NAME: ROLL NUMBER:

Data Structures (IT205) 2015-16
Final Exam
30^{th} **November, 2015**

Time: 2 hours, 30 minutes
12 Each Question each worth 4 marks (total: 48); Two pages back-to-back.
Wrong answers result in negative marking, so beware of guessing answers. (minus 1)
Attempt all questions.
Write your name and roll number on both pages.
You may use rough sheets but DO NOT submit them
Answers must be written on question paper itself and returned.
The exam is open book and open notes; no exchange of material between students.

1. Consider an open address table with 4 slots. The universe consists of 96 elements say $\{1,\ldots,96\}$. The insert priority is $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$ for the elements 1,2,3 and 4. The insert priority is $1 \rightarrow 2 \rightarrow 4 \rightarrow 3$ for the next four elements 5,6,7 and 8. This continues with the insert priorities changing in lexicographic order and valid for four elements each. Thus elements 93,94,95 and 96 have insertion priority $4 \rightarrow 3 \rightarrow 2 \rightarrow 1$. The search time for any element in the table is either 1 or 2 or 3 or 4 and 5 if it is missing from the table in the worst case. Write a sequence of inserts and deletes starting from an initialised empty table such that the table is full and the total search time for all the four keys present in the table is 12. Fill your answer in the blanks below: Insert(______), Insert (______), Insert (______),Insert(______), Delete (______), Insert(______).

2. BFS is run on an undirected graph U with source node s. The graph U is connected if and only if:

   for each node v, $d[v] \neq$ __________ at the end of the algorithm.

3. Recall that a binary tree can be represented as a set of prefix closed binary strings with the root being represented by ϵ and the left child of a node x getting the label $label(left(x)) \leftarrow label(x).0$. Similarly $label(right(x)) \leftarrow label(x).1$. Consider the tree $T = \{\epsilon, 0, 1, 00, 001, 01, 010, 0100, 0101, 11\}$. Say this is a binary search tree with the keys $1,\ldots,10$ built using the standard insert algorithm.

   The lexicographically earliest order of inserts of these 10 keys that would result in this tree is: ____.

   ____.

   The lexicographically last order of inserts of these 10 keys that would result in this tree is: ____.

   ____.

4. Consider a 2,3,4-tree which is initially empty. Consider the following four sequences of inserts into the tree. Arrange these orders according to the increasing number of inserted elements before the heights of the corresponding trees become 2.

   (a) 1,3,5,7,9,11,13,6,15,4,2,8,12,10,16,14
   (b) 16,15,14,13,12,11,10,9,8,7,6,5,4,3,2,1
   (c) 16,13,11,9,7,5,3,10,14,15,12,8,1,6,2,4
   (d) 15,13,11,9,7,5,3,10,16,2,1,14,8,12,6,4

   ____________ < ____________ < ____________ < ____________

5. Consider the connected undirected graph C_{10}; this is the cycle on 10 vertices. For concreteness let the edge set consist of the edges $E = \{(v_1, v_2), (v_2, v_3), \ldots, (v_9, v_{10}), (v_{10}, v_1)\}$. Here the vertex set is $V = \{v_1, \ldots, v_{10}\}$. The graph is weighted and the weights of the edges are $w(v_1, v_2) = w(v_2, v_3) = w(v_3, v_4) = w(v_4, v_5) = w(v_5, v_6) = 2$ and $w(v_6, v_7) = w(v_7, v_8) = w(v_8, v_9) = w(v_9, v_{10}) = w(v_{10}, v_1) = 1$.

   Weight of Prim's Tree run from node v_1 as the source is ______ and weight of Dijkstra's Tree run from the node v_1 as the source is ______.

   The two trees will have identical cost if the source node is ____.
   (Write all the correct answers and no wrong answers)

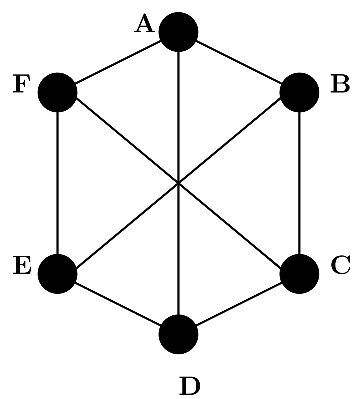
6. Consider an array $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31\}$.

   The elements in this array which will be detected in the same number of steps in both linear search and binary search are _______________.

   The elements which will be detected in fewer steps by linear search than binary search are:_______________.

Write the input list of numbers in the order of the difference in number of search steps between linear search and binary search to detect that element. If this difference is identical for two elements list them in the same order as they appear in the input. ____.
____

7. For the graph drawn below label each of its 9 edges with weights either 3,4 or 5 such that **no matter** which vertex we run both Prim's and Dijkstra's algorithms from the resulting trees in the two cases will be different.



8. For an arbitrary positive-weighted, connected, undirected graph there is always at least one source node such that running Dijkstra's algorithm and Prim's algorithm from that node result in the same tree. (TRUE/FALSE)? ________

9. Consider a 15 node undirected graph and the following 15 vectors are the discovery time, finish time pairs of the 15 vertices: $(1,30),(2,19),(20,23),(3,16),(17,18),(4,9),(10,15),(5,8),(6,7),(11,12),(13,14),(21,22),(24,25),(26,29),(27,28)$.
   (a) Is this DFS-tree a binary tree (ignore left right and just consider whether every internal node has at most 2 children). ________ (YES/NO).
   (b) This DFS tree has ______ leaves, ______ nodes with one child each and _______ nodes with two children each and _______ nodes with more than 2 children each.
   (c) The height of this DFS tree is _______.

10. Let H_n be the number of valid arrangements of n distinct keys in an n node binary max heap. Let A_n be the number of valid arrangements of n distinct keys into an n node alternating heap. For an alternating heap, the max. heap and min heap property should be interchanged at every alternate level. However, at the root level there is no restriction on whether it is a max heap or min heap.
    (a) $A_n > H_n, \forall n \geq 2$
    (b) $A_n < H_n, \forall n \geq 2$
    (c) $A_n = H_n, \forall n \geq 2$
    (d) The answer is different for different values of $n \geq 2$.

11. The comparison based sorting algorithm best suited to a virtual memory environment is ______________________. This is because the number of ________ ____________ is minimised during the execution of this algorithm in a virtual memory environment.

12. A stack of n elements can be reconfigured to any permutation using one auxiliary stack in worst case asymptotic time
    (a) $\Theta(n^2)$
    (b) $\Theta(n^n)$.
    (c) $\Theta(\log n)$
    (d) $\Theta(n^3)$