

1. Evaluate

(a) $\int (r^2 + \vec{r} \cdot \vec{a} + a^2) \delta^3(\vec{r} - \vec{a}) dV$ over the whole space where \vec{a} is a fixed vector.

(b) $\int_V |\vec{r} - \vec{b}|^2 \delta^3(5\vec{r}) dV$ over a cube of side 2, centered at the origin, and $\vec{b} = 4\hat{y} + 3\hat{z}$

2. The electric field in a region is given as

$$\begin{aligned} \vec{E} &= \frac{\sigma}{2\epsilon_0} \hat{i}; & \text{for } x > 0 \\ &= -\frac{\sigma}{2\epsilon_0} \hat{i}; & \text{for } x < 0 \end{aligned}$$

Find the charge distribution in the region using the differential form of Gauss's law.

3. The electric field in a region is cylindrically symmetric, given as follows:

$$\begin{aligned} \vec{E}(\vec{r}) &= \frac{c\hat{s}}{s}; & \text{when } s \geq a \\ &= 0; & \text{when } s < a \end{aligned}$$

Find the charge distribution in the region using Gauss' law.

4. We have seen that $\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right) = 4\pi \delta^3(\vec{r})$. In a similar manner justify that

$$\vec{\nabla} \cdot \left(\frac{\hat{s}}{s} \right) = 2\pi \delta^2(\vec{s})$$

Here s is the distance from the z axis in cylindrical coordinates and $\delta^2(\vec{s})$ is a two dimensional delta function on the xy plane.

5. Prove that $\delta(r) = 4\pi r^2 \delta^3(\vec{r})$ and $\delta(s) = 2\pi s \delta^2(\vec{s})$.

Here $\int_0^\epsilon \delta(r) dr = 1$ for any $\epsilon > 0$. The integral is 0 otherwise. $\delta(s)$ is defined likewise.

6. The electric field in a region is given as

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 s} \hat{s}$$

Find the charge densities in the region.