SET A

Calculus SC-107

Full marks 30, Time 1 Hour

Choose the correct answer from the given choices for each of the questions.

1. Find the value of x that maximizes the value of the integral

$$g(x) = \int_x^{x+3} t(5-t)dt$$

(a) $x = -3$

(b) $x = 5$

(c) $x = 1$

(d) $x = 0$

Correct Answer: (c)

2. Find the volume of the solid generated by revolving the following region about the y-axis.

The region in the first quadrant bounded above by the parabola $y = x^2$, below by the x-axis, and on the right by the line $x = 2$.

(a) 2π

(b) 4π

(c) 6π

(d) 8π

Correct Answer: (d)

3. Which of the following function is not Riemann integrable on $[0, 1]$?

(a) $f(x) = \dfrac{1}{x+2}$

(b) $f(x) = [10x]$

(c) $f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ -x & \text{if } x \in \mathbb{R} - \mathbb{Q} \end{cases}$

(d) $f(x) = x|x|$

Correct Answer: (c)

4. Let $f(x) = 1 - x + \frac{x^2}{2} - \frac{x^3}{3} + \cdots + (-1)^n \frac{x^n}{n}$. Which of the following is true?

   (a) $f(x)$ has no real root for any value of n.

   (b) $f(x)$ has one real root if n is odd and no real root if n is even.

   (c) $f(x)$ has one real root if n is odd and one real root if n is even.

   (d) $f(x)$ has more than one real root if $n > 1$.

   Correct Answer: (b)

5. Let $a_1, a_2, \cdots, a_n$ be real numbers and let f be defined on $\mathbb{R}$ by

$$f(x) = \sum_{i=1}^{n} (a_i - x)^2 \text{ for } x \in \mathbb{R}.$$

   Then the point of local minimum of $f(x)$ is

   (a) Unique and is $\frac{a_1 + a_2 + \cdots + a_n}{n}$.

   (b) Unique and is $a_1 + a_2 + \cdots + a_n$.

   (c) not unique. There are n number of local minimum.

   (d) Unique and is $\frac{a_1 + a_2 + \cdots + a_n}{2}$.

   Correct Answer: (a)

6. A particular solution to the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x} \sec 2x$$

   is

   (a) $\frac{1}{2} x e^{-x} \sin 2x + \frac{1}{4} x e^{-x} \cos 2x \ \log(\cos 2x)$

   (b) $\frac{1}{2} x e^{-x} \sin 2x + \frac{1}{4} e^{-x} \cos 2x \ \log(\cos 2x)$

   (c) $\frac{1}{2} e^{-x} \sin 2x + \frac{1}{4} e^{-x} \cos 2x \ \log(\cos 2x)$

   (d) $\frac{-1}{2} x e^{-x} \sin 2x + \frac{1}{4} x e^{-x} \cos 2x \ \log(\cos 2x)$

   Correct Answer: (b)

7. The solution of the initial value problem is

$$\frac{dy}{dx} + y \cos x = \sin 2x, \ y(\pi) = 0$$

is

(a) $y = 2(\sin x - 1 + e^{-\sin x})$

(b) $y = 2(\cos x - 1 + e^{-\sin x})$

(c) $y = 2(1 - \sin x + e^{\sin x})$

(d) $y = 2(\sin x - 1 + e^{-\cos x})$

Correct Answer: (a)

8. Consider the function

$$f(x) = [x]|x|sgn(x)$$

where $|.|$ is absolute value function, $[.]$ is greatest integer function, sgn is sign function. Then $f(x)$ has discontinuities

(a) at all real numbers.

(b) at 0 and 1 only.

(c) at natural number only.

(d) at all integer points of its domain.

Correct Answer: (d)

9. The value of the limit

$$\lim_{h \to 0} \int_{-1}^{1} \frac{h^3}{h^2 + x^2} dx$$

equals

(a) 0

(b) e

(c) π

(d) does not exist

Correct Answer: (a)

10. A long rectangular sheet of metal, 12 inches wide is to be made into a rain gutter by turning up two sides so that they are perpendicular to the sheet. How many inches should be turned up to give the gutter its greatest capacity?

(a) 1

(b) 4

(c) 3

(d) 2

Correct Answer: (c)