

1. Calculate the laplacian of the following:

(i)  $F = x^2 + 2xy + 3z + 4$       (ii)  $F = \sin(\hat{\mathbf{k}} \cdot \vec{\mathbf{r}})$       (iii)  $F = \frac{1}{r}$

2. Evaluate  $(\hat{\mathbf{r}} \cdot \vec{\nabla})r$  and  $(\hat{\mathbf{r}} \cdot \vec{\nabla})\hat{\mathbf{r}}$

3. Find the volume of an ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  using the tripple integral  $\int \int \int dx dy dz$  with appropriate limits.

4. Consider  $\vec{\mathbf{A}} = x^2\hat{\mathbf{i}} + y^2\hat{\mathbf{j}} + z^2\hat{\mathbf{k}}$

(a) Evaluate  $\oint_S \vec{\mathbf{A}} \cdot d\vec{\mathbf{a}}$  where  $S$  is a cubical surface given by the planes  $x = a \pm l$ ;  $y = b \pm l$ ;  $z = c \pm l$ .

(b) Verify that at the point  $(a, b, c)$ ,

$$\vec{\nabla} \cdot \vec{\mathbf{A}} = \lim_{l \rightarrow 0} \frac{1}{8l^3} \oint_S \vec{\mathbf{A}} \cdot d\vec{\mathbf{a}}$$

5. Evaluate  $\int_P^Q \vec{\mathbf{A}} \cdot d\vec{\mathbf{l}}$  for  $\vec{\mathbf{A}} = y\hat{\mathbf{i}} - x\hat{\mathbf{j}}$  along the following paths :  
 $P \equiv (-a, 0)$ ;  $Q \equiv (a, 0)$ .

(a)  $(-a, 0) \rightarrow (0, a) \rightarrow (a, 0)$

(b)  $(-a, 0) \rightarrow (0, -a) \rightarrow (a, 0)$

(c) a loop, forward along (a) and backward along (b)

(d) Let  $I$  be the value of the loop integral evaluated in (c). Let  $S$  be the flat area enclosed by the loop. Verify that at the origin

$$(\vec{\nabla} \times \vec{\mathbf{A}}) = \left[ \lim_{a \rightarrow 0} \frac{I}{S} \right] (-\hat{\mathbf{k}})$$

(e) Can we find a scalar function  $F$  such that  $\vec{\nabla}F = y\hat{\mathbf{i}} - x\hat{\mathbf{j}}$  ?