Data Structures (IT205)
Midsemester-semester Re-Exam

2^{nd} November, 2011

Time: 2 hours **marks: 60**

1. Consider the a recursive algorithm for sorting an array of n elements as follows. The algorithm sorts the first three-fourths of the array, by a recursive call to itself. It then sorts the last three-fourths of the array resulting from the first recursive call. Then it finally sorts the first three-fourths again.

   SORT(A,1,n)
   1 SORT(A,1,$\lfloor\frac{3n}{4}\rfloor$)
   2 SORT(A,n-$\lfloor\frac{3n}{4}\rfloor, n$)
   3 SORT(A,1,$\lfloor\frac{3n}{4}\rfloor$)

   This algorithm sorts by brute force (no recursive calls) for an array of size less than 5, and is assumed to take constant time, $\theta(1)$ for such a small array.

   Write a recurrece for the running time, and solve this recurrence to get the running time for the algoorithm **SORT** described, as a function of n.

2. Consider a binary minimum heap with 31 **distinct** elements stored in the standard array form in an array $A[1 \ldots 31]$. Consider the element $A[5]$ in the array. What are the possible legal values for its rank?

   [NOTE: The rank of an element in an n element set can take any value in the range $1, \ldots n$, with the minimum element having rank 1, and the maximum element having rank n]

3. Consider two arrays A and B of integers with n elements each.

   It is given that for $2 \leq i \leq n$, if i is even then $A[i] > A[k]$, for $1 \leq k < i$ and if i is odd, then $A[i] < A[k]$ for $1 \leq k < i$.

   For array B, the following informatioon is available. For $2 \leq i \leq n$, if i is odd then $B[i] > B[k]$, for $1 \leq k < i$ and if i is even, then $B[i] < B[k]$ for $1 \leq k < i$.

   Give an optimal algorithm to sort the entire set of $2n$ elements of A and B into a combined array, and analyse the running time of your algorithm.

   [You may use any amount of extra memory as you need]

4. What is the smallest and largest number possible number of nodes in a 2-3-4 tree (B-Tree with parameter t=2) of height 3. In each of these cases, state also the smallest and largest possible number of keys stored in the corresponding trees. Justify your answers by drawing an example for each, and explaining why it is the correct maximum/minimum possible value.