Change the order of integration

$$\iint_{R} f(x,y)\, dy\, dx$$

Integrate w.r.t y first then w.r.t x.

<u>Step-1</u> Sketch the region of integration and label the boundary curves.

<u>Step-2</u> Imagine a vertical line L cutting through R in the direction of increasing y. Mark the y-values where L enters and leaves. These are y-limits of integration. They are usually functions of x.

<u>Step-3</u> Choose x-limits that include all the vertical lines through R.

$$\iint_R f(x,y)\,dx\,dy$$

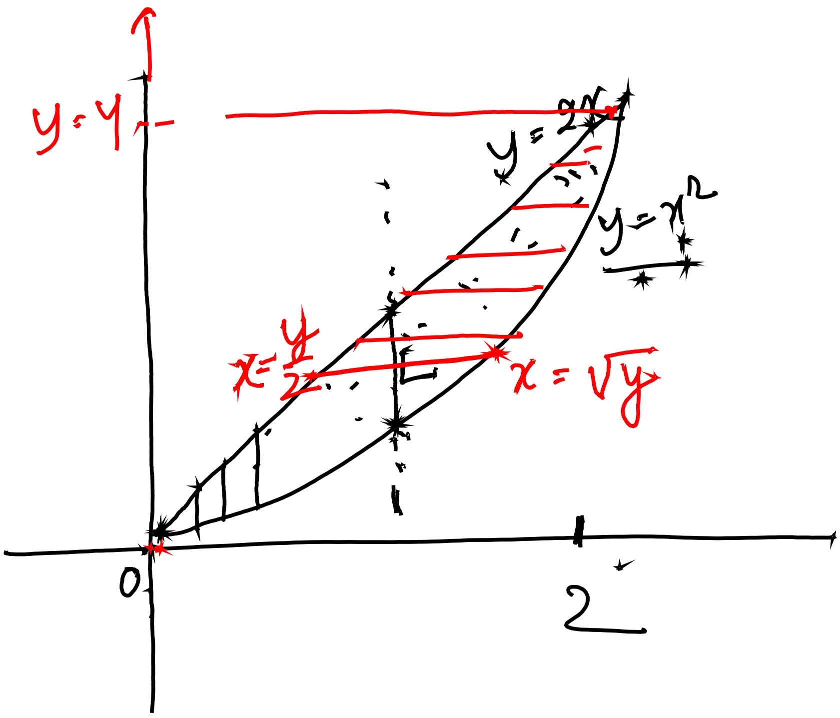
Step-1 same.

Step-2 Imagine a horizontal line L cutting through R in the direction of increasing x.
Mark the x-values where L enters and leaves the region R.
These are x-limits of integration and they are usually functions of y.

Step-3 Choose y-limits of integration that include all the horizontal lines through R.

Exp Change the order of integration of

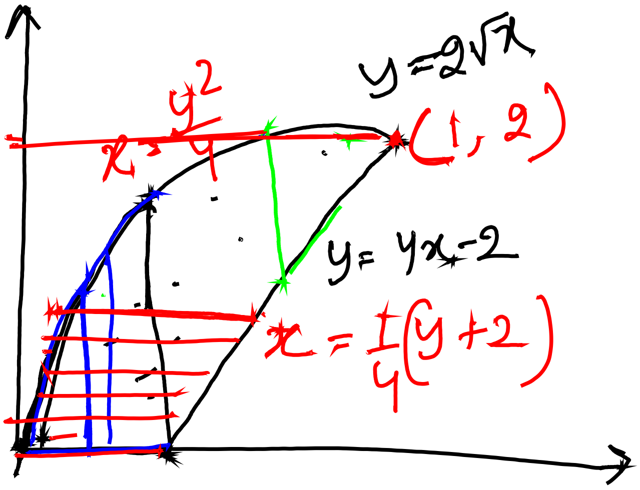
$$\int_{x=0}^{x=2}\int_{y=x^2}^{y=2x}(4x+2)\,dy\,dx = \boxed{}$$



$$\int_{y=0}^{y=4}\int_{x=\frac{y}{2}}^{x=\sqrt{y}}(4x+2)\,dx\,dy = \boxed{}$$

Exp Find the volume of the solid that lies beneath the surface $z = 16 - x^2 - y^2$ and above the region R bounded by the curve $y = 2\sqrt{x}$ and the line $y = 4x - 2$ and the x-axis.

Sol^n



$$\int\int (16 - x^2 - y^2)\, dy\, dx$$

$$\boxed{\int_{y=0}^{y=2}\int_{x=\frac{y^2}{4}}^{x=\frac{1}{4}(y+2)} (16 - x^2 - y^2)\, dx\, dy}$$

Area by double integrals

The area of a closed bounded plane region R is

$$A = \iint_R dx\,dy$$

$$= \iint_R dy\,dx$$

The volume of a solid using triple integral

$$V = \iiint_D dV$$

$\cancel{D \subset \mathbb{R}^3}$

$$= \iiint_D dx\,dy\,dz$$

Find the limits of integration in the order $dz\,dy\,dx$

$$\iiint_D dz\,dy\,dx$$

Step-1 Sketch the region D along with its shadow R in the xy-plane. Label the upper and lower boundary surfaces of D and the upper and lower boundary curves in R.

Step-2 Find z limits of integration.

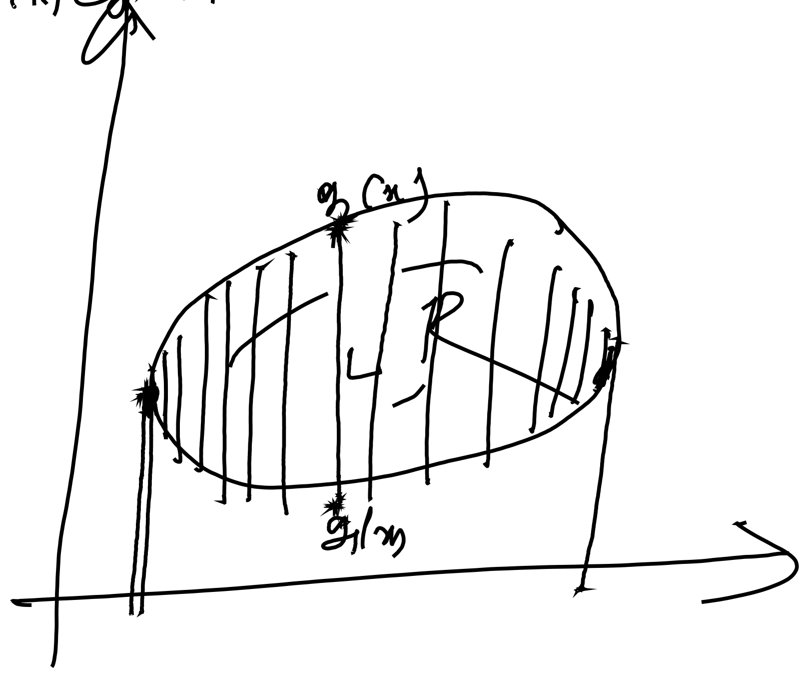
Draw a line M passing through a typical point (x,y) in R, parallel to z-axis. As z increases M enters D at $z = f_1(x,y)$ and leaves D at $z = f_2(x,y)$.

There are z limits of integration.

Step 3   Finding the y-limits of integration

Draw a line L through xy plane parallel to y-axis. As y-increases L enters R at $y = g_1(x)$ and leaves R at $y = g_2(x)$. These are y-limits of integration.

Step-4   Choose x-limit that include all the lines L through R parallel to y-axis. These are the x-limits of integration.

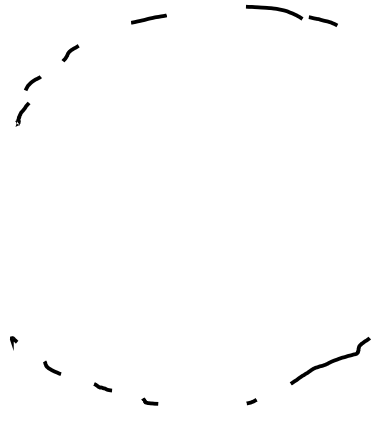


Exp   Find the volume of the solid D enclosed by the surfaces $z = x^2+3y^2$ and $z = 8-x^2-y^2$.

$$V = \iiint_D dz\,dy\,dx$$

z limits of integration.

$$\int_{x^2+3y^2}^{8-x^2-y^2} dz$$



Y- limits of integration

$$x^2+3y^2 = 8-x^2-y^2$$

$$\Rightarrow 2x^2+4y^2 = 8$$

$$\Rightarrow x^2+2y^2 = 4 \qquad y^2 = \frac{4-x^2}{2}$$

$$\Rightarrow y_1(x) = -\sqrt{\frac{4-x^2}{2}} \qquad y_2(x) = +\sqrt{\frac{4-x^2}{2}}$$

x- limits of integration

letting $y = 0$

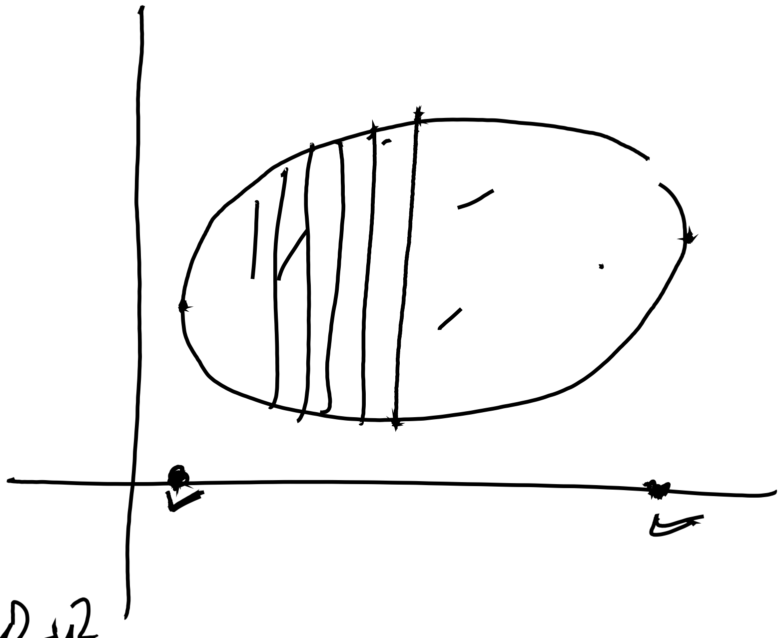
$$4 - x^2 = 0$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

$x_1 = -2$, $x_2 = +2$

$V =$



$$V = \int_{-2}^{2} \int_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}} \int_{x^2+3y}^{8-x^2-y^2} dz \; dy \; dx$$