Probability theory

Sample Space: It is the set of all possible outcomes of an experiment. It is denoted by S

Examples:

Flipping a coin, S = {T, H}

Rolling a dice, S = {1,2,..,6}

Probability Space (PS)

PS = (S, Pr), where :

S is the sample space

Pr is a function from 2^S to [0, 1]

Event: Each subset of S is called an event. It is denoted by A.

Example: Rolling a dice, S= {1,2, ...,6}

Odd number appears, A={1,3,5}

Axioms of a PS = (S, Pr)

1. $Pr(\Phi) = 0$
2. $Pr(S)=1$
3. $0 <= Pr(A) <= 1$, where $A \subseteq S$
4. If $A, B \subseteq S$ and $A \sqcap B = \Phi$, then

   $Pr(A \cup B) = Pr(A) + Pr(B)$

Independence

- Two events A and B are independent if

$$Pr(A \sqcap B) = Pr(A).\ Pr(B)$$

Conditional Probability

The probability that A happens, given that B has already happened is denoted by $Pr(A|B)$.

$Pr(A|B) = Pr(A \sqcap B)/ Pr(B)$

Random Variable X

It is a function X that maps each element of S to a real number

X: $S \to R$

Indicator Random Variable:

X: $S \to \{0, 1\}$

Expected value

- The expected value of a random variable X is denoted by E(X) where

$$\mathrm{E}(X) = \sum_x (x.\Pr(X = x))$$

Example

- Let X be a random variable that assigns the outcomes of the roll of two fair dice the sum of the number on the two dices. What is the expected value of X.

- E(X)= ?

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- $E(X)= 7$

Linearity of Expectation

- Let X and Y be two random variables

- $E(X + Y) = E(X) + E(Y)$
- $E(c.X) = c.E(X)$ where c is a real number.

Example

- What is the expected number of times a person must toss a coin to get a head.

Example

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- Answer is 2