

1. In the spherical polar system:

- (a) Evaluate $\frac{\partial \hat{r}}{\partial \theta}, \frac{\partial \hat{\theta}}{\partial \theta}, \frac{\partial \hat{\phi}}{\partial \theta}, \frac{\partial \hat{r}}{\partial \phi}, \frac{\partial \hat{\theta}}{\partial \phi}, \frac{\partial \hat{\phi}}{\partial \phi}$
- (b) Using the above partial derivatives evaluate $\vec{\nabla} \cdot \hat{r}, \vec{\nabla} \cdot \hat{\theta}$ and $\vec{\nabla} \cdot \hat{\phi}$ where $\vec{\nabla} = \hat{r} \frac{\partial}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}$

Now you can evaluate the expression for $\vec{\nabla} \cdot \vec{A}$ in spherical polar co-ordinates using the result of part (b) and using the product rules. Try it and see whether you get the expression for divergence.

2. Cylindrical system of co-ordinate is specified by three variables (s, ϕ, z) given by

$$x = s \cos \phi; \quad y = s \sin \phi; \quad z = z$$

Find the unit vectors $\hat{s}, \hat{\phi}, \hat{z}$ in this co-ordinate system. Find h_s, h_ϕ and h_z and write down the expression for $\vec{\nabla} F$ for a scalar function F in this system.

3. (a) Evaluate $\vec{\nabla} \cdot \vec{r}, \vec{\nabla} \cdot (r^2 \hat{r})$ and $\vec{\nabla} \cdot (\frac{\hat{r}}{r^2})$ for $r \neq 0$.
(b) Evaluate $\vec{\nabla} \cdot \vec{r}, \vec{\nabla} \cdot (r^2 \hat{r})$ and $\vec{\nabla} \cdot (\frac{\hat{r}}{r^2})$ at $r = 0$.
4. (a) Evaluate $\vec{\nabla} \times \hat{\phi}, \vec{\nabla} \times \frac{1}{r \sin \theta} \hat{\phi}$ and $\vec{\nabla} \times r \sin \theta \hat{\phi}$ for $0 < \theta < \pi$.
(b) Evaluate $\vec{\nabla} \times \hat{\phi}, \vec{\nabla} \times \frac{1}{r \sin \theta} \hat{\phi}$ and $\vec{\nabla} \times r \sin \theta \hat{\phi}$ for $\theta = 0$.
5. If $\vec{\nabla} \cdot \vec{B} = 0$ show that there exists a vector function \vec{A} such that $\vec{\nabla} \times \vec{A} = \vec{B}$