SC217: Electromagnetic Theory **Assignment 10**

DA-IICT, B.Tech, Sem II June 27 Winter2021

1. **Let $\vec{\mathscr{r}} = \vec{r} - \vec{r}'$. Then verify that $\vec{\nabla}'\frac{1}{\mathscr{r}} = \frac{\hat{\mathscr{r}}}{\mathscr{r}} = -\vec{\nabla}\frac{1}{\mathscr{r}}$ where $\vec{\nabla}'$ is the gradient with respect to the primed coordinates.**

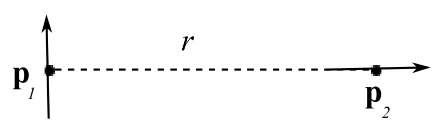
2. **A sphere of radius R, centered at the origin, carries charge density**
$\rho(r,\theta) = k(R-r)\cos\theta$. Find the approximate potential on the z axis far from the sphere.

3. **A dipole $\vec{\mathrm{p}}$ is at a distance r from a point charge q and oriented so that $\vec{\mathrm{p}}$ makes an angle θ with the vector $\vec{\mathrm{r}}$ from q to $\vec{\mathrm{p}}$.**

   (a) **What is the force on $\vec{\mathrm{p}}$?**

   (b) **What is the force on q ?**

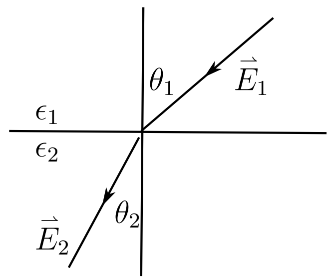
4. **$\vec{\mathrm{p}}_1$ and $\vec{\mathrm{p}}_2$ are perfect dipoles a distance r apart. $\vec{\mathrm{p}}_2$ is along $\vec{\mathrm{r}}$ while $\vec{\mathrm{p}}_1$ is orthogonal to $\vec{\mathrm{r}}$. Calculate the torque on the dipoles. Are they equal and opposite?**



5. **A sphere of radius R carries a polarization $\vec{\mathbf{P}}(\vec{\mathbf{r}}) = k\vec{\mathbf{r}}$**
   (a) Calculate the bound charges ρ_b and σ_b and the electric field caused due to them inside and outside the sphere.
   (b) Find the electric field using the Gauss' law for the displacement vector $\vec{D}$ given as $\oint_S \vec{D}\cdot\hat{n}da = Q_{f(enc)}$.

6. **A point charge q is imbedded at the center of a sphere of linear dielectric material with susceptibility χ_e and radius R. Find the electric field, the polarization, and the bound charge densities, ρ_b and σ_b. What is the total bound charge on the surface? Where is the compensating negative bound charge located?**

7. **At the interface between one linear dielectric and another the electric field lines bend. Show that** $\tan\theta_2/\tan\theta_1 = \epsilon_2/\epsilon_1$ **assuming there is no free charge at the boundary. Refer to fig.1 below.**



8. **Suppose the field inside a large piece of dielectric is** $\vec{\mathbf{E}}_0$**, so that the electric displacement is** $\vec{\mathbf{D}}_0 = \epsilon_0\vec{\mathbf{E}}_0 + \vec{\mathbf{P}}$**.**

   (a) **If we have a narrow cylindrical(needle-like) cavity inside the material running parallel to** $\vec{\mathbf{P}}$ **find the field near the center of the cavity in terms of** $\vec{\mathbf{E}}_0$ **and** $\vec{\mathbf{P}}$**. Also find the displacement at the center of the cavity in terms of** $\vec{\mathbf{D}}_0$ **and** $\vec{\mathbf{P}}$**.**

   (b) **Do the same for a thin wafer shaped cavity perpendicular to** $\vec{\mathbf{P}}$**.**