GREEDY ALGORITHMS

EXAMPLE: 1

COIN CHANGING PROBLEM

GIVEN US CURRENCY DENOMINATIONS :

1 CENT : PENNY
5 CENTS : NICKEL
10 CENTS: DIME
25 CENTS: QUARTER
100 CENTS: DOLLAR

WE HAVE STOCK OF ALL KINDS OF COINS.

PROBLEM

SUPPOSE YOU OWE SOMEONE AMOUNT x.
DEVISE A METHOD TO RETURN THE AMOUNT SUCH THAT YOU END UP PAYING AS FEW COINS AS POSSIBLE.

NOTE: THE ~~COI~~ COIN TYPES ARE $C_1, C_2, C_3, \ldots, C_n$

WHERE $0 < C_1 < C_2 < C_3 \ldots < C_n$

ALGO(x, $0 < c_1 < c_2 \ldots < c_n$)

MULTISET $S \leftarrow \phi$

WHILE $x > 0$

IF c_k IS THE LARGEST COIN s.t $c_k \leq x$ THEN

$S = S \cup \{c_k\}$

$x = x - c_k$

END IF

END WHILE

RETURN S

- AN OPTIMAL ALGO IS ONE WHICH RETURNS AS FEW COINS AS POSSIBLE.

- IS THE DESCRIBED ALGO OPTIMAL. FOR THE US CURRENCY.

LEMMA :

ANY OPTIMAL SOLUTION WILL NOT HAVE MORE THAN 4 PENNIES.

LEMMA :

ANY OPTIMAL SOLUTION WILL NOT HAVE MORE THAN 1 NICKEL

LEMMA :

ANY OPTIMAL SOLUTION WILL NOT HAVE MORE THAN 3 QUARTERS.

LEMMA :

ANY OPTIMAL SOLUTION WILL NOT HAVE MORE THAN 2 NICKEL PLUS DIME.

CONSTRAINTS

(1) $P \leq 4$

(2) $N \leq 1$

(3) $N + D \leq 2$

(4) $Q \leq 3$

(5) $\$ \leq \infty$

WHAT IS THE MAX AMOUNT YOU CAN PAY OPTIMALLY WITHOUT USING DOLLARS

LEMMA

If amount $x \geq 100$, then the optimal solution has to use dollars.

LEMMA :

If the amount $25 \leq x \leq 99$, then the optimal solution has to use quarters.

LEMMA :

If the amount $10 \leq x \leq 24$, then the optimal solution has to use Dimes.

LEMMA : If the amount $5 \leq x \leq 9$, then the optimal solution has to use Nickels.

THEOREM

THE PROPOSED ALGO IS OPTIMAL.

Proof : It satisfies all the lemmas just stated.

GREEDY ALGORITHMS

(1) A GREEDY ALGORITHMS SOLVES SOME OPTIMIZATION PROBLEM

(2) IT BUILDS SOLUTION IN SMALL STEPS.

(3) AT EACH STEP A DECISION IS TAKEN MYOPICALLY.

GREEDY ALGORITHMS

(1) THERE CAN BE MANY GREEDY ALGORITHMS FOR THE SAME PROBLEM.

(2) EACH ALGO WILL TRY TO OPTIMIZE SOME MEASURE ON ITS WAY TO FINAL SOLUTION.

GREEDY ALGORITHMS

(1) GREEDY ALGORITHMS MAY WORK IN SOME CASES ONLY.

(2) OUR JOB IS TO FIRST INVENT A GREEDY ALGO AND THEN PROVE IT TO BE CORRECT

PROBLEM

WILL THE GREEDY ALGORITHM WORK IF THE COINS ARE VALUED AS FOLLOWS:

(a) \$.30, \$.20, \$.05, \$.01

(b) \$.32, \$.08, \$.01

(c) \$.07, \$.08, \$.09