Data Structures (IT205) 2013-14
Midsemester-semester re-exam
18th November, 2013

Time: 2 hours

marks: 60

This question paper consists of three questions each worth 20 marks. Attempt all questions.

PART *A*

1. (a) Write an efficient code to find out the numbers of pairs of positions in an n node binary maximum heap whose key values can be exchanged without violating the key value property of the heap. Analyse the running time of your algorithm. Assume for this part as well as the next part of this question that all key values are distinct.

   (b) For any given n derive as good a lower bound as possible on the number of pairs of such interchangeable positions. Derive also as good an upper bound as you can for the number of exchangeable pairs.

2. Suppose we relax the structural requirement that a heap is a complete binary tree, except possible for the last level, where the nodes are filled from left to right. In this case we can no longer store it implicitly as an array but need to use a pointer based structure. Now given a skeletal rooted binary tree on n nodes (without key values written), and key values $1, \ldots, n$ with which you must label this tree:

   (a) How many such labellings are there to satisfy the binary search tree property?

   (b) Write a code to efficiently compute the number of labelings to satisfy the binary maximum heap property.

   (c) Write efficient code to determine the smallest and largest possible number of positions in which the labelling for BST and binary maximum heap coincide in their values.

3. Write a routine, almost identical to bubble sort such that the resulting sequence is in alternating order. That is if the input sequence is $a1, \ldots, an$ then the output permutation $b1, \ldots, bn$ should be such that $b1 < b2, b2 > b3, b3 < b4$ etc. In symbolic notation for all odd values of i, $b_{i-1} > b_i < b_{i+1}$ and for even values of i, $b_{i-1} < b_i > b_{i+1}$. The expected structure off the pseudocode is a double nested for loop. You may assume that all elements have distinct key values. What is the running time of your routine?