SET C

Calculus SC-107

Full marks 30, Time 1 Hour

Choose the correct answer from the given choices for each of the questions.

1. If $f(x) = \int_{\cos x}^{\sin x} \frac{1}{1-t^2} dt$. Then a critical point of $f(x)$ is

   (a) $x = \pi/2$

   (b) $x = 0$

   (c) $x = \pi/4$

   (d) $x = \pi$

   Correct Answer: (c)

2. Find the volume of the solid generated by revolving the region bounded by the parabola $y = x^2$ and the line $y = 1$ revolved about the line $y = -1$.

   (a) $\frac{68}{15}\pi$

   (b) 4π

   (c) $\frac{64}{15}\pi$

   (d) 3π

   Correct Answer: (c)

3. Which of the following function is not Riemann integrable on $[0, 1]$

   (a) $f(x) = \begin{cases} x \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

   (b) $f(x) = \begin{cases} 2x & \text{if } x \in [0, 1/4] \\ 1 - x & \text{if } x \in (1/4, 1/2] \\ 1 + x & \text{if } x \in (1/2, 1] \end{cases}$

   (c) $f(x) = \begin{cases} 0 & \text{if } x = 0 \text{ or } x = 1 \\ 1 & \text{if otherwise} \end{cases}$

   (d) None of the above

Correct Answer: (d)

4. Let $f(x) = 1 - x + \dfrac{x^2}{2} - \dfrac{x^3}{3} + \cdots + (-1)^n \dfrac{x^n}{n}$. Then $f(x)$ has

   (a) one real root if n is even.

   (b) no real root if n is even.

   (c) n real roots.

   (d) even number of real roots.

   Correct Answer: (b)

5. Let $a_1, a_2, \cdots, a_n$ be real numbers and let f be defined on $\mathbb{R}$ by

$$f(x) = \sum_{i=1}^{100} (x - a_i)^2 \text{ for } x \in \mathbb{R}.$$

   Then the point of local minimum of $f(x)$ is

   (a) $\dfrac{a_1 + a_2 + \cdots + a_{100}}{100}$

   (b) $-\Big(\dfrac{a_1 + a_2 + \cdots + a_{100}}{100}\Big)$

   (c) $\dfrac{a_1 + a_2 + \cdots + a_{100}}{2}$

   (d) $-\Big(\dfrac{a_1 + a_2 + \cdots + a_{100}}{2}\Big)$

   Correct Answer: (a)

6. A particular solution to the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x} \log x$$

   is

   (a) $\dfrac{-1}{2} x e^{-x} \log x + \dfrac{3}{4} x^2 e^{-x}$

   (b) $\dfrac{1}{2} x^2 e^{-x} \log x - \dfrac{3}{4} x^2 e^{-x}$

   (c) $\dfrac{3}{4} x^2 e^{-x} \log x - \dfrac{1}{2} x^2 e^{-x}$

   (d) $\dfrac{-1}{2} x^2 e^{-x} \log x - \dfrac{3}{4} x e^{-x}$

   Correct Answer: (b)

7. The general solution of

$$x\frac{dy}{dx} + y = y^2 \log x$$

is

(a) $y(1 + cx + \log x) = 1$

(b) $y(cx + \log x) = 1$

(c) $y^2(1 + cx + \log x) = 1$

(d) $y^2(1 + cx + \log x) = -1$

Correct Answer: (a)

8. Let $f(x) = \begin{cases} \frac{1}{4^n} & \text{if } x = \frac{1}{2^n},\ n = 1, 2, 3, ... \\ 0 & \text{otherwise} \end{cases}$

Then

(a) f is not continuous at $x = 0$.

(b) $\lim_{x\to 0} f(x)$ does not exist.

(c) f is continuous at $x = 0$.

(d) f is not defined at $x = 0$.

Correct Answer: (c)

9. The value of the $\lim_{x\to 1} \frac{1 + \cos \pi x}{\tan^2 \pi x}$ equals

(a) 0

(b) π

(c) $\pi/2$

(d) $1/2$

Correct Answer: (d)

10. A circular cylindrical metal container, open at the top, is to have a capacity of $24\pi\ cm^3$. The cost of the material used for the bottom of the container is 15 rupees per cm^2, and that the cost of the material used for the curved part is 5 rupees per cm^2. If there is no waste of material, find the dimension of the container that will minimize the cost of the material.

(a) base radius = 2 cm, height of the cylinder = 6 cm

(b) base radius = $2\sqrt{2}$ cm, height of the cylinder = 3 cm

(c) base radius = 3 cm, height of the cylinder = $\frac{24}{9}$ cm

(d) None of the above will minimize the cost.

Correct Answer: (a)