Vector Space V, over an underlying field F. has two basic operations and must satisfy the following defining axioms (+ stands for vector addition; . stands for multiplication of a vector by a scalar):

1.Closure:
(a) for all u,v in V, u+v is also in V
(b) For all v in V and t in F, tv is also in V

2. Associativity:
for all u,v,w in V, (u+v)+w=u+(v+w)

3. Commutativity:
for all u,v in V, u+v=v+u

4. Additive identity: there exists a vector, usually denoted by 0, such that:
for all v in V, v+0=0+v=v.

5. Multiplicative identity usually enoted by the scalar 1, such that,
for all v in V, 1.v=v

6. Additive inverse: for each v in V, there exists v', such that v+v'=v'+v=0

7. Distributive properties:
(a) for all t_1,t_2 in F; v in V, (t_1+t_2).v=t_1.v+t_2.v
(b)for all t in F; v,w in V, f.(v+w)=f.v+f.w

The field F is usually either R, the set of real numbers, or C the set of complex numbers.

1. Vector spaces over R are called real vector spaces

2. Vector spaces over C are called complex vector spaces

$F^S$, is a notation where F is a field, S is a set.

Sequence: is a function with domain being $Z^+$ and codomain being arbitrary, and typically finite.

$S=Z^+$; $F^S$ is a vector space where vector addition is done by component-wise addition over the underlying field, and scalar multiplication is done by component-wise scalar multiplication of that component with the scalar multiplier.

It can be verified that the set together with the operations we defined, satisfy the axioms of the vector space definition, and thus form a vector space.