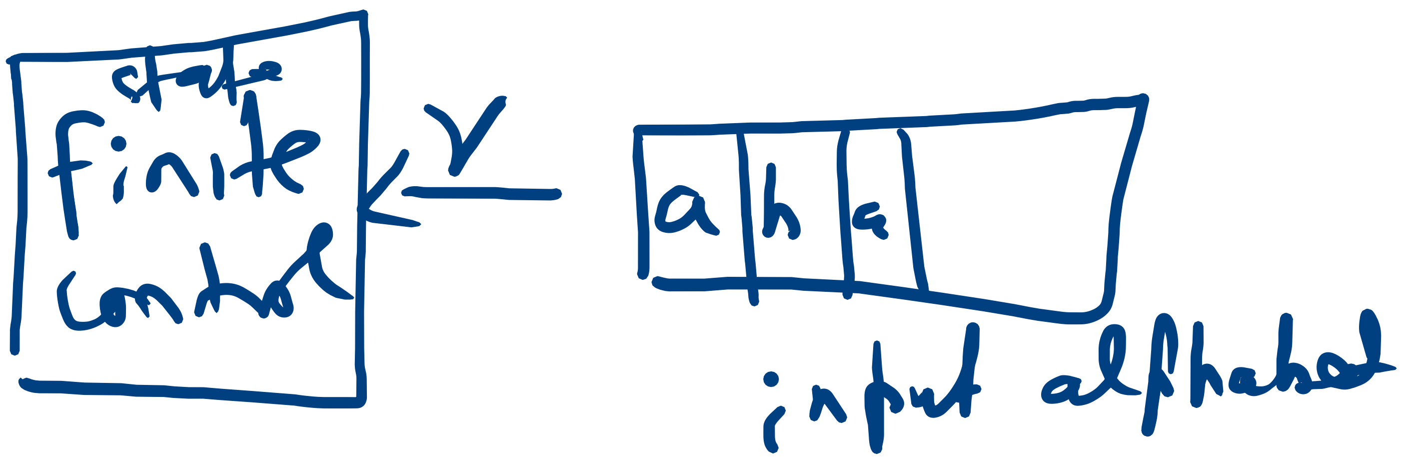
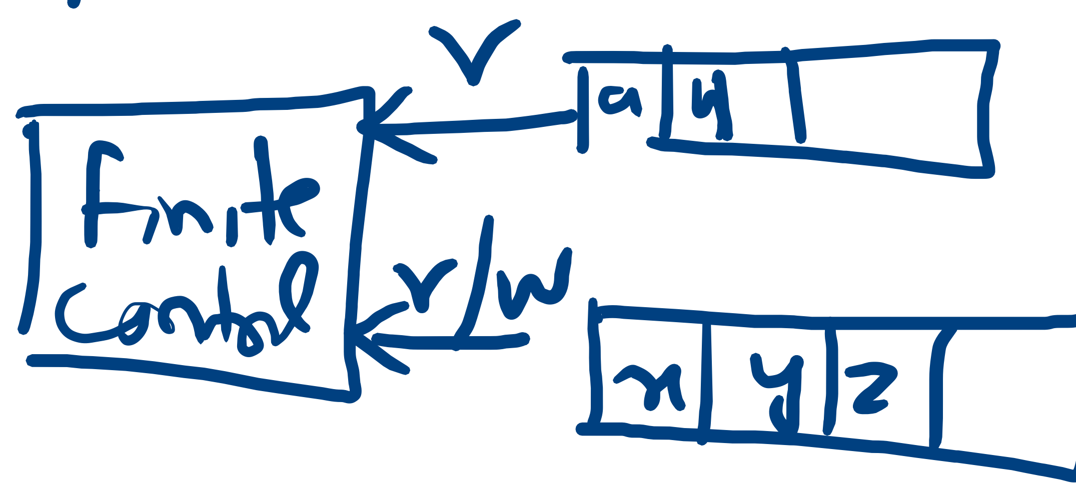
Pushdown Automata

PDA <-> NFA

CFG <-> regular expresn



NFA



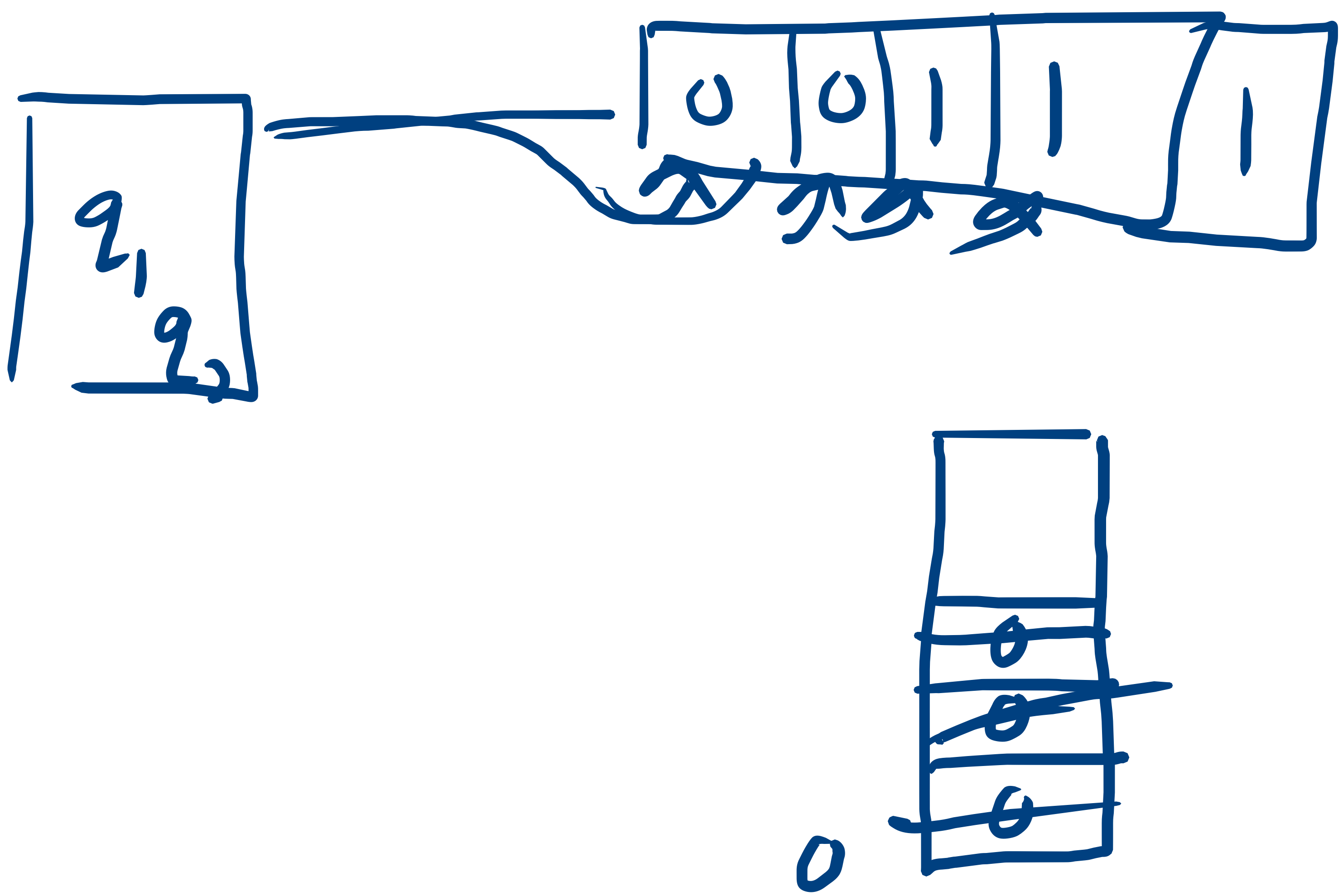
$L(M) = \{ 0^n 1^n \mid n \geq 0 \}$

0011

00111

00011

Σ



Γ

Def^n A pushdown automaton M is a tuple $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

1. Q finite set of state
2. Σ input alphabet
3. Γ stack alphabet
4. $q_0 \in Q$ the initial state
5. $Z_0 \in \Gamma$ stack symbol called the start symbol
6. $F \subseteq Q$ set of final state

7. $\delta : Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma$

$\rightarrow$ finite subset of $Q \times \Gamma^*$

$\delta : Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma \to$ finite subsets of $Q \times \Gamma^*$

1. $\delta(q, \underline{a}, \underline{Z}) = \{(p_1, \underline{\gamma_1}), (p_2, \gamma_2), \ldots (p_m, \gamma_m)\}$

$\underline{a \in \Sigma}$

$|\gamma| = \underline{1}$

$|\gamma| = \underline{0}$

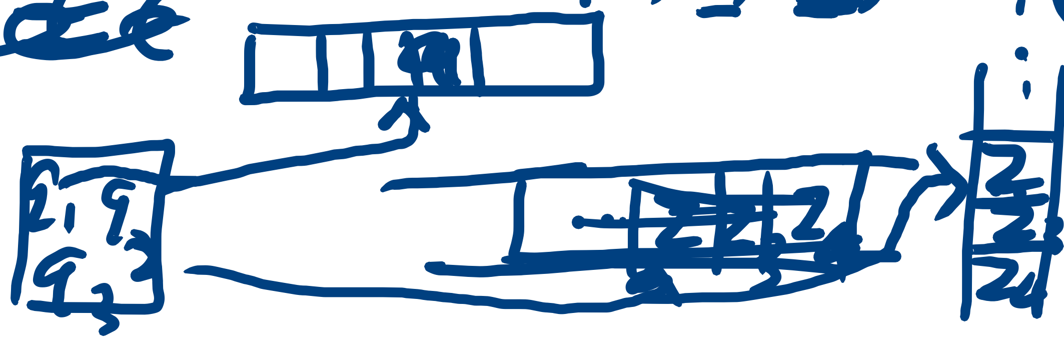
$|\gamma| \geq \underline{1}$

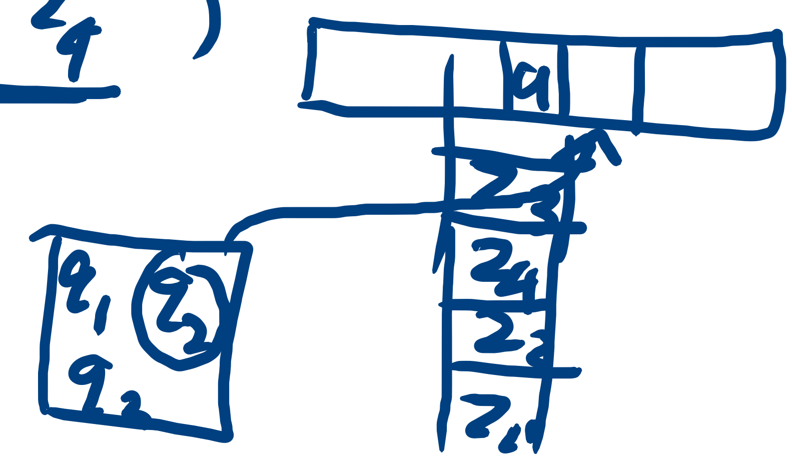
2

γ_i's are string of stack symbol

~~a ∈ ε~~

$\delta(q_1, \underline{a}, \underline{Z}) = (q_2, \underline{Z_3 Z_4})$





Exp

$$L = \{ w c w^R \mid w \in (0+1)^* \}$$

c

$$M = \{ \{q_1, q_2\}, \{0, 1, c\}, \{R, B, G\}, \delta, q_1, R, \phi \}$$

$Q \qquad \Sigma \qquad \Gamma$

$\delta(q_2, \epsilon, R) = (q_2, \epsilon)$

$\delta(q_2, 0, B) = (q_2, \epsilon)$

$\delta(q_2, 1, G) = (q_2, \epsilon)$

$0 \rightarrow B$

$1 \rightarrow G$

$\delta(q_1, 1, G) = (q_1, GG)$

✓ $\delta(q_1, c, R) = (q_2, R)$

$\delta(q_1, c, B) = (q_2, B)$

$\boxed{q_1, q_2}$

$\delta(q_1, c, G) = (q_2, G)$

0	1	c	1	0	1

1 ↑ ↑ ↑

B G

c	0	1	1

$\delta(q_1, 0, R) = (q_1, BR)$

$\delta(q_1, 1, R) = (q_1, GR)$

$\delta(q_1, 0, B) = \{(q_1, BB)\}$

$\delta(q_1, 1, B) = \{(q_1, GB)\}$

$\delta(q_1, 0, G) = \{(q_1, BG)\}$

$\delta(q_1,$

Instantaneous Description

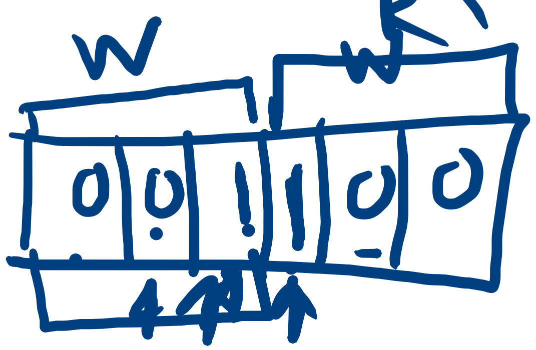
$B \to 0$

$G \to 1$

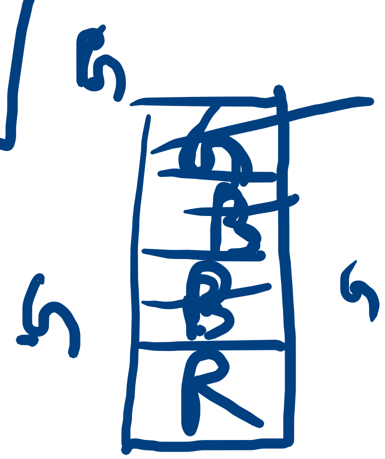
Exp

$$L = \{ ww^R \mid w \in (0+1)^* \}$$

$$M = (\underbrace{\{q_1, q_2\}}_{Q}, \underbrace{\{0,1\}}_{\Sigma}, \underbrace{\{R, B, G\}}_{\Gamma}, \delta, q_1, R, \phi)$$



q_1, q_2



$\delta(q_1, 0, R) = \{(q_1, BR)\}$

$\delta(q_1, 1, R) = \{(q_1, GR)\}$

$\delta(q_1, 0, B) = \{(q_1, BB), (q_2, \epsilon)\}$

$\delta(q_1, 0, G) = \{(q_1, BG)\}$

$\delta(q_1, 1, B) = \{(q_1, GB)\}$

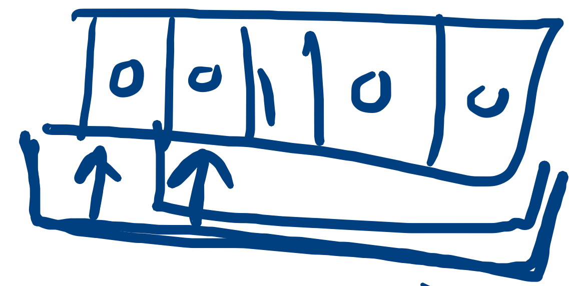
$\delta(q_1, 1, G) = \{(q_1, GG), (q_2, \epsilon)\}$

$\delta(q_2, 0, B) = (q_2, \epsilon)$

$\delta(q_2, 1, G) = (q_2, \epsilon)$

$\delta(q_1, \epsilon, R) = (q_2, A)$

≡ Acceptance by final state



Acceptance by empty stack

$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$\underline{\delta} : Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma$

$\to$ finite subset of $Q \times \Gamma^*$

Instantaneous Description

An $\underline{ID}$ is a triple $(q, \underline{w}, \gamma)$

q state, w string of input symbol

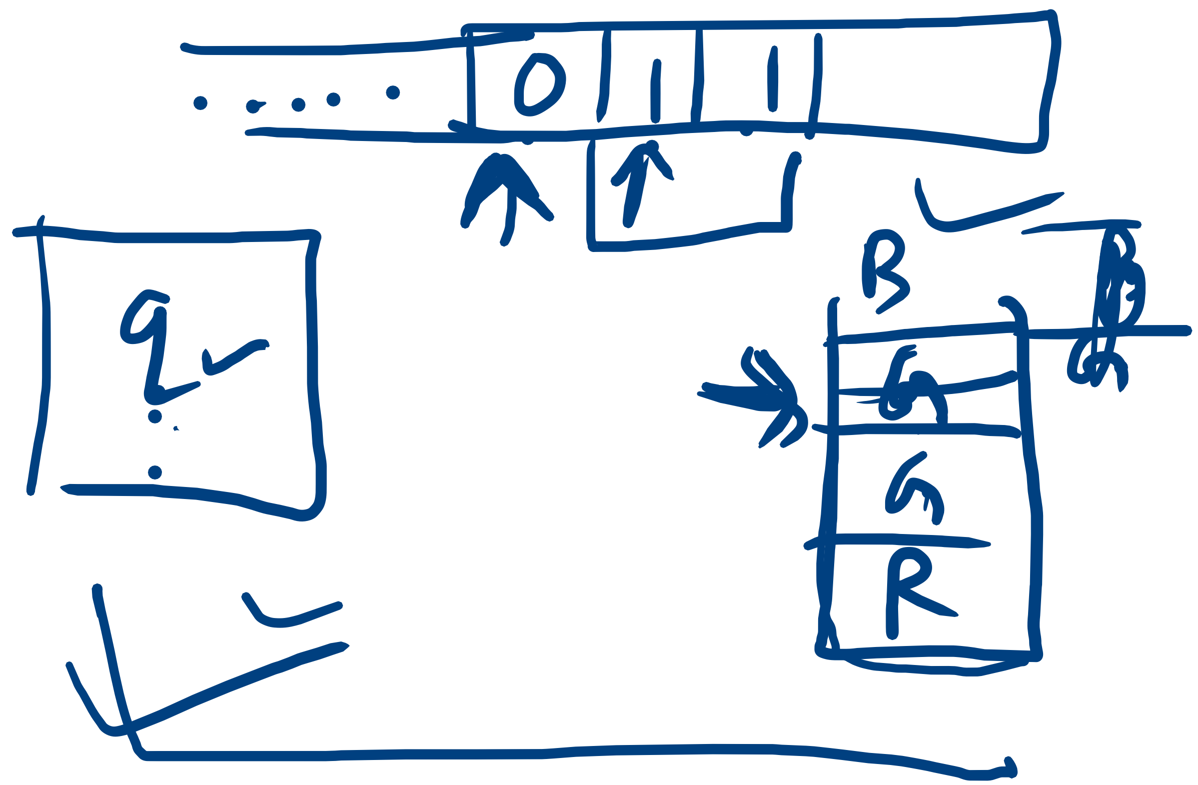
γ string of stack symbol

we say $(q, aw, Z\alpha) \vdash_M (p, w, \beta\alpha)$

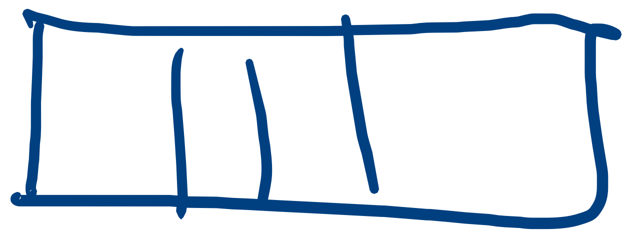
if $\delta(q, a, Z)$ contains (p, β)

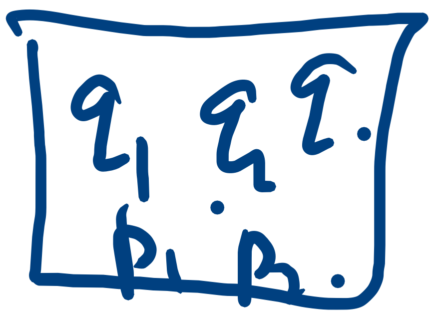
If $\delta(q, 0, G) = (q, BG)$

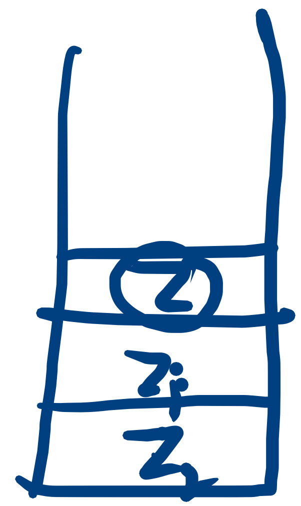
then $(q, 011, GGR) \vdash (q, 11, BGGR)$



2. $\delta(q, \epsilon, Z) = \{(p_1, \gamma_1), (p_2, \gamma_2), \ldots (p_m, \gamma_m)\}$







γ_1 $Z_3 Z_4$

γ_2 $Z_1 Z_4 Z_3$

$|\gamma| = 1$

Accepted Language

$L(M) = \{ ww^R \mid w \in (0+1)^* \}$

$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ $\quad wcw^R$

1. The language accepted by final state

$L(M) = \{ w \mid (q_0, w, Z_0) \vdash^* (p, \epsilon, \gamma) \}$

for some $p \in F$, $\gamma \in \Gamma^*$

2. Language accepted by empty stack

$N(M) = \{ w \mid (q_0, w, Z_0) \vdash^* (p, \epsilon, \epsilon) \}$

These two def's are equivalent