Data Structures (IT205) 2014-15 Final Exam 27th November, 2014

Time: 2 hours

12 Each Question each worth 4 marks (total: 48); Two pages back-to-back. Wrong answers result in negative marking, so beware of guessing answers. Attempt all questions.

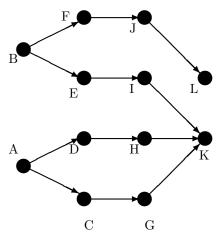
Write your name and roll number on both pages.

You may use rough sheets but DO NOT submit them

Answers must be written on question paper itself and returned.

- 1. Consider binary search (which assumes that the list is sorted in ascending order) and it repeatedly searches for the element at the middle position of the subarray. If the subarray consists of an even number of elements it searches at the **lower median** position. The elements present in the sequence 2, 8, 3, 9, 6, 1, 4, 5, 7 which will not be detected by binary search are ______
- 2. Consider linear search on a linked list, where the algorithm **assumes** the elements are sorted in increasing order. The elements which will not be detected by linear search in the list: 9, 6, 5, 1, 3, 2, 8, 4, 7 are ______.
- 3. In Question 2, an element will not be detected if and only if a _____ (smaller/larger) number appears to its _____ (left/right).
- 4. Consider a sequence of n numbers a_1, \ldots, a_n , such that $a_1 < a_2 > a_3 < a_4 > a_5 < \cdots < a_n$. Which of the following is true:
 - (a) The maximum element is in an even position.
 - (b) The minimum element is in an odd position.
 - (c) Both (a)&(b).
 - (d) Neither (a) nor (b)
 - (e) It cannot be determined.
- 5. Insertion sort works best on an already sorted input and worst on a reverse sorted input. Consider a composite algorithm wherein you are willing to get output in either increasing or decreasing sorted order. Thus the algorithm has two copies of the array and it performs one comparison and data movement (if needed) of insertion sort in the first array for increasing sort and then one comparison and data movement (if needed) of insertion sort in the second array for decreasing sort. It then alternates back and forth between these two processes and terminates when either of them is sorted. Clearly in this case neither the increase sorted nor the reverse sorted order will elicit the worst case behaviour. Which of the following permutation of the elements 1, 2, 3, 4, 5, 6, 7, 8, 9 will make this alternating algorithm perform at its worst.
 - (a) 6, 7, 8, 9, 5, 1, 2, 3, 4
 - (b) 1, 3, 5, 7, 9, 8, 6, 2, 4
 - (c) 9, 5, 2, 1, 3, 7, 8, 6, 4
 - (d) 9, 8, 7, 6, 1, 2, 3, 4, 5
- 6. Assume a composite list data structure which behaves like a queue whenever there are at most five items and for six or more items it behaves like a stack. Notice that in this case the element deleted is according to FIFO or LIFO but the actual policy depends on the number of elements present in the list **at the time of deletion**. Don't worry about how the data structure keeps track of the order in which the elements have arrived. For the following sequence of inserts and deletes label the element being deleted. Insert 1, insert 2, insert 3, insert 4, insert 5, delete (______), delete (______), insert 6, insert 7, delete (______), insert 8, delete(______).
- 7. Consider a conected weighted simple undirected graph G. Suppose both Kruskal's algorithm and Prim's algorithm are run on G. Which of the following statements is possibly true.
 - (a) Weight of Kruskal's tree=15. Weight of Prim's tree=18.
 - (b) Weight of Kruskal's tree=18. Weight of Prim's tree=15.
 - (c) Weight of both trees is 17.
 - (d) Both scenarios (a)&(b) are possible.

- 8. For the directed graph below if we run DFS, the number of DFS trees in the DFS forest would depend on the node on which DFS visit is called first.
 - (a) The nodes on which DFS visit must be called first, in order that the DFS forest has exactly one DFS tree are (______).
 - (b) We can modify the graph by adding exactly four directed arcs, so that the resulting graph will have only one DFS tree in the DFS forest, independent of which node DFS visit is first called. The four arcs which should be added are (______), (______) and (______).
 - (c) Consider an n-node directed graph. The number of choices for first call of DFS visit which result in the DFS forest having exactly one DFS tree is either: (______), (______) or (_______). (Fill in the blanks with appropriate numbers from the set 0, 1, 2, ..., n).



- 9. Consider connected simple undirected graphs which have the same BFS and DFS tree (assuming the source vertex of BFS is the same as the first vertex on which DFS visit is called by DFS). In addition to the trees being the same, the order in which the nodes are visited is also the same. What is the maximum degree of the graph? (______). What is the minimum degree of the graph? (______). The unique graph on n vertices which achieves this is the (______).
- 10. Consider an *n*-node binary search tree with distinct key values. Consider two nodes with ranks i and j. Assume without loss of generality that i < j.
 - (a) What is the maximum possible size (number of vertices) of the smallest subtree containing these two nodes? (_____).
 - (b) What is the minimum possible size (number of verties) of the smallest subtree containing these two nodes? (______).
 - (c) Is every value between the maximum and minimum realisable in some BST? (_____) (YES/NO).
- 11. Consider a binary minimum heap with $2^{h+1} 1$ nodes consisting of distinct keys $1, \ldots, 2^{h+1} 1$. Consider a node x at level l, $0 \le l \le h$, where h is the height of the heap. What are the possible values of the rank of x? (______)
- 12. Consider randomised quick-sort running on an input array consisting of 127 elements $\{1, \ldots, 127\}$.
 - (a) In the best case execution, how many of these elements will be used as pivot during the execution of the algorithm? (_____) Which specific elements are used as pivot? (_____).
 - (b) In the worst case execution, how many of these elements will be used as pivot during the execution of the algorithm?