Electrostatics

1 Forces caused due to electric charges

We idealise charges to exist at a point called point charge. If we have charges q_1 and q_2 then the magnitude of the force between them is given as

$$F = K\frac{q_1 q_2}{r^2}$$

where r is the distance between the charges. In the SI units the measure of a charge is coulomb. It is defined as an amount of charge, which when placed 1m apart exerts a force of 9×10^9N on each other.

$$\begin{aligned} \text{So} \;\; 9 \times 10^9 &= K\frac{1\text{C} \cdot 1\text{C}}{1\text{m}^2} \\ \therefore \;\; K = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} & \end{aligned}$$

It is natural to imagine that every point charge create an effect in its 3 dim surrounding which is spherically symmetric and hence the strength of this effect would be distributed over an area of $4\pi r^2$ at a distance r. So

$$F = \alpha\frac{q_1}{4\pi r^2}q_2 = K\frac{q_1 q_2}{r^2}$$

So $\alpha = 4\pi K$
Generally α is expressed as $\frac{1}{\epsilon_0}$ where ϵ_0 is a fundamental constant called permittivity of free space.

$$\text{So}\epsilon_0 = \frac{1}{4\pi K} = 8.85 \times 10^{-12}\text{C}^2/\text{Nm}^2$$

In C.G.S units charges are measured in e.s.u (electrostatic unit), which is defined as the amount of charge that , when separated by a distance of 1cm would exert a force of 1 dyne. We can then evaluate

$$1\text{e.s.u} = \frac{1}{3 \times 10^9}\text{C}$$

In this units, the expression for electrostatic force is

$$F = \frac{q_1 q_2}{r^2}$$

The permittivity ϵ_0 will be given as

$$\epsilon_0 = \frac{1}{4\pi}(\text{e.s.u})^2/\text{dyne cm}^2$$

As far as electrostatic is concerned Coulomb doesn't seem to be an appropriate unit of charge But it is adopted due to certain current based devices which measures magnetic forces rather than electrostatic forces. Typical values of currents and potential differences are in ampere and volts which are naturally expressed in terms of Coulombs.

A wire carrying a current of 5 amperes carry 5 C of charges across in 1 s. How does it survive the enormous electrostatic repulsion?
The net current density of a wire is zero due to the positively charges nucleus which are stationary.

2 Electric Field

The electric force on q_2 due to q_1 is

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{\mathscr{r}^2}\hat{\mathscr{r}}$$

where $\vec{\mathscr{r}} = \vec{r}_2 - \vec{r}_1$.
$\vec{r}_i$ is the position vector of charge q_i, $i = 1, 2$. $\hat{\mathscr{r}}$ is the unit vector along $\vec{\mathscr{r}}$.
Now let us consider a number of charges $q_1, q_2, ..., q_n$ in space. The force due to these on a charge q is

$$\vec{F} = \sum_{i=1}^{n} \frac{q_i q}{4\pi\epsilon_0 \mathscr{r}_i^2}\hat{\mathscr{r}}_i$$

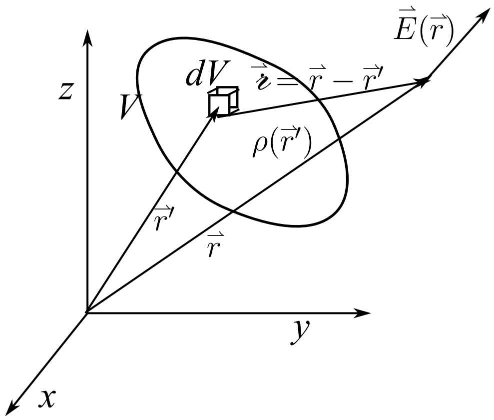
where $\vec{\mathscr{r}}_i = \vec{r} - \vec{r}_i$.
Assumption : The force on q due to q_i is unaffected by the presence of q_j.

We can write

$$\vec{F} = q\sum_{i=1}^{n} \frac{q_i}{4\pi\epsilon_0 \mathscr{r}_i^2}\hat{\mathscr{r}}_i = q\vec{E}\left(\vec{r}\right)$$

$$\vec{E}\left(\vec{r}\right) = \sum_{i=1}^{n} \frac{q_i}{4\pi\epsilon_0 \mathscr{r}_i^2}\hat{\mathscr{r}}_i$$

is the 'electric field' at the point $\vec{r}$ due to the given configutration of charges. It is the force per unit charge. We feel the presence of the electric field at $\vec{r}$ only when we place a charge q at $\vec{r}$. For a neutral particle this force doesn't exist. But we believe that the field $\vec{E}\left(\vec{r}\right)$ exist. So the picture is that the charge configuration carries the field around it. wherever it goes. Note that this picture is valid only in electrostatics. Whenever we move or change a configuration , the message of this change (disturbance) is not felt instantaneously at a distance. It takes afinite time and this has to be dealt with in Electrodynamics. In electrostatics we can assume instantaneous action at a distance. Since in reality we know that the electric



effect travels in time, we believe that the electric field is indeed a physical entity and not just a mathematical convenience. In fact we can as well take the view that it is the electric field which is the physical reality and the idea of a charge is an element of our imagination. Possibly nothing wrong with this view since now we know that electromagnetic waves travels through vacuum where no charge exist.

Continuous charge distributions:
We can extend the idea of electric field due to a number of discrete charges to a field due to continuous charge distribution. If the charge is situated over a volume V with density $\rho(\vec{r}')$, then the charge within an infinitesimal volume element dV at the location $\vec{r}'$ is $dq = \rho\left(\vec{r}'\right) dV$. Then the electric field at the point $\vec{r}$ is given as

$$\vec{E}\left(\vec{r}\right) = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho\left(\vec{r}'\right) dV}{\mathscr{r}^2}\hat{\mathscr{r}}$$

If the charge is distributed over a surface S with surface charge density $\sigma(\vec{r}')$ then

$$\vec{E}\left(\vec{r}\right) = \frac{1}{4\pi\epsilon_0}\int_S \frac{\sigma\left(\vec{r}'\right) da}{\mathscr{r}^2}\hat{\mathscr{r}}$$

and if we have a linear charge density $\lambda(\vec{r}')$ along a curve C then

$$\vec{E}\left(\vec{r}\right) = \frac{1}{4\pi\epsilon_0}\int_C \frac{\lambda\left(\vec{r}'\right) dl}{\mathscr{r}^2}\hat{\mathscr{r}}$$