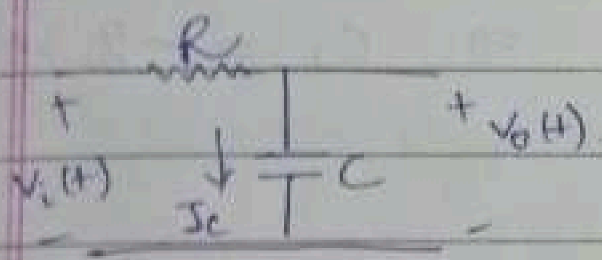
Signal : $x(t) \longrightarrow$ signal.

eg :- Speech signal, Image

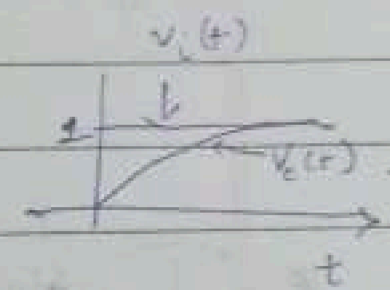
Integrator can be considered as low-pass filter.

04/08/22

1) $y(t) = 2x(t) \Rightarrow$ Amplifier
2) $y(t) = \frac{1}{3}x(t) \Rightarrow$ Accumulator.
3) $y(t) = x(t-3) \Rightarrow$ Delay system.
4) $y(t) = x(2t) \rightarrow$ Signal compression
5) $y(t) = x(t/4) \Rightarrow$ " Expansion.

* 

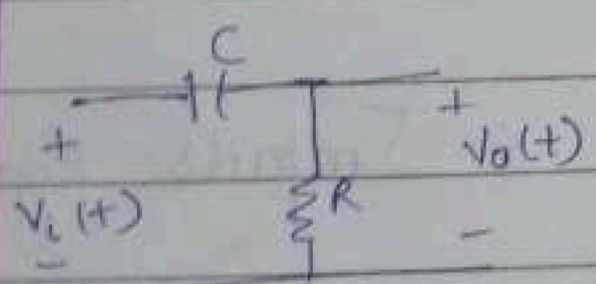

$$I_c = C\frac{dV_c}{dt} \Rightarrow$$

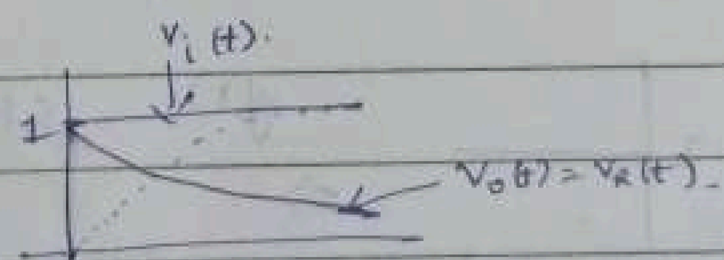



Using KVL, $V_i(t) = I_C R + V_0(t)$

$$= RC\frac{dV_0}{dt} + V_0(t)$$

$$V_i(t) = RC\frac{dV_0}{dt} + V_0(t).$$

* 

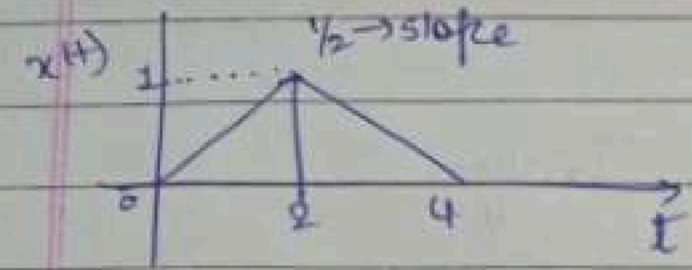



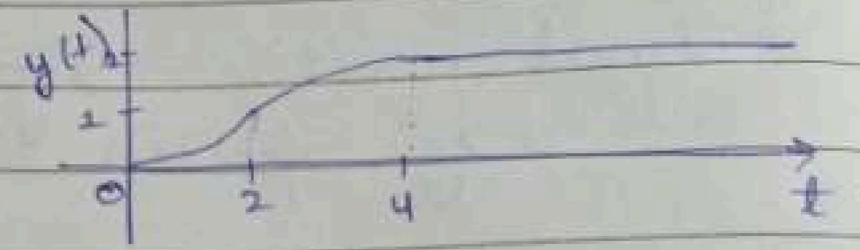

$$\therefore V_c(t) = 1 - e^{-t/RC}$$

$$V_R(t) = e^{-t/RC}$$

* Integrator :-

* $x(t) \rightarrow \boxed{\int} \rightarrow y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau.$




$t \to \infty$ $\quad y(t) = \frac{1}{2} \times 4 \times 1 = 2$

$0 \le t \le 2 \quad x(t) = t/2$

$y(t) = \frac{t^2}{4}$

$y(2) = 1$

$2 \le t \le 4 \quad x(t) = -\frac{1}{2}t + 2$

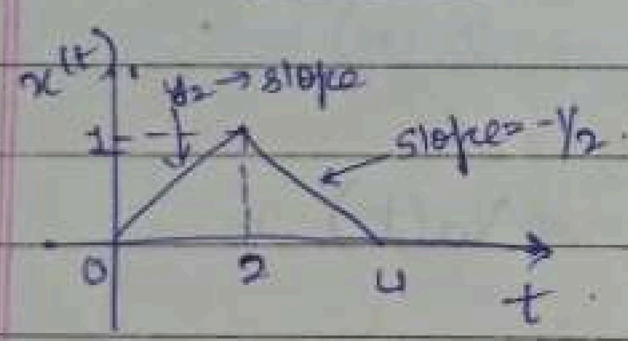
$y(t) = -\frac{1}{2}\frac{t^2}{2} + 2t + C_1$

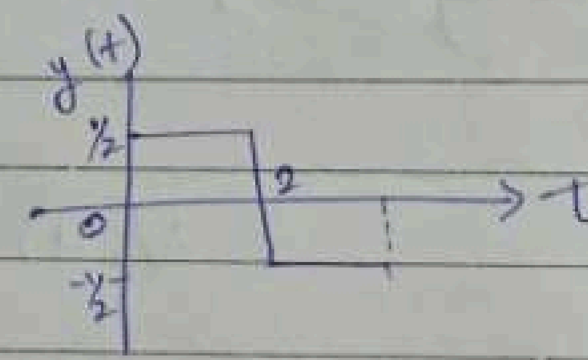
$y(2) = \frac{-4}{4} + 4 + C_1 = 1$

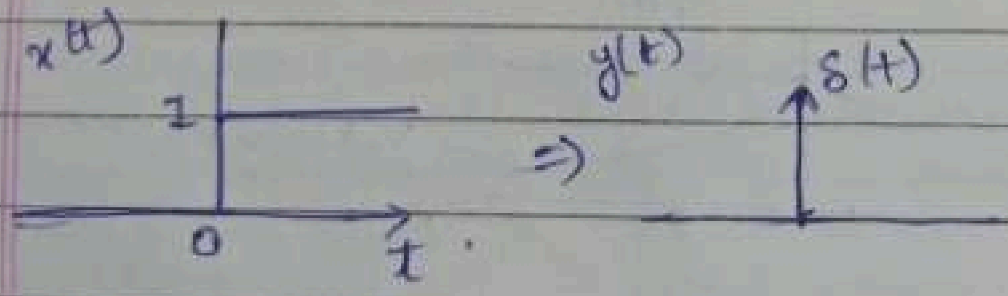
$\Rightarrow C_1 = -2$

## * Differentiator

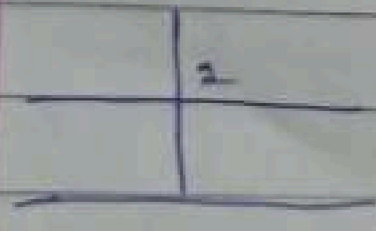
$x(t) \xrightarrow{} \boxed{d/dt} \xrightarrow{} y(t) = \frac{dx}{dt}$




$\Rightarrow$ $\leftarrow$ Impulse signal.




$\Rightarrow$ $y(t) = 0$

08/08/22

# Signals

1. Few standard signals
   (Test signal)

2. Operations on signal
   1) Time shifting
   2) Time scaling
   3) Time reversal
   4) Addition
   5) Multiplication.

3. Types of signal

# Systems.

1. Types of systems
   a) linear & non-linear
   b) time-invariant & time-varying
   c) Causal and non-causal.
   d) memory less and memory
   e) Stable and unstable.
   f) Static and dynamic
   g) Invertible " Non-invertible

2. LTI system (in detail).
   (causal)

* Standard Signals :

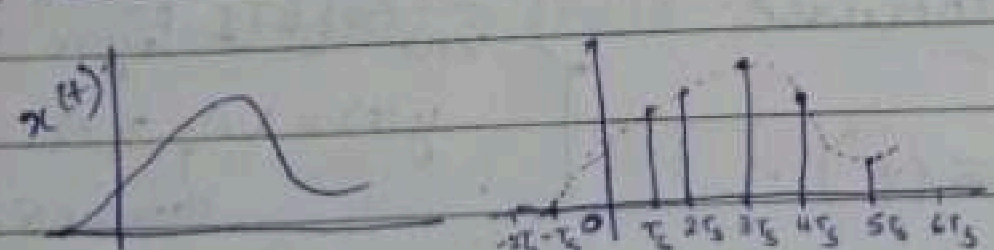
1) Unit Step Signal :-

$u(t) = 1$ for $t \geq 0$ or $1$ for $t > 0$ or $= 1, \ t > 0$

$= 0$ " $t < 0$. $\quad$ $0$ for $t \leq 0$ $\quad$ $= 0, \ t < 0$

$\frac{1}{2}$ for $t = 0$

Discrete




$\frac{1}{T_s} = f_s$

if $f_s \geq 2B$ (band width) then we can convert continuous from discrete signals perfectly using low pass filter.

$u(n) = 1$ for $n \geq 0$
$= 0$ otherwise.

2> Unit ramp signal.

$r(t) = t \quad t \geq 0$
$= 0 \quad t \leq 0$

Discrete
$r(n) = n \quad n \geq 0$
$0 \quad n < 0$

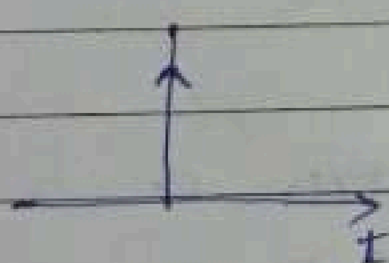
10/8

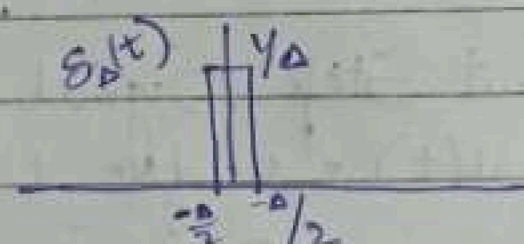
$\therefore \int_{-\infty}^{t} u(\tau)\, d\tau = r(t)$

3> Unit impulse signal.

$\delta(t) = 0$ at $t \neq 0$
$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

(we can't say $\delta(0) = \infty$, but we can say magnitude of $\delta(t)$ at 0 is very large s.t area under the curve is 1.)




$\delta_\Delta(t) = \frac{1}{\Delta}$ for $|t| \leq \frac{\Delta}{2}$.
$= 0$ otherwise.

$$\underset{x(t)}{\xrightarrow{\delta(t)}} \boxed{LTI} \underset{y(t)}{\xrightarrow{h(t)}}$$ impulse response

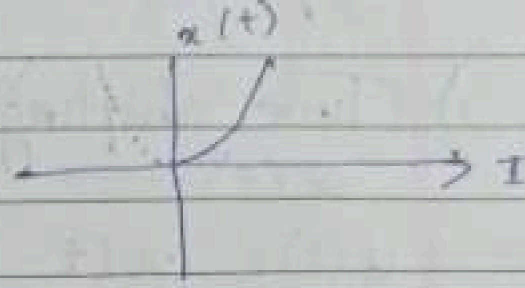
$y(t) = h(t) * x(t)$
convolution

$\lim_{\Delta \to 0} \delta_\Delta(t) = \delta(t)$.

$\delta(t) \xrightarrow{\int} u(t)$

4) Unit parabolic system signal

$x(t) = \frac{1}{2}t^2 \quad t \geq 0$
$\quad\; = 0 \quad$ otherwise




$r(t) \xrightarrow{\int} x(t)$

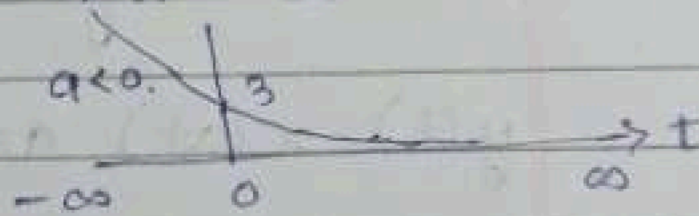
10/8/22

5) Exponential signal:

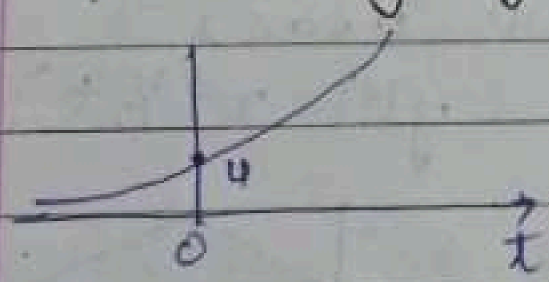
$x(t) = \underset{\text{amp}}{K} e^{at}$

a) exponential decaying signal

$x(t) = 3e^{-2t}$




b) expo. rising signal.




$x(t) = 4e^{2t}$

- $x(t) = 4e^{j\omega_0 t}$
$= 4\cos\omega_0 t + j4\sin\omega_0 t$

6) Sinusoidal signals:

$x(t) = \underset{\text{amp}}{3} \sin(\underbrace{60\pi t}_{\omega_0 \to rad/s = 2\pi f_0 \to Hz})$

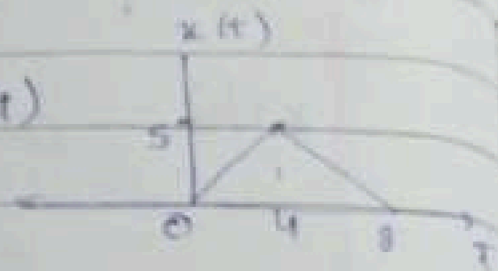
## * Operations on signals :

1) Time shifting

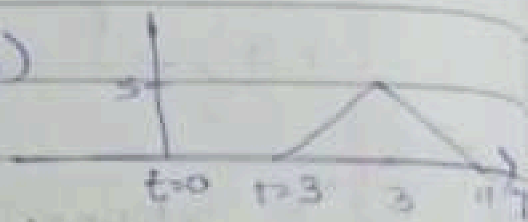
$x(t) \longrightarrow$ signal

$y(t) = x(t - \tau)$

$y(t) = x(t)$




$y(t) = x(t-3)$

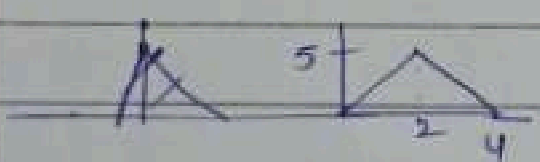



2) Time scaling

$y(t) = x(at) \quad a \geq 0.$

i) $a > 1$

$y(t) = x(2t)$




Contract.

ii) $0 < a < 1$

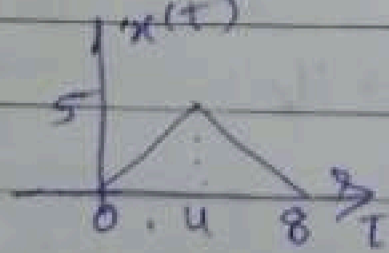
$y(t) = x(\frac{1}{2}t)$

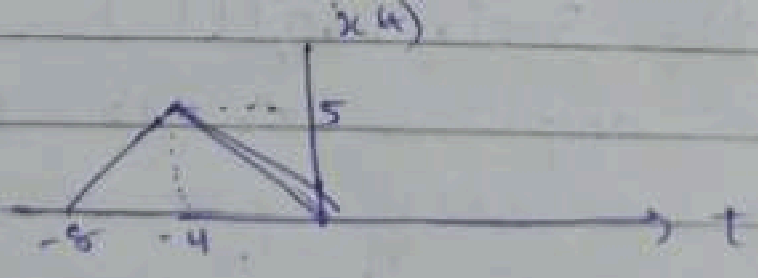

Expansion.

3) Time reversal.

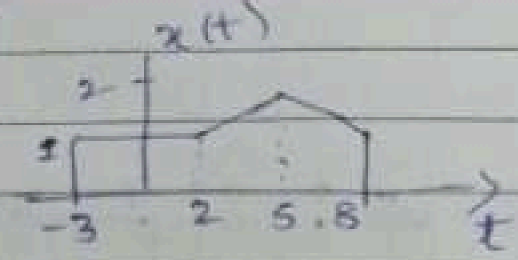
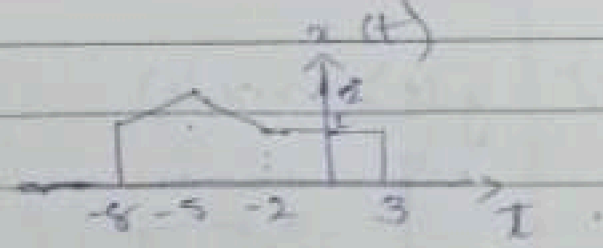
$x(t) \longrightarrow$ signal.

$y(t) = x(-t)$



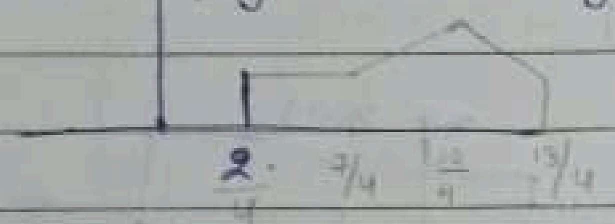




Folding.

eg:-  

* $y(t) = x(4t-5)$.

1) Shifting 2) Scaling



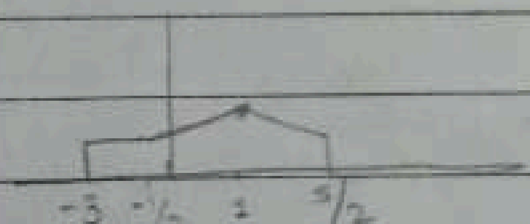
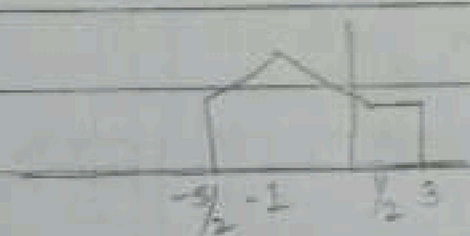
i) Scaling ii) Shifting

$t \to 4t \Rightarrow y(t) = x(t)$

$\rightarrow y(t) = x(4t)$

Scaling $y(t) = x(t - \frac{5}{4})$

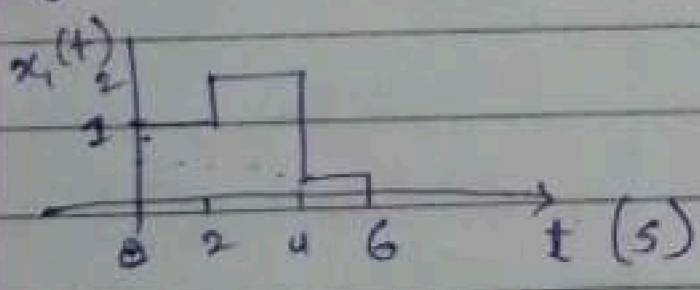
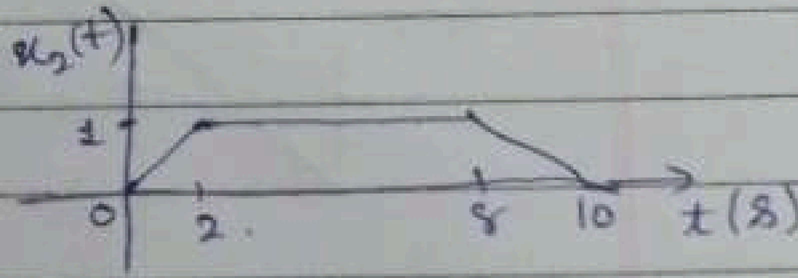
* $y(t) = x(3-2t)$

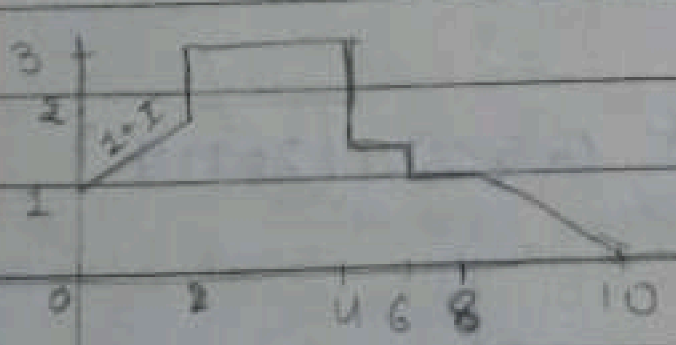
 → 

17/08/23

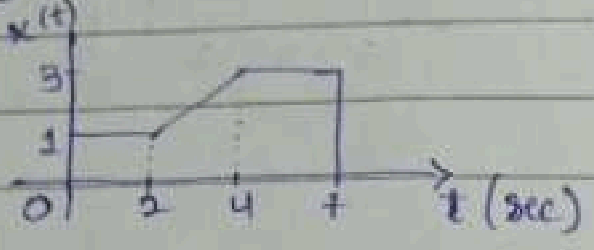
## * Addition of two signals :-

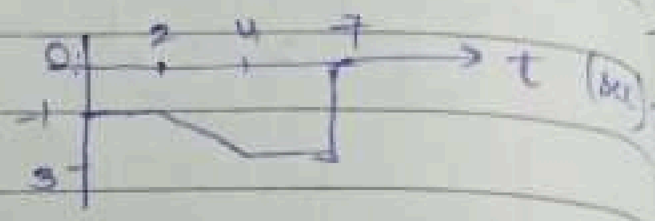
1) $y(t) = x_1(t) + x_2(t)$.

eg-  

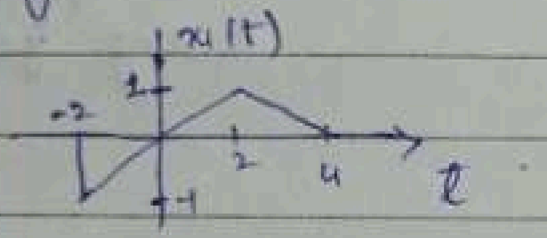


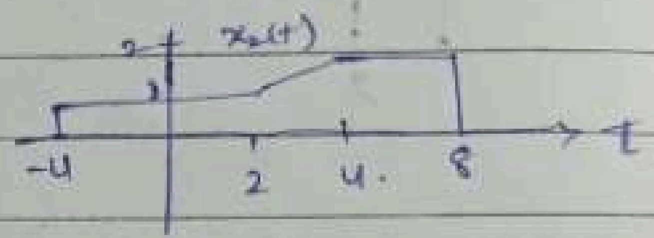
2) $y(t) = -x(t)$

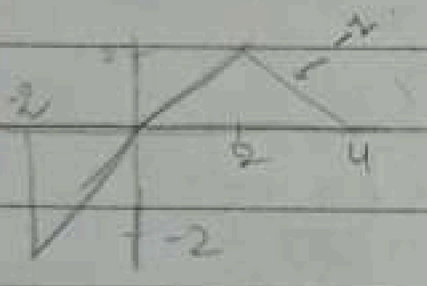


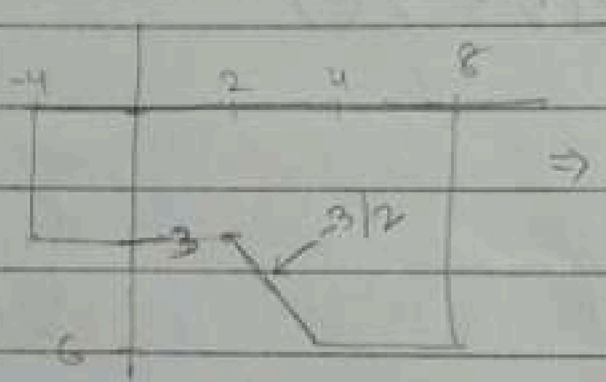



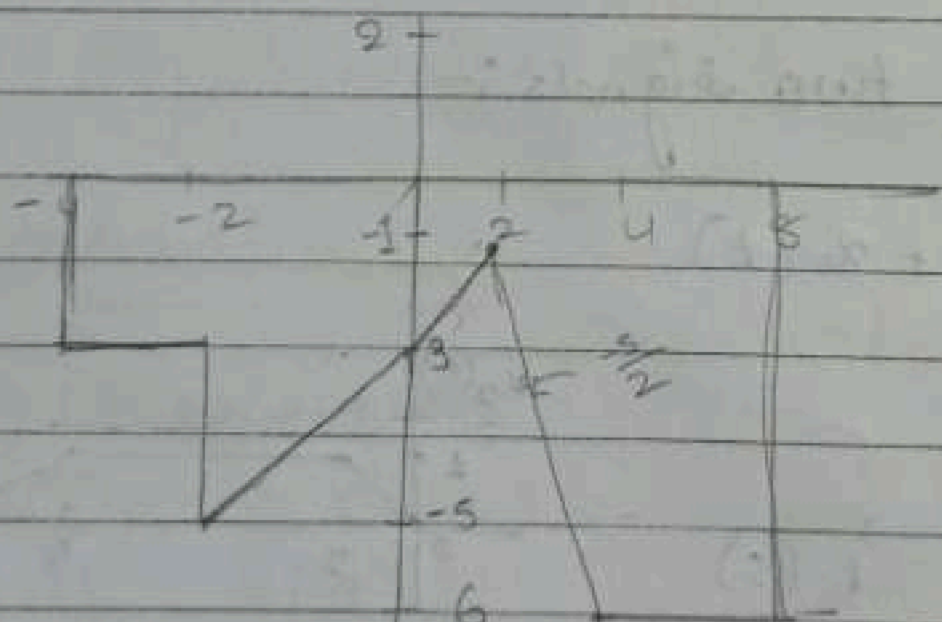

eg-2) $y(t) = 2x_1(t) - 3x_2(t)$



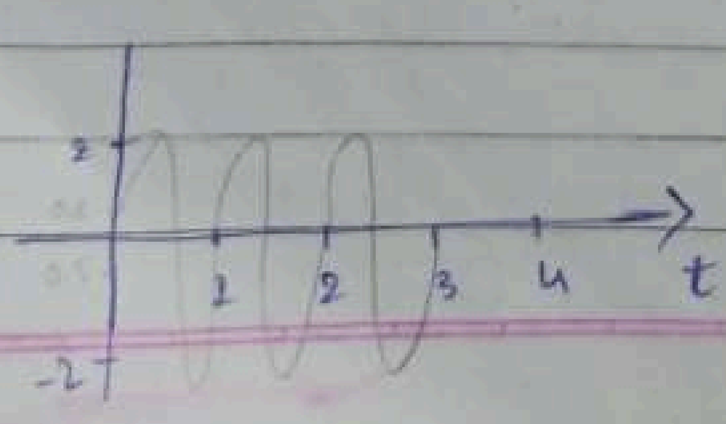



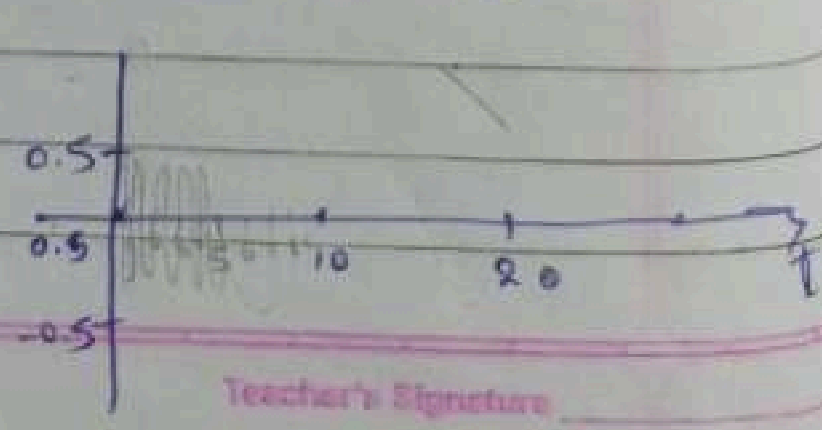



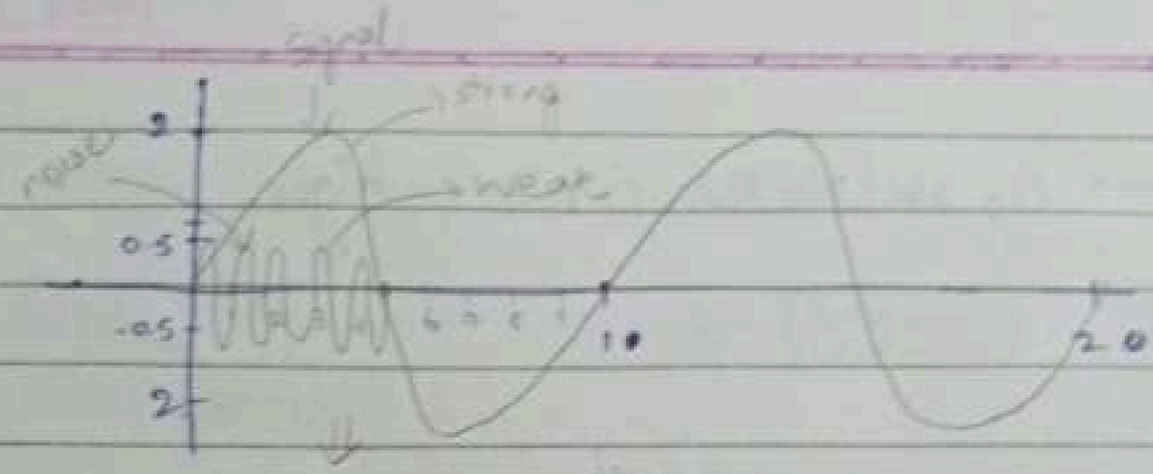



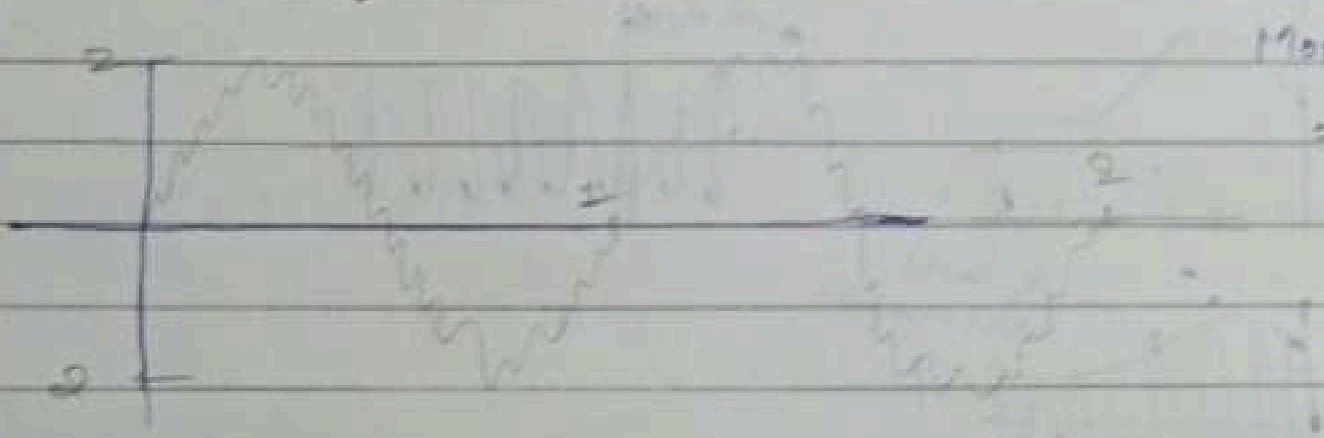

eg-3) $x(t) = 2\sin(2\pi t) + 0.5\sin(20\pi t)$



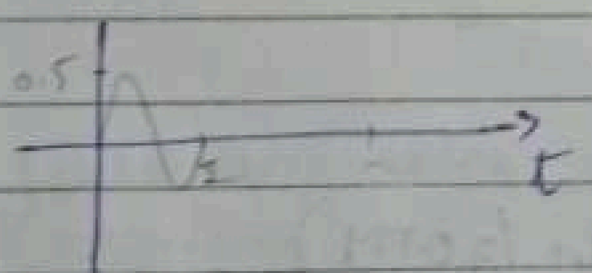

Teacher's Signature

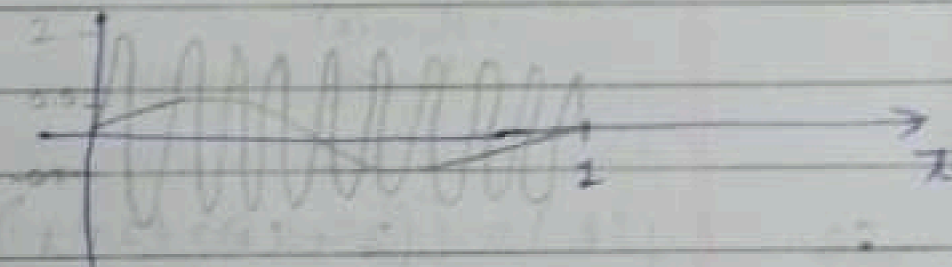


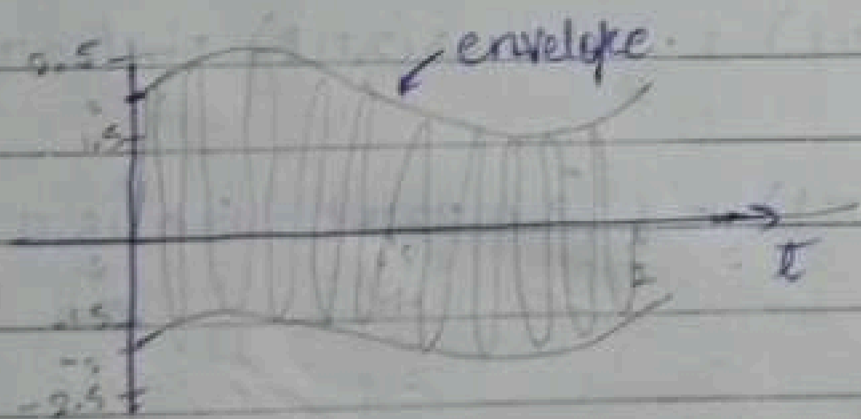



More Amplitude
= strong signal

eg-4) $y(t) = 0.5\sin(2\pi t) + 2\sin(20\pi t)$

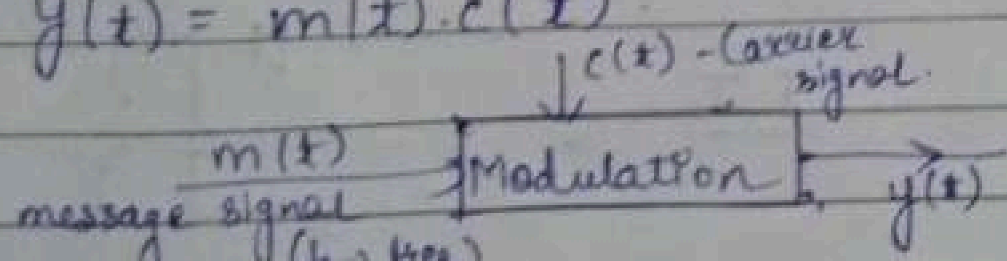



18/08 Multiplication :-

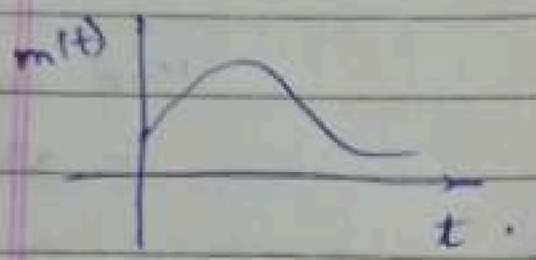
$y(t) = m(t) . c(t)$ speech - 0-4 kHz.

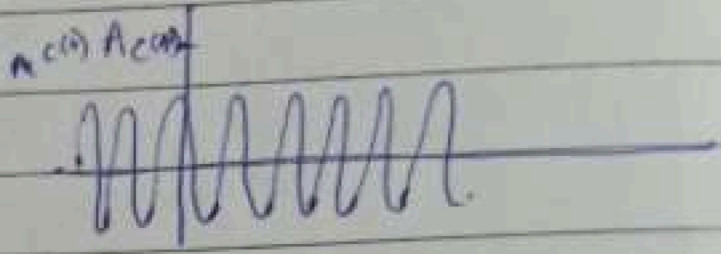


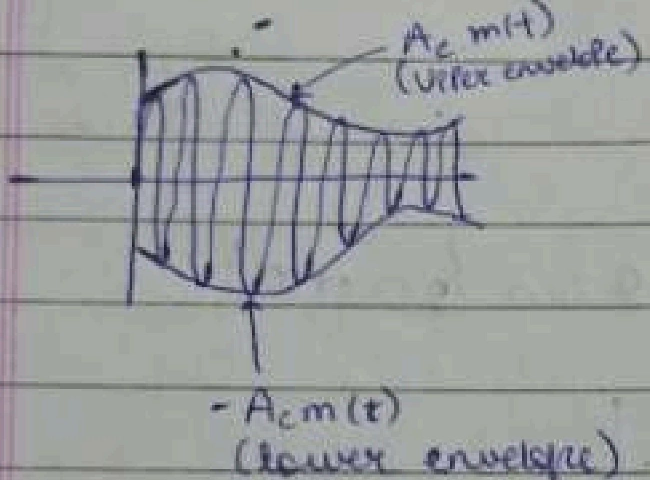

f → freq ↓ → λ ↑ → length of antenna ↑

$c(t) \longrightarrow A_c \sin(\omega_c t) \quad \omega_c \rightarrow \text{high}$

AM



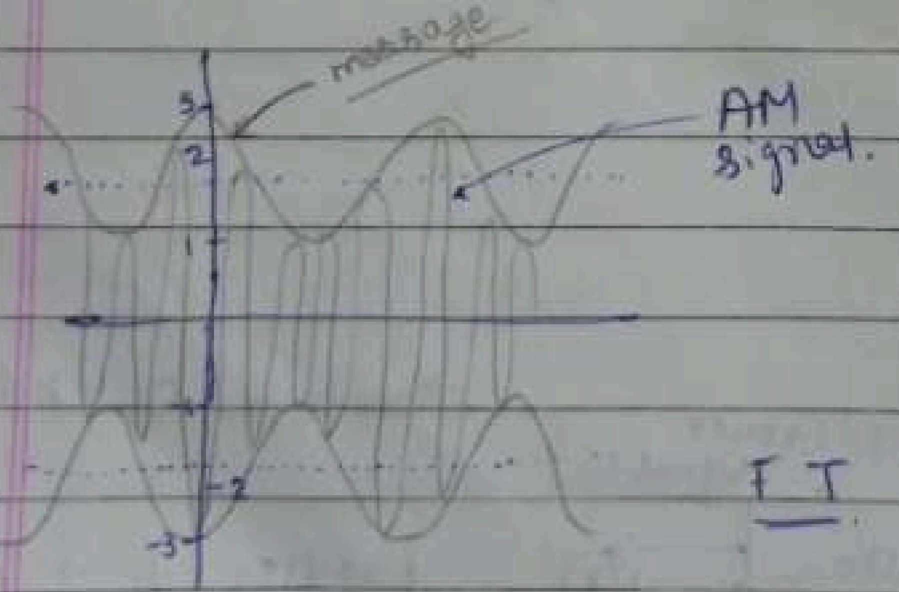




eg. $y(t) = (2 + \cos(2\pi t))\sin(20\pi t)$

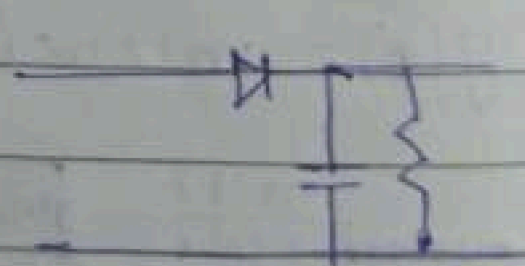
$= 2\sin(20\pi t) + \frac{1}{2} \times 2\cos(2\pi t)\sin(20\pi t)$

$= \underset{10\,Hz}{2\sin(20\pi t)} + \frac{1}{2}\left[\sin \underset{11\,Hz}{22\pi t} - \sin \underset{9\,Hz}{18\pi t}\right]$

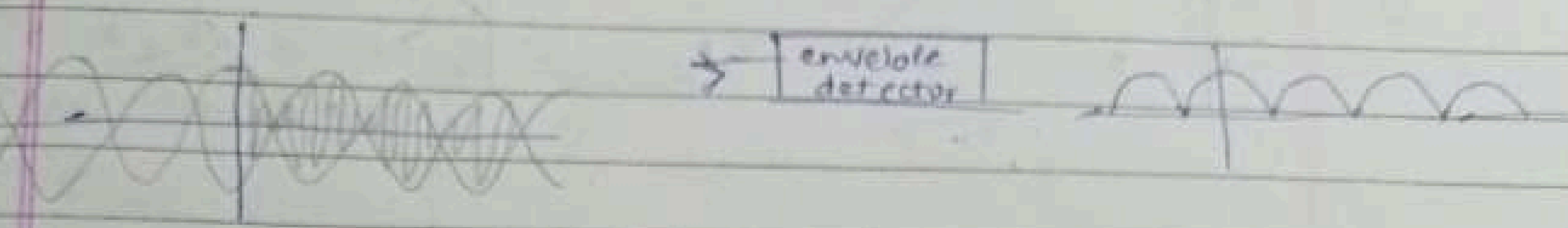



Double side band with carrier (DSB)

FT

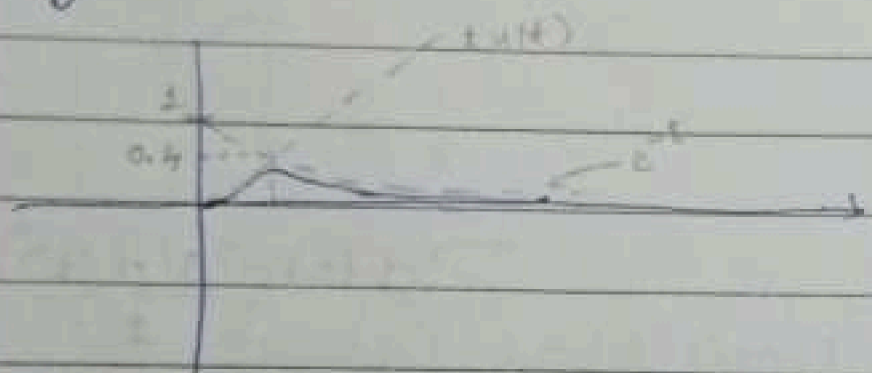


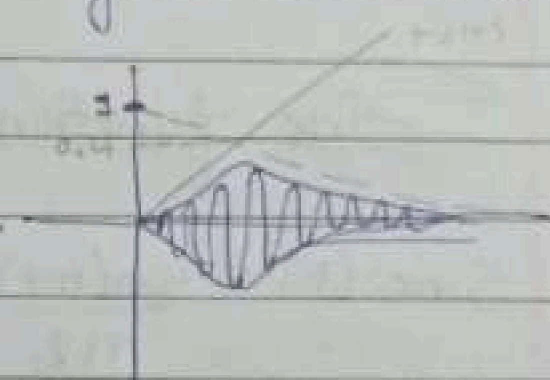
$y(t) = (\cos 2\pi t) \sin(20\pi t)$




eg $y(t) = te^{-t} u(t)$.

$y(t) = te^{-t} \sin(20\pi t) u(t)$

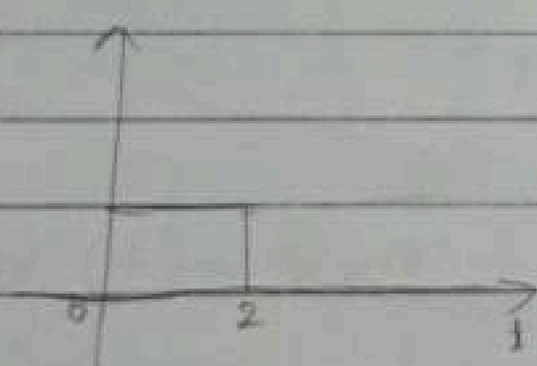


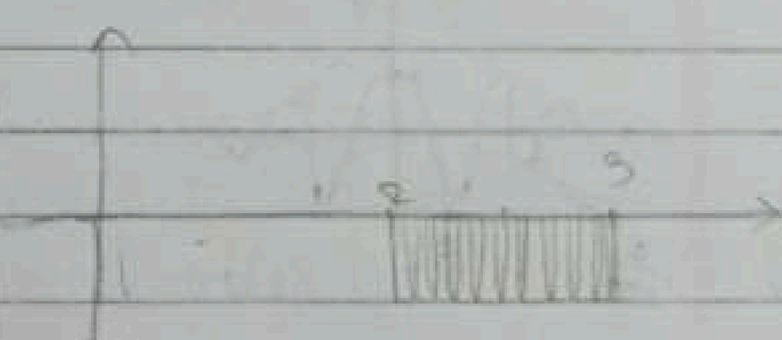

1) $y(t) = u(t) - u(t-2)$

2) $y(t) = \sin 10\pi t \cdot [u(t-3) - u(t-2)]$

1)

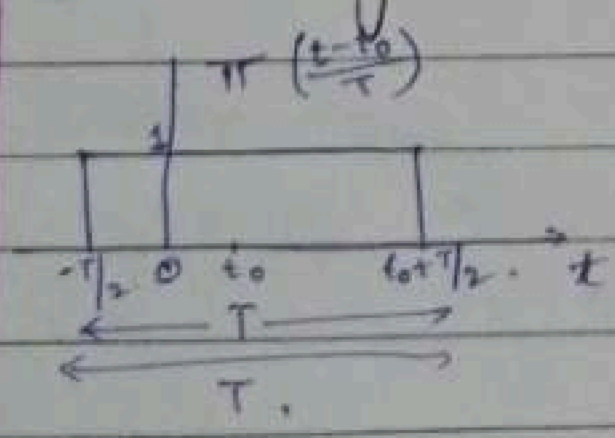


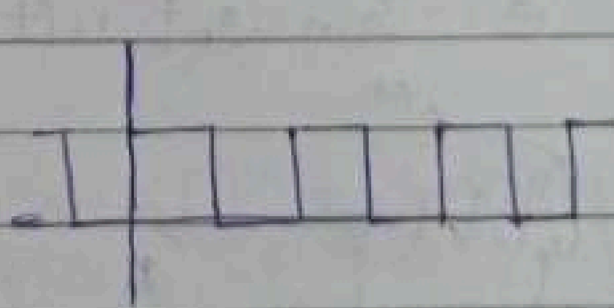

2)

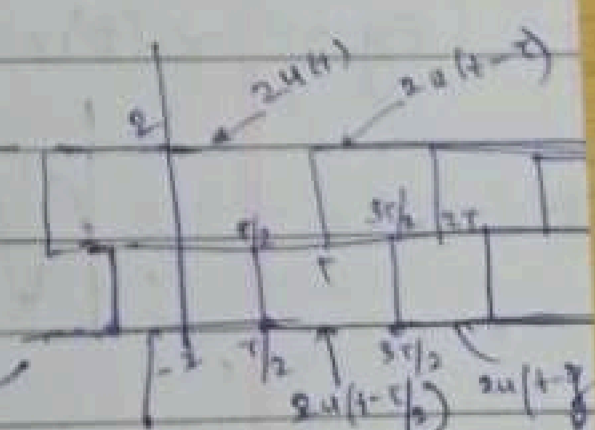



22/08

## Gate signal:

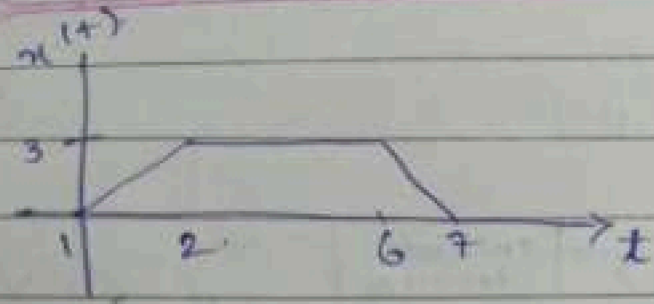


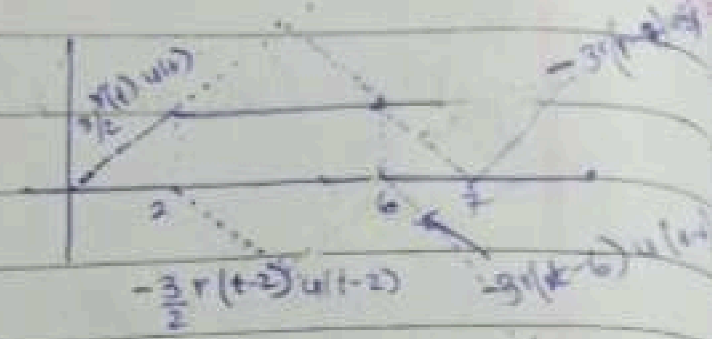






$2u(t) - 2u\left(t - T/2\right) + 2u(t-T) - 2u\left(t - \frac{3T}{2}\right)$

eg:-



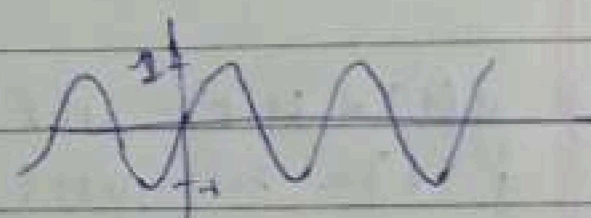



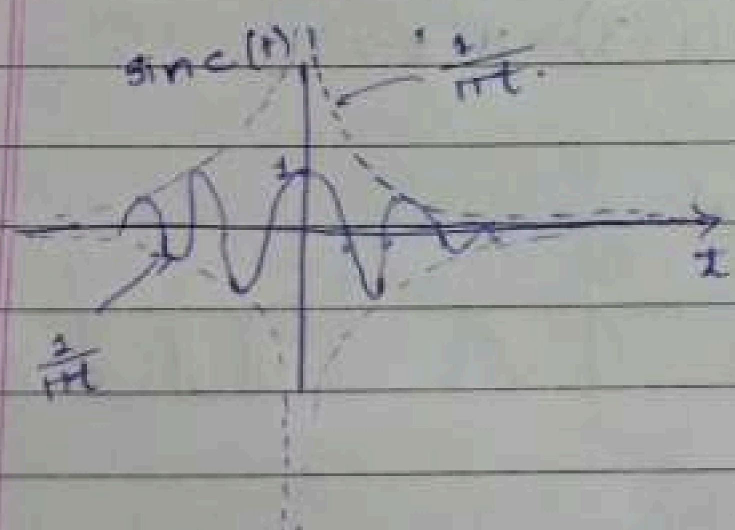

$$x(t) = \frac{3}{2} r(t)u(t) - \frac{3}{2} r(t-2)u(t-2) - 3r(t-6)u(t-6) + 3r(t-7)u(t-7)$$

eg:- **Sinc function**

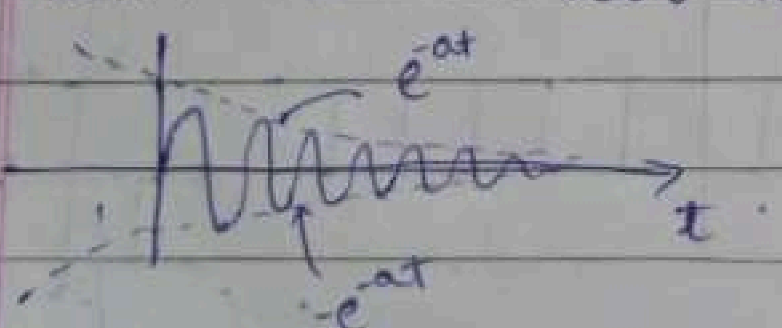
$$\text{Sinc}(t) = \frac{\sin(\pi t)}{\pi t} = \left(\frac{1}{\pi t}\right)(\sin \pi t)$$

$$S_a(t) = \frac{\sin(t)}{t}.$$




eg:- $x(t) = e^{-at} \sin \omega_0 t \, u(t).$



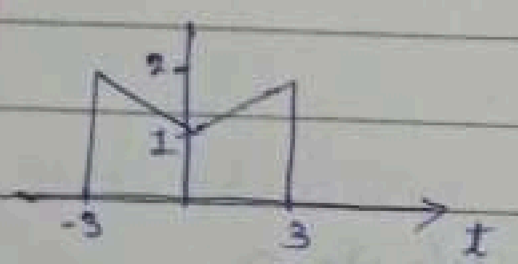
# Types of signals:-

1) Even & Odd Signals

**Even signal:-**

$x_e(t) \rightarrow$ even signal iff $x_e(-t) = x_e(t)$.

eg:-



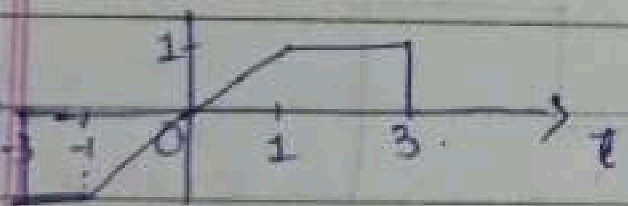

**Odd signal:-**

$x_o(t) \rightarrow$ odd signal iff $x_o(-t) = -x(t)$

$x_o(-0) = x_o(0)$

$x_o(0) = -x_o(0) \Rightarrow 2x_o(0) = 0 \Rightarrow x_o(0) = 0$.

eg




$\underset{\text{given}}{x(t)} = x_e(t) + x_o(t)$ ———— (i)

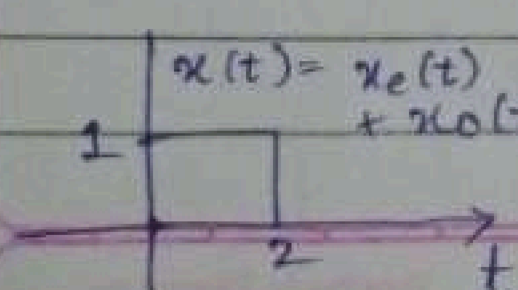
(1) Put $t = -t$. $\Rightarrow x(-t) = x_e(-t) + x_o(-t)$

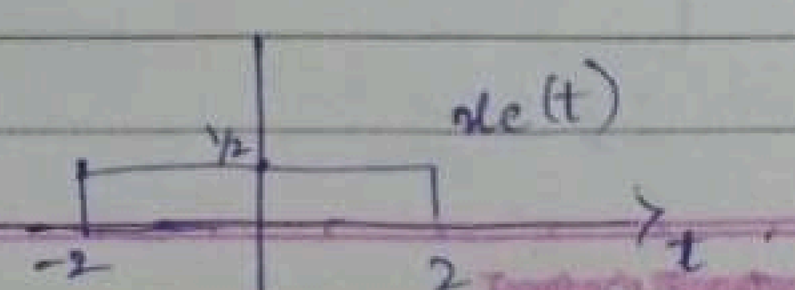
$= x_e(t) - x_o(t)$. ———— (ii)

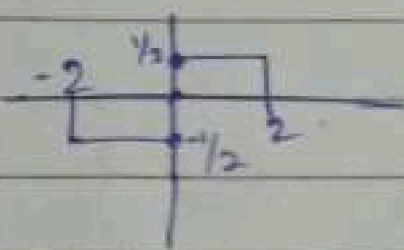
(i) (i) + (ii) $\Rightarrow x(t) + x(-t) = 2\,x_e(t) \Rightarrow x_e(t) = \frac{1}{2}\boxed{x(t) + x(-t)}$

$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

eg:-






2) Energy and power signals:-

$x(t) \rightarrow$ signal. $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$P_{av} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$

$\rightarrow P(av) = 0$

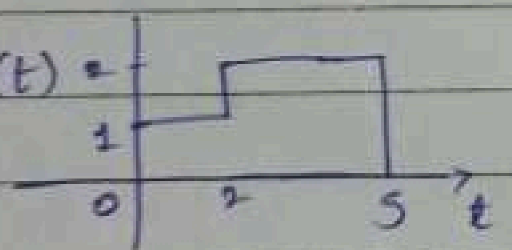
$E \rightarrow$ finite $\Rightarrow$ Energy signal

$P_{av} \rightarrow$ " $\Rightarrow$ Power signal $\rightarrow E = \infty$

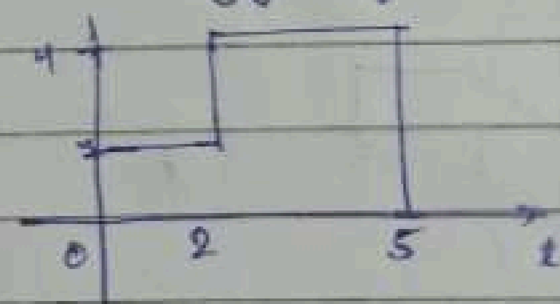
Limited time signals are energy signals.

25/08

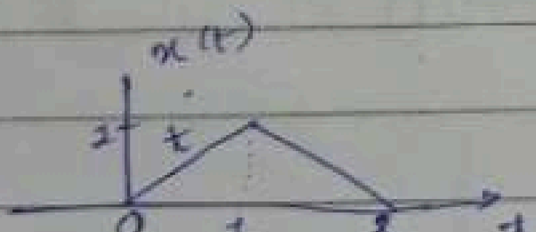
1) $x(t) =$




$x^2 =$

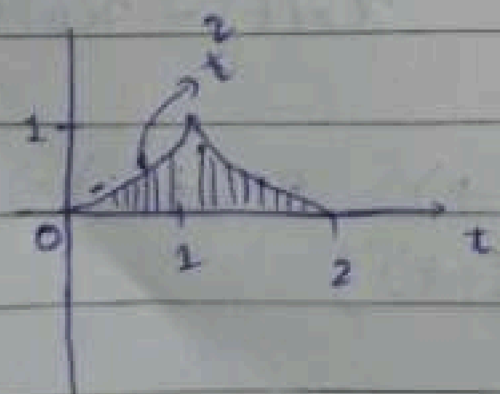



2) $x(t)$



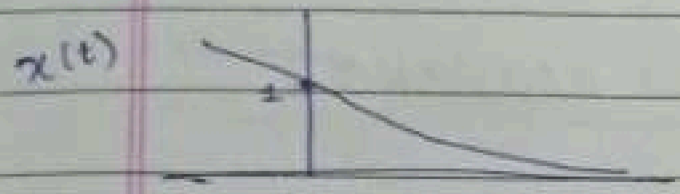

$E =$


$x^2 =$




$E = 2 \times \int_0^1 t^2 dt = 2\left[\frac{t^3}{3}\right]_0^1 = \frac{2}{3}$

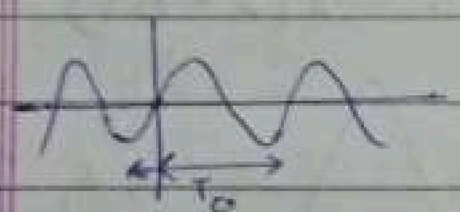
3) $x(t) = e^{-at} u(t) \quad a > 0.$




$$E = \int_0^{\infty} \left[e^{-at}\right]^2 dt = \int_0^{\infty} e^{-2at} dt = \left[\frac{e^{-2at}}{-2a}\right]_0^{\infty}$$

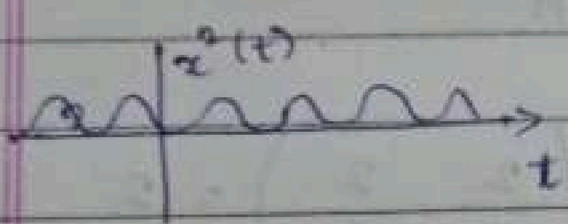
$$= \frac{1}{2a}.$$

4) $x(t) = A \sin \omega_0 t$ ← Not energy signal.




$$E = \int_{-\infty}^{\infty} A^2 \sin^2 \omega_0 t \, dt = A^2 \int_{-\infty}^{\infty} \frac{1}{2} - \int_{-\infty}^{\infty} \frac{\cos 2\omega_0 t}{2}$$

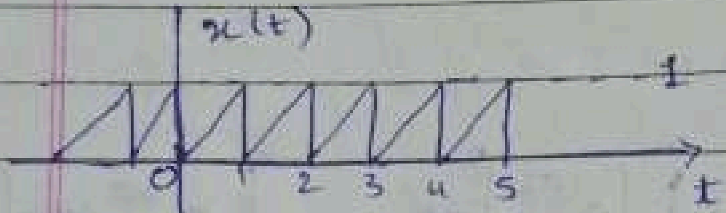
$$= A^2 \left(\frac{1}{2} A - \frac{\sin 2\omega_0 t}{2 \times 2\omega_0}\right)_{-\infty}^{\infty} \Rightarrow A^2 \left(\frac{\infty}{2} - \frac{\sin 2\omega_0(-\infty)}{4\omega_0} - \frac{\sin 2\omega_0(\infty)}{4\omega_0}\right)$$




$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x^2(t) dt = \frac{1}{T_0} \int_0^{T_0} A^2 \sin^2 \omega_0 t \, dt.$$

$$= \frac{A^2}{2T_0} \left[T_0 - \sin 2\omega_0 T_0\right]$$ $= 0$ (Negative & Positive area cancels each other)

$$= \frac{A^2}{2T_0} \times T_0 = \boxed{\frac{A^2}{2}}$$



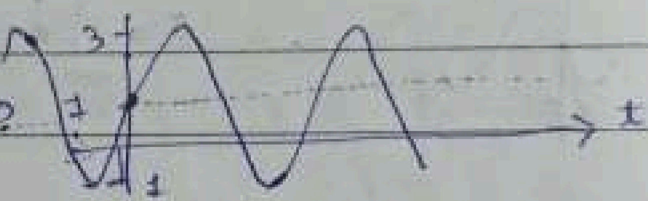
$P_{av} = ?$

## * Periodic & aperiodic signals :-

**Periodic :-**

Time Period

$$x(t) = x(t-T) = x(t-2T) = \ldots \; . \; x(t-nT), \; n = 0,1,2 \ldots$$
$$= x(t+T) = x(t+2T) = \ldots = x(t+nT)$$

eg :- 1) $x(t) = 1 + 2\sin 4\pi t \rightarrow$ 

→ By adding a dc to periodic signal it continues to remain periodic.

2) $x(t) = 3\sin \underset{T=2/5}{5\pi t} + 2\cos(\underset{T=2/9}{9\pi t})$

$T = 4/45$ sec or $T = 2$ sec $= LCM\left(\dfrac{2}{5}, \dfrac{2}{9}\right) = 2$

Summation of sinosoidal signal can be periodic if we can find feasible solution (common T) for both

3) $x(t) = \sin 2t + 2\sin 4\pi t$

$T = \pi$ sec $\qquad T = 1/2$ sec

Possable

Not Periodic

$x(t) = \sin \omega_0 t + 3 \sin \omega_1 t$ If $\frac{\omega_0}{\omega_1}$ = rational then Periodic

29/08

# Systems

$\xrightarrow{x(t)} \boxed{\text{System}} \xrightarrow{y(t)}$

1) $y(t) = x(t-3)$ → Delay system
2) $y(t) = x^2(t)$
3) $y(t) = \frac{dx}{dt}$
4) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$.

## Types :-

1) **Linear and non-linear system**

$\xrightarrow{x(t)} \underset{\text{Linear}}{\boxed{\text{sys}}} \xrightarrow{y(t)}$

1) $a x(t) \rightarrow \boxed{\text{sys}} \xrightarrow{a y(t)}$ (Homogenity)
2) $x_1(t) \longrightarrow y_1(t)$

$x_2(t) \longrightarrow y_2(t)$.

$x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t)$ (Super-Position / Additivity)

$a x_1(t) + b x_2(t) \longrightarrow a y_1(t) + b y_2(t) = y(t)$ → Superposition Principle.

eg:- 1) $\xrightarrow{x(t)} \boxed{\text{system}} \xrightarrow{y(t) = x^2(t)}$.

$x_1(t) = a_1 x(t)$ $\quad y_1(t) = x_1^2(t) = (a_1 x(t))^2 = a_1^2 x^2(t) = a^2 y(t) \neq a y(t)$

2) $\xrightarrow{x(t)} \boxed{\phantom{sys}} \xrightarrow{} y(t) = x(t) + 5$

$x_1(t)$ $\quad y_1(t) = x_1(t) + 5$

$x_2(t)$ $\quad y_2(t) = x_2(t) + 5$.

$x(t) = ax_1(t) + bx_2(t) \xrightarrow{y(t)} x(t)+5 = ax_1(t) + bx_2(t)$

$ay_1(t) + by_2(t) = a[x_1(t)+5] + b[x_2(t)+5]$
$= ax_1(t) + bx_2(t) + 5a + 5b$

$y(t) \neq ay_1(t) + by_2(t)$

3) $\xrightarrow{x(t)} \boxed{Sys} \xrightarrow{} y(t) = tx(t)$

$x_1(t) \longrightarrow y_1(t) = tx_1(t)$
$x_2(t) \longrightarrow y_2(t) = tx_2(t)$

$x(t) = ax_1(t) + bx_2(t) \longrightarrow y(t) = tx(t)$
$= t[ax_1(t) + bx_2(t)]$
$= a[tx_1(t)] + b[tx_2(t)]$
$= ay_1(t) + by_2(t)$

Any other signal of variable t

* $y(t) = g(t)x(t)$ is linear

4) $\xrightarrow{x(t)} \boxed{System} \xrightarrow{} y(t) = \frac{dx}{dt}$

$\frac{d}{dt}[ax_1(t) + bx_2(t)]$

$= a\frac{d}{dt}x_1(t) + b\frac{d}{dt}x_2(t)$

$= ay_1(t) + by_2(t)$

5) $\xrightarrow{x(t)} \boxed{\int} \xrightarrow{} y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

6) $\xrightarrow{x(t)} \boxed{System} \xrightarrow{} y(t)$

$a_n \frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1} y(t)}{dt^{n-1}} \dots a_1\frac{dy(t)}{dt} + a_0 y(t)$

$= b_m \frac{d^m x}{dt^m} + \dots + b_1 \frac{dx(t)}{dt} + b_0 x(t)$

→ Coefficients can be of time.
→ If Power of each term should be one to make given signal linear.

eg:- 1) $\frac{d^2y}{dt^2} + 4\left(\frac{dy}{dt}\right)^2 + 4y(t) = x(t)$

2) $t\frac{dy}{dt} + 5y(t) = \sin t \; x(t)$

2) Time invariant & time-varying system :-

$x(t) \rightarrow \boxed{\text{Systems}} \rightarrow y(t)$

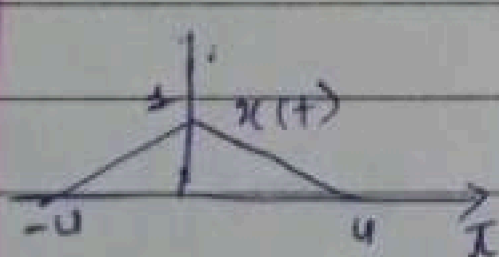
$x_1(t) = x(t-T) \longrightarrow y_1(t) = y(t-T)$

eg:- $y(t) = x^2(t) \longrightarrow$ Time invariant
$y(t) = t\,x(t) \longrightarrow$ Time varying

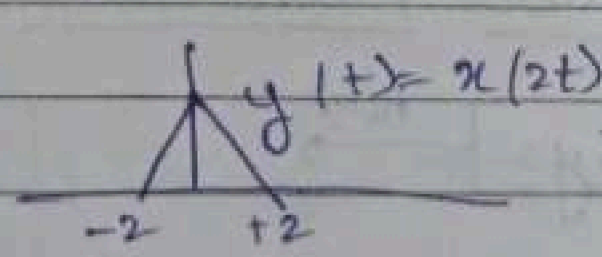
01/09

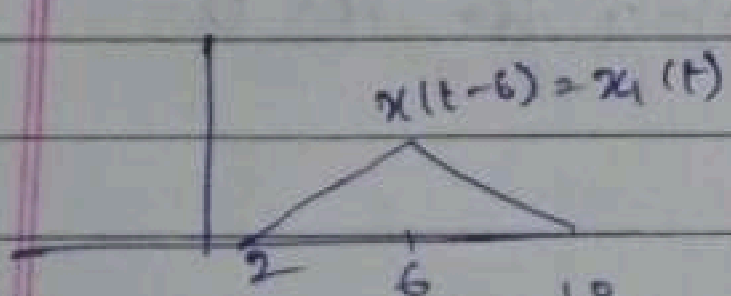
ex $x(t) \rightarrow \boxed{\text{System}} \rightarrow y(t) = g(t)x(t) \rightarrow$ Time varying.

ex $x(t) \rightarrow \boxed{\text{System}} \rightarrow y(t) = x(2t)$ $\quad y(t) = g(x(g_1(t))) \rightarrow$ Time varying
$g_1(t) \neq t$
$y(t) = g(x(t)) \rightarrow$ Time invariant

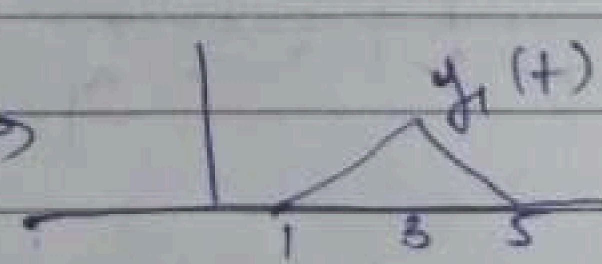


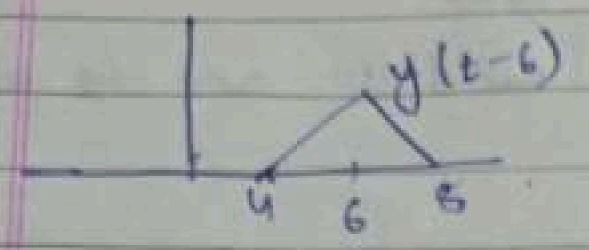

→




↔






eg:- $y(t) = \sin(x(t)) \longrightarrow$ Non-linear
$\longrightarrow$ Time invariant

$x_1(t) = x(t-T) \rightarrow y_1(t) = \sin(x_1(t)) = \sin(x(t-T))$
$y(t) = \sin(x(t-T))$

3) Causal and Non-Causal system.

$\longrightarrow$ If the output depends on current input, past input & past output = Causal.

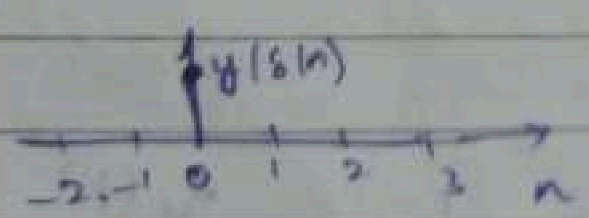
Non Causal $\rightarrow$ Output depends on future input

eg:- $x(n) \rightarrow$ Sys $\rightarrow y(n)$

$y(n) = 0.2y(n-1) + x(n) - x(n-1) \rightarrow$ Causal
$y(n) = \cancel{y}(n+2) + x(n-2) \longrightarrow$ Non-Causal

* Impulse Response.

$\delta(n) \rightarrow$ Sys $\rightarrow h(n)$




$h(n) = 0$ for $n < 0$.

4) Static and Dynamic system.

Static system: output depends on only current input (Memoryless !!)

eg:- $y(t) = x^3(t) \rightarrow$ Static

$y(t) = x^2(t) + 3x(t-1) \rightarrow$ Dynamic.

5) Stable and Unstable system

$x(t) \rightarrow \boxed{Sys} \rightarrow y(t)$

BIBO → (Bounded input bounded output)

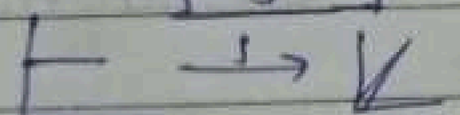
For every bounded input, if there will be bounded output then system is called stable.

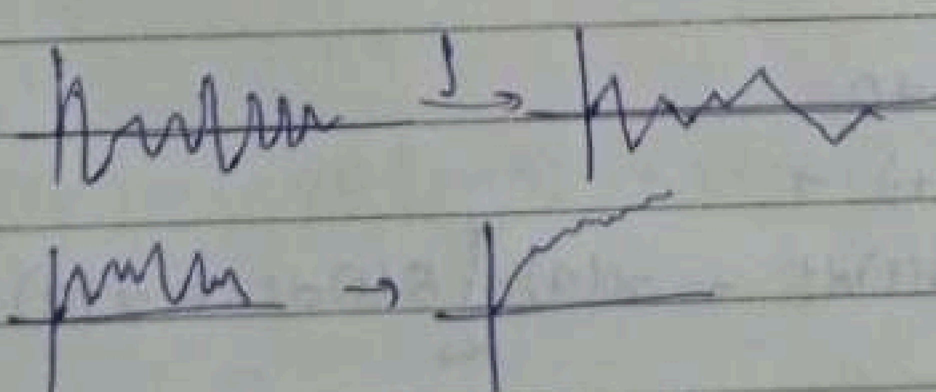
eg:- $y(t) = x^2(t)$ let $|x(t)| < M \rightarrow |y(t)| < M^2 \rightarrow$ Stable

eg $y(t) = \sin(x(t)) \rightarrow$ stable.

eg $y(t) = \tan(x(t)) \rightarrow$ Unstable

$x(t) \rightarrow \boxed{\int} \rightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)\, d\tau$ Unstable.





→ Stable.

# Invertible & Non-invertible system:-

$$\xrightarrow{x(t)} \boxed{sys} \xrightarrow{y(t)} \boxed{sys\ 1} \xrightarrow{x(t)}$$

Inverse system

eg:- $\xrightarrow{x(t)} \boxed{\int} \xrightarrow{y(t)} \longrightarrow \boxed{d/dt} \xrightarrow{x(t)}$

Integrator $= \int_{-\infty}^{t} x(\tau)d\tau$ Differentiator.

→ Differentiator → Integrator is not perfect invertible.

$$\xrightarrow{x(t)} \boxed{(\ )^{1/2}} \xrightarrow{y(t) = \sqrt{x(t)}} \boxed{(\ )^2} \xrightarrow{x(t)}$$

$\xrightarrow{x(t)} \boxed{(\ )^2} \xrightarrow{y(t) = x^2(t)} \boxed{(\ )^{1/2}} \longrightarrow$ Non-invertible.

## * Unit Impulse Signal.

**Cont.**
(Dirac Delta $f^n$).
$\delta(t) = 0$ for $t \neq 0$
$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$.

**Discrete**
(Kronecker Delta $f^n$).
$\delta(n) = 1$ for $n = 0$.
$= 0$ otherwise

**Prop.**

1) $x(t).\delta(t) = 0 \quad t \neq 0$

$x(t).\,\delta(t) = x(0).\delta(t) \rightarrow$

2) $\int_{-\infty}^{\infty} x(t).\delta(t) = \int_{-\infty}^{\infty} x(0)\delta(t)dt = x(0)\int_{-\infty}^{\infty}\delta(t)dt = x(0).1 = x(0)$

3) $x(t)\delta(t-T) = x(T)\,\delta(t-T)$

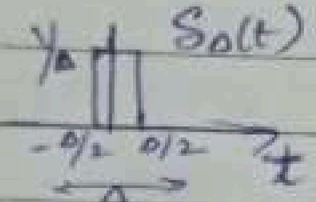
4) $\int_{-\infty}^{\infty} x(t)\delta(t-T)\,dt = x(T)$

5) $\delta(-x) = \delta(x)$

6) $\delta(2x) = \frac{1}{2}\delta(x) \quad \therefore \quad \delta(at) = \frac{1}{|a|}\delta(t)$

1) $\int_{-2}^{2} \delta(t-4)\,dt = 0.$ 2) $\int_{-\infty}^{\infty} [t^2 \delta(t-2) + t\,\delta(t-3)]\,dt$

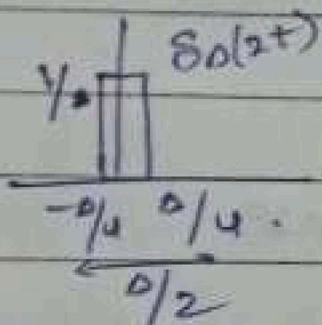
$= (2)^2 + 3 = 7.$




$\lim_{\Delta \to 0} \delta_\Delta(t) = \delta(t)$

$\star \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau = \int_{-\infty}^{\infty} x(t)\,\delta(t-\tau)\,d\tau$

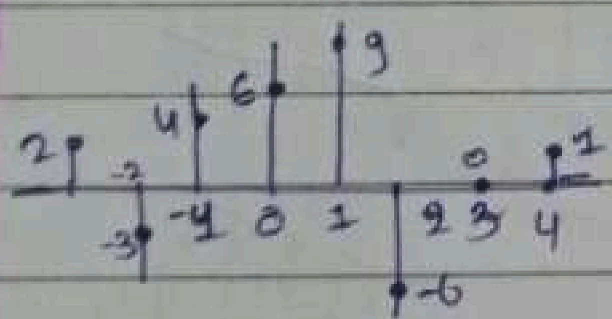
$= x(t).$

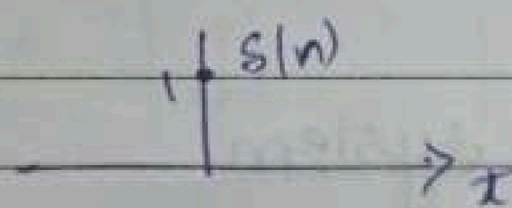



$\lim_{\Delta \to 0} \delta_\Delta(2t) = \frac{1}{2}\delta(t)$

## ✱ Discrete sequence.

$x(n) = \{\overset{}{2}, \overset{x(-2)}{-3}, \overset{x(-1)}{4}, \overset{x(0)}{\underset{\uparrow}{6}}, \overset{x(1)}{9}, \overset{x(2)}{-6}, \overset{x(3)}{0}, \overset{x(4)}{1}\}$




$x(n) = 2\delta(n+3) - 3\delta(n+2) + 4\delta(n+1) + 6\delta(n) + 9\delta(n-1)$
$\quad - 6\delta(n-2) + \delta(n-4).$

$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau.$

$x(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot \delta(n-k)$

## * LTI system
(Linear & Time Invariant).

$x(t) \rightarrow \boxed{LTI} \rightarrow y(t)$

eg. 1) $y(t) = 3x(t)$ - Amplifier

2) $y(t) = x(t-4) \rightarrow$ Delay system.

3) $y(t) = 0.5\, x(t-10) \rightarrow$ Distortion less system

4) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$.

5) $y(t) = \frac{dx}{dt}$.

$\xrightarrow[x(t)]{\delta(t)} \underset{\text{system}}{\boxed{LTI}} \xrightarrow[y(t)]{h(t)}$ Impulse response

convolution operator.

$y(t) = h(t) * x(t)$.

$\xrightarrow[x(n)]{\delta(n)} \boxed{LSI} \xrightarrow{h(n)}$ $y(n) = h(n) * x(n)$.

## * LSI system

$$x(n) = \sum_{K=-\infty}^{\infty} x(K)\,\delta(n-K)$$

$\delta(n) \longrightarrow h(n)$

$\Rightarrow \delta(n-K) \longrightarrow h(n-K) \rightarrow$ TI or SI.

$\Rightarrow x(K)\,\delta(n-K) \longrightarrow x(K)\,h(n-K) \rightarrow$ Linear system

$x(n)$ $\Rightarrow \sum_{k=-\infty}^{\infty} x(k)\cdot\delta(n-k) \longrightarrow \sum_{k=-\infty}^{\infty} x(k)h(n-k) \xrightarrow{y(n)}$ Linear system.

$\xrightarrow[x(n)]{\delta(n)}$ [ LSI ] $\longrightarrow h(n)$

system $\quad y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$

$= x(n) * h(n).$

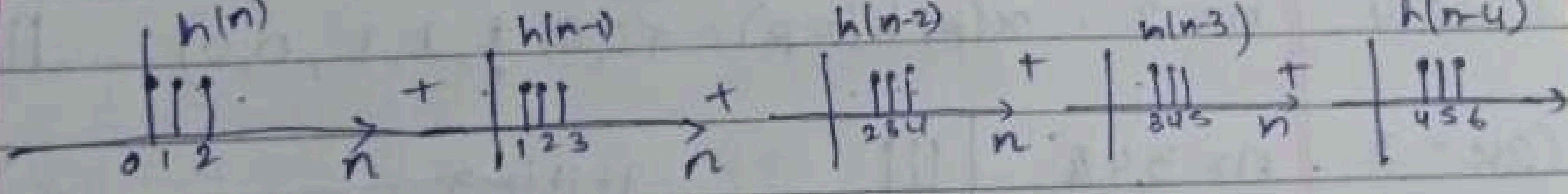
eg: $h(n) = \{1, 1, 1\} = \delta(n) + \delta(n-1) + \delta(n-2) \rightarrow$ FIR.

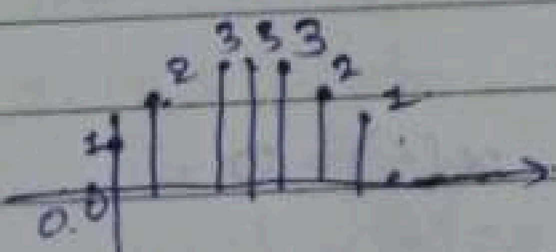
Finite Impulse Response

IIR $\rightarrow$ Infinite Impulse system. $\rightarrow h(n) = \left(\frac{1}{2}\right)^n \cdot u(n)$

$x(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\} = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3) + \delta(n-4) \rightarrow y(n) = h(n) + h(n-1) + h(n-2) + h(n-3) + h(n-4)$ $\quad h(n) = \{\underset{\uparrow}{1}, 1, 1\}$

$y(n) = \sum_{k=-\infty}^{k=\infty} x(k)h(n-k)$




$y(n) = \{0, 0, \underset{\uparrow}{1}, 2, 3, 3, 3, 2, 1, 0\}$.

$y(n) = \sum_{k=0}^{4} x(k)h(n-k)$ $\qquad x(k) = 1$ for $k = 0, 1, 2, 3, 4$.

$= 0$ otherwise.

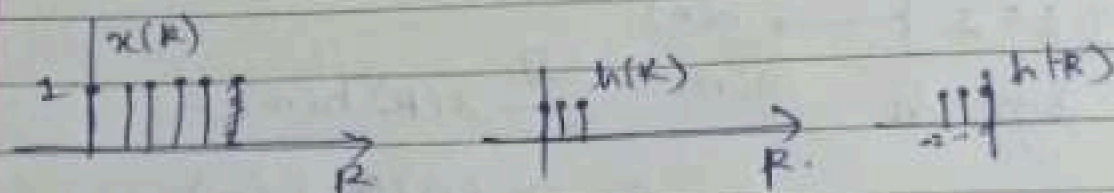
$= x(0)h(n) + x(1)h(n-1) + x(2)h(n-2) + \dots + x(4)h(n-4)$

$= h(n) + h(n-1) + h(n-2) + \dots h(n-4).$

$x(n) \longrightarrow x(k)$ time inversion/folding

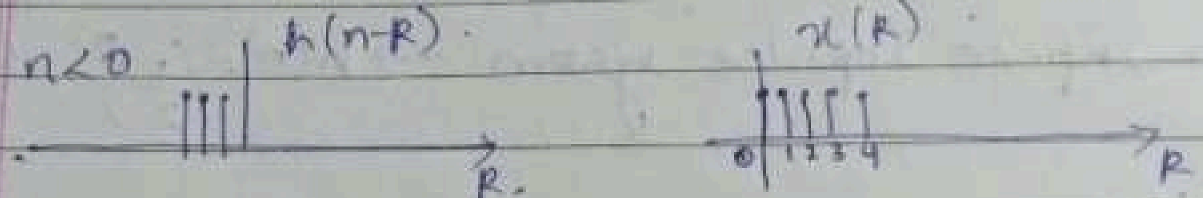
$h(n) \xrightarrow{n \to k} h(k) \xrightarrow{k \to -k} h(-k) \xrightarrow{k \to k-n} h(n-k)$.

Replacing $n \to m$




$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k).$$

Case 1 - $n<0$



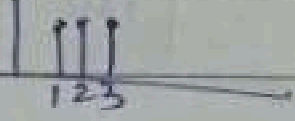

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n+k) = 0$$

Case 2 : $n=0$ $\quad x(k)h(-k) = \{0, 0, \underset{\uparrow}{1}, 0, 0\}$

Case 3 $\quad n=1 \quad x(k)h(1-k) = \{0, 0, \underset{\uparrow}{1}, 1, 0, 0\}$

Case 4 $\quad n=2. \quad x(k)h(2-k) = \{0, 0, \underset{\uparrow}{1}, 1, 1, 0, 0\}$

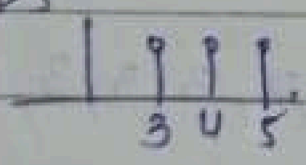
Case - 5 $\quad n = 3, 4, 5$




$= 1+1+1 = 3$

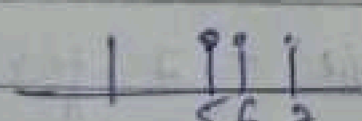
$n>5$

Case - 7




$= 1+1 = 2$ as $x(5) = 0$.

$n=7$




$\rightarrow$ No overlap $= 0$.

eg:- $x(n) = \{\overbrace{a_0, a_1, \dots, a_7}^{8 = N_1}\}$.

$h(n) = \{\underbrace{b_0, b_1, \dots, b_{10}}_{11 = N_2}\}$

$y(n) =$

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

Replacing $n-k=m$

$$= \sum_{m=-\infty}^{\infty} h(m)x(n-m) = h(m) * x(m) = h(n) * x(n).$$

---

$x(-k) \longrightarrow x(n-k) \qquad \sum h(k)x(n-k).$

$x(n-k) = \{\underset{\uparrow}{0} \ldots 0, a_7, a_6, \ldots a_0\}$.

$h(k) = \{0, b_0, b_1, \ldots, b_{10}, 0, 0, 0\}$.

$n = 17 \overset{N=18}{=} N_1 + N_2 - 1$.

$y(n) = 0$ for $n > 17$

$\quad = 0$ for $n < 0$.

Total $N = 18 = N_1 + N_2 - 1$.

## For continuous system

$\xrightarrow{x(t)} \boxed{h(t)} \xrightarrow{y(t)} = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau.$

$\delta(t) \longrightarrow h(t)$

$\delta(t-\tau) \longrightarrow h(t-\tau) \Rightarrow$ TI.

$x(\tau)\delta(t-\tau) \longrightarrow x(\tau)h(t-\tau) \Rightarrow$ homogeneous.

$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau \longrightarrow \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = y(t)$