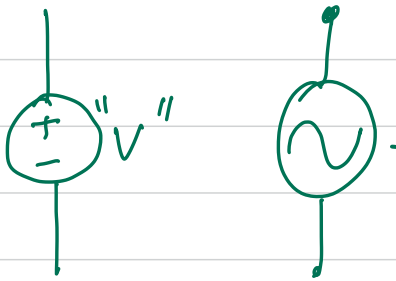
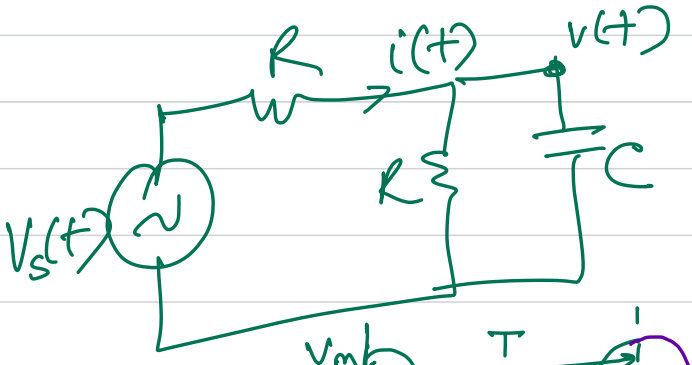
Lecture 16



all i/p are sinusoidal with same freq.

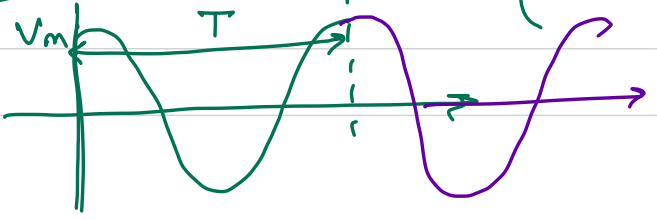


$$v(t) = V_m \cos \omega t$$

peak: "V_m"

angular fr: rad/s

$(\omega = 2\pi f)$ $\quad T = \left(\frac{1}{f}\right)$



$v(t) = V(t+T)$

$V(0) = V(T)$

$\{v(t) = v(t+nT)$ periodic

$v(t) = V_m \cos \omega t$

$t \rightarrow (t + t_a)$

$v(t + t_a) = V_m \cos[\omega(t + t_a)]$

$= V_m \cos(\omega t + \omega t_a)$

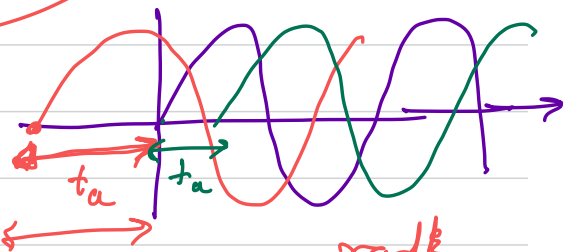
$= V_m \cos(\omega t + \phi)$

ϕ → phase

$v(t - t_a) = V_m \cos[\omega t - \omega t_a]$

$\phi = -\omega t_a$

If $x(t)$ → $x(t - t_a)$



radians

$\phi = \omega t_a$

$\phi = \frac{2\pi}{T} t_a$

$t_a = \left\{ \left(\frac{\phi T}{2\pi} \right) \right\}$

$t_a = \left(\frac{-\phi T}{2\pi} \right)$

$V(t) = V_m \cos\left(\frac{2\pi}{T} t\right)$ → ref: signal → blue

$V_2(t) = V_m \cos\left(\frac{2\pi}{T} t + \pi/4\right)$ → $+\pi/4$ green

$V_3(t) = V_m \cos\left(\frac{2\pi}{T} t - \pi/4\right)$ → red

phase lag , phase lead

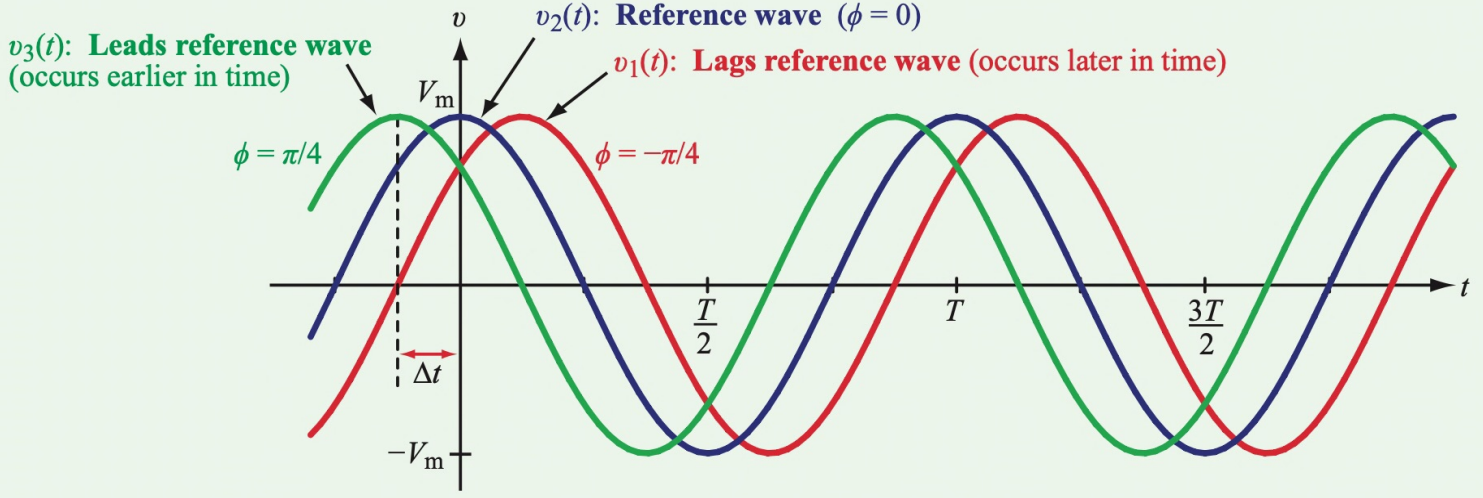


Figure 7-2: Plots of $v(t) = V_m \cos[(2\pi t/T) + \phi]$ for three different values of ϕ.

$$\sin x = \pm\cos(x \mp 90°) \tag{7.7a}$$
$$\cos x = \pm\sin(x \pm 90°) \tag{7.7b}$$
$$\sin x = -\sin(x \pm 180°) \tag{7.7c}$$
$$\cos x = -\cos(x \pm 180°) \tag{7.7d}$$
$$\sin(-x) = -\sin x \tag{7.7e}$$
$$\cos(-x) = \cos x \tag{7.7f}$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \tag{7.7g}$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \tag{7.7h}$$

$$2\sin x \sin y = \cos(x - y) - \cos(x + y) \tag{7.7i}$$
$$2\sin x \cos y = \sin(x + y) + \sin(x - y) \tag{7.7j}$$
$$2\cos x \cos y = \cos(x + y) + \cos(x - y) \tag{7.7k}$$

$V_1(t) = 10\cos(200t + 45^\circ)$ $\quad V_2(t) = 8\sin(200t + 15^\circ)$

$V_2(t)$ is advanced or delayed with w.r.to $V_1(t)$

$$V_2(t) = 8\cos(200t + 15 - 90^\circ) = 8\cos(200t - 75^\circ)$$

$\phi_2 = -75$ $\quad \phi_1 = 45^\circ$

$$\phi_2 - \phi_1 \equiv -120^\circ = \left[-\frac{2\pi}{T} f_d\right]$$

$$t_d = \frac{2\pi}{600} = 10.4\text{ms}$$

$$-200 f_d = -2\pi/3$$

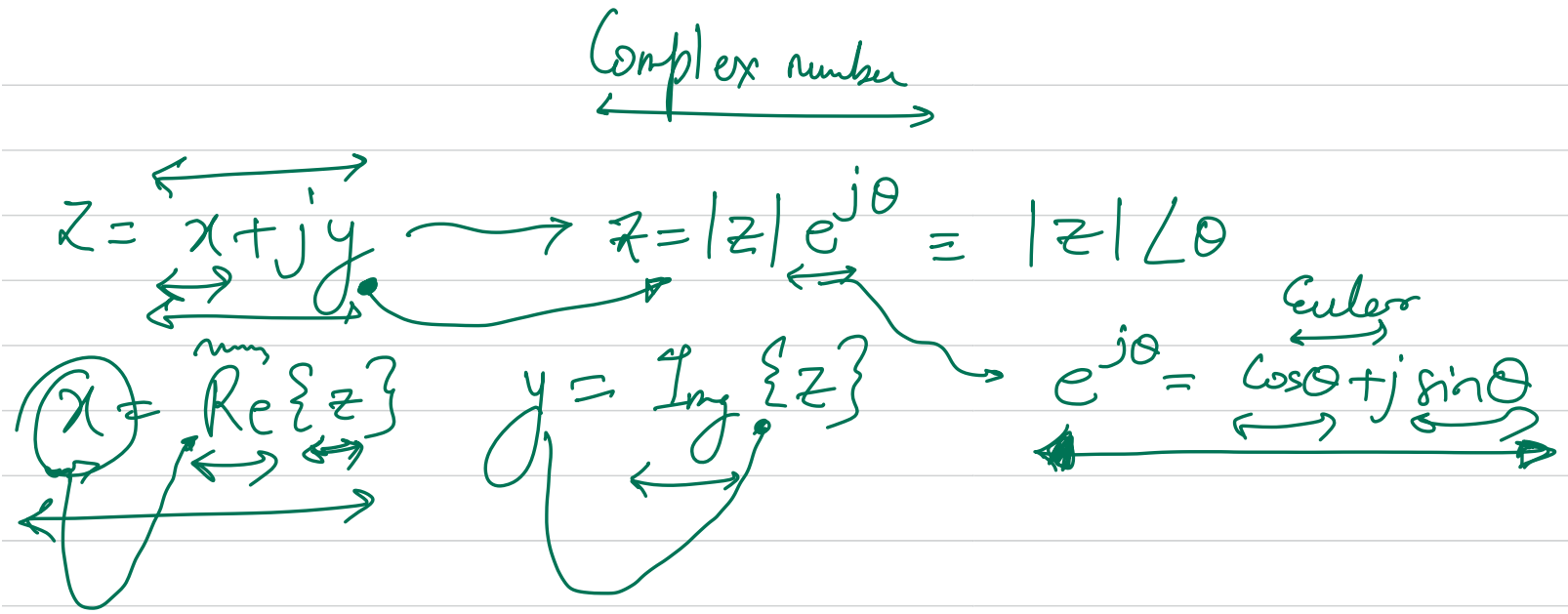
Q $i_1(t) = -8\cos(\omega t - 30^\circ)$ $i_1(t)$ leads i_2

$i_2(t) = 12\sin(\omega t + 45^\circ)$ $[-180^\circ, 180^\circ]$

$i_1(t) = 8\cos(\omega t - 30 + 180^\circ) = 8\cos(\omega t + 150^\circ)$

$i_2(t) = 12\cos(\omega t + 45 - 90^\circ) \equiv 12\cos(\omega t - 45^\circ)$

$\Delta\phi = \phi_1 - \phi_2 = 195^\circ$ $195^\circ - 360^\circ \equiv -165^\circ$

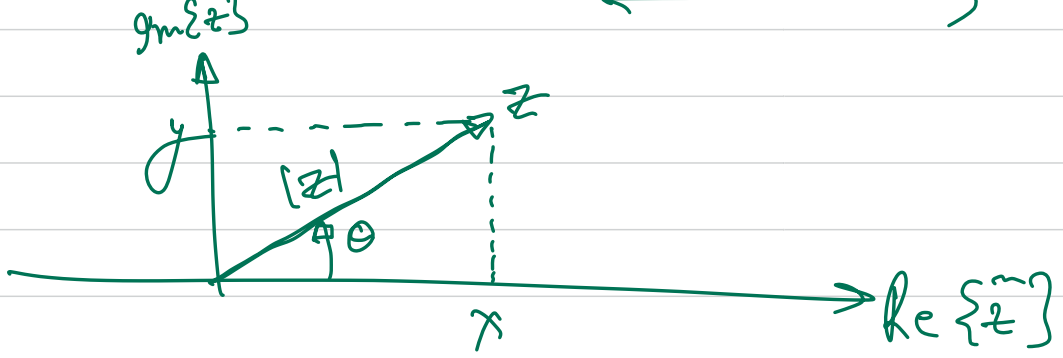


$$z = |z| \{\cos\theta + j\sin\theta\} \equiv |z|\cos\theta + j|z|\sin\theta$$

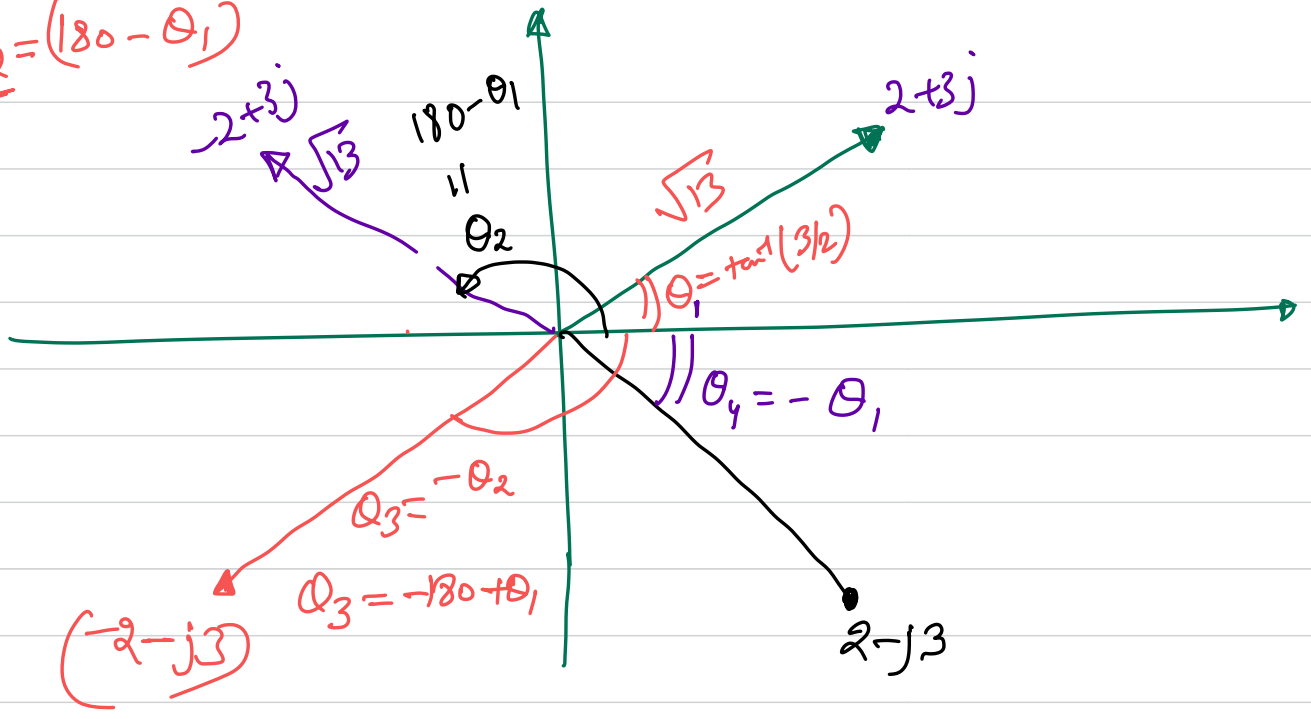
$$x = |z|\cos\theta, \quad y = |z|\sin\theta$$

$$|z| = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(y/x\right)$$



$\theta_2 = (180 - \theta_1)$



$(2+3j)$ $-2+j3$ $2-j3$ $-2-3j$

$$z^* = (x+jy)^* = x-jy = |z| \angle -\theta$$

$$|z| = \sqrt{z \cdot z^*}$$

Complex

$z_1 = x_1 + jy_1 \qquad z_2 = x_2 + jy_2$

$z_1, z_2 \qquad z = z_1 \pm z_2 = (x_1 \pm x_2) + j(y_1 \pm y_2)$

$$z_1 \cdot z_2 = (x_1+jy_1)(x_2+jy_2) = (x_1x_2 - y_1y_2) + j(x_1y_2 + x_2y_1)$$

$$z_1 \cdot z_2 = |z_1| e^{j\theta_1} \; |z_2| e^{j\theta_2}$$

$$= |z_1||z_2| \; e^{j(\theta_1+\theta_2)}$$

$$z_1 z_2 = |z_1||z_2| \{\cos(\theta_1+\theta_2) + j\sin(\theta_1+\theta_2)\}$$

$$\left(\frac{z_1}{z_2}\right) = \frac{|z_1| e^{j\theta_1}}{|z_2| e^{j\theta_2}} = \left\{\frac{|z_1|}{|z_2|} e^{j(\theta_1-\theta_2)}\right\}$$

$$= \frac{(x_1x_2+y_1y_2) + j(x_2y_1 - x_1y_2)}{(x_2^2+y_2^2)}$$

$$z^n = \left(|z| e^{j\theta}\right)^n \equiv |z|^n \, e^{jn\theta}$$

$$\equiv |z|^n \{\cos n\theta + j \sin n\theta\}$$

$$V = 3 - j4 = 5 \angle -53.1$$

$$-(2+j3) \Rightarrow \sqrt{13}\left(-\pi + \tan^{-1} 3/2\right)$$

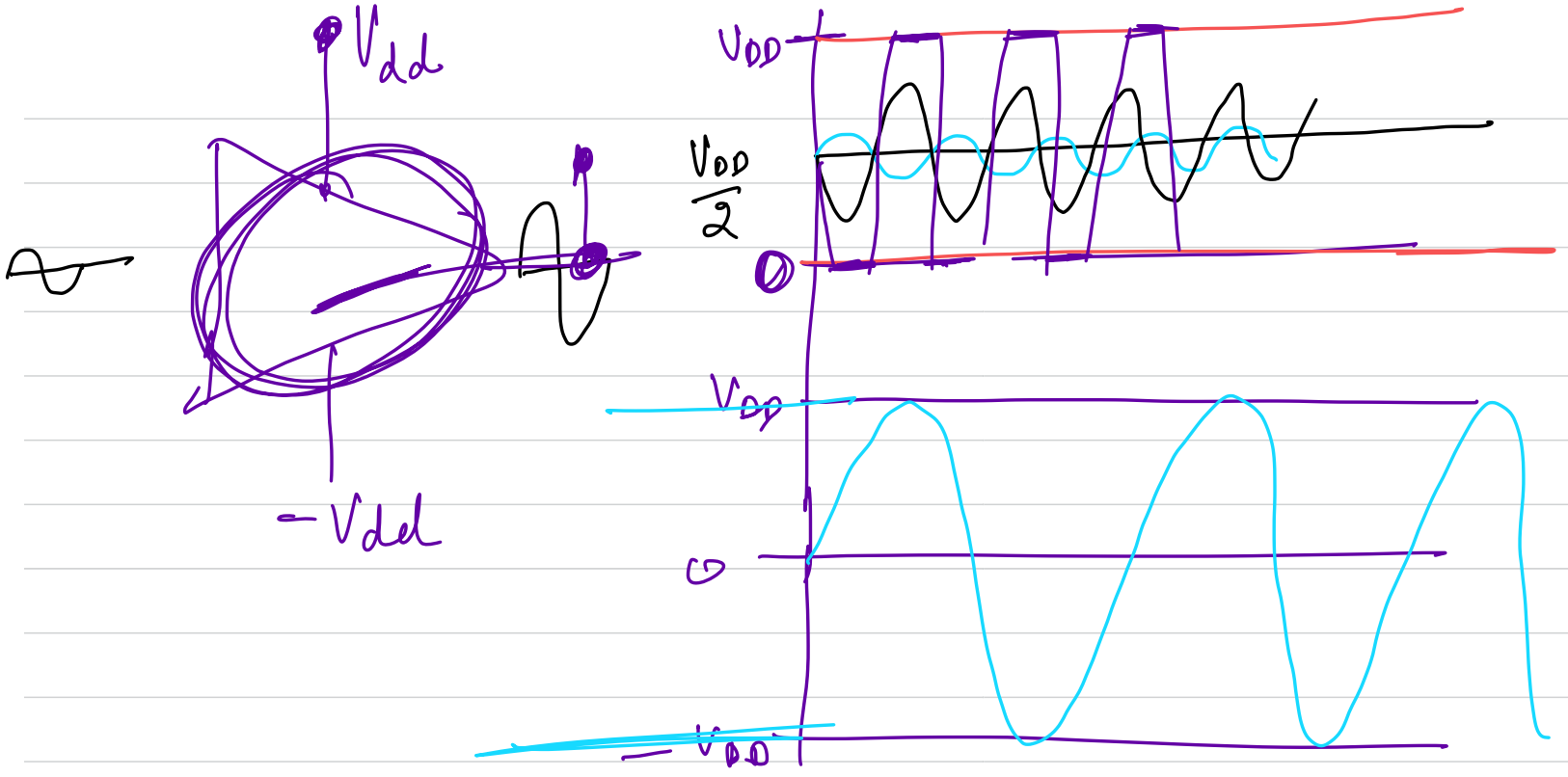
$$123.6$$

$$R \angle \theta \equiv$$

$$\begin{cases} V_1 = -1.796 + j\,3.852 = 4.25\angle 115 \\ V_2 = -4 + j3 \equiv 5\angle 143^\circ \end{cases}$$

$$V_1 + V_2 = [-5.796 + j6.852]$$

$$\begin{cases} V_1 \cdot V_2 = (5 \times 4.25)\angle 115 + 143 \equiv \_\_\_\_\angle -102 \\ \dfrac{V_1}{V_2} = \dfrac{4.25}{5}\angle 115 - 143^\circ = \_\_\_\_\_ \end{cases}$$


V_{dd}
$-V_{dd}$
V_{DD}
$\frac{V_{DD}}{2}$
0
V_{DD}
$-V_{DD}$