SET B

Calculus SC-107

Full marks 30, Time 1 Hour

Choose the correct answer from the given choices for each of the questions.

1. Let $f(x) = \int_{1/x}^{x} \frac{1}{t} dt$. Then what is the value of $f''(1)$?

   (a) $f''(1) = -2$

   (b) $f''(1) = -1$

   (c) $f''(1) = 1$

   (d) $f''(1) = 2$

   Correct Answer: (a)

2. Find the volume of the solid generated by revolving the region in the first quadrant bounded above by the curve $y = x^2$, below by the x-axis, and on the right by the line $x = 1$, revolved about the line $x = -1$.

   (a) $\frac{3}{7}\pi$

   (b) $\frac{6}{7}\pi$

   (c) $\frac{7}{3}\pi$

   (d) $\frac{7}{6}\pi$

   Correct Answer: (d)

3. Which of the following function is not Riemann integrable on $[-1, 1]$?

   (a) $f(x) = \frac{1}{x+2}$

   (b) $f(x) = [15x]$

   (c) $f(x) = |x|$

   (d) $f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ -1 & \text{if } x \in \mathbb{R} - \mathbb{Q} \end{cases}$

   Correct Answer: (d)

4. Let $f(x) = 1 - x + \frac{x^2}{2} - \frac{x^3}{3} + \cdots + (-1)^n \frac{x^n}{n}$. Then $f(x)$ has

   (a) no real root if n is odd.

   (b) n number of real roots if n is even.

   (c) one real root if n is odd.

   (d) 3 real roots.

   Correct Answer: (c)

5. Let $a_1, a_2, \cdots, a_n$ be real numbers and let f be defined on $\mathbb{R}$ by

$$f(x) = \sum_{i=1}^{n} (a_i + x)^2 \text{ for } x \in \mathbb{R}.$$

   Then the unique local minimum of $f(x)$ is

   (a) $\frac{a_1 + a_2 + \cdots + a_n}{n}$

   (b) $-\Big(\frac{a_1 + a_2 + \cdots + a_n}{n}\Big)$

   (c) $a_1 + a_2 + \cdots + a_n$

   (d) $\frac{a_1 + a_2 + \cdots + a_n}{2}$

   Correct Answer: (b)

6. A particular solution to the differential equation

$$\frac{d^2y}{dx^2} + 4y = 2\tan 2x$$

   is

   (a) $\frac{-1}{2} \sin 2x \ \log(\sec 2x + \tan 2x)$

   (b) $\frac{-1}{2} \cos 2x \ \log(\sec 2x + \tan 2x)$

   (c) $\frac{1}{2} \ log(\sec 2x + \tan 2x)$

   (d) $\frac{1}{2} \sin 2x \ log(\sec 2x + \tan 2x)$

   Correct Answer: (b)

7. The general solution of

$$\frac{dy}{dx} + y \tan x = y^3 \sec x$$

   is

(a) $y^2 = \dfrac{\cos^2 x}{c - 2\sin x}$

(b) $y^2 = \dfrac{\cos x}{c - 2\sin x}$

(c) $y^2 = \dfrac{\sin^2 x}{c - 2\cos x}$

(d) $y^2 = \dfrac{\sin^2 x}{c + 2\cos x}$

Correct Answer: (a)

8. Let $f(x) = e^x$, $g(x) = e^{-x}$, and $h(x) = g(f(x))$, where x is real number. Then $\dfrac{dh}{dx}$ at $x = 0$ is

(a) $\dfrac{1}{e}$

(b) -1

(c) 1

(d) $\dfrac{-1}{e}$

Correct Answer: (d)

9. Let $A(t)$ be the area of the region enclosed by the curve $e^{-|x|}$ and the portion of the x-axis between $x = -t$ and $x = +t$. Then $\lim\limits_{t\to\infty} A(t)$ equals

(a) ∞

(b) 0

(c) 2

(d) does not exist

Correct Answer: (c)

10. Find the maximum volume of a right circular cylinder that can be inscribed in a cone of altitude 12 inch and base radius 4 inch, if the axes of the cylinder and cone coincide.

(a) $\dfrac{64}{9}\pi$

(b) $\dfrac{256}{9}\pi$

(c) $\dfrac{128}{9}\pi$

(d) None of the above

Correct Answer: (b)