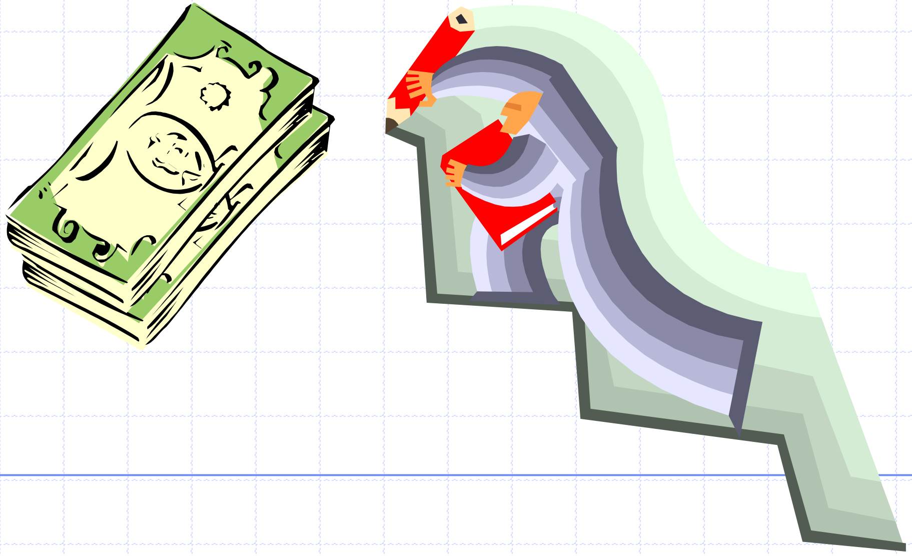
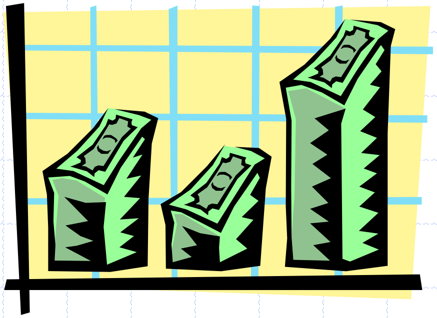
The Greedy Method



Outline and Reading

- The Greedy Method Technique
- Fractional Knapsack Problem
- Task Scheduling
- Minimum Spanning Trees [future lecture]

The Greedy Method Technique



- **The greedy method** is a general algorithm design paradigm, built on the following elements:
  - **configurations**: different choices or collections
  - **objective function**: a score assigned to configurations, which we want to either maximize or minimize
- It works best when applied to problems with the **greedy-choice** property:
  - a globally-optimal solution can always be found by a series of local improvements from a starting configuration.

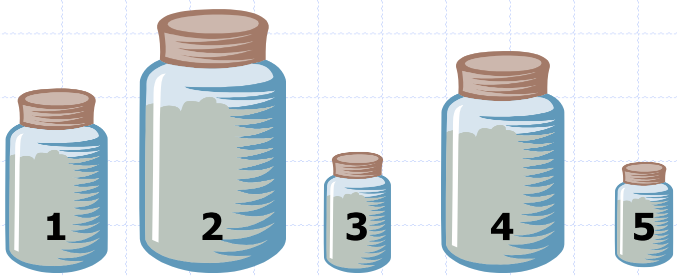
The Fractional Knapsack Problem

- Given: A set S of n items, with each item i having
  - b_i - a positive benefit
  - w_i - a positive weight
- Goal: Choose items with maximum total benefit but with weight at most W.
- If we are allowed to take fractional amounts, then this is the **fractional knapsack problem**.
  - In this case, we let x_i denote the amount we take of item i
  - Objective: maximize $\sum_{i \in S} b_i (x_i / w_i)$
  - Constraint: $\sum_{i \in S} x_i \leq W$

Example

- Given: A set S of n items, with each item i having
  - b_i - a positive benefit
  - w_i - a positive weight
- Goal: Choose items with maximum total benefit but with weight at most W.

Items:



Items:	1	2	3	4	5
Weight:	4 ml	8 ml	2 ml	6 ml	1 ml
Benefit:	\$12	\$32	\$40	\$30	\$50
Value: (\$ per ml)	3	4	20	5	50

“knapsack”

10 ml

Solution:
- 1 ml of 5
- 2 ml of 3
- 6 ml of 4
- 1 ml of 2

The Fractional Knapsack Algorithm

- Greedy choice: Keep taking item with highest **value** (benefit to weight ratio)
  - Since $\sum_{i \in S} b_i (x_i / w_i) = \sum_{i \in S} (b_i / w_i) x_i$
  - Run time: ?
- Correctness: ?

Algorithm *fractionalKnapsack*(S, W)

Input: set S of items w/ benefit b_i and weight w_i; max. weight W

Output: amount x_i of each item i to maximize benefit with weight at most W

for *each item i in S*

$x_i \leftarrow 0$

$v_i \leftarrow b_i / w_i$ {value}

$w \leftarrow 0$ {total weight}

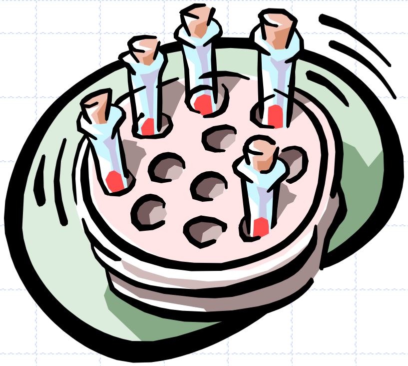
while $w < W$

remove item i with highest v_i

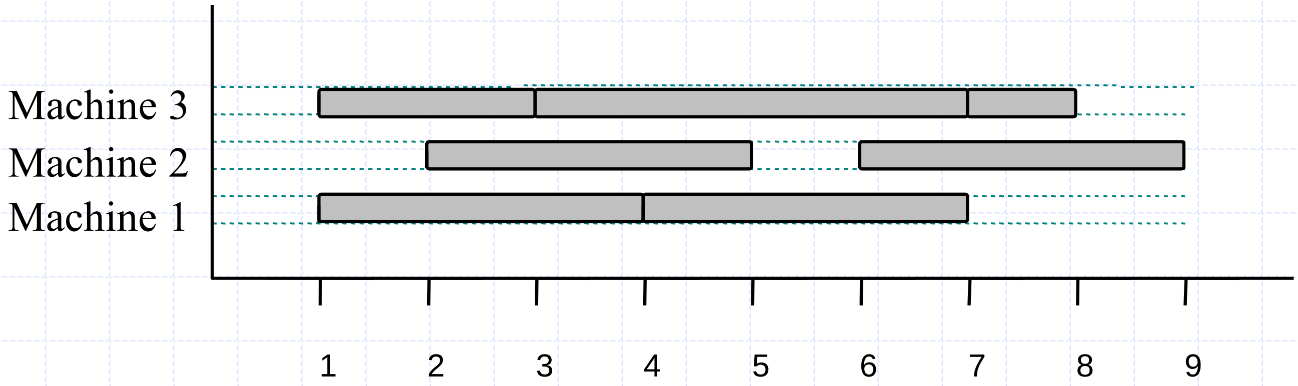
$x_i \leftarrow \min\{w_i, W - w\}$

$w \leftarrow w + \min\{w_i, W - w\}$

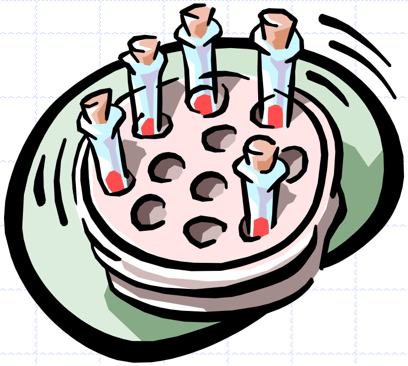
Task Scheduling



- Given: a set T of n tasks, each having:
  - A start time, s_i
  - A finish time, f_i (where $s_i < f_i$)
- Goal: Perform all the tasks using a minimum number of "machines."



Task Scheduling Algorithm



- Greedy choice: consider tasks by their start time and use as few machines as possible with this order.
  - Run time: O(n log n). Why?
- Correctness:

Algorithm ***taskSchedule***(*T*)

  Input: set *T* of tasks w/ start time s_i and finish time f_i

  Output: non-conflicting schedule with minimum number of machines

  $m \leftarrow 0$ {no. of machines}

  while *T is not empty*

    remove task i w/ smallest s_i

    if ***there's a machine j for i*** **then**

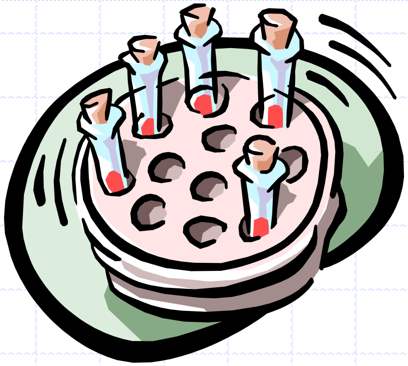
      schedule i on machine j

    else

      $m \leftarrow m + 1$

      schedule i on machine m

Task Scheduling Algorithm



- Greedy choice: consider tasks by their start time and use as few machines as possible with this order.
  - Run time: O(n log n). Why?
- Correctness: Suppose there is a better schedule.
  - We can use k-1 machines
  - The algorithm uses k
  - Let i be first task scheduled on machine k
  - Machine i must conflict with k-1 other tasks
  - But that means there is no non-conflicting schedule using k-1 machines

Algorithm ***taskSchedule***(***T***)

Input: set ***T*** of tasks w/ start time s_i and finish time f_i

Output: non-conflicting schedule with minimum number of machines

$m \leftarrow 0$ {no. of machines}

while ***T is not empty***

remove task i w/ smallest s_i

if ***there's a machine j for i*** **then**

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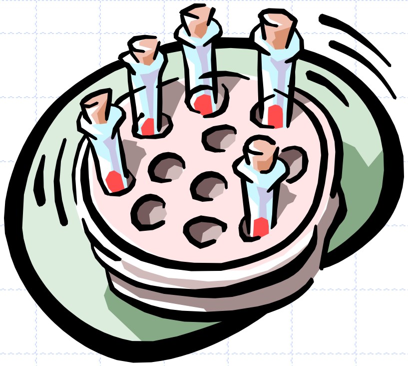
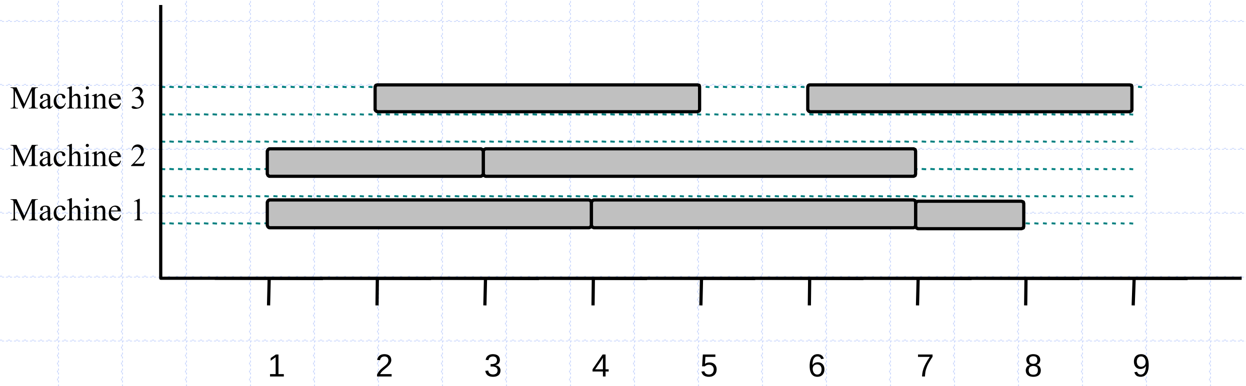
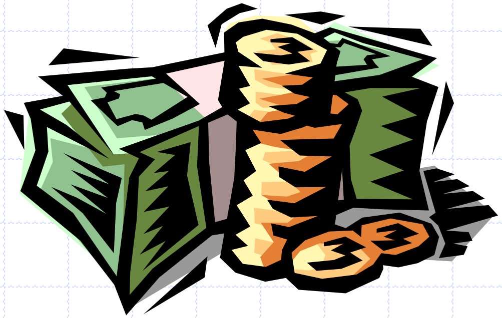
Example



- Given: a set T of n tasks, each having:
  - A start time, s_i
  - A finish time, f_i (where $s_i < f_i$)
  - [1,4], [1,3], [2,5], [3,7], [4,7], [6,9], [7,8] (ordered by start)
- Goal: Perform all tasks on min. number of machines



Making Change



- Problem: A dollar amount yet to return to a customer.
- Objective function: Minimize number of coins returned.
- Greedy solution: Always return the largest coin you can
- Example 1: Coins are valued \$.32, \$.08, \$.01
  - Has the greedy-choice property, since no amount over \$.32 can be made with a minimum number of coins by omitting a \$.32 coin (similarly for amounts over \$.08, but under \$.32).
- Example 2: Coins are valued \$.30, \$.20, \$.05, \$.01
  - Does not have greedy-choice property, since \$.40 is best made with two \$.20's, but the greedy solution will pick three coins (which ones?)