$m \times n$ Matrix as a function $R^n \to R^m$

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We saw in earlier lectures how an $m \times n$ matrix with real number entries is a representation of coefficients of a real values system of m linear simultaneous equations in n identical variables, aligned. When multiplied by a column vector with the variables, it results in the column vector consisting of the constant terms on the right hand sides of the equations. We will look today at the matrix alone and allow the two column vectors to be variable.

In this framework the column vector on the right hand side can be thought of as a scalar linear combination of the columns of the matrix. The number of linearly independent columns must be exacty m in order to generate any possible column vector b. It is not possible to have more than m linearly independent columns of dimension m vectors. There can however, be fewer than m. In this case, it is not possible to generate all possible columns. Those that may be generated constitut what is called the column space of the matrix. The column space vectors all belong to R^m.

When we consider the homogeneous equatios $Ax = 0$, there is always a trivial solution for x involving the all zero vector. All other solutions constitute what is called the null space. The null space vectors are all from R^n.

An inconsistent system of equatons always has two equations (either originally or after simplifying and eliminating some), which are scaled on the variable coefficients, but not so on the right hand constants. This leads to the absurd conclusion 1=0 in all such cases.

The number of linearly independent columns in the matrix is called the **column rank** of the matrix. No vector in a list of linearly independent vectors can be generated by a linear combination of the other vectors in the list. Conversely, any vector not generatable by a list of linearly independent vectors can be added to that list to get a bigger list of linearly independent vectors.