Turing Machine

CFG $\rightarrow$ Pushdown Automata

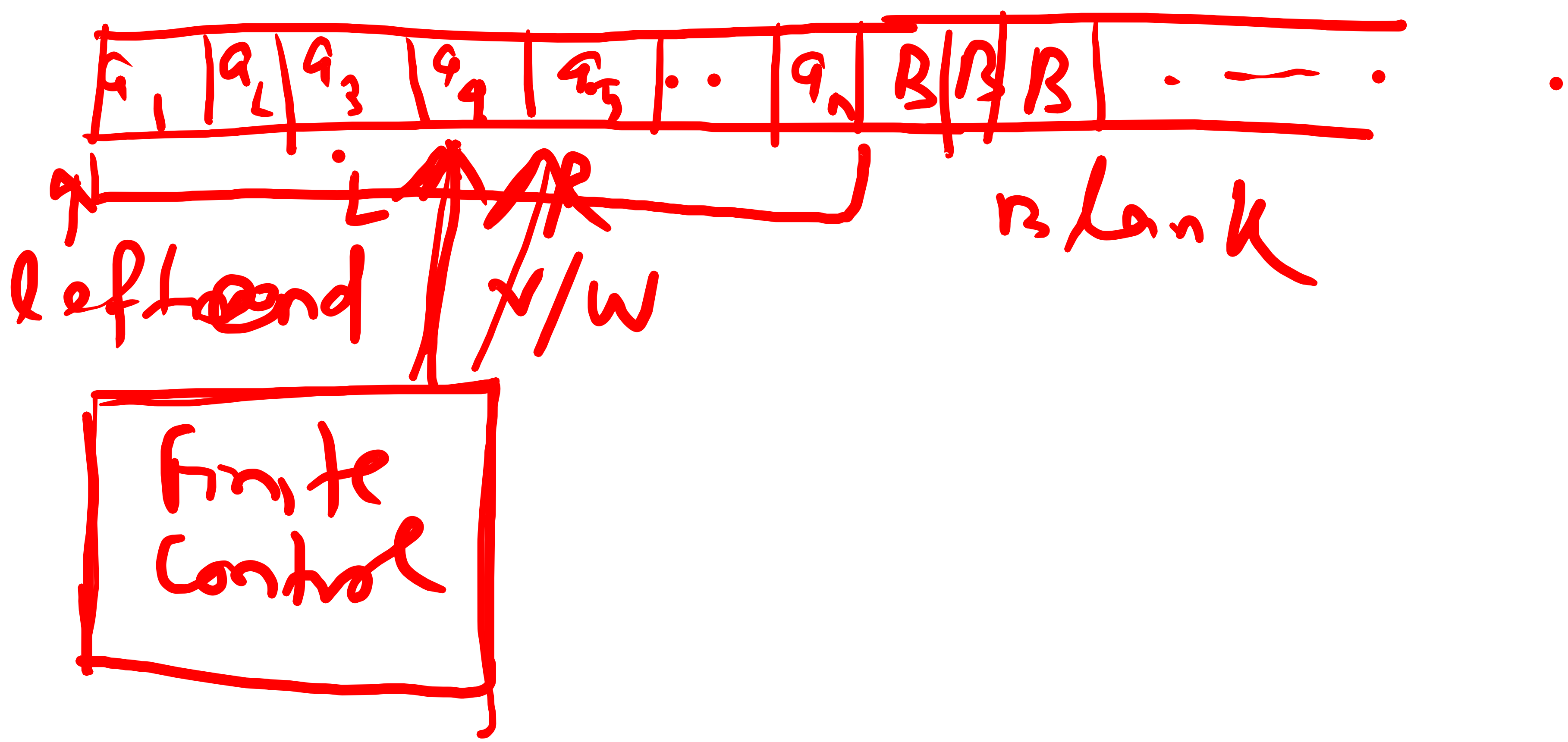
CFL $\rightarrow$

Turing Machine

Recursively enumeran

Basic Model

Turing Machine



TM versus FA

1. TM both read/write
   FA only read
2. FM read/write head
   can move left or right
   FA can only move in one direction

Def^n

A Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

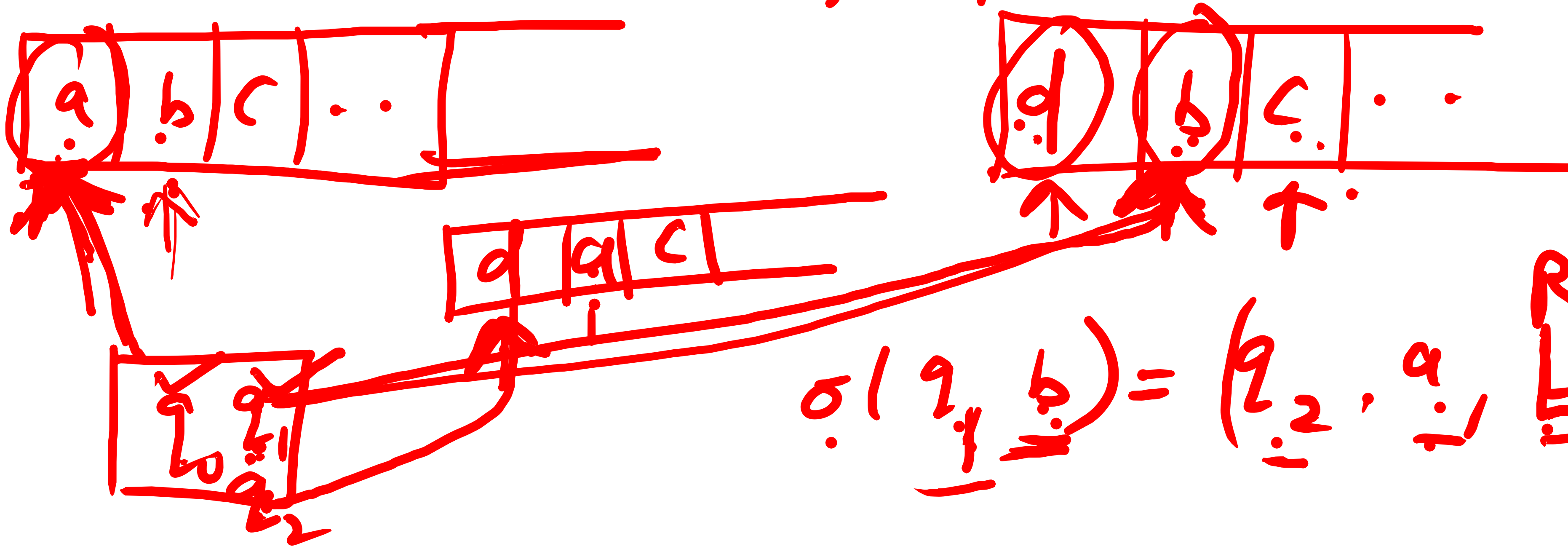
1. Q finite set of states
2. Γ tape alphabet
3. $B \in \Gamma$, B is the blank symbol
4. $\Sigma \subseteq \Gamma$, $B \notin \Sigma$, Σ set of input symbol
5. $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$
6. $q_0 \in Q$ initial state
7. $F \subseteq Q$ set of final state

1. $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$

1. if $\delta(q, a) = (r, b, L)$

2. $\delta(q, a) = (r, b, R)$

$\delta(q, a) = (q, d, R)$



$\delta(q_1, b) = (q_2, a, L)$

Instantaneous Description

Denote ID of M $\quad \alpha_1 \, q \, \alpha_2$

$\alpha_1 \quad \alpha_2 \in \Gamma^*$

q state

$X_1 X_2 \cdots X_{i-1} \, q \, X_i \cdots X_n$ be an ID

$\delta(q, X_i) = (p, Y, L)$

$X_1 X_2 \cdots X_{i-1} \, q \, X_i \cdots X_n \vdash X_1 X_2 \cdots X_{i-2} \, p \, X_{i-1} \, Y \, X_{i+1} \cdots X_n$

$$\delta(q, X_i) = (p, Y, L)$$

$$X_1 X_2 \cdots X_{i-1}\, q\, X_i X_{i+1} \cdots X_n \vdash X_1 X_2 \cdots p\, X_{i-1}\, \overset{\cancel{X_i}}{Y}\, X_{i+1} \cdots X_n$$

p

ID

If $\delta(q, X_i) = (p, Y, R)$

$$X_1 X_2 \cdots X_{i-1}\, q\, X_i X_{i+1} \cdots X_n \vdash X_1 \cdots X_{i-1}\, Y\, p\, X_{i+1} \cdots X_n$$

p

$L = \{0^n 1^n \mid n \geq 1\}$

2. $\overset{x}{\cancel{0}}\ \overset{x}{\cancel{0}}\ \overset{x}{\cancel{0}}\ \overset{Y}{\cancel{1}}\ \overset{Y}{\cancel{1}}\ \overset{Y}{\cancel{1}}$ B B B . . . $0^3 1^3$

$\overset{x}{\cancel{0}}\ \overset{x}{\cancel{0}}\ \overset{x}{\cancel{0}}\ \overset{Y}{\cancel{1}}\ \overset{Y}{\cancel{1}}\ \overset{Y}{\cancel{1}}\ 1$ B B B . . . $0^3 1^4$ X

$\overset{x}{\cancel{0}}\ \overset{x}{\cancel{0}}\ \overset{x}{\cancel{0}}\ \overset{x}{\cancel{0}}\ \overset{Y}{\cancel{1}}\ \overset{Y}{\cancel{1}}\ \overset{Y}{\cancel{1}}$ B B B . . . X

$0^4 1^3$

Turing Machine for functions

Represent integers by unary notation

i represented as 0^i

$5 \qquad 00000$

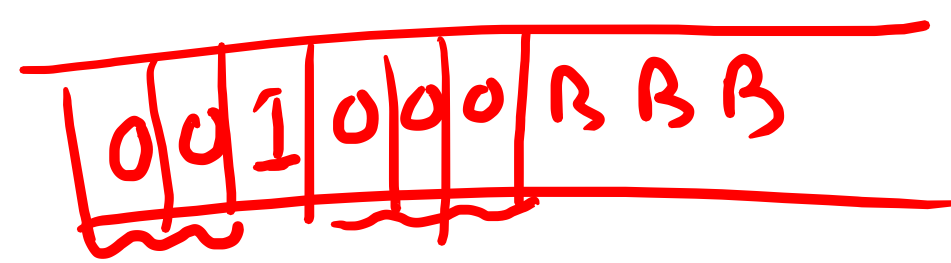
$7 \qquad 0000000$

$f(m, m$

$f(\overset{m_1}{2}, \overset{m_2}{3}) = 5$

$f(2, 5, 7)$

$0^2 \, 1 \, 0^5 \, 1 \, 0^7 \, BBB\ldots$



$L = \{ 0^n 1^n \mid n > 0 \}$

$\delta(q_0, 0) = (q_1, X, R)$

	0	1	X	Y	B
q_0	(q_1, X, R)			(q_1, Y, R)	
q_1	$(q_1, 0, R)$	(q_2, Y, L)		(q_1, Y, R)	
q_2	$(q_2, 0, L)$		(q_0, X, R)	(q_2, Y, L)	
q_3				(q_3, Y, R)	q_4, B, R
q_4					

$q_0\ 0\ 0\ 0\ 1\ 1\ 1 \vdash X\ q_1\ 0\ 0\ 1\ 1\ 1 \vdash X\ 0\ q_1\ 0\ 1\ 1\ 1 \vdash X\ 0\ 0\ q_1\ 1\ 1\ 1$

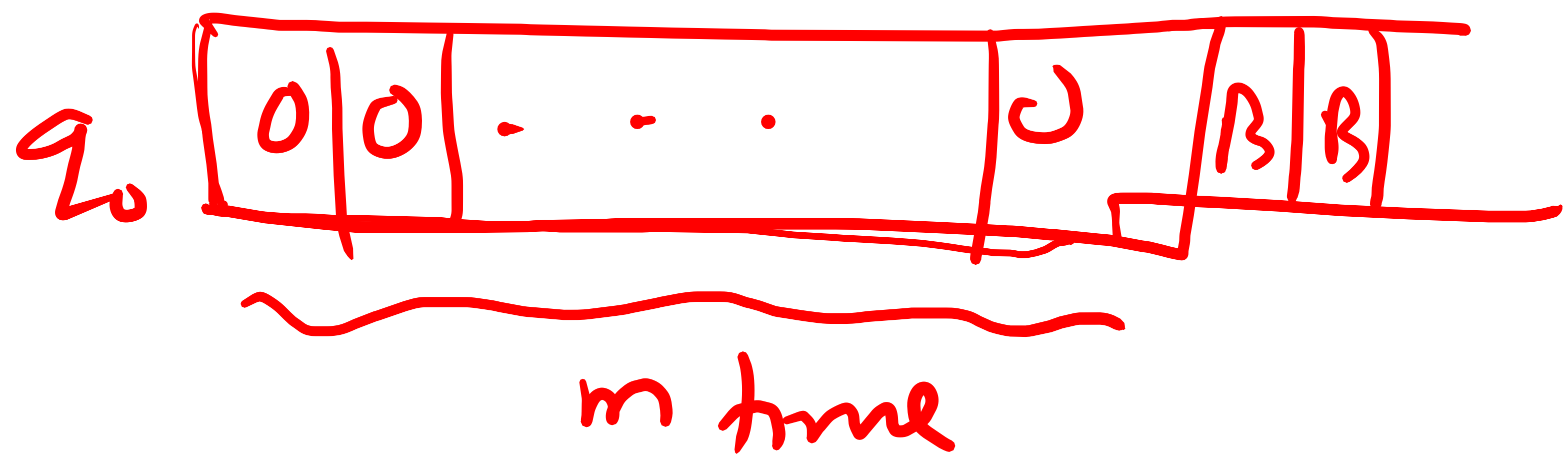
$\vdash X\ 0\ q_2\ 0\ Y\ 1\ 1 \vdash X\ q_2\ 0\ 0\ Y\ 1\ 1$

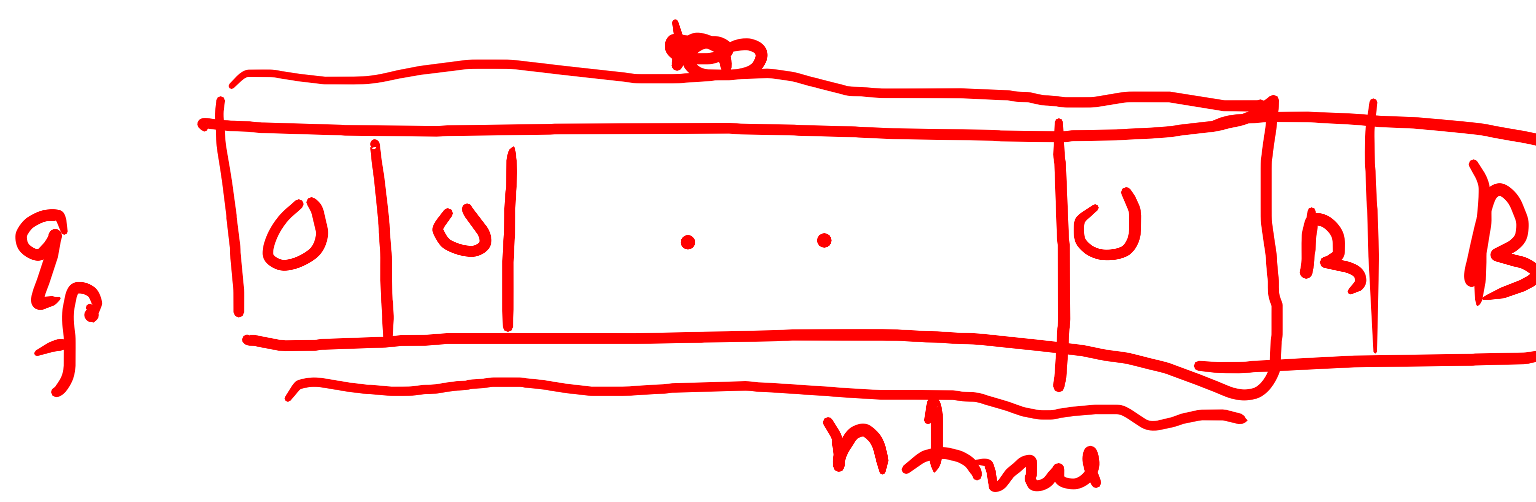
$\vdash q_2\ X\ 0\ 0\ Y\ 1\ 1 \vdash X\ q_0\ 0\ 0\ Y\ 1\ 1$

$f(x) = 2^x$

$f(m) = n$

0^m





$$f(i_1, i_2 \ldots i_m) = m$$

$$q_0\, 0^{i_1} 1\, 0^{i_2} 1 \cdots 1\, 0^{i_m}\, BB \cdots$$

$$q_f\, 0^m\, BB \cdots$$

Addition

$$f(m,n) = \underline{m+n}$$

$~~2+3~~$

$$f(2,3) = 2+3 = 5$$

ID $q_0 0 0 1 0 0 0$

	0	1	B
q_0	$(q_1, 0, R)$		
q_1	$(q_1, 0, R)$	$(q_2, 0, R)$	
q_2	$(q_2, 0, R)$		(q_3, B, L)
q_3	(q_4, B, R)		

q_0 0 0 ~~1~~ 0 0 ~~0~~ B B . . .

q_f

$\underbrace{0\,0\,0\,0\,0}$ B B B .

$\vdash 0\, q_1\, 0\, 1\, 0\, 0\, 0$

$\vdash 0\, 0\, q_1\, 1\, 0\, 0\, 0\, B \vdash 0\, 0\, 0\, q_2\, 0\, 0\, 0\, B$

$\vdash 0\, 0\, 0\, 0\, q_2\, 0\, 0\, B \vdash 0\, 0\, 0\, 0\, 0\, q_2\, 0\, B$

$\vdash 0\, 0\, 0\, 0\, 0\, 0\, q_2\, B \vdash 0\, 0\, 0\, 0\, 0\, q_3\, 0\, B$

$\vdash \underbrace{0\,0\,0\,0\,0}\, R\, q_4\, R$