1. Hopcroft, Ullman
2. Sipser

Alphabets A finite set of symbols

$\{a, b, \ldots z\}$

$\{0, 1, 2 \ldots 9\}$

$\$, \#$

String A string over an alphabet is a finite sequence of symbols

$\{a, b, \ldots z\}$ taxi

$\{a, b\}$

aba, bba

Empty string has no symbols denoted $\underline{\epsilon}$

λ

u, v, w, x, y, z and greek letters denote strings

$u = abb$

$$\frac{\{a, b, c\}}{abba}$$

Let $\Sigma = \{0, 1\}$ 0110, 001

$\underline{\Sigma^*} = \{ \epsilon, 0, 1, 00, 11, 01, 10, 000 \ldots \}$

kleene closure

Language : A set of strings of symbols from some alphabet ?

$\Sigma = \{a, b, c, d\}$

$L_1 = \{abb, bba, baa, cbdc, \}$

3 4

Length : The length of a string is its length as a sequence.

w $|w|$

$|\epsilon| = 0$

Concatenation

$w_1 = \underline{abc}$

$w_2 = bcd$

$w_1 . w_2 = \underbrace{abc}\underbrace{bcd}$

$\epsilon w = w \epsilon = w$ for each string w.

Substring

A string $\underline{v}$ is a substring of a string $\underline{\underline{w}}$ iff $\exists$ x, y s.t.

$w = x v y$

Prefix

If $w = vy$ for some y then v is a prefix of w

$abcd$

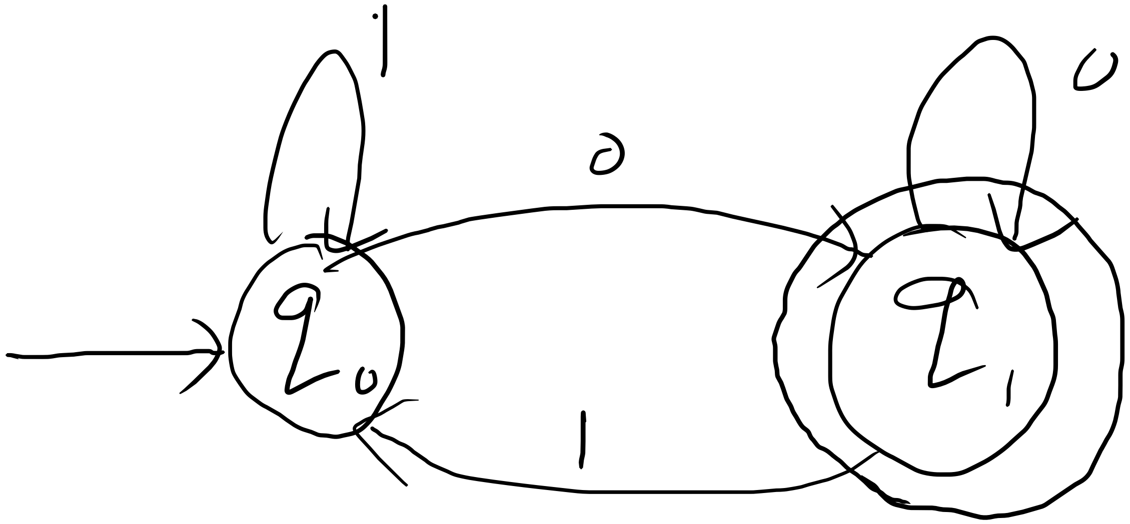
suffix

$abba$

Formal Def^n

A finite automaton is a 5 tuple $(Q, \Sigma, \delta, q_0, F)$
where

1. Q is a finite set called states
2. Σ is a finite set called alphabet
3. $\delta : Q \times \Sigma \to Q$ is the transition fn.
4. $q_0 \in Q$ is the start state or initial state
5. $F \subseteq Q$ is the set of ~~accep~~ final states.



Transition diagram

$$\delta(q_0, 0) = q_1$$
$$\delta(q_1, 0) = q_1$$
$$\delta(q_0, 1) = q_0$$
$$\delta(q_1, 1) = q_0$$

q_0 initial state

q_1 final state

$Q = \{q_0, q_1\}$

$\Sigma = \{0, 1\}$

δ	q_0	q_1
0	q_1	q_1
1	q_0	q_0

Each node is a state

An arrow going from one state to another is labeled by a symbol

Concatenation

$\Sigma = \{0, 1\}$

011

01

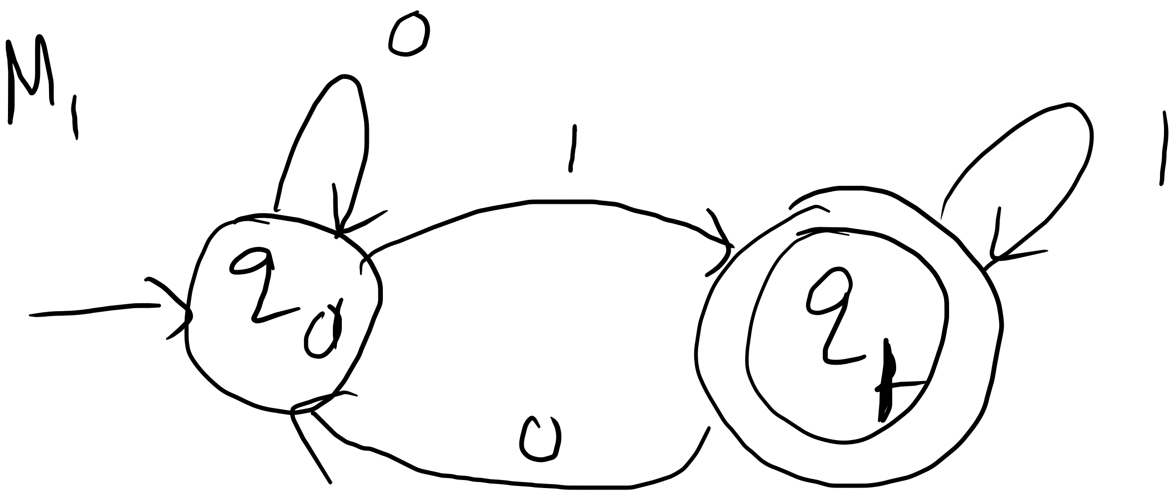
$Q = \{q_0, q_1, q_2\}$ $\delta(q_0, \underline{01}) = \delta((q_0, 0), 1)$

$\Sigma = \{0, 1\}$

q_0 initial start

δ	q_0	q_1	q_2
0	q_1	q_1	q_2
1	q_2	q_2	q_1

M_1



01101

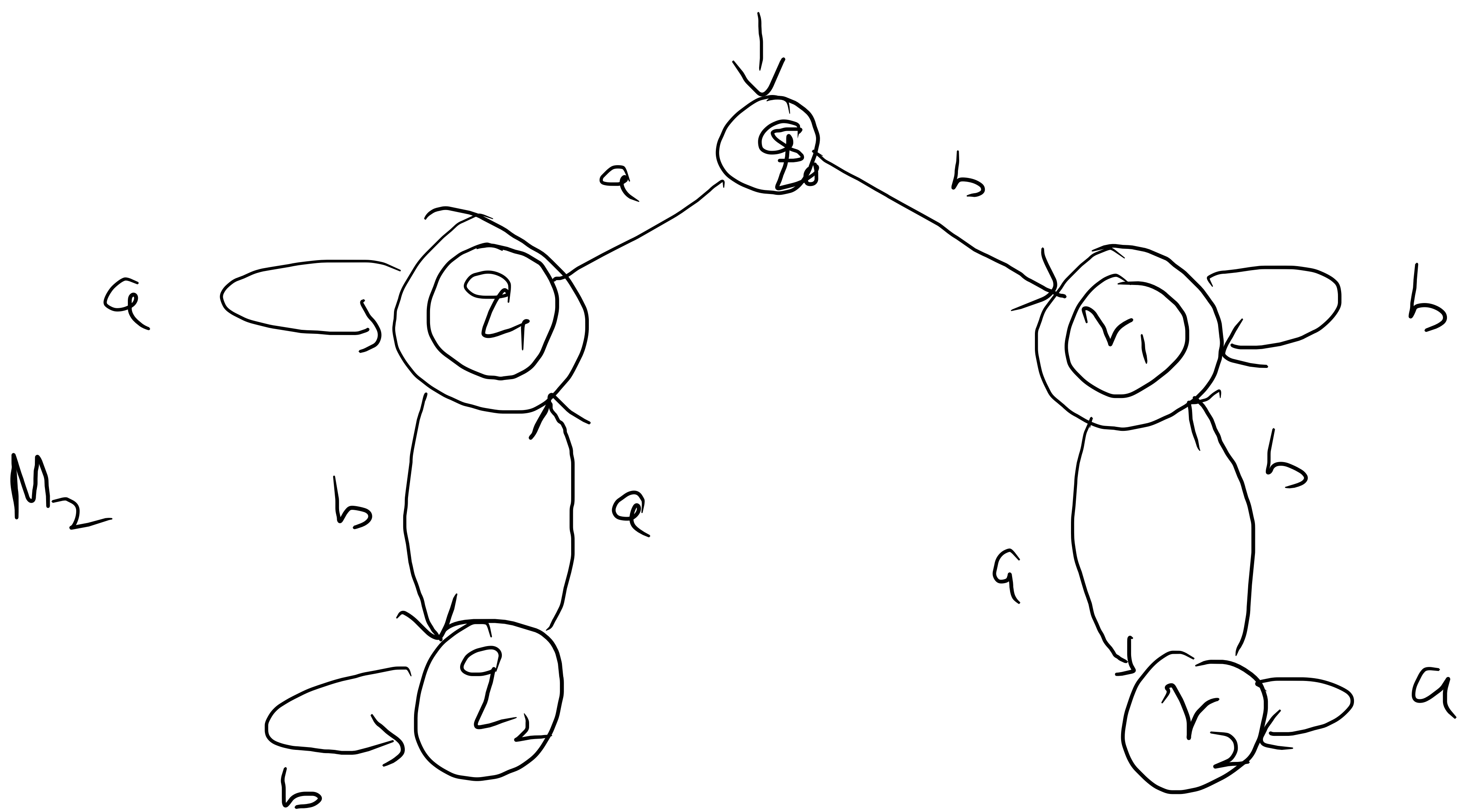
$\underline{0}$ $\quad \delta(q_0, 0) = q_0$ $\quad$ x

$\underline{01}$ $\quad \delta(q_0, 01) = q_1$

000 $\quad \delta(q_0, 000) = q_0$ $\quad$ x

00011 $\quad \delta(q_0, 00011) = q_1$

$L(M_1) = \{ w \mid w \text{ ends in } 1 \}$



$\delta : Q \times \Sigma \to Q$

$\delta(q_0, a) = q_1$

$\delta(q_0, \underline{abba}) = q_1$

aa ✓ bba ✗

aab

$L(M_2) = \{ w \mid w$ start and end with a or then w start and end with b $\}$

$a = \epsilon a$

$\delta : Q \times \Sigma \to Q$

$\hat{\delta}(q, a) = \delta(\hat{\delta}(q, \epsilon), a)$
$= \delta(q, a)$

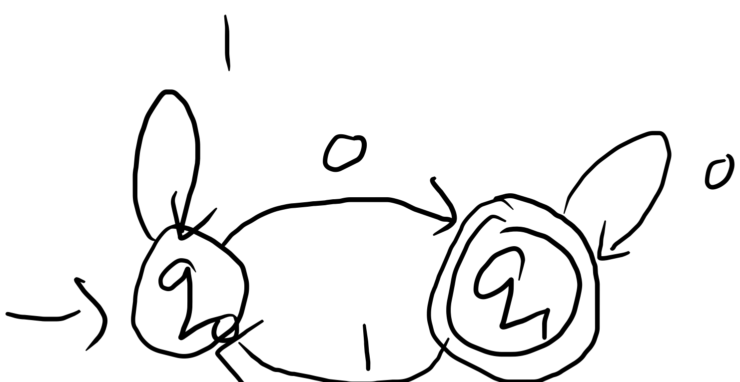
Extend to strings

Define $\hat{\delta} : Q \times \Sigma^* \to Q$

by (i) $\hat{\delta}(q, \epsilon) = q$

(ii) for all string w and input symbols a

$$\hat{\delta}(q, wa) = \delta(\hat{\delta}(q, w), a)$$



$\hat{\delta}(q_0, 011) = \delta(\hat{\delta}(q_0, 01), 1)$

$= \delta(\delta(\underbrace{\delta(q_0, 0)}_{q_1}, 1), 1)$

$= \delta(\underbrace{\delta(q_1, 1)}_{q_1}, 1) = \delta(q_0, 1) = q_0$

Regular operations

Def^n A string $\underline{x}$ is said to be accepted by a finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ if

$\delta(q_0, x) = \underline{p}$ for some $p \in F$

The language accepted by M

$\underline{L(M)} = \{ \underline{x} \mid \delta(q_0, x) \in \underline{F} \}$

Regular language