Guass Jordan Elimination and computing inverse of a matrix

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The pseudocode and data structural and algorithmic details of the Guass Jordan elimination are almost identical to the Guassian Elimination. Here again we proceed systemmatically column by column starting from right and proceeding left, making all entries above the diagonal 0. Note that, here, when we say right, we mean the right most column of the augmented matrix that comes from the coeffiicient matrix. We do not consider the extra columns coming into the augmented matrix. At the end of this algorithm, the $n \times n$ submatrix of the augmented matrix, consisting of the first n columns, is a diagonal matrix. Thus, we are left, in effect, with a system of n equations, each in one variable, spanning all the original n variables. Thus we can solve the system trivially.

We also outlined an identical algorithm for computing the inverse of an invertible square matrix. For this, we view the equation

$$AA^{-1} = I$$

In the same way we created an augmented matrix in the case of $Ax = b$, here we let augment the right hand side constant. The difference is that in this case it is not a single column, but a square matrix. That is We append the $n \times n$ identity matrix to the right of our original square matrix A. Now we perform Guass Jordan Elimination on this augmented matrix until we reach a diagonal matrix on the first n columns. Now we multiply each row of the resulting matrix by the reciprocal of the diagonal entry on that row. At the end of this we will have the identity matrix on the first n columns. And the matrix consisting of the last n columns is the inverse of the original matrix.

An example of this will be presented in the next lecture.