

①

GIVEN $\vec{G} = (V, E)$

- $s \in V$ is the start vertex.
- $\forall u \in V$: $D[u]$ is the length of the best path we have found so far from s to u
- $\forall v_1, v_2 \in V$: $d(v_1, v_2)$ is the length of the shortest distance from v_1 to v_2

(2)

Theorem : In Dijkstra's

Algo, whenever a vertex u is pulled into the cloud the label $D[u]$ is equal to $d[s, u]$.

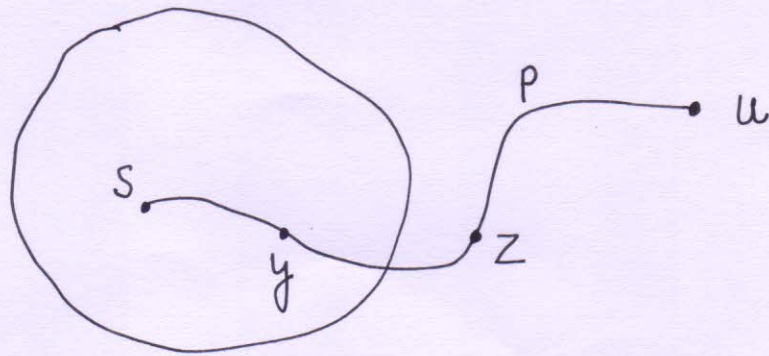
Proof: Proof by Contradiction

Say Not.

Let u be the 1st vertex which is pulled into the cloud s.t $D[u] > d(s, u)$.

③

The following Scenario should essentially exist when the vertex u is about to be pulled into the cloud.



- P is the shortest path from s to u .
- z is the 1st vertex on P that is not in the cloud C
- y is the predecessor of z on P [y could be s also]

(4)

- $D[y] = d(s, y)$
- $D[z] \leq D[y] + w(y, z)$
- $D[z] \leq d(s, y) + w(y, z)$
- $D[z] \leq d(s, z)$
- $D[z] \geq d(s, z)$
- $D[z] = d(s, z)$
- $D[u] \leq D[z]$
- $D[u] \leq d(s, z) < d(s, z) + d(z, u)$
 $= d(s, u)$
- $D[u] < d(s, u)$

A CONTRADICTION !