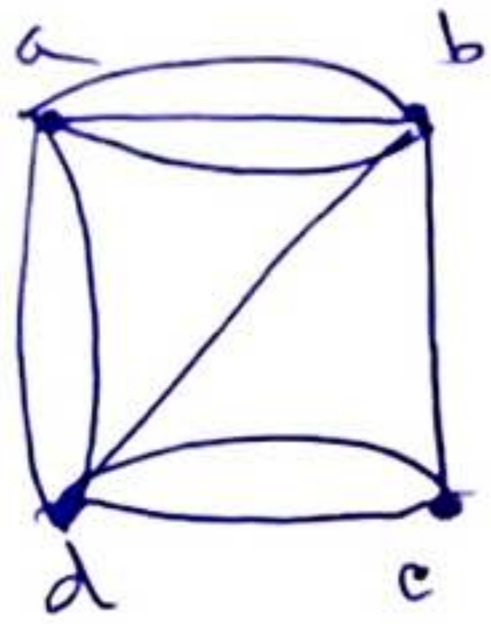


$$A = \begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix}$$

A Pseudograph

Adjacency list Vs Adjacency Matrix

G simple graph
sparse (# of edges less)

Adjacency list
is prefered

G simple graph
dense (# of edges more)

Adj'acency matrix

INCIDENCE MATRICES

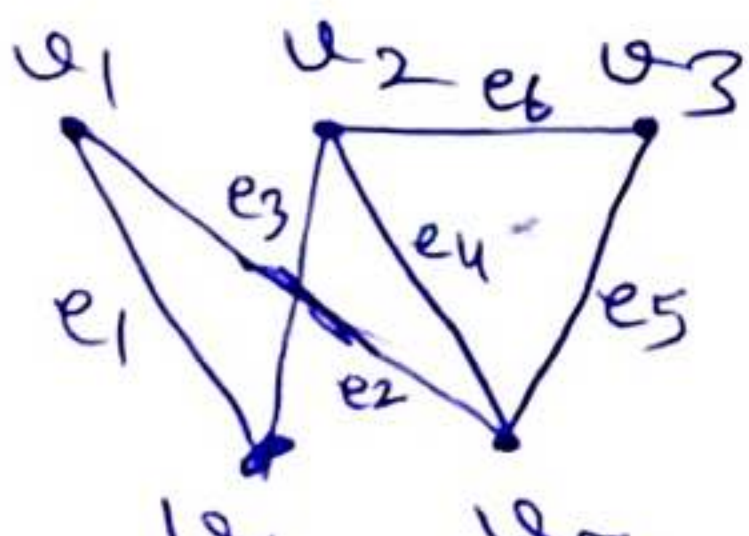
$G = (V, E)$ undirected graph

$V = \{1, 2, \ldots, n\}$ verticles $E = \{e_1, e_2, \ldots, e_m\}$
Edges

$M = (m_{ij})_{n \times m}$ matrix

$$m_{ij} = \begin{cases} 1 & \text{when edge } e_j \text{ is incident with } i \\ 0 & \text{otherwise} \end{cases}$$

Example

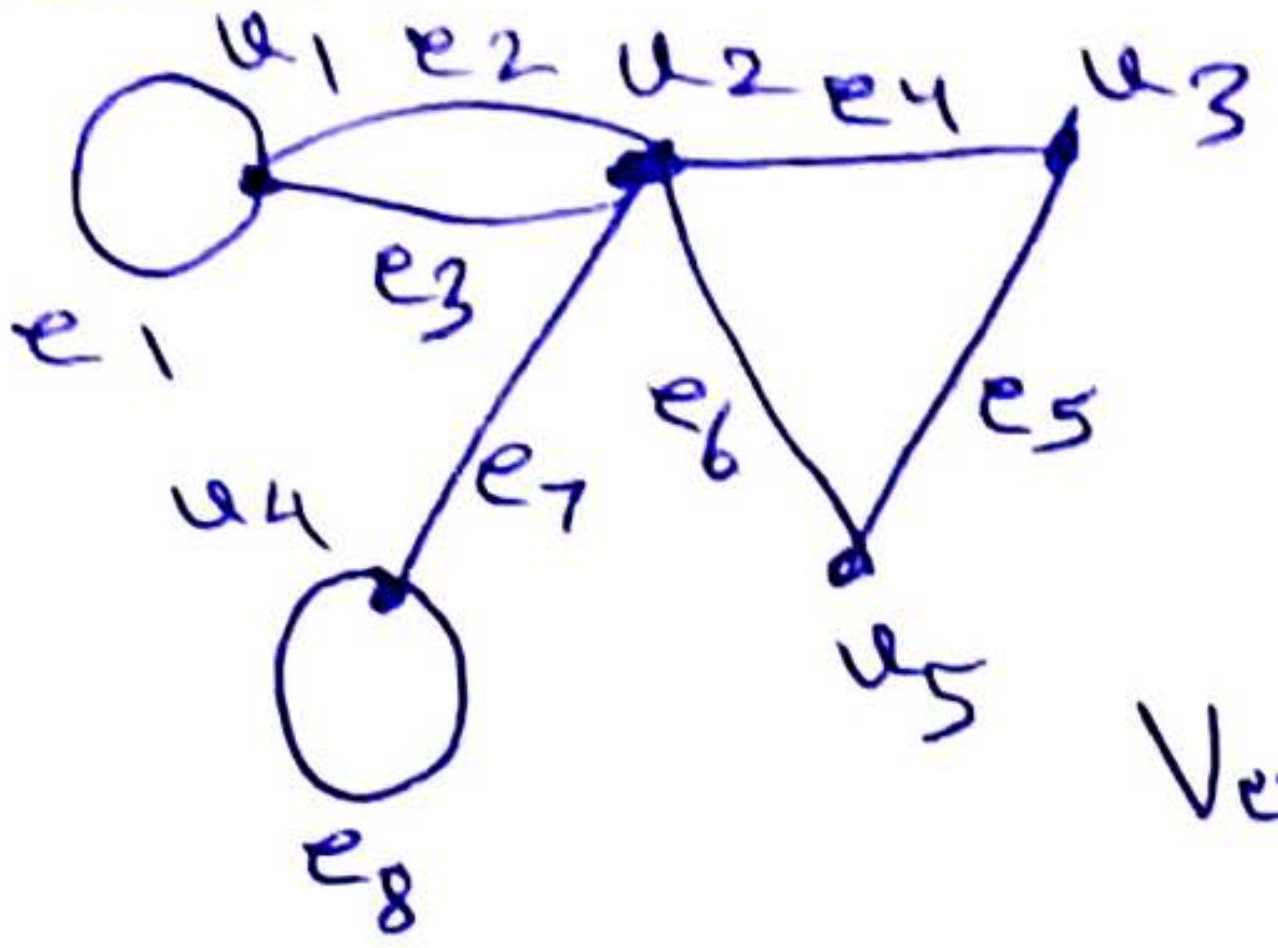


Undirected graph
G

$$\text{Vertices}\quad \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Incidence matrix

Example



$$
\begin{array}{cc}
 & \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\
\text{Vertices} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0
\end{bmatrix}
\end{array}
$$

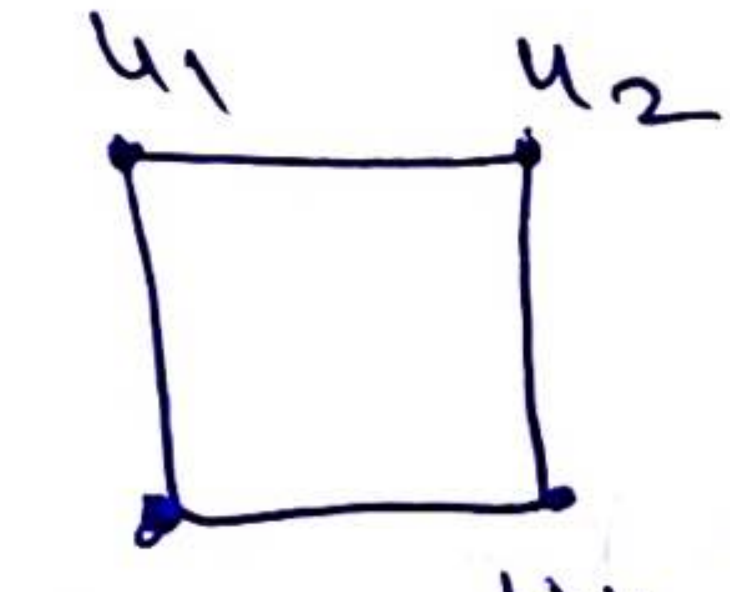
ISOMORPHISM OF GRAPHS

$G_1 = (V_1, E_1)$ simple graph $\qquad$ $G_2 = (V_2, E_2)$ simple graph

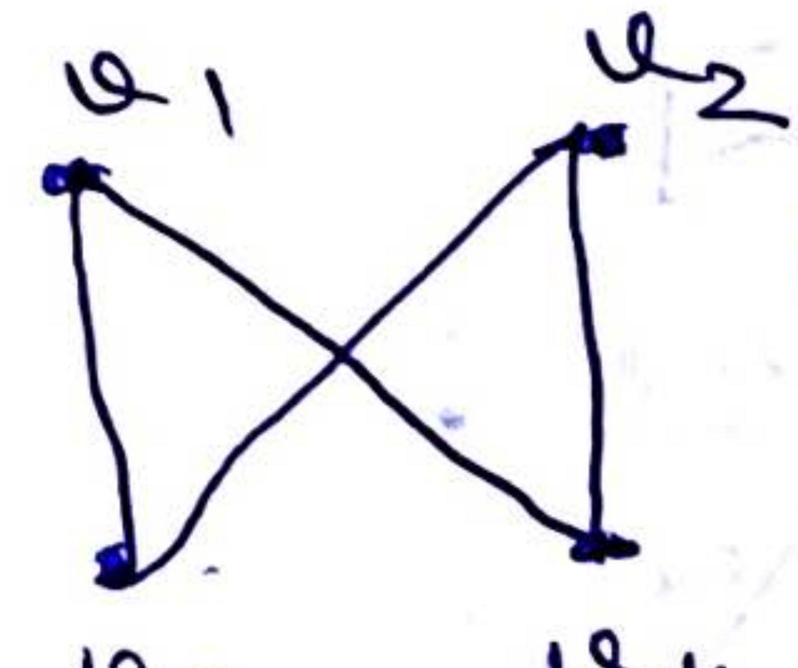
- $G_1 \cong G_2$ (isomorphic) if $\exists$ a 1-1 & onto fn. f from V_1 to V_2 with the property that a & $b \in V_1$ are adjacent in G_1 iff $f(a)$ & $f(b) \in V_2$ are adjacent in G_2 $\forall\, a, b \in V_1$
- f is called isomorphism.
- $\cong$ is an equivalence relation.

Example

$G = (V, E)$ $\qquad$ $H = (W, F)$



$G \cong H$ (isomorphic)

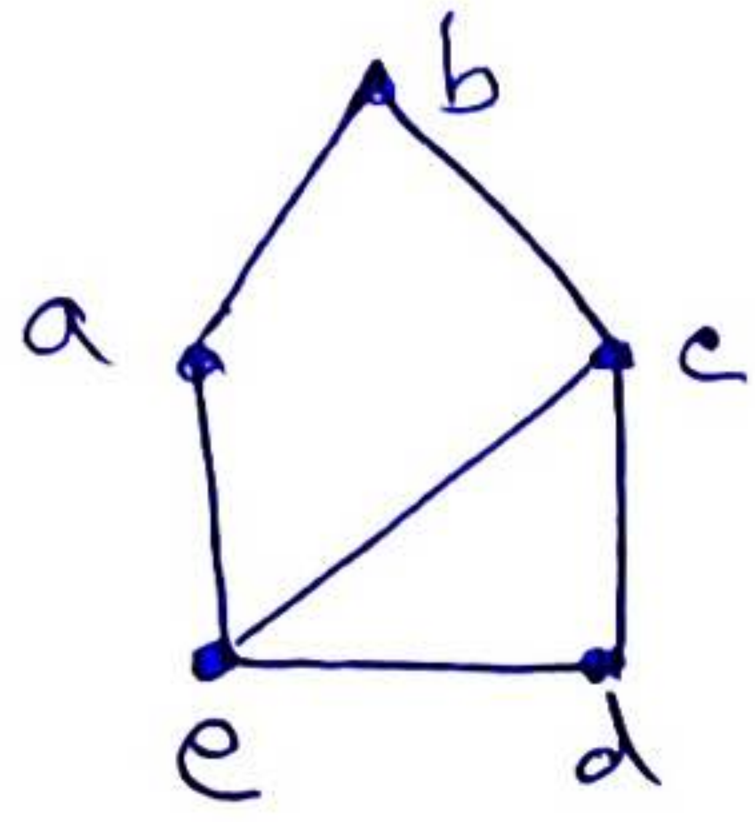


let a fn. f is defined as

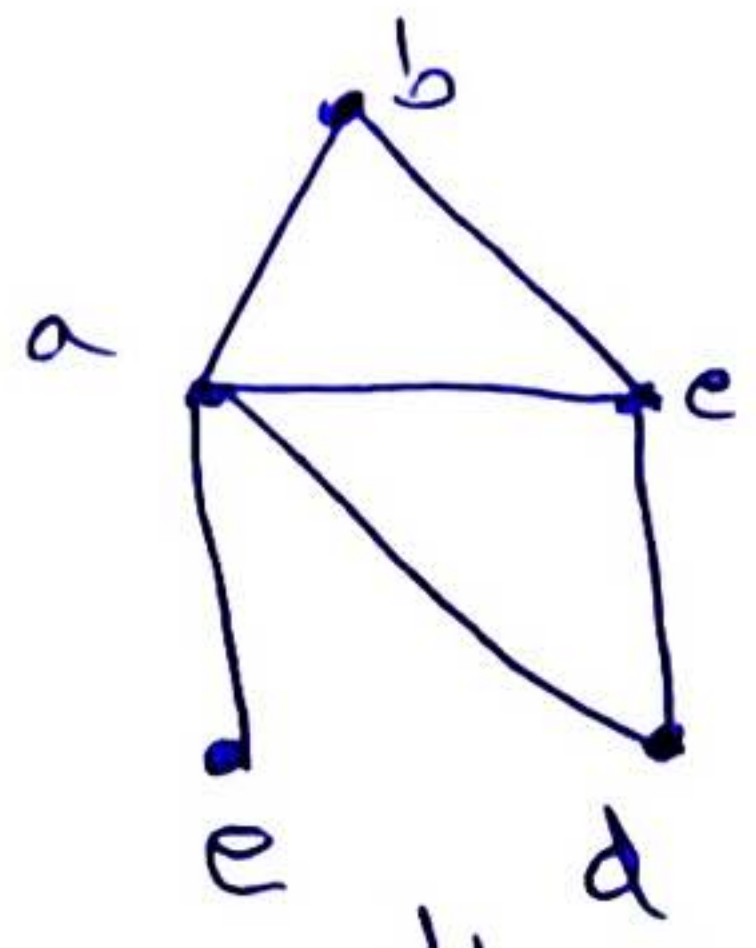
$f(u_1) = v_1 \quad f(u_2) = v_4$ is 1-1 & onto

$f(u_3) = v_3 \quad f(u_4) = v_2$

Example



G



H

$G \not\cong H$
(not isomorphic)

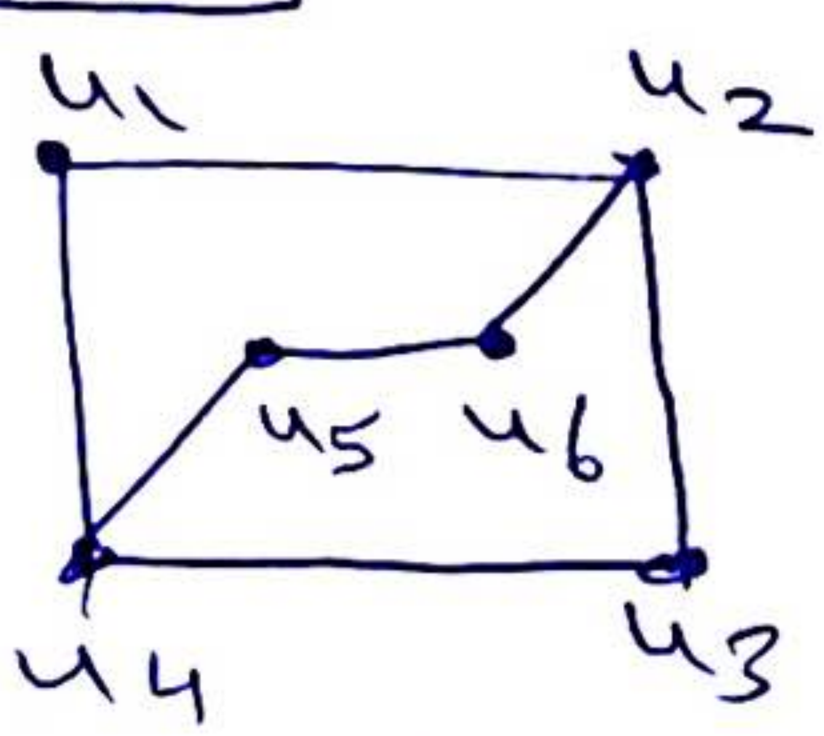
$|V| = 5$
$|E| = 6$ for both G & H

$\deg(e) = 1$ in H whereas G has no vertices of deg 1

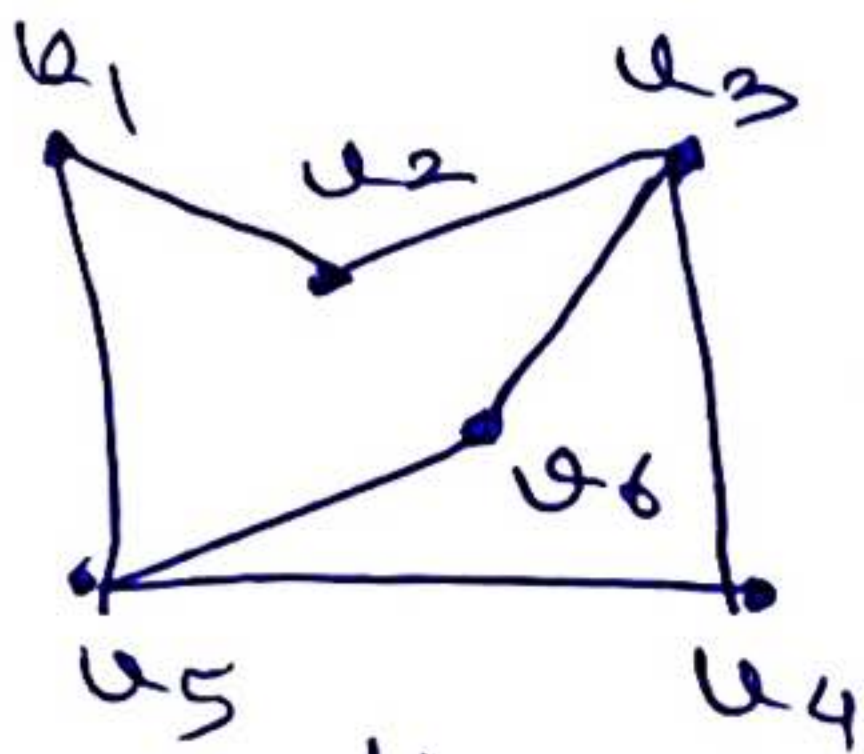
$\Rightarrow G \not\cong H$.

NAUTY is used to check isomorphism.

Example



G



H

$|V| = 6$, $|E| = 7$ for both G & H
deg. seq. is also same for both G & H
$\therefore$ f may exist

$f(u_1) = v_6$ $f(u_2) = v_3$ $f(u_3) = v_4$

$f(u_4) = v_5$ $f(u_5) = v_1$ $f(u_6) = v_2$

Compare adjacency matrices f is 1-1 & onto

PATH for undirected graph G | **CONNECTIVITY**

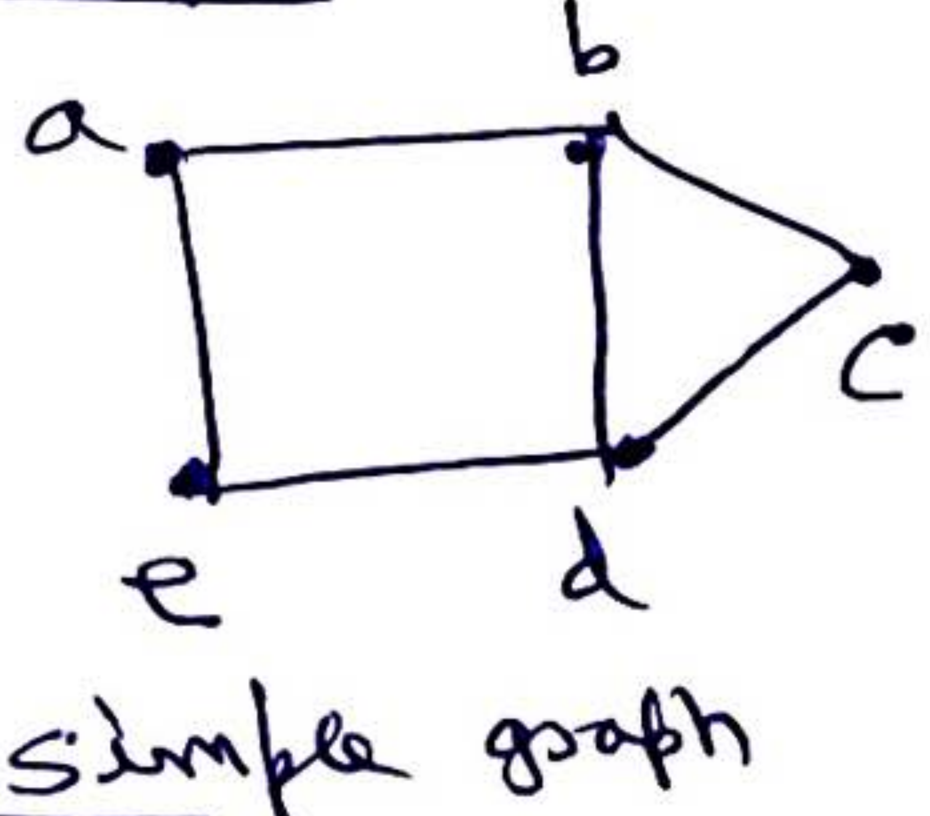
$n \in \mathbb{Z}^+ \cup \{0\}$ G undirected graph

- A path of length n from u to v in G is a seq. of n edges $e_1, e_2, \ldots, e_n$ of G s.t.

e_1 is associated with $\{x_0, x_1\}$ $x_0 = u$
e_2 " " " $\{x_1, x_2\}$ $x_n = v$
...
e_n " " " $\{x_{n-1}, x_n\}$

- If G is simple Path — $x_0, x_1, x_2, \ldots, x_n$
- Path is a circuit if $u = v$ & $n > 0$
- The path or circuit is said to pass through the vertices $x_1, x_2, \ldots, x_{n-1}$ or traverse the edges $e_1, e_2, \ldots, e_n$
- A path or circuit is simple if it does not contain the same edge more than once.
- circuit is also called cycle in G.

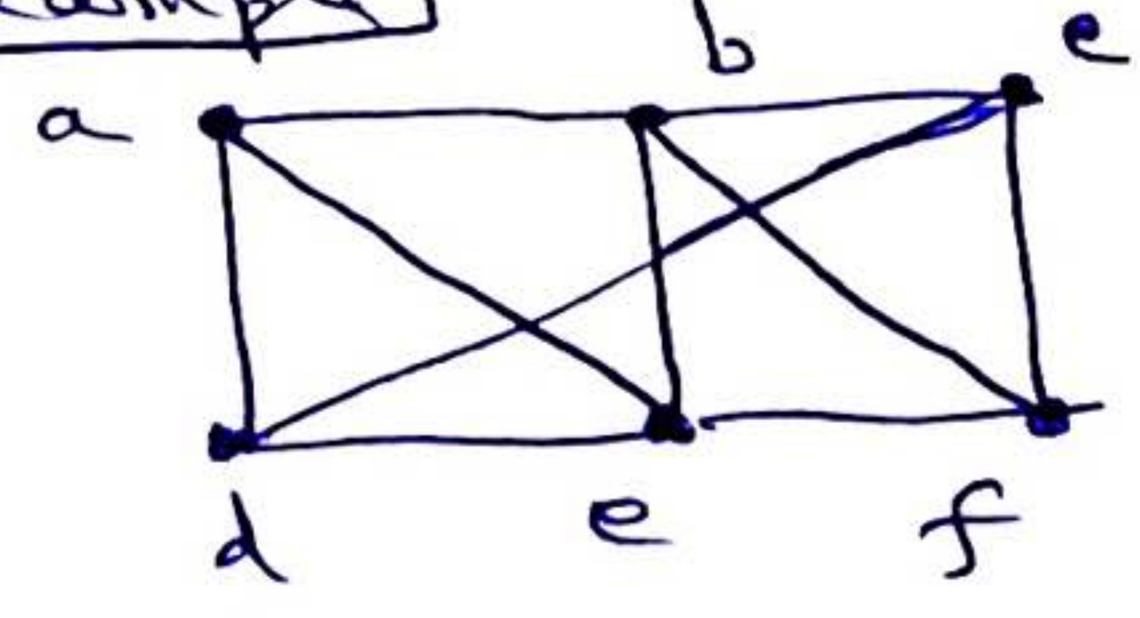
Examples



simple graph

a, b, d, e is cycle
a, b, c, d, e is also a cycle
b, c, d is a cycle

Examples



simple graph

a, d, c, f, e simple path of length 4
d, e, c, a not a path
b, c, f, e, b circuit of length 4
a, b, e, d, a, b path is not simple
4 not as q $\{a, b\}$ twice