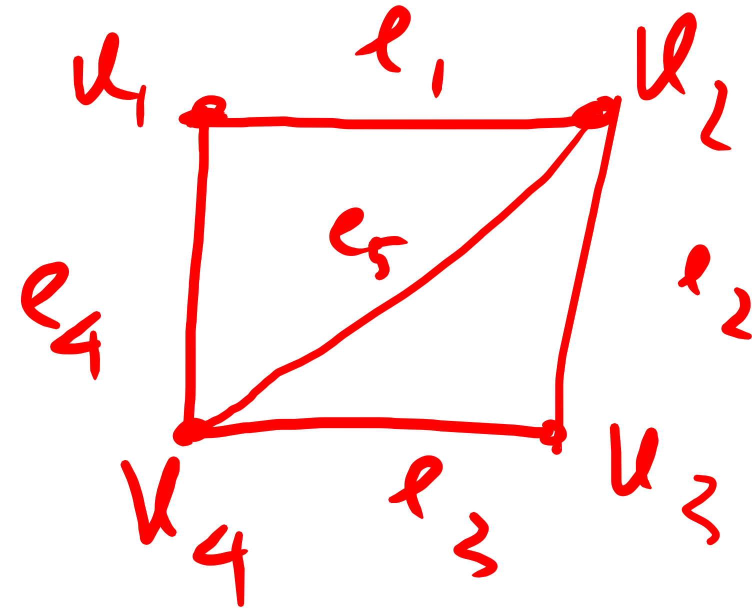
Graphs

Defn

$G = (V, E)$

V = set of vertices

E = set of edges

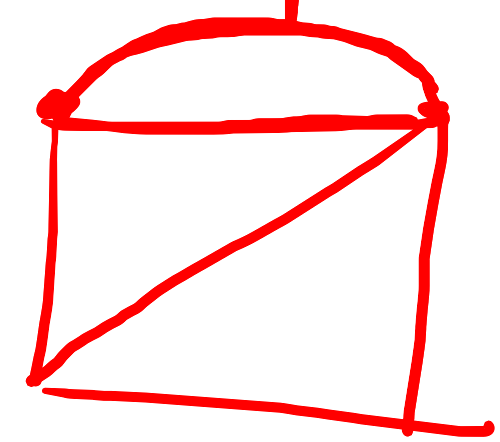


$V = \{v_1, v_2, v_3, v_4\}$

$E = \{e_1, e_2, e_3, e_4, e_5\}$

Def^n simple graph

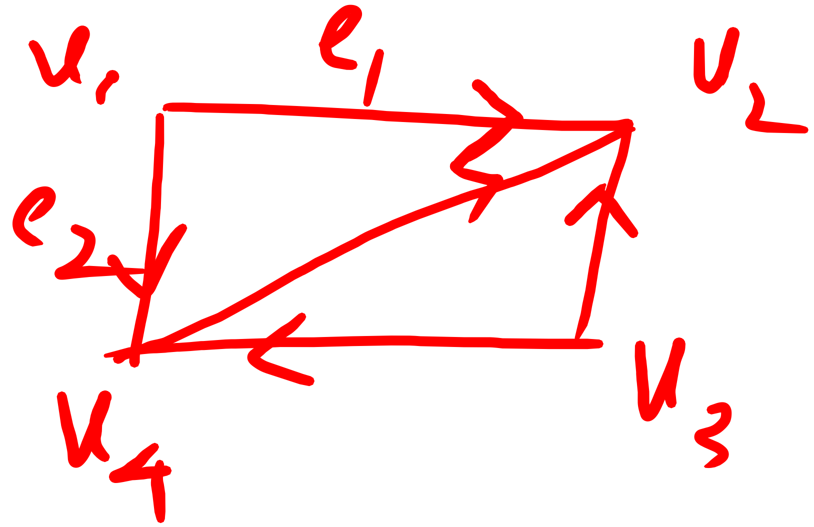
Each edge connects two different vertices and No two edges connect the same pair of vertices



No simple

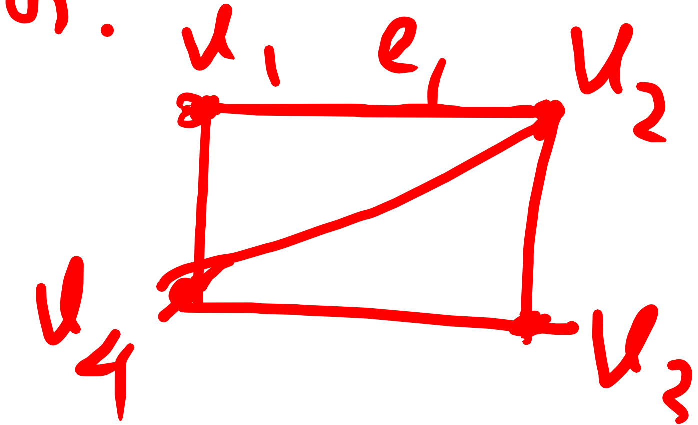
Defⁿ A directed graph (V, E) consist of a set of vertices V, set of edges E. each edge is ordered pair of elements of V.

$$e_1 = (v_1, v_2)$$
$$e_2 = (v_1, v_4)$$



(Handshaking Th^m)

Def^n Two vertices u and v are adjacent in an undirected graph if (u, v) is an edge of G.



$e = (u, v)$ edge e is incident with the vertices u and v.

v_1, v_2 are end point of the edge (v_1, v_2)

Def^n

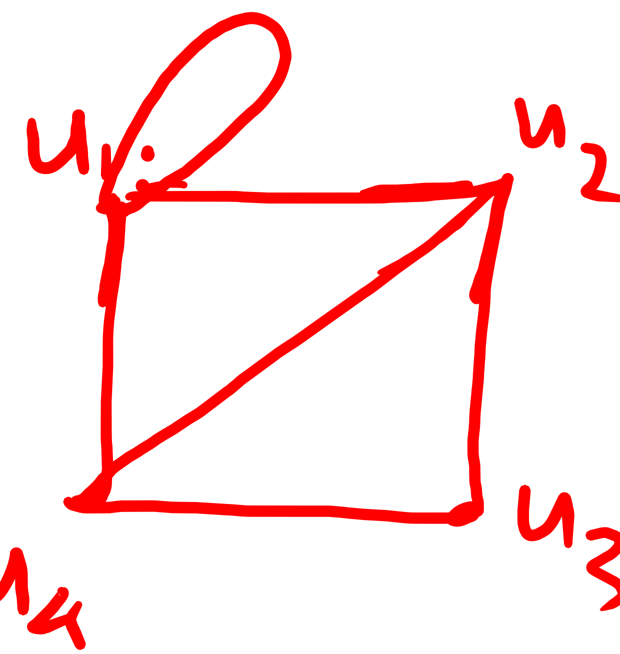
Degree of a vertex = in an undirected graph

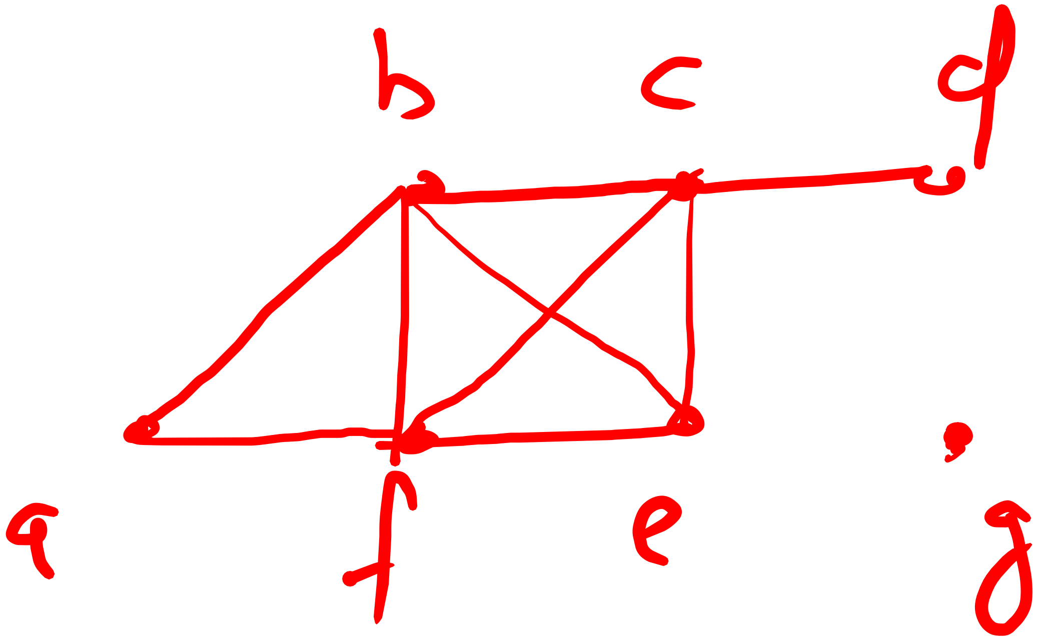
= no. of edges incident with it

A loop contributes twice to the degree of the vertex

$\deg(u_1) = 4$

$\deg(u_2) = 3$





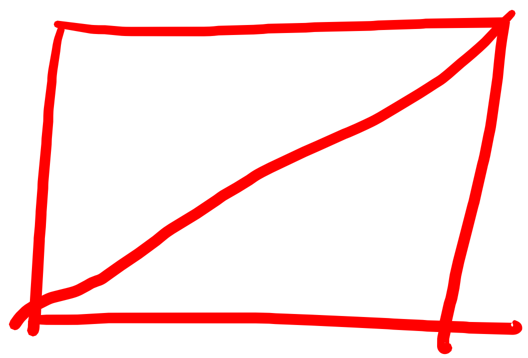
$\deg(a) = 2$, $\deg(b) = 4$ $\deg(g) = 0$

g is isolated

$\underline{\text{Th}^{m}}$ Let $G=(V,E)$ be an undirected graph with e edges. Then

$$2e = \sum_{v\in V} \deg(v)$$

This holds for loops or multiple edges.



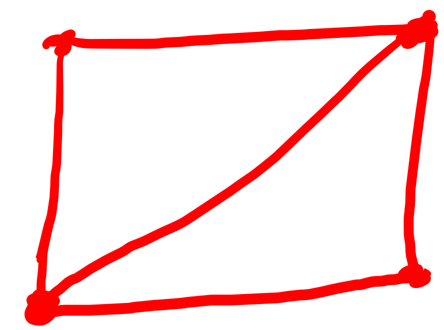
Exp How many edges are there in a graph with 10 vertices each of degree 6?

Total degree $= 10 \times 6 = 60$

$60 = 2e \Rightarrow e = 30$

Thm An undirected graph has an even number of vertices of odd degree.

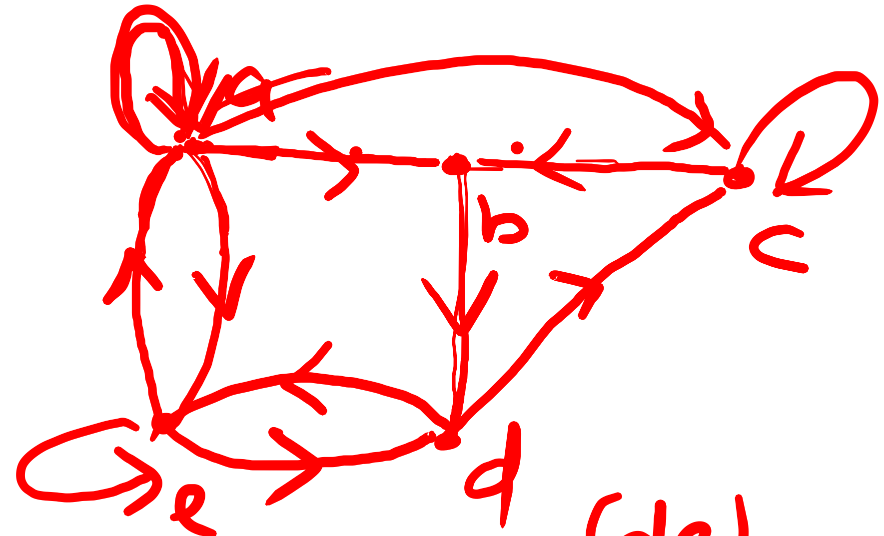
Pf Let V_1 and V_2 be the set of vertices of even degree and the set of vertices of odd degree respectively.



$$2e = \sum_{v \in V} \deg(v) = \underbrace{\sum_{v \in V_1} \deg(v)}_{\text{even.}} + \underbrace{\sum_{v \in V_2} \deg(v)}_{\text{even}}$$

Def^n G Directed graph.

(u, v) an edge of G



u is said to be adjacent to v

v is said to be adjacent to u

(de)

(e, d)

(a, b)

u is called an initial vertex a - initial ver

v is called a terminal vertex b - terminal vertex

The initial and terminal vertex of a loop are same

$deg^-(a) = 2$ $deg^-(b) = 2$ $deg^+(a) = 4$

Defn The in-degree of a vertex v

$\underline{\deg^{-}(v)}$ = no. of edges with v as their terminal vertex

$\deg^{+}(v)$ = no. of ~~xen~~ edges with v as their initial vertex.

Note A loop contributes 1 to both the indegree and out degree of this vertex

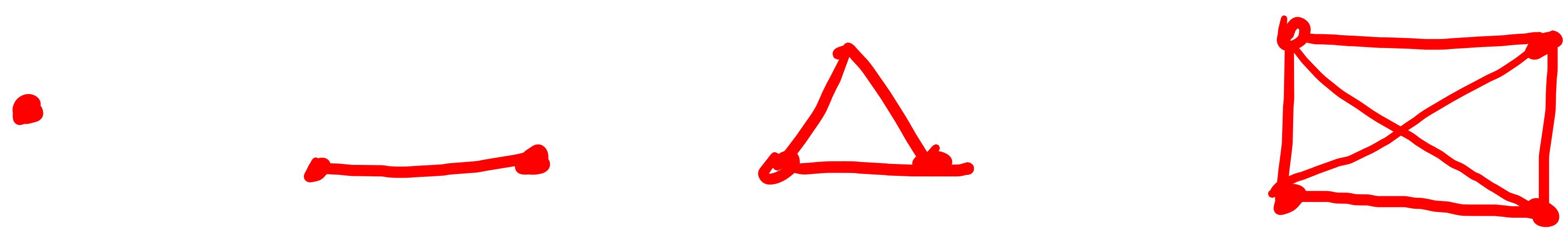
$u \longrightarrow v$ $\qquad \deg^{+}(u) = 1, \quad \deg^{-}(v) = 1$

Thm Let $G = (V, E)$ be a graph with directed edges.

$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|$$

Example   Complete Graphs K_n

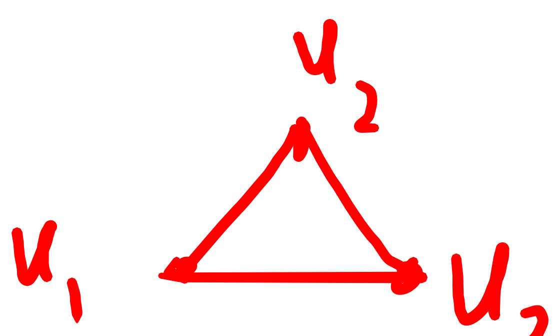
A complete graph is a simple graph that contains exactly one edge between each pair of distinct vertices.



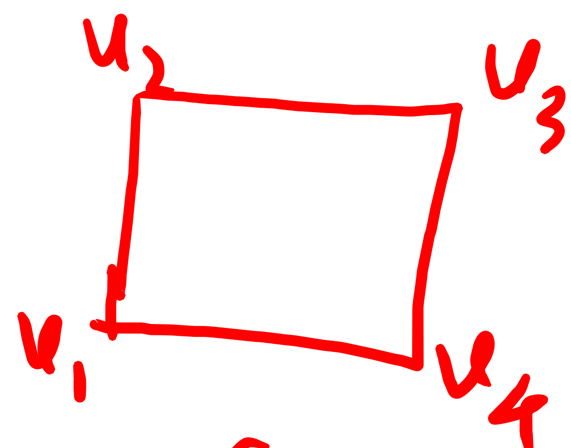
K_1 K_2 K_3 K_4

Example Cycle C_n $\underline{n \geq 3}$

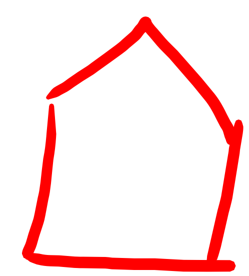
It consists of n vertices $v_1, v_2, \ldots v_n$ and edges $\underline{(v_1, v_2)}$, $\underline{(v_2, v_3)}$ ... $\underline{(v_{n-1}, v_n)}$, $\underline{(v_n, v_1)}$

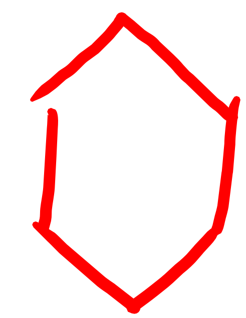


C_3



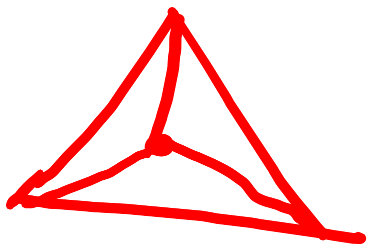
C_4


C_5

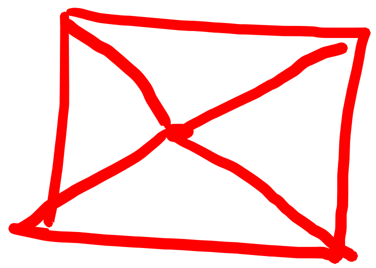

C_6

Wheels

$W_n \quad n \geq 3$



W_3

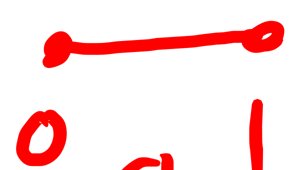


W_4

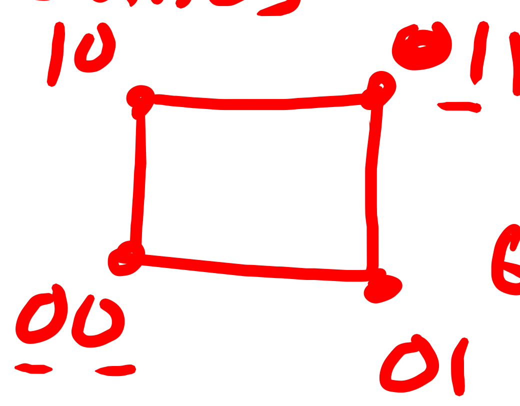
Example

n-cubes $\quad Q_n$

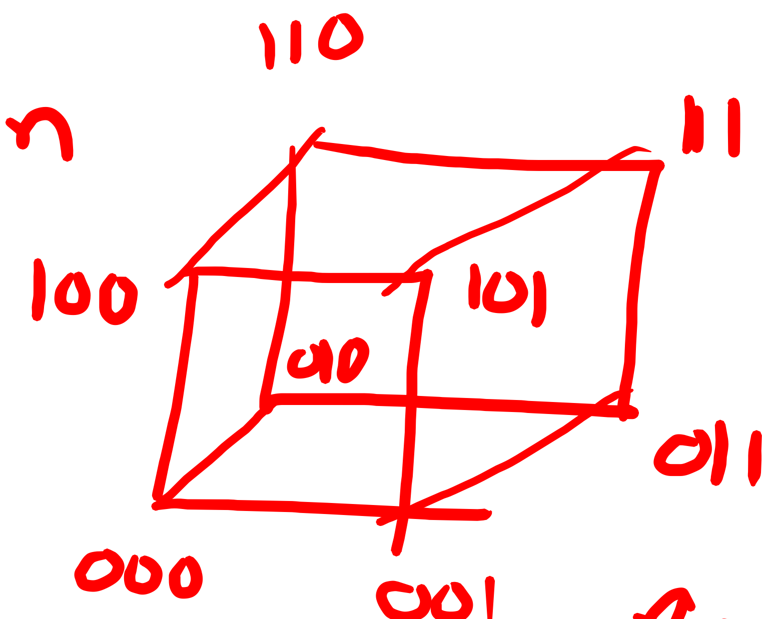
2^n strings



Q_1



Q_2



Q_2

Bipartite Graphs

Def^n A simple graph G is called bipartite if its vertex set V can be partitioned into two disjoint nonempty sets V_1 and V_2 s.t. every edge in the graph connect a vertex in V_1 and a vertex in V_2.
(No two vertices in V_1 are adjacent and no two vertices in V_2 are adjacent). (V_1, V_2) a bipartition.

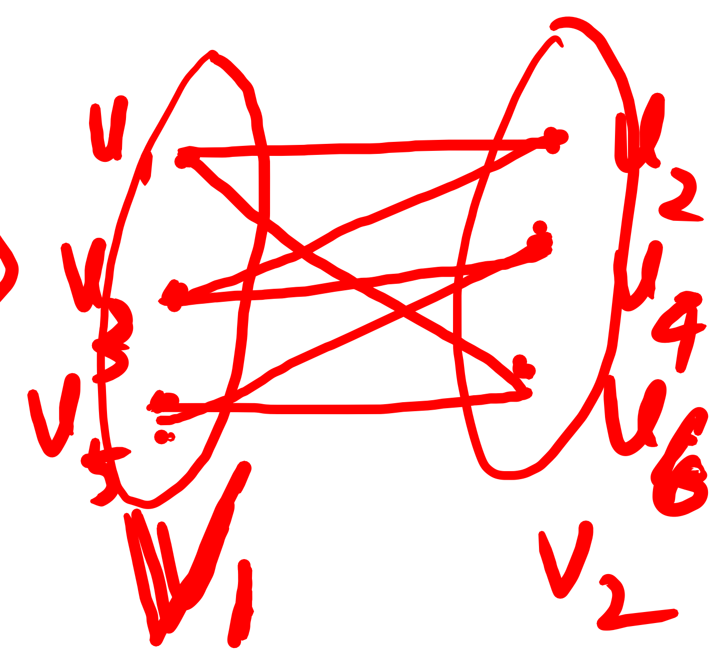
$V = V_1 \cup V_2$ $\quad V_1 \cap V_2 = \phi$

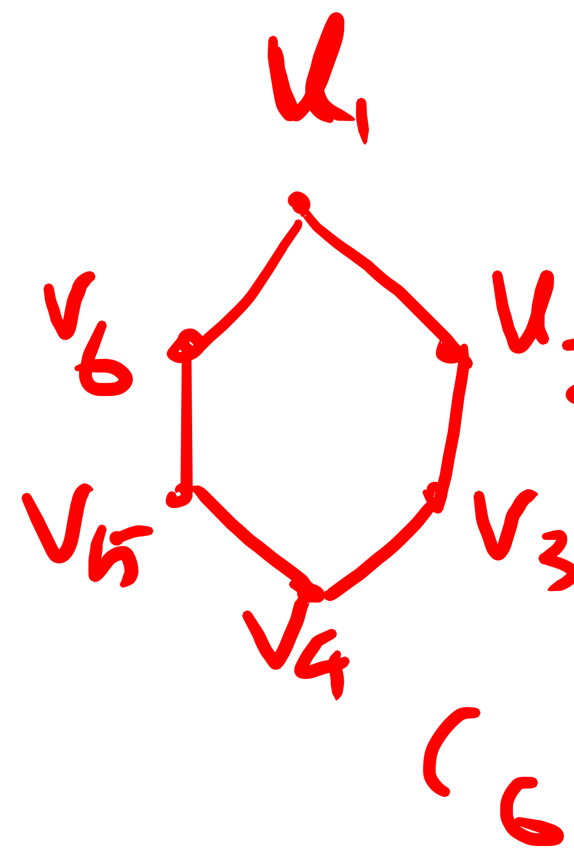
each edge connects a vertex in V_1 and a vertex in V_2

Example C_6

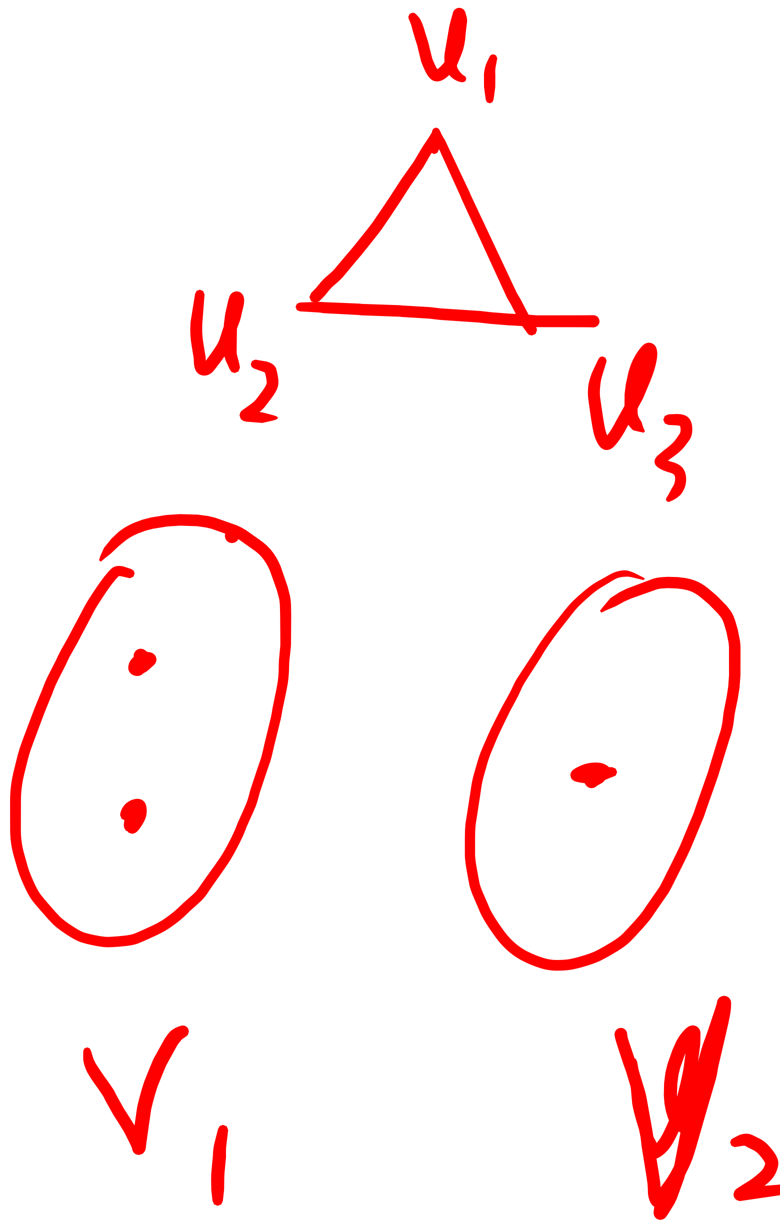
$V_1 = \{v_1, v_3, v_5\}$

$V_2 = \{v_2, v_4, v_6\}$

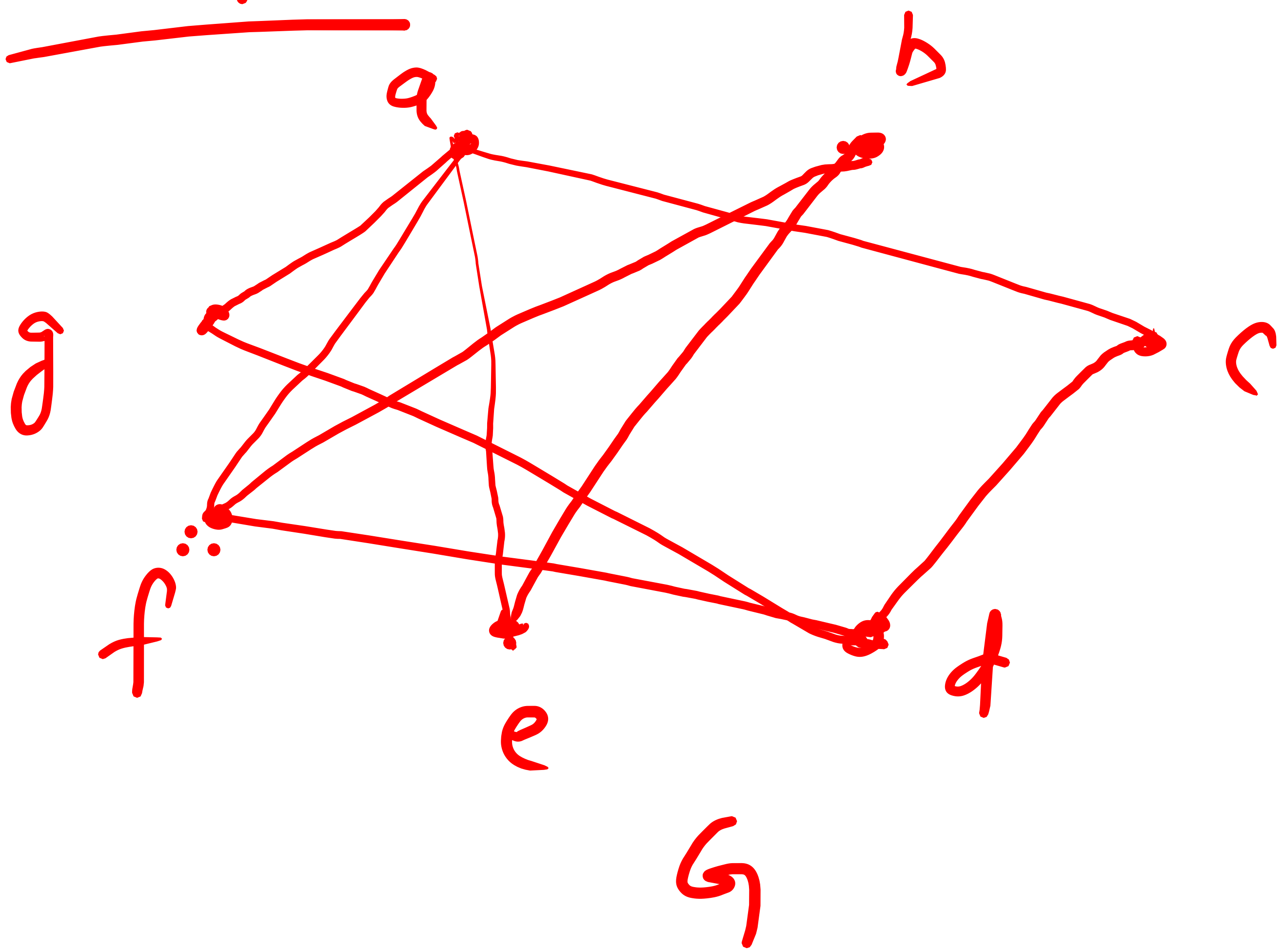




<u>Exp</u> K_3 is not bipartite



Example



$V_1 = \{a, b, d\}$

$V_2 = \{c, e, f, g\}$

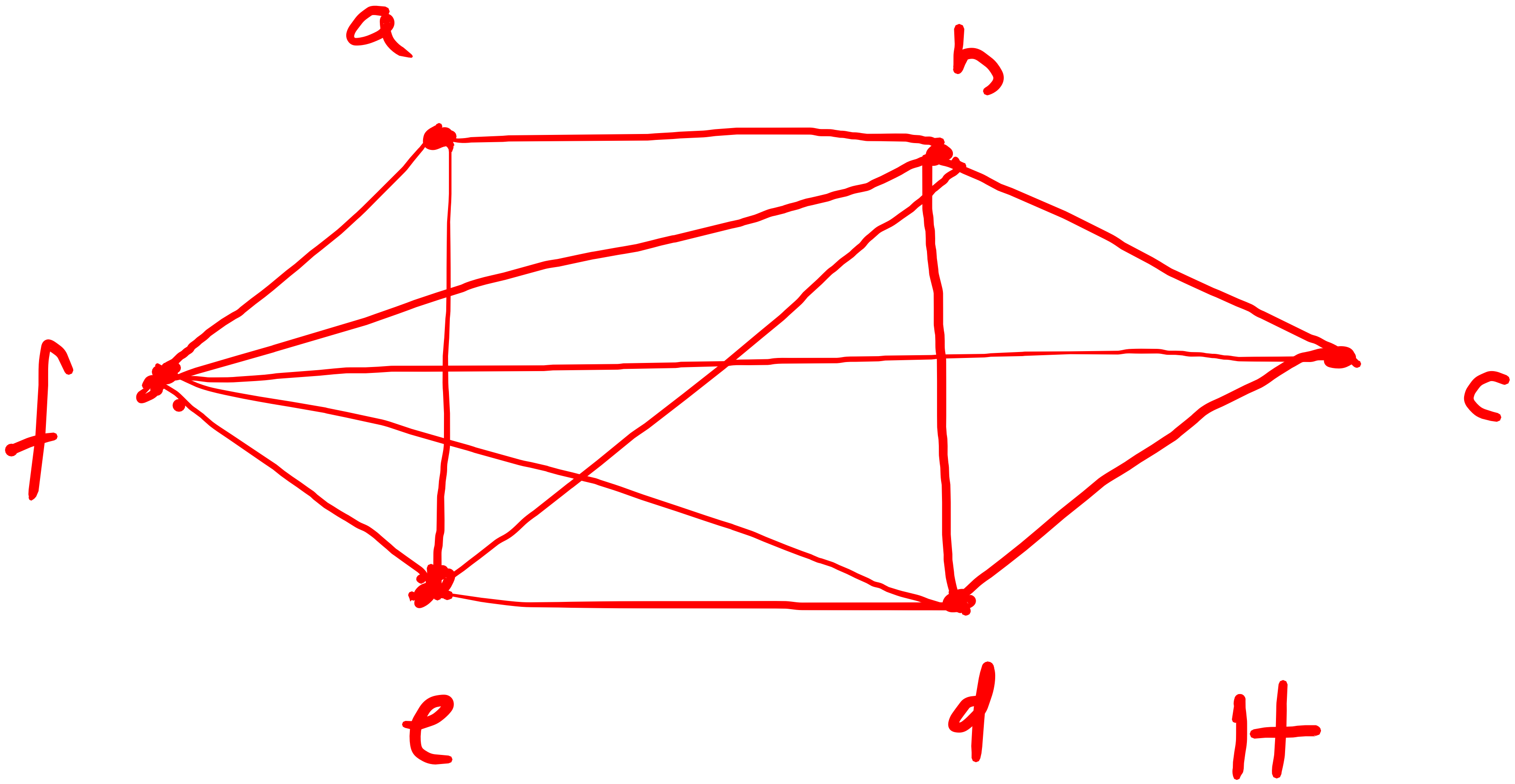
bipartite

blue red

a → red

g, f, e, c → blue

b, d, – red



not bipartite

a → red

f, e, b → blue

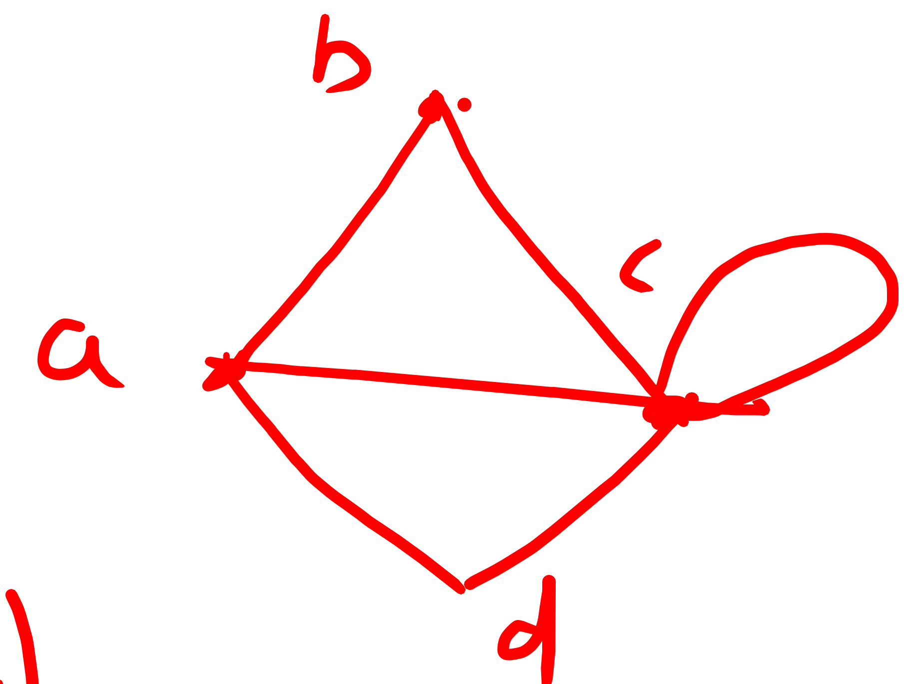
Thm A simple graph is bipartite iff it is possible to assign one of two different colours to each vertex of the graph so that no two adjacent vertices are assigned the same colour.

Pf $V = V_1 \cup V_2$ $\quad V_1 \cap V_2 = \phi$.

Degree sequence

The degree sequence of a graph is the sequence of the degrees of the vertices of the graph in non increasing order.

$(5, 3, 2, 2)$



$(3, 3, 2, 2)$