Conductors

1 Conductors

All material is electrically neutral. However there are a large (enormously large) number of point charges within a material with equal amount of positive and negative types. These are the electrons and the positive ions in the material. Generally these charges are tightly bound to each other and hence they don't move even under the influence of electric field. But in metals the electrons are free to wander around. There are certain liquids where positive and negative ions are free to move around. They are called electrolytes. These kind of substances which have enormously ($\sim 10^{23}$) number of charged particles to freely move around are conductors. Ideally conductors are considered as substances with an infinite supply of positive and negative charges.

When we place a conductor in a region which has no electric field, every part of the conductor is neutral, i.e, positive and negative charges are found in equal amount in every part. Once we place the conductor in an electric field , the free charge start moving. The charges will keep moving till the net electric field everywhere within the conductor is 0. As long as there is evena tiny electric field somewhere, an infinite amount of electric charges will respond to it and move. The motion can stop only when every region in the conductor is devoid of any electric field.

Due to the movement of the electric charges the conductor no longer remain neutral everywhere. The positive charges move along the direction of the electric field and the negative charges move opposite to the electric field. This leads to accumulation of charges as shown in the figure 1.

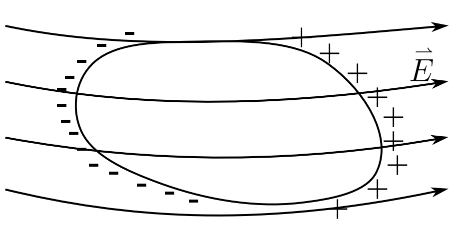


Figure 1: Conductor in an external Electric field

Since the charges can't leave the surface of the conductor, the charges accumulate on the surface and create a surface charge distribution on the conductor. The final configuration of these charges is such that there is no electric field inside the conductor i.e, the configuration of surface charges on the conductor produces an electric field which is opposite to the external electric field in which the conductor is placed.

These surface charges that gets produced on the conductor are called incuced charge on the conductor due to the electric field $\vec{E}$. Now, since the elctric field in a conductor is always 0, the conductor has a constant potential throughout. This is a very important characteristic of a conductor. The surface of the conductor becomes a very convenient surface to specify the boundary condition of an electrostatic problem. These surfaces are equipotential. Let a and b be two points within the conductor. Then the potential difference between a and b is

$$V_b - V_a = -\int_a^b \vec{E} \cdot \vec{dl} = 0 \qquad \text{since } \; \vec{E} \; \text{ is } 0$$

$\therefore \quad V_b = V_a$.
This shows that potential everywhere is same.

2 Cavity inside a conductor

What happens if we have a cavity inside a conductor ?
We can show that the electric field inside the cavity is also 0.

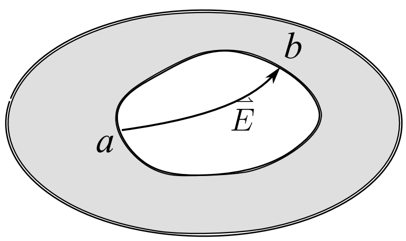


Figure 2:

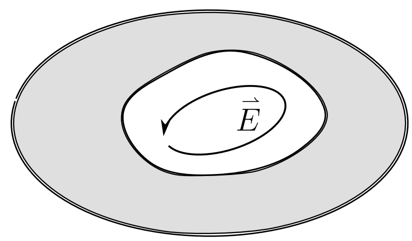


Figure 3:

The whole conductor is at the same potential. The wall of the cavity is also at this potential. We can not have any electric field lines starting from one point of the cavity wall to another point as shown in the figure 2 since $\int_a^b \vec{E} \cdot \vec{dl}$ will lead to a potential difference between point a and point b. The only way, if at all, we can have electric field inside the cavity is to have a loop of electric field line as shown in figure 3.

But this is impossible too. This is because in such a case $\oint \vec{E} \cdot \vec{dl} \neq 0$ along the closed curve. This would mean $\vec{\nabla} \times \vec{E} \neq 0$ which is absurd for an electrostatic field. Hence we cannot have any electric field inside the conductor. So the cavity is also an equipotential region. The potential inside the cavity is the same as the potential of the conductor.

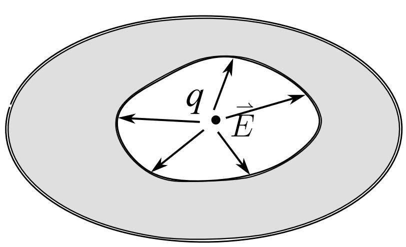
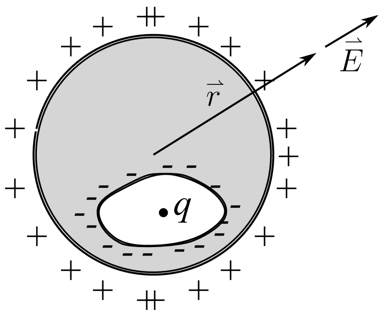


Figure 4:

All these is true when we don't have charges inside the cavity. If we have charges isolated inside the cavity, we can have electric field inside the cavity. These electric fields are no more divergenceless. The fields can diverge from the charge sources and terminate on the walls of the cavity as shown in the figure 4

Eg.

Consider a solid spherical conductor with a cavity inside. A point charge q is situated inside the cavity. What will be the electric field outside the outer surface of the spherical conductor.



Due to the point charge q in the cavity, a charge $-q$ is induced on the inner surface of the cavity. This is because if we consider a Gaussian surface enclosing the cavity but lying completely within the conductor then

$$\oint_S \vec{E} \cdot \hat{n} da = 0$$

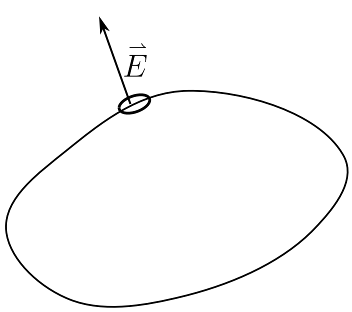
since $\vec{E} = 0$. So the charge enclosed in the Gaussian surface is 0. The only charges are the point charge q and the surface charge on the inner surface of the cavity. So the total surface charge on the wall of the cavity is $-q$. Since the conductor was chargeless, a charge $+q$ will accumulate over the outer surface of the conductor. Since there is no electric field in the conductor, this charge distributes uniformly over the whole sphere. The only electric field outside the conductor is due to this surface charge since the internal charges have cancelled each others effect. So the electric field will be

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}$$

So from outside the spherical conductor we only know the total charge q placed within the cavity. All other information about the position of this charge and the shape of the cavity is lost.

If we now place this conductor in an external electric field, only the outer surface charge $+q$ will readjust to maintain the electric field inside the conductor to be 0. No change occurs on the surface charge over the walls of the cavity. This phenomenon is described as shielding. "The internal region of a conductor is shielded from any electric field in the outside world". Such an enclosure is called the Faraday's cage.

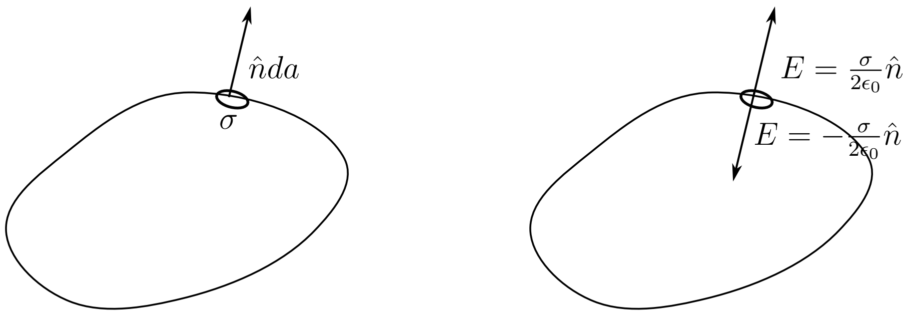
Force on the surface of a conductor



As the charges accumulate over the surface of a conductor, the conductor achieves a surface charge density σ. These surface charges tends to move outward. This creates a force on the surface of the conductor. Before we calculate this force let us calculate the electric field near the surface of the conductor. This will be normal to the surface of the conductor. This is because the surface of the conductor is equipotential. So there cannot be any tangential electric field. To calculate the normal component we apply the boundary condition on $E_\perp$. Since $E_{in} = 0$ inside, we have

$$\begin{aligned} E_{out} - E_{in} &= \frac{\sigma}{\epsilon_0} \\ \therefore \quad \vec{E}_{out} &= \frac{\sigma}{\epsilon_0}\hat{n} \end{aligned}$$

If σ is positive $\vec{E}$ is along $\hat{n}$. If σ is negative, $\vec{E}$ is along $-\hat{n}$.
Due to this electric field outside the conductor, the surface charge over an element of area da experiences a force



If we didn' t have the rest of the charges over the surface of the conductor, the electric field on either side of the small element da would be $\frac{\sigma}{2\epsilon_0}$. Since the conductor had no electric field on the inside, we conclude that the effect of the rest of the charges is to produce exactly an equal and opposite electric field to that of the inside field due to the small element da. So the electric field due to the rest of the charges is

$$\vec{E}_{rest} = \frac{\sigma}{2\epsilon_0}\hat{n}$$

This electric field due to the rest of the charges exerts a force over the element da. This force is given as

$$\vec{f} = \vec{E}_{rest} \times \sigma da = \frac{\sigma^2}{2\epsilon_0}\hat{n}$$

Note that this force is always towards the outward normal to the surface of the conductor since σ^2 is always positive, irrespective of whether σ is positive or negative.