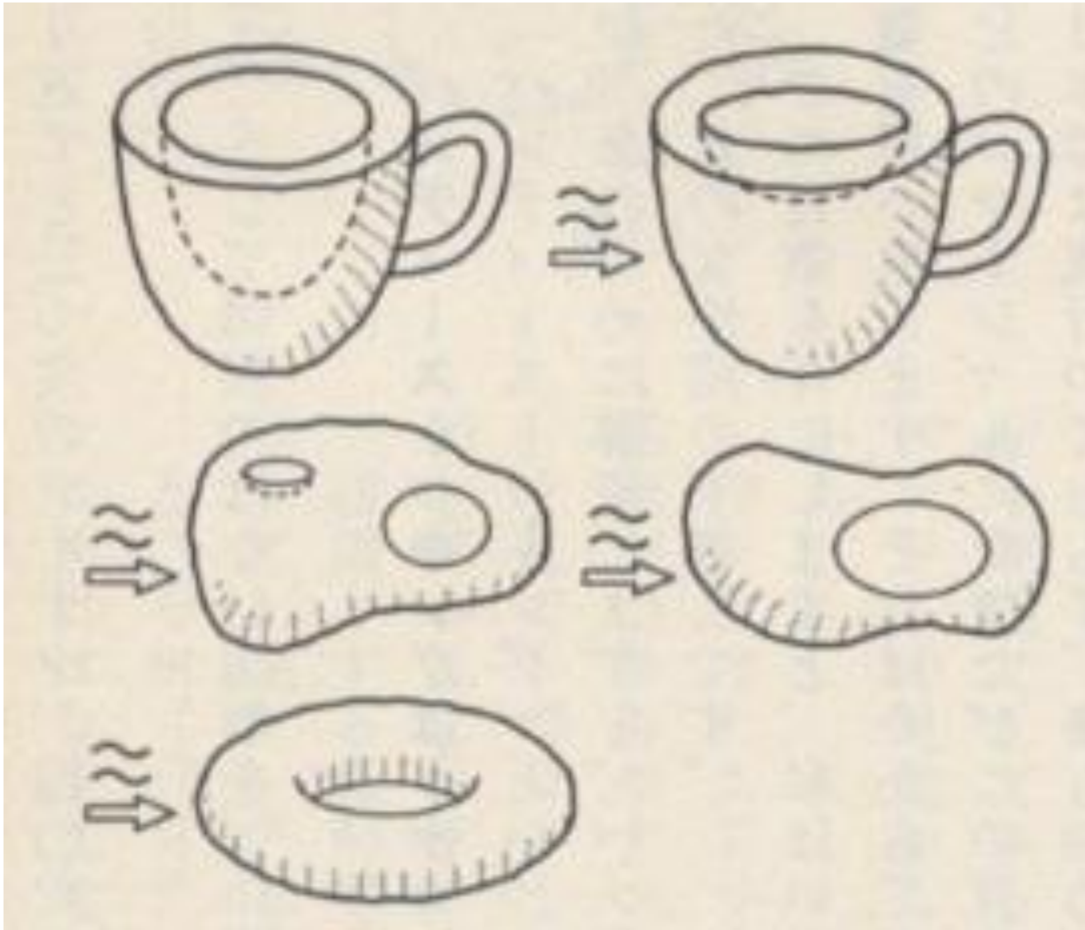
TOPOLOGY

Suppose we could study objects that could be stretched, bent, or otherwise distorted without tearing or scattering. This is topology (also known as "rubber sheet geometry"). Topology investigates basic structure like number of holes or how many components.

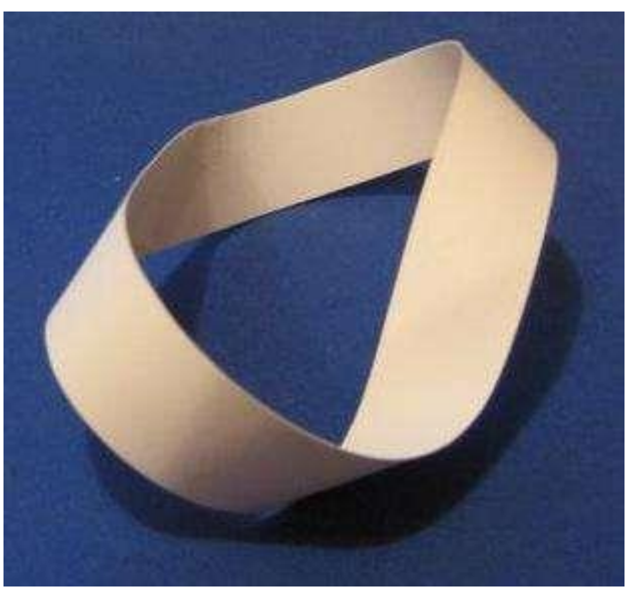
Topologically equivalent

- A donut and a coffee cup are equivalent while a muffin and coffee cup are not.





Interesting Topological Surfaces



Moebius Strip

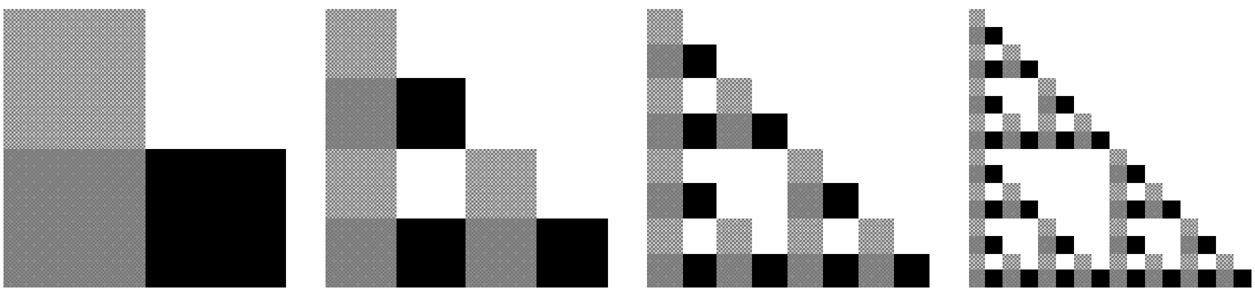
Klein Bottle

ORIENTABILITY AND GENUS

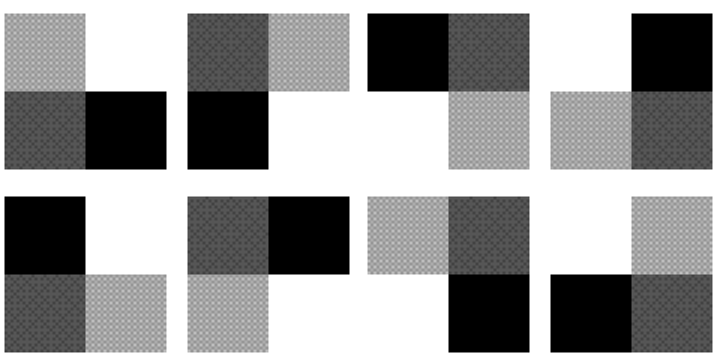
A topological surface is orientable if you can determine the outside and inside.
Any orientable, compact (finite size) surface is determined by its number of holes (called the genus).

SIERPINSKI RELATIVES

The Sierpinski Gasket (right triangle version) can be obtained via the three maps which map the unit square to three smaller squares, and repeating those maps ad infinitum:

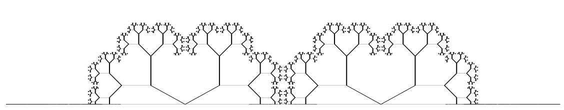


Could include symmetries of the square:
4 rotations (a = 0, b =90, c=180, d= 270)
4 reflections (e = horizontal, f = vertical, g = diagonal top right, h = diagonal top left)

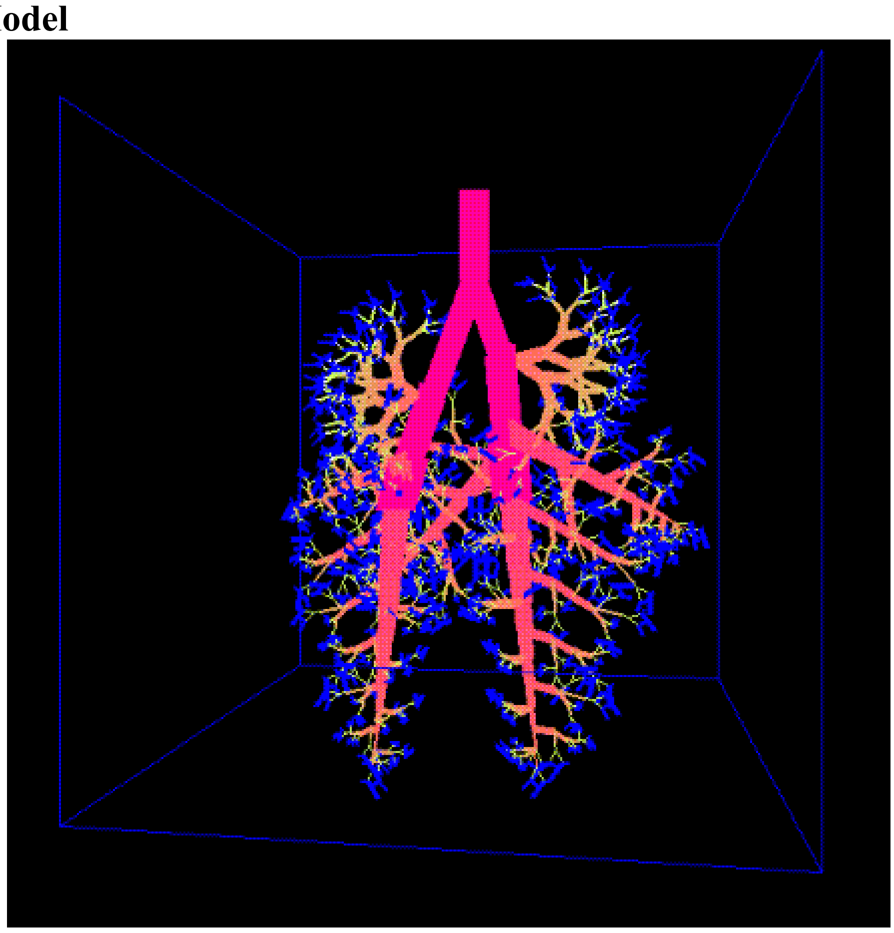


FRACTAL TREES

Compact, connected subsets that exhibit some kind of branching pattern. There are different types of fractal trees. Many natural systems can be modeled with fractal trees.



Rat Lung Model

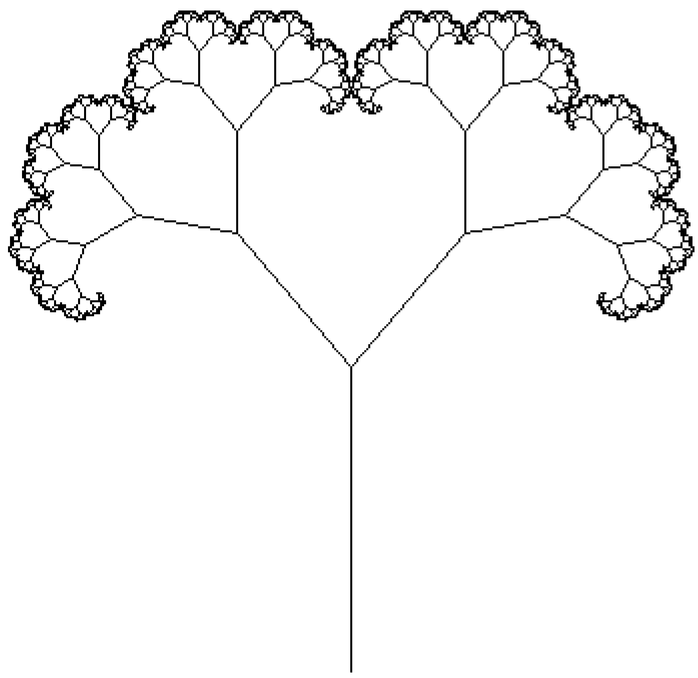


SYMMETRIC BINARY FRACTAL TREES

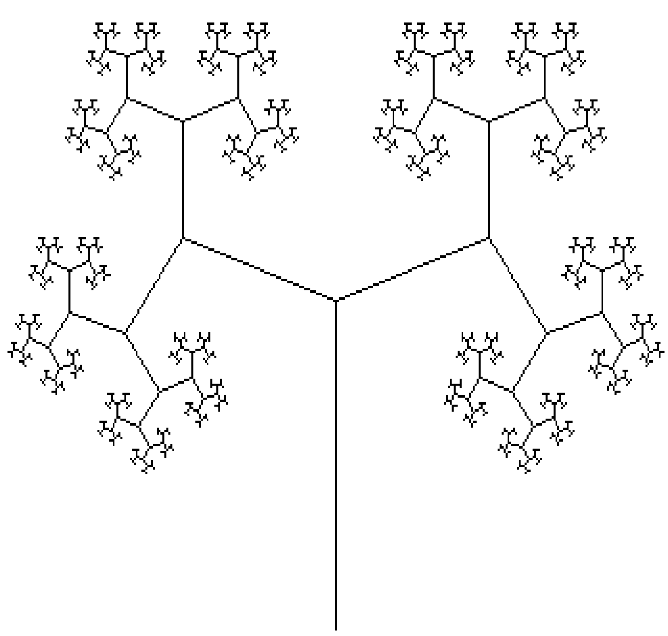
$T(r,\theta)$ denotes tree with scaling ratio r (some real number between 0 and 1) and branching angle θ (real-valued angle between 0º and 180º). Trunk splits into 2 branches, each with length r, one to the right with angle θ and the other to the left with angle θ.
Level k approximation tree has k iterations of branching.

- A symmetric binary tree can be seen as a representation of the free monoid with two generators
- Two generator maps m_R and m_L that act on compact subsets
- Addresses are finite or infinite strings with each element either R or L.

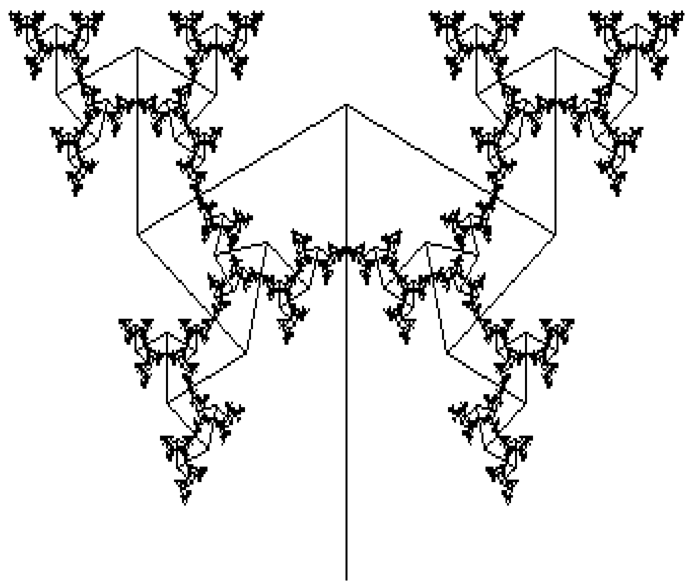
Examples:



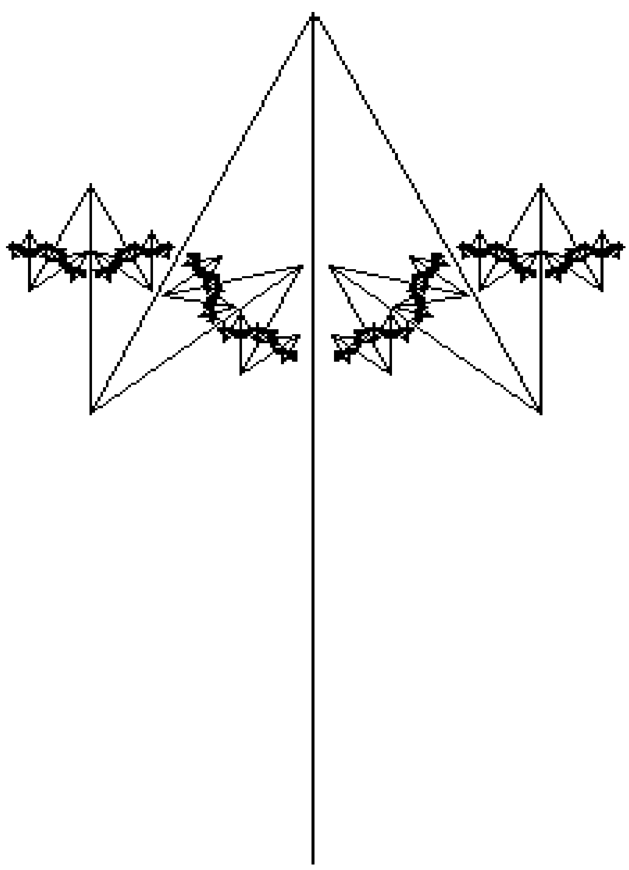
$T(.55, 40º)$



$T(.6, 72º)$



$T(.615, 115^{o})$



$T(.52, 155^{o})$

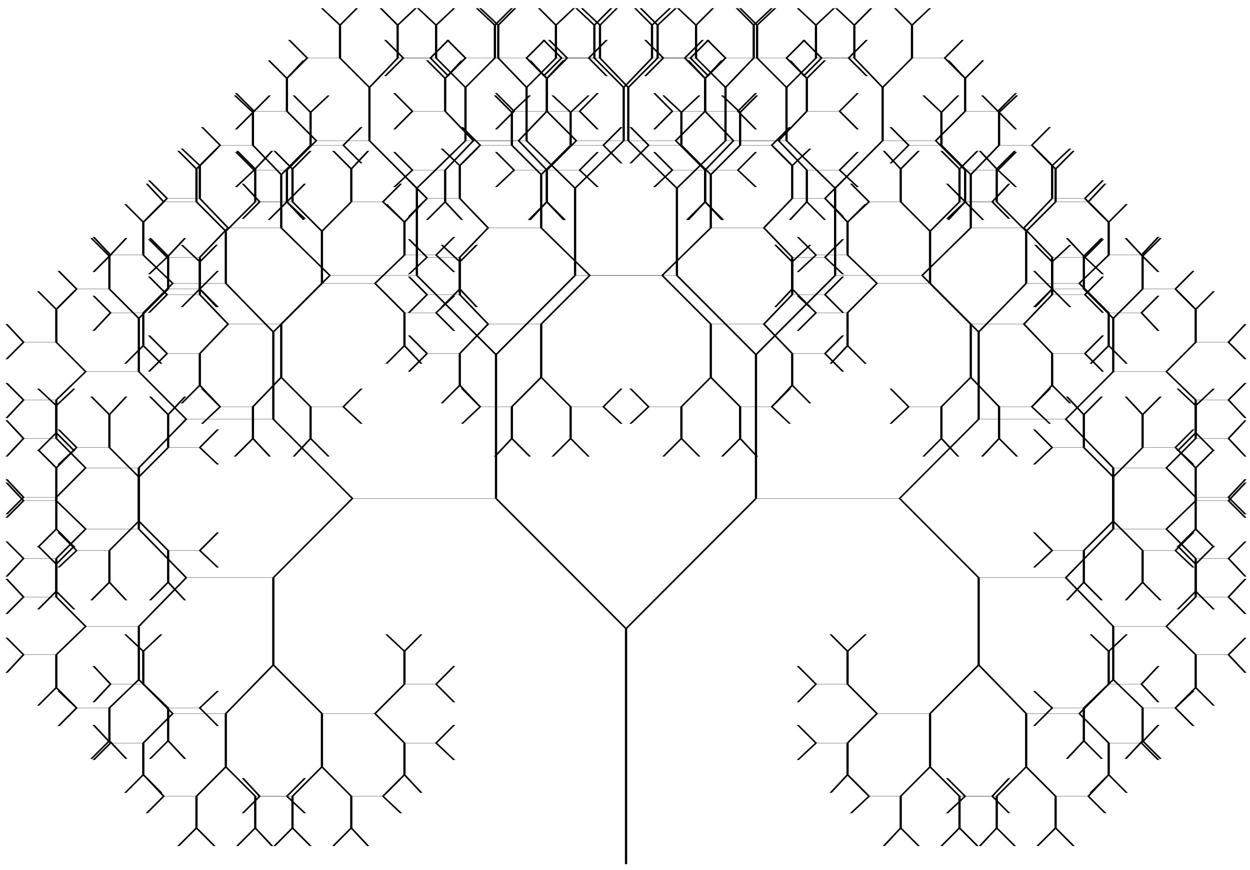
SELF-CONTACT

For a given branching angle, there is a unique scaling ratio such that the corresponding symmetric binary tree is "self-contacting". We denote this ratio by r_{sc}. This ratio can be determined for any symmetric binary tree.

If $r < r_{sc}$, then the tree is self-avoiding.

If $r > r_{sc}$, then the tree is self-overlapping.

OVERLAPPING TREE



$T(r_{sc}, 120°)$

What is the self-contacting scaling ratio for the branching angle 120°?
It must satisfy:

$$1 - r_{sc} - r_{sc}^2 = 0$$

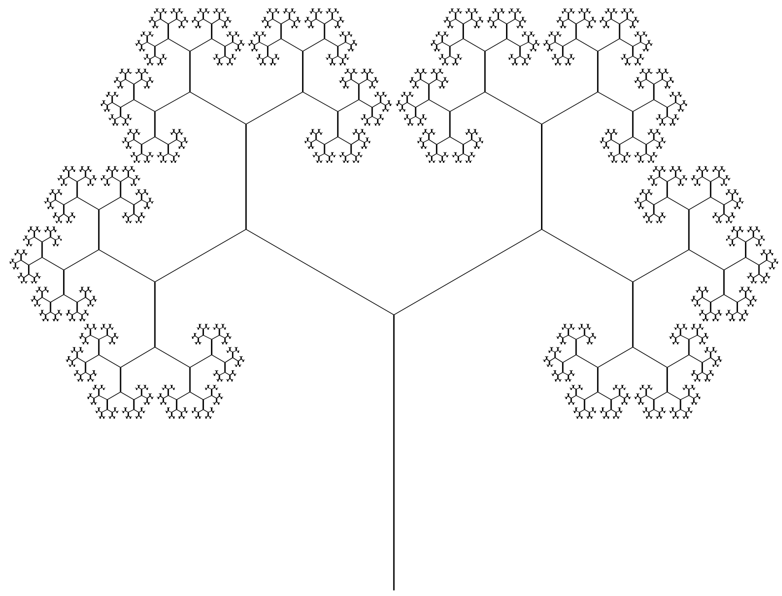
Thus

$$r_{sc} = (-1 + \sqrt{5})/2$$

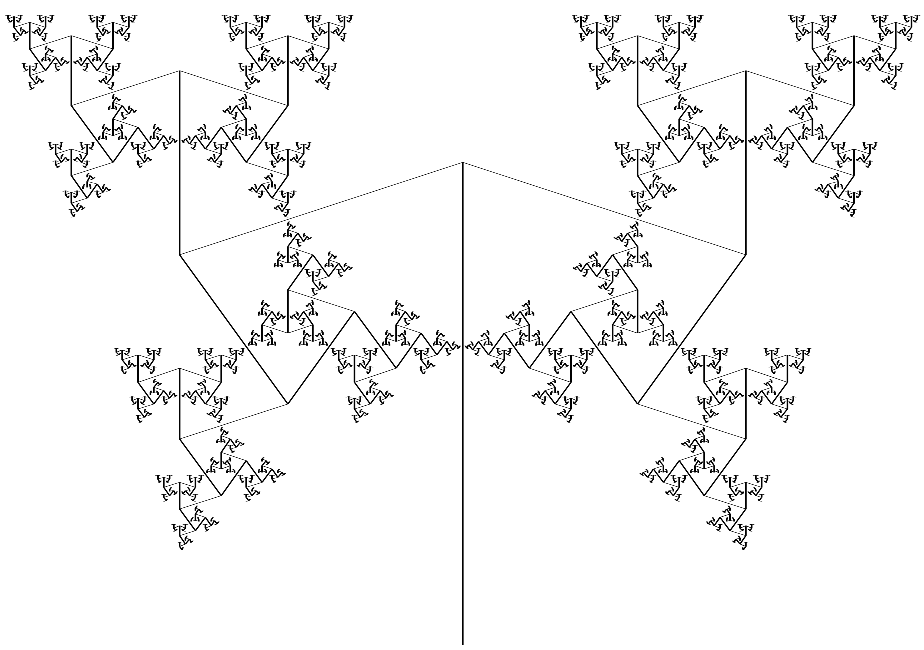
THE GOLDEN TREES

Four self-contacting trees have scaling ratio $1/\Phi$
Each of these trees possesses extra symmetry, they seem to "line up"
The four angles are 60°, 108°, 120° and 144°

GOLDEN 60

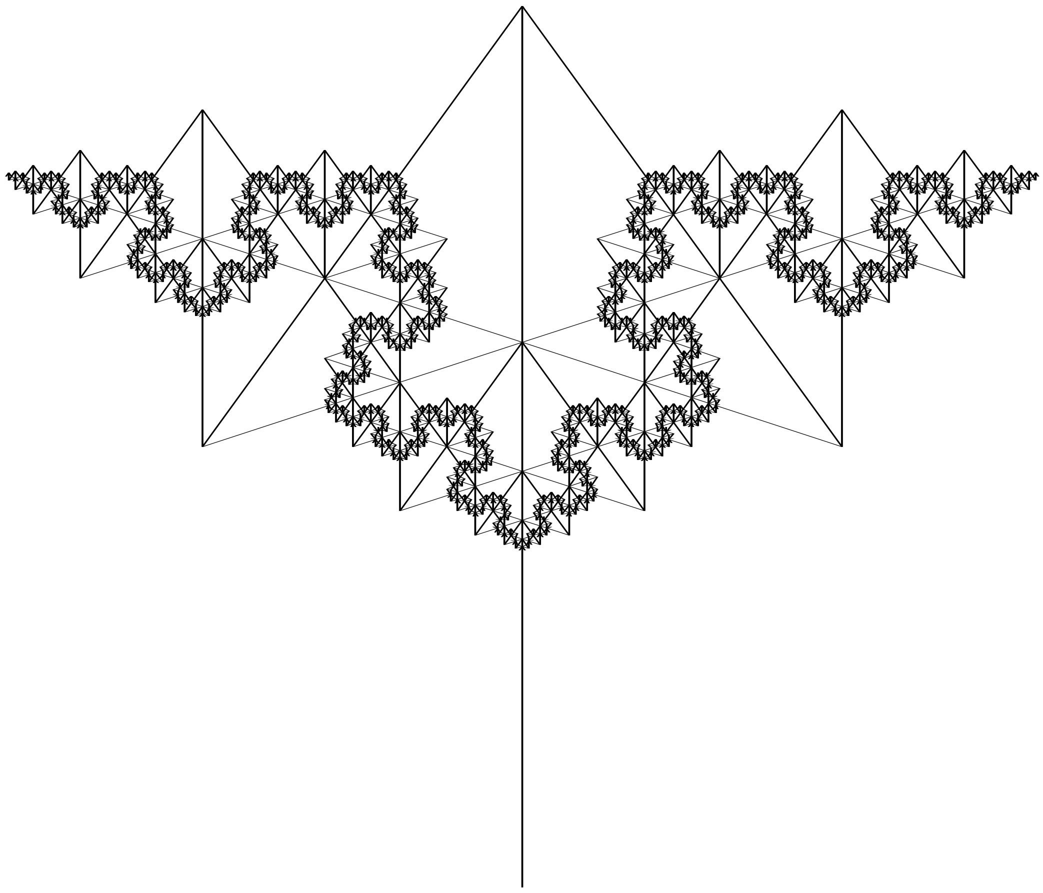


GOLDEN 108



GOLDEN 120

GOLDEN 144



V. SUMMARY

Patterns exist in nature with different types, either in color, shape, number and object repetition. There are four types of pattern namely; Logic patter, number pattern, geometrical pattern and word pattern. Isometry is a way of transforming the plane that preserves geometrical properties such as length. There are also four types of isometry or transformation according to Euclidian isometry; translation, reflection, rotation and dilation. Sometimes rigid transformation is added if there is a combination of three transformation that is translation, reflection and rotation.

VI. ENRICHMENT VIDEOS

https://www.youtube.com/watch?v=J7An1mcFHBU
https://www.youtube.com/watch?v=o68FAFj04Vg
https://www.youtube.com/watch?v=ax8mLKp6ouU
https://www.youtube.com/watch?v=gB9n2gHsHN4
https://www.youtube.com/watch?v=XwWyTts06tU

VII. **REFERENCE**:

http://www.learner.org/teacherslab/math/patterns/number.html

http://www.scottcamazine.com/personal/DesignNature/

Natures Number – Ian stewart