Guassian elimination

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For this, and the next few lectures, the focus is on **square** matrices. That is, matrices with the number of rows equal to the number of columns. Thus A of dimensions $n \times n$.

The **coefficients matrix** for a system of n linear equations in n variables is an $n \times n$ matrix. If we append an extra column consisting of the constants on the right hand side of the equations, we get a matrix of dimensions $n \times n + 1$, called the **augmented mateix**.

We will describe the **Guassian elimination** algorithm, now.

1. Proceed column-wise, starting from column 1 and ending in column $n - 1$.

2. In the i^{th} column, start processing from row number $i+1$. It is assumed that the entry $A[i, i]$ is non-zero. If this is not, then exchange this row with a row below, that has a non-zero entry on the i^{th} column.

3. For each row below row i, which has a non-zero entry on column i multiply the entire row by the factor $\frac{A[i,i]}{A[t,i]}$, where t is the row number. Now subtract entry wise, the row i from this row.

The algorithmic complexity of this algorithm is $\theta(n^3)$.

We describe the resulting matrix by the term **upper triangular matrix**.

An upper triangular matrix is one where all entries below the diagonal are 0. A lower triangular matrix is one where all entries above the diagonal are 0. It is known that every invertible square matrix has an LU decomposition. That is, the matrix can be written as the product of an upper triangular matrix and a lower triangular matrix.