Data Structures (IT205)
Final Exam

25^{th} November, 2011

Time: 2 hours **marks: 80**

1. (a) Consider a set of n elements stored on a stack S_1. In addition, you are provided two empty stacks S_2 and S_3. Write routines for:
   - EXCHANGE(i, j) which exchanges the positions of element i and element j, leaving the others in their original positions. (the other elements may move during the course of the algorithm, but must eventually return to their original positions on the stack).
   - REVERSE(i, j) which reverses the elements occurring between positions i and j inclusive and leaves the other elements undisturbed (at the end). In both cases, $1 \leq i < j \leq n$.

   (b) Repeat the above two codes for n elements stored in a queue Q_1 where you have recourse to two other empty queues Q_2 and Q_3.

   You may not use any other memory and only use the operations of PUSH, POP, ENQUEUE, DEQUEUE. Analyse the running times of each of your algorithms in terms of the total number of elementary operations (PUSH, POP, ENQUEUE, DEQUEUE) required in the worst case, as a function of n, i, j.

2. For the following graph represented by the adjacency matrix below, simulate the working of Breadth First Search (BFS) starting from vertex A. In particular,

   (a) Draw the graph and draw the BFS tree

   (b) list the cross edges and specify which of the cross edges are horizontal and which are vertical.

   (c) Compute the maximum size of the queue (in terms of number of nodes it holds) during the entire course of the algorithm's execution on this input graph.

   (d) Is there any cycle, using **only** cross edges? Specify such a cycle by listing its vertices in cyclic order and its length.

Table 1: $Q2$

	A	B	C	D	E	F	G	H	I	J
A	0	1	0	0	1	1	0	0	0	0
B	1	0	1	0	0	0	1	0	0	0
C	0	1	0	1	0	0	0	1	0	0
D	0	0	1	0	1	0	0	0	1	0
E	1	0	0	1	0	0	0	0	0	1
F	1	0	0	0	0	0	0	1	1	0
G	0	1	0	0	0	0	0	0	1	1
H	0	0	1	0	0	1	0	0	0	1
I	0	0	0	1	0	1	1	0	0	0
J	0	0	0	0	1	0	1	1	0	0

3. Consider the Depth First Search (DFS) of a connected undirected graph G on n vertices.

   (a) Specify the smallest and largest values of $f[u] - d[u]$ the difference between finishing and discovery times of a vertex.

   (b) If vertex r is the root of the DFS-Tree and has 4 children in the DFS-Tree, then assuming that the graph represents a communication network, if the node r fails or is shut down for maintenance, into how many pieces is the rest of the network broken.

   (c) What if r is NOT the root of the DFS tree?

   [Hint: this can be solved using the concept of cut vertices and number of components]

4. (a) Draw the graph represented by the following weighted adjacency matrix.

   (b) How many minimum spanning trees does it have?

   (c) List the edge sets of each of its MSTs.

   (d) Specify the number of edges in the union of these individual edge sets.

Table 2: $Q4$

	A	B	C	D	E	F
A	0	1	0	0	0	6
B	1	0	7	0	0	5
C	0	7	0	4	2	0
D	0	0	4	0	3	0
E	0	0	2	3	0	7
F	6	5	0	0	7	0