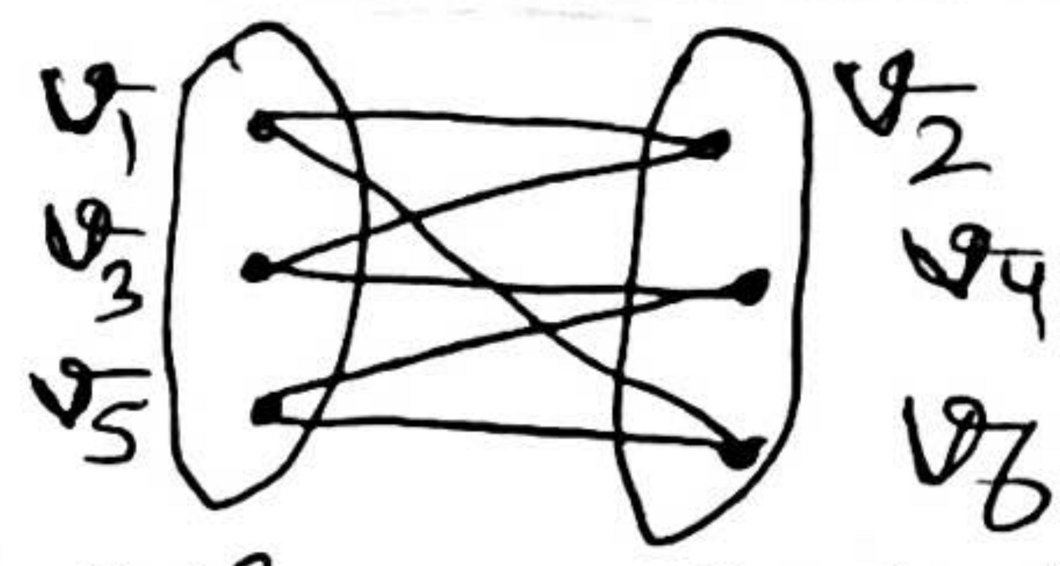
Example C_6 is bipartite
K_3 is not bipartite.
(Verify!)



$V_1 = \{1, 3, 5\}$ $V_2 = \{2, 4, 6\}$

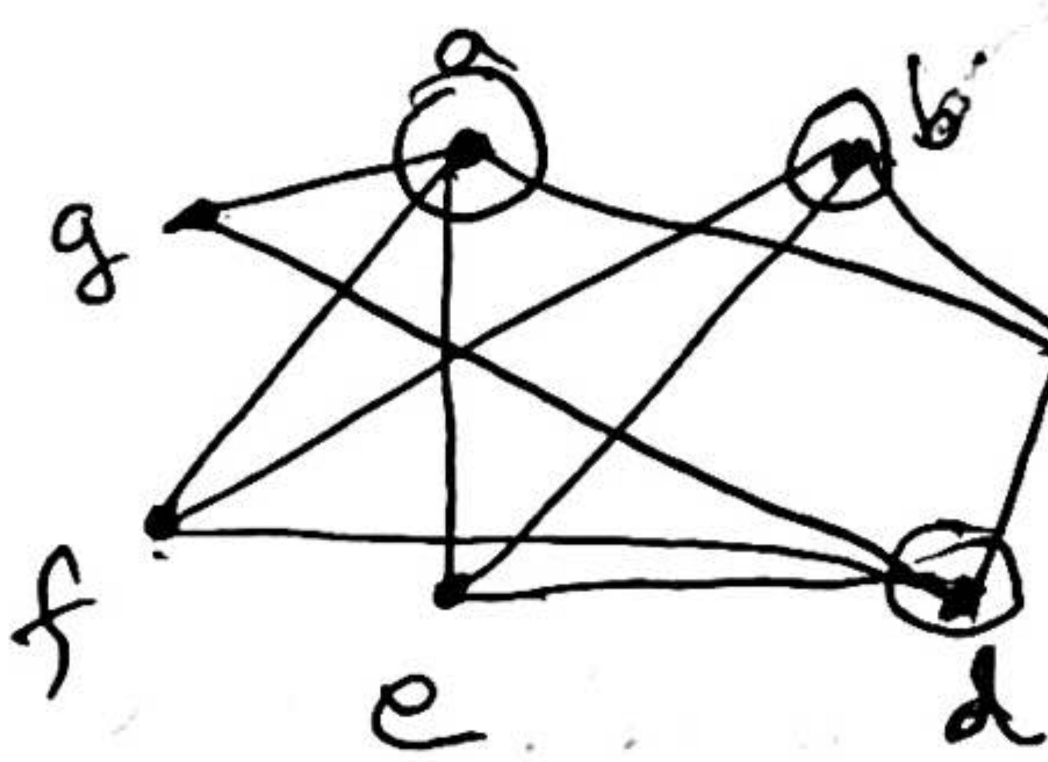
Theorem

A simple graph is bipartite iff it is possible to assign one of two different colors to each vertex of the graph so that no two adjacent vertices are assigned the same color.

($\Rightarrow$) $\square$ $G = (V, E)$ bipartite simple graph. $V = V_1 \cup V_2$, $V_1 \cap V_2 = \emptyset$
$\Rightarrow$ Every edge in E connects a vertex in V_1 & a vertex in V_2.
Assign one color in V_1 & one in V_2.

($\Leftarrow$) Suppose it is possible to assign colors to vertices using 2 colors so that no two adjacent vertices have same color. V_1 = set of vertices with one color & V_2 = set of vertices with other color. Then $V = V_1 \cup V_2$, $V_1 \cap V_2 = \emptyset$
$\Rightarrow$ G is bipartite graph. ▨

Example



G

Assign $a \to$ red

$c, e, f, g \to$ blue

assign red to all vertices adjacent to c, e, f, g

$\Rightarrow$ b & d red

$\Rightarrow$ a must be red

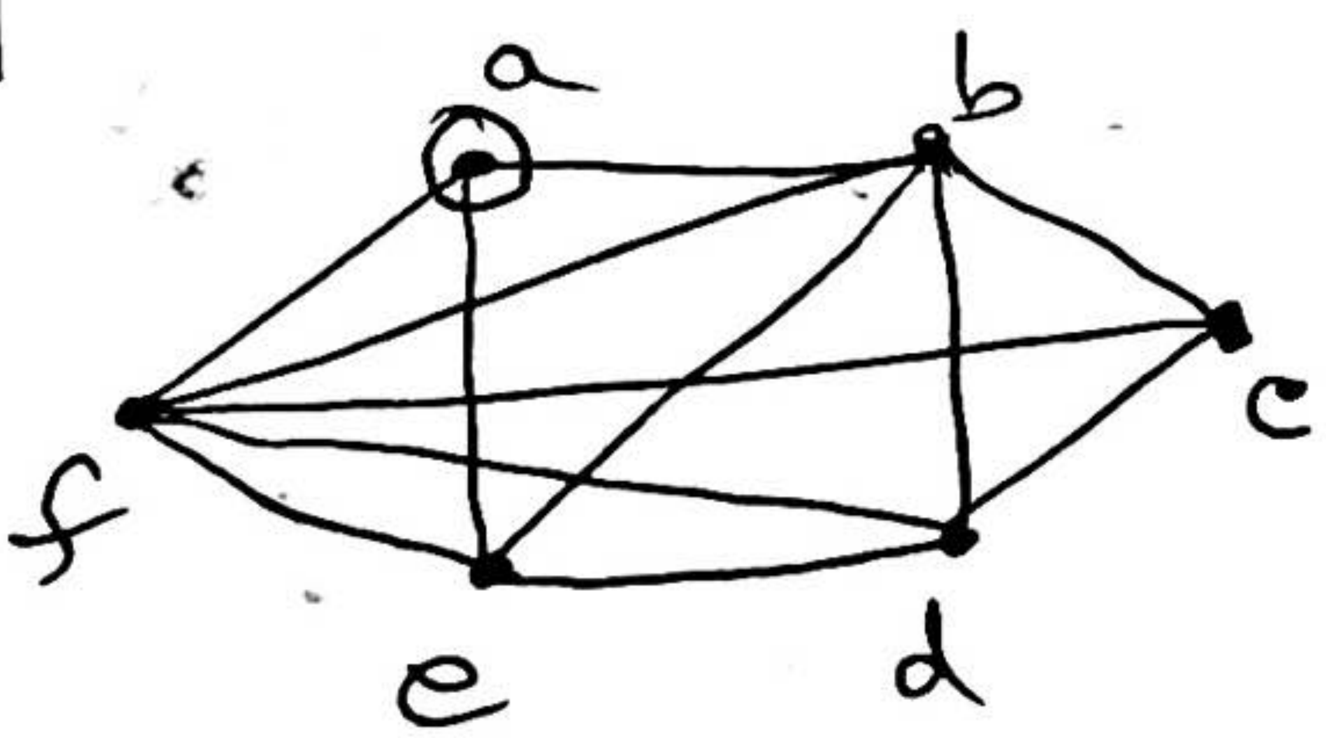
$\therefore V_1 = \{a, b, d\} \to$ Red

$V_2 = \{c, e, f, g\} \to$ Blue

$\Rightarrow$ G is bipartite

Example

H



H is not bipartite

Assign $a \to$ red

$\Rightarrow$ f, e & b blue

$\Rightarrow$ d & c should be red

$\Rightarrow$ d & c adjacent vertices assign same color.

f & e are blue adjacent

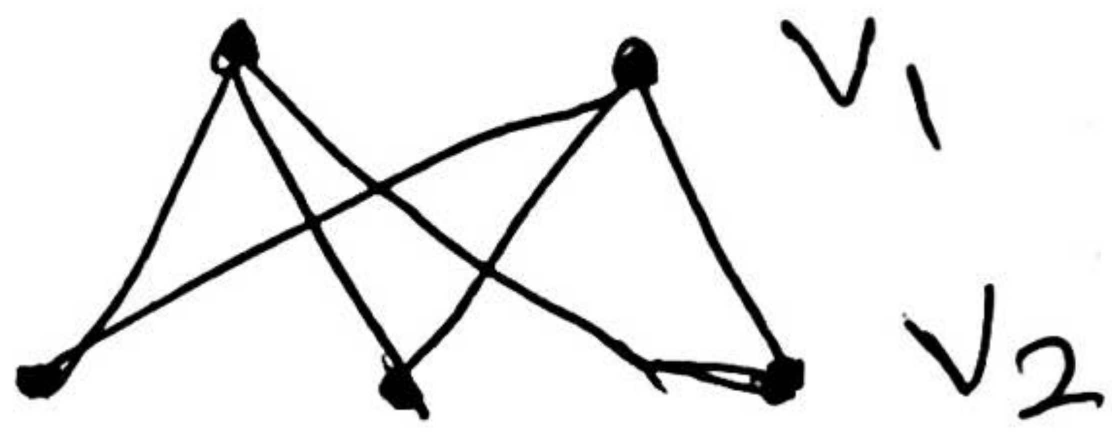
Complete Bipartite graph

$K_{m;n}$ $\quad V = V_1 \cup V_2$

$|V_1| = m$ & $|V_2| = n$

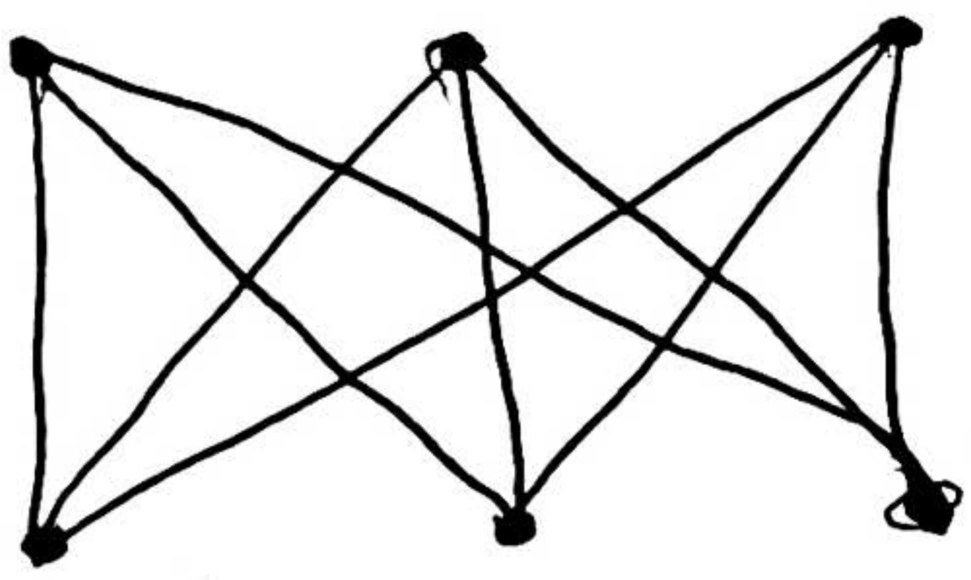
There is an edge between two vertices iff one vertex is in V_1 & other in V_2

Example



$K_{2;3}$

Job assignments & Local Area Networks



$K_{3;3}$

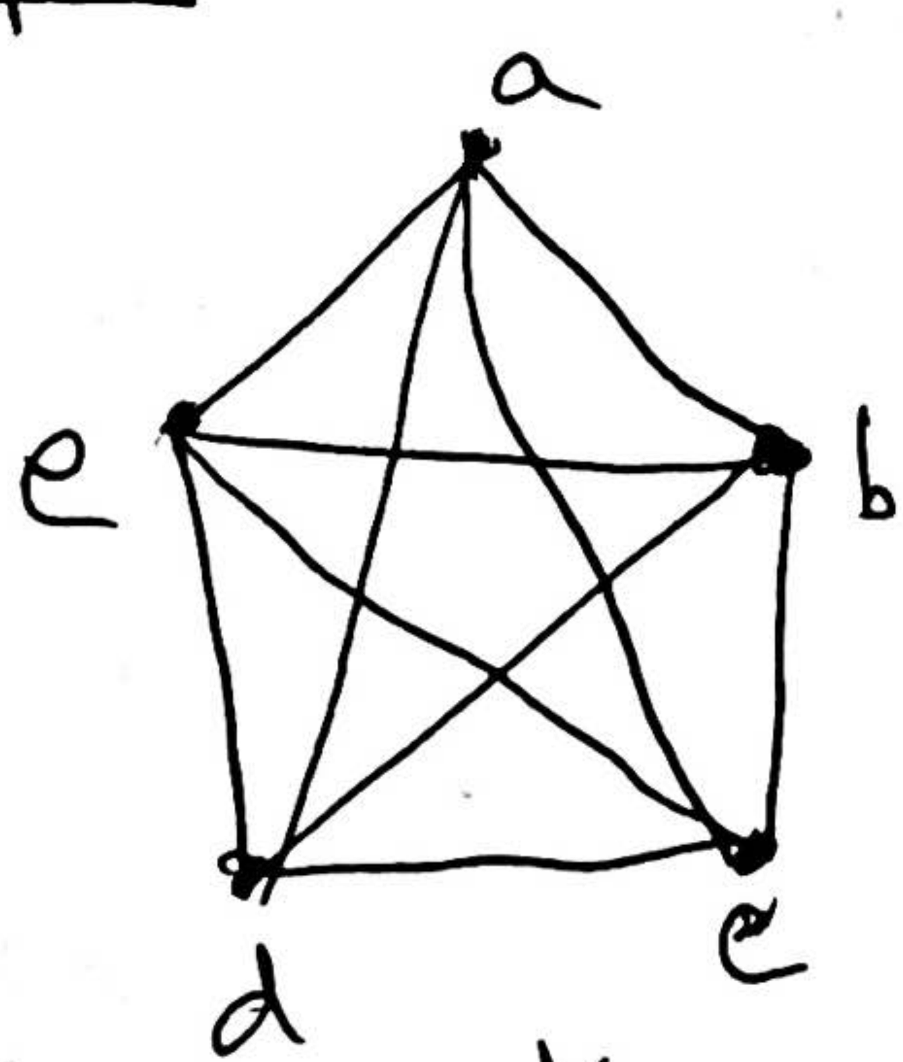
Subgraph

$G = (V, E)$ graph

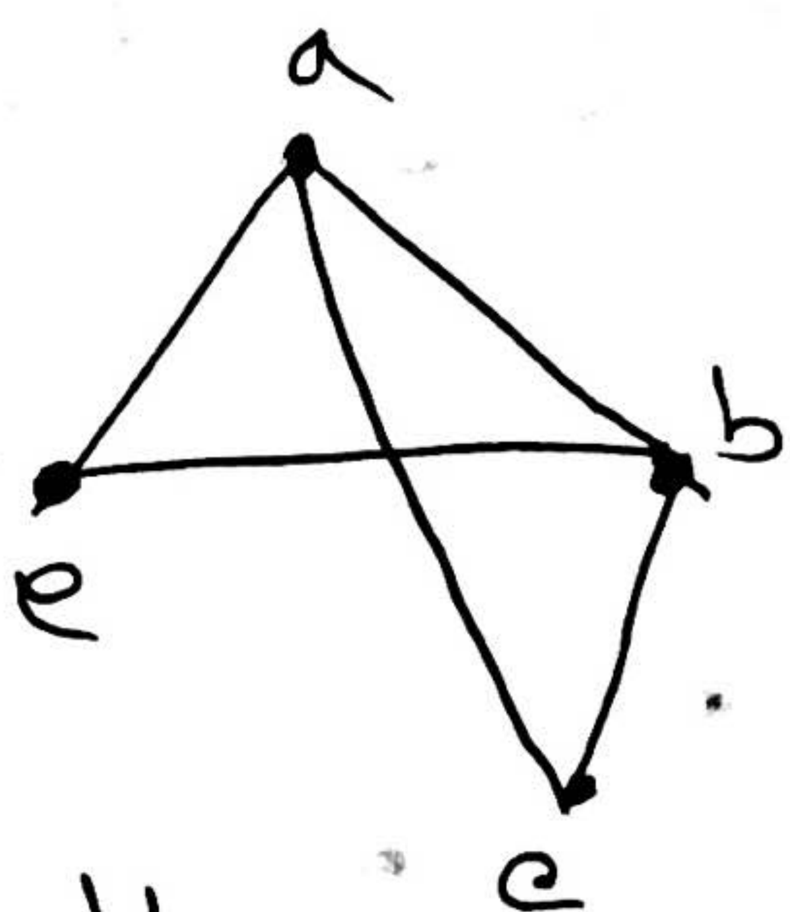
$H = (W, F)$ is a subgraph if $W \subseteq V$ & $F \subseteq E$

A subgraph H is proper if $H \neq G$

Example



$G = K_5$



H

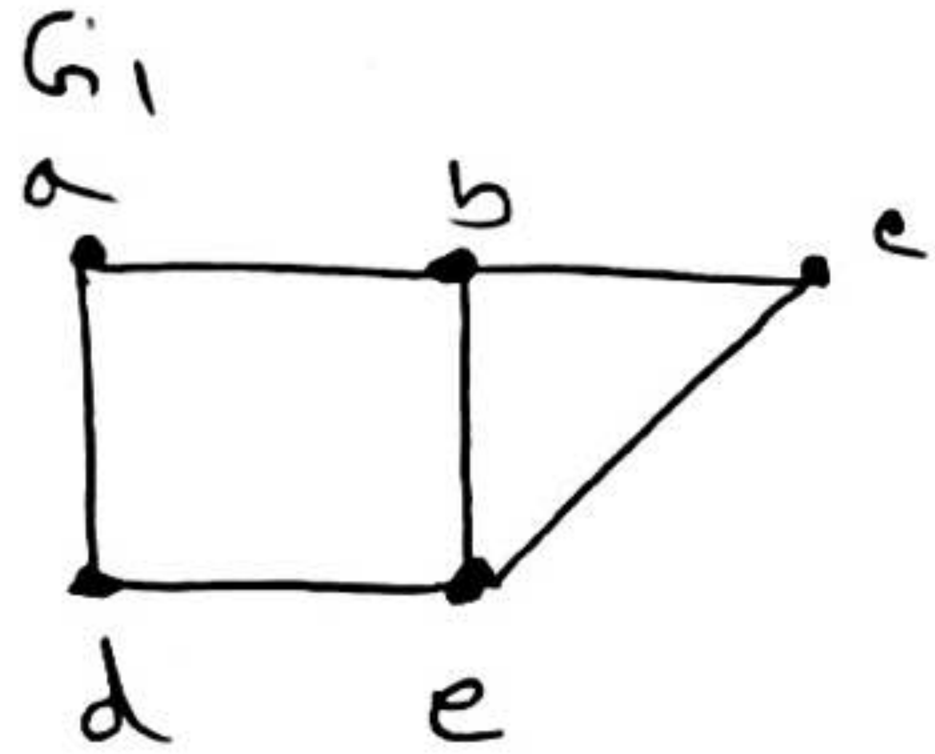
$H \leq G$

Union of two simple graphs G_1 & G_2

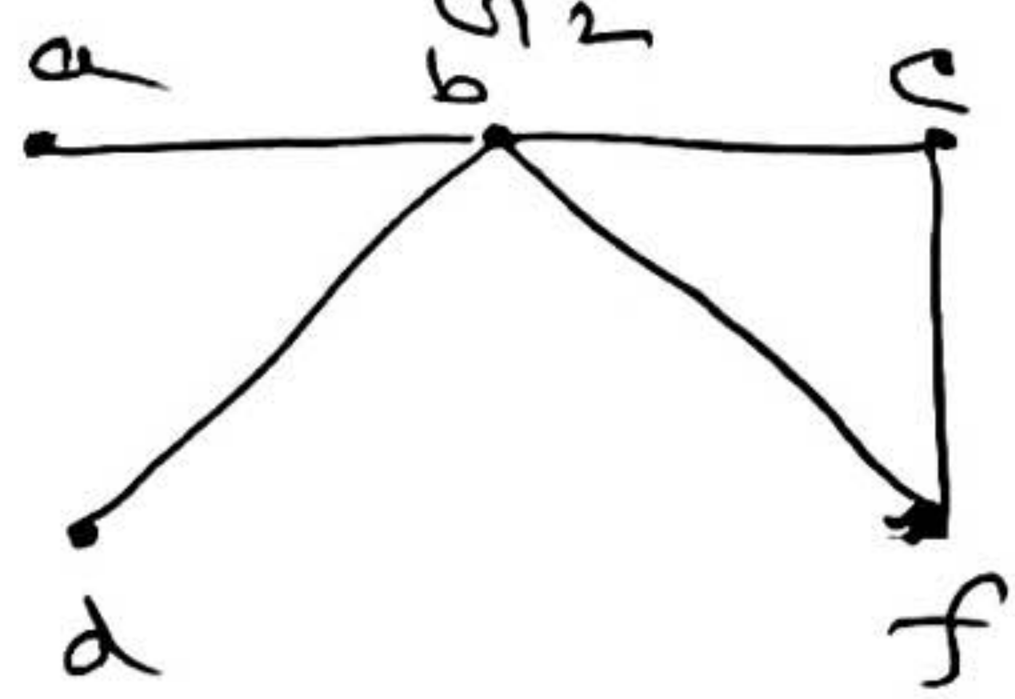
$G_1 = (V_1, E_1)$ & $G_2 = (V_2, E_2)$

Union graph $G_1 \cup G_2$ $= (V_1 \cup V_2,\ E_1 \cup E_2)$

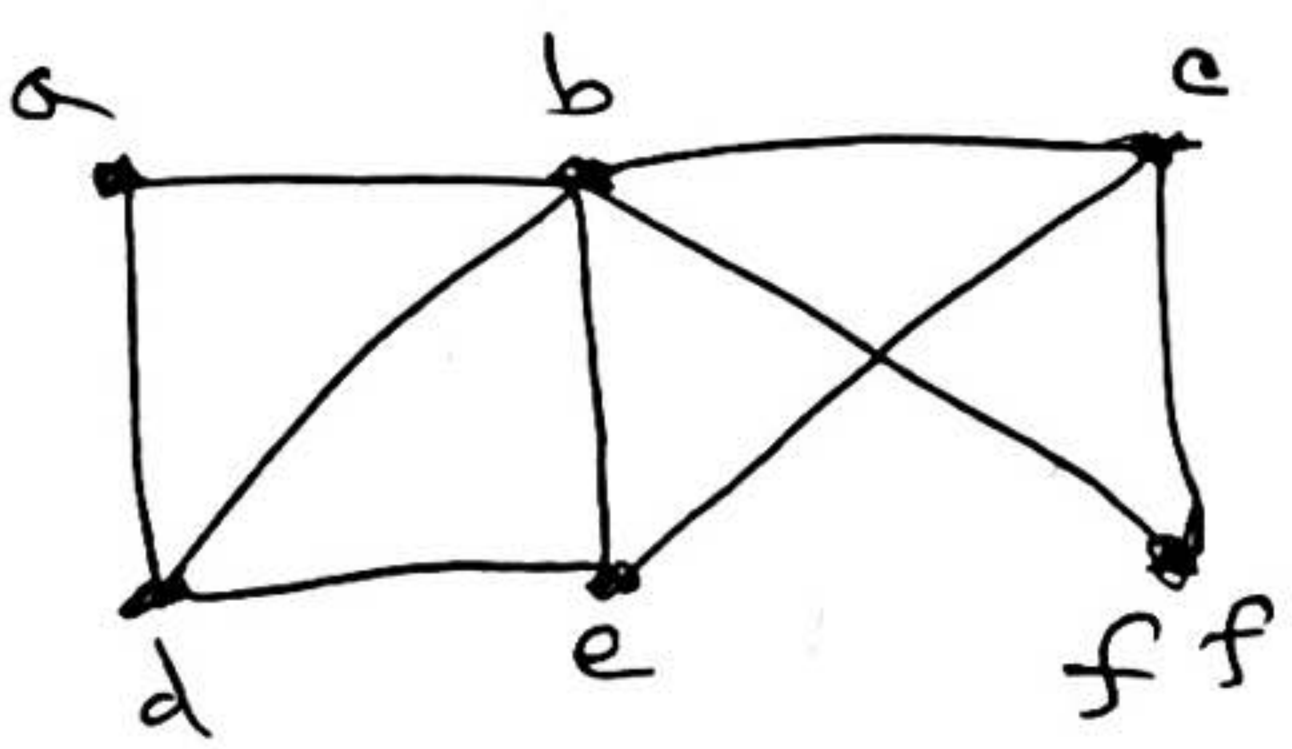
G_1



G_2

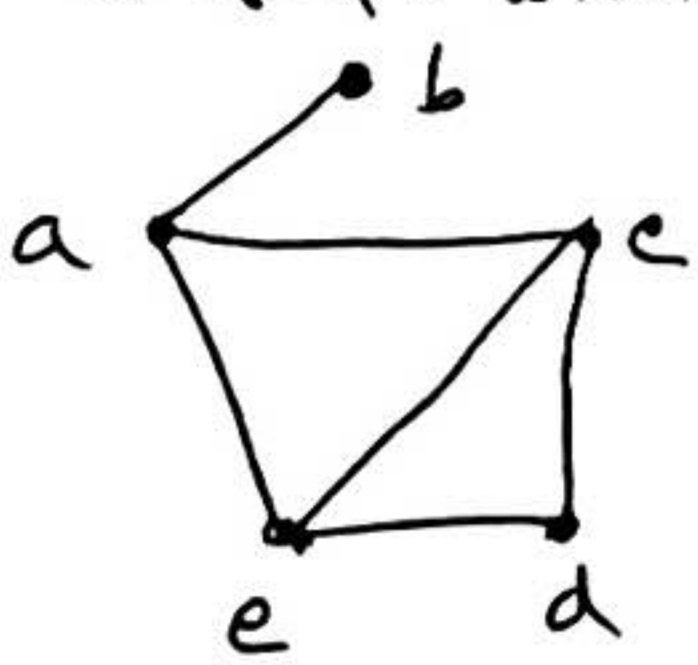


$\Rightarrow G_1 \cup G_2$



Representing graphs

(1) A graph without multiple edges

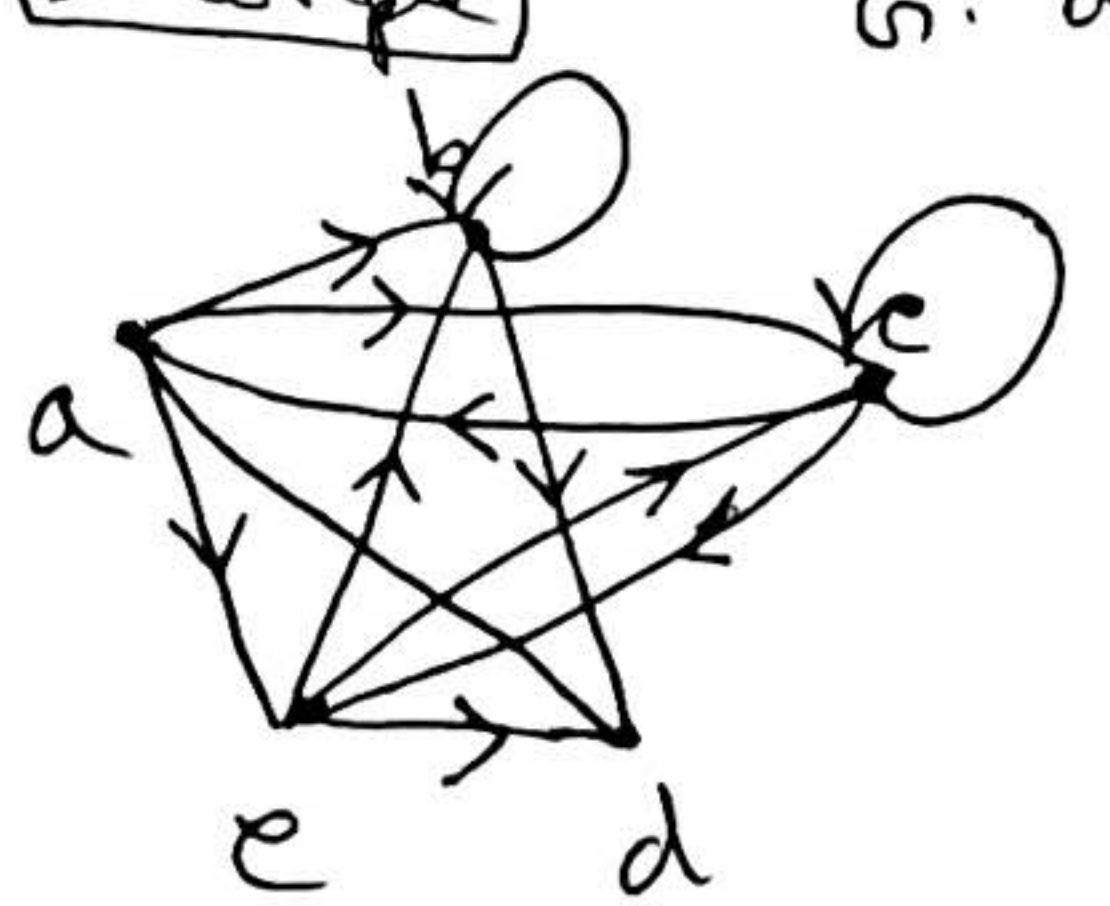


Simple graph

Adjacency list

Vertex	Adjacent Vertices
a	b, c, e
b	a
c	a, d, e
d	c, e
e	a, c, d

Example G: directed graph



Adjacency list

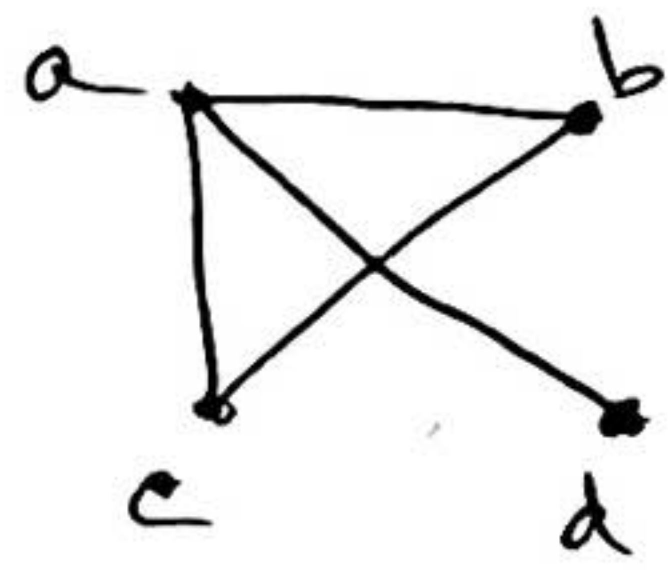
Initial Vertex	Terminal Vertices
a	b, c, d, e
b	b, d
c	a, c, e
d	
e	b, c, d

ADJACENCY MATRICES

$G = (V, E)$ $\quad |V| = n \quad V = \{1, 2, 3, \ldots, n\}$ listed arbitrarily

$A = (a_{ij})$ Binary matrix $\quad a_{ij} = \begin{cases} 1 & \text{if } \{i, j\} \text{ is an edge of } G \\ 0 & \text{otherwise} \end{cases}$

Example

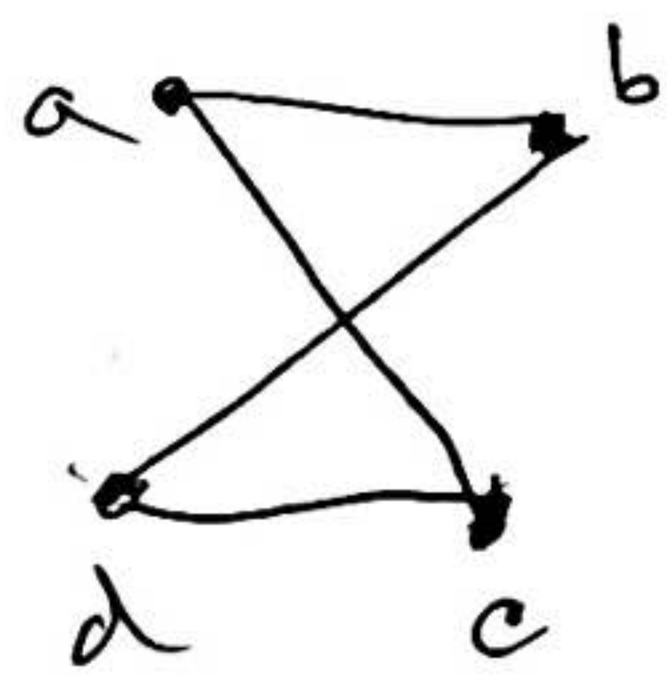


$V = \{a, b, c, d\}$

$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Example let $A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ find G.

w.r. to ~~order~~ ordering of vertices a, b, c & d



Note: A is symm. for simple graphs