Course overview

August 4, 2022

We saw today an example of application of multiplication of matrices. The columns of a matrix could represent various ingredients used in bakery products, while the rows could represent different products baked using the ingredients. A particular cell represents the quantity of the ingredient corresponding to the column used in baking one unit of the product in that row. When this is multiplied by a column vector whose entries correspond to the unit cost of the various ingredients (listed in the same order as in the columns of the matrix), the result gives the cost of manufacturing one unit of the product. If instead of one list of prices, we have lists from various vendors, the right vector would also be replaced by a matrix.

We looked at linear equations in one two and three variables. These get increasingly complex. We observed that apart from being interesting in their own right. linear equations in one variable may arise while solving a problem of a system of linear equations in several variables.

A system of linear equations always has either 0, 1 or infinitely many solutions. A system which has zero solutions is said to be inconsistent. A system of equations is definitely inconsistent if one equation is obtained from another by multiplying by a non zero constant and adding a non zero constant to the result. However, for example, a system of three equations could be inconsistent, but each two of the three equations may well be consistent. Thus inconsistency is a subtler concept when we move to three or more variables. In two variables, it was straight forward.

We can represent a system of linear equations over the same variables as a matrix multiplication problem. Assume the equations all list the variables in the same order with their corresponding coefficients and the constant term is on tee other side. One creates a matrix with the coefficients and multipliec it with a column vector of the variables and then the result is equated to a column vector of the constants on the right hand side of the equations.

The adhoc rules we apply to solving these systems of equatons typically consist of scaling adding subtracting to eliminate a variable from two equations. We iteratively do this getting fewer and fewer variables in fewer and fewer equations. The process ends when we get one equation with one variable.

A aytem of two equations with a large number of variables could be inconsistent. However, if consistent the number of solutions is infinitely many. Only a system of equations with number of variables equal to or less than the number of equations can possibly have unique solutions.

The standard manipulations we do on a system of equations are called **elementary row operations** when performed on the corresponding matrix equation.