Negating Quantified Expressions

Every student in your class has taken a course in calculus

$P(x)$: x has taken a course in calculus

$\forall x\, P(x)$

Negation

It is ~~that~~ not the case that every student in your class has taken a course in calculus

Negating Quantified Expressions

There is a student in your class who has Not taken a course in calculus

$$\boxed{\neg \forall x P x \equiv \exists x \neg P x}$$

Exp 2

There is a student in this class who has taken a course in calculus

$\neg \exists x P x$

It is not the case that there is a student in this class who has taken a course in calculus

Every student in this class has not taken a course in calculus

$$\boxed{\neg \exists x\, P x \equiv \forall x\, \neg P x}$$

$$\boxed{\neg \forall x\, P x \equiv \exists x\, \neg P x}$$

Nested quantifiers

Two quantifiers are nested if one is within the scope of the other

$\forall x \, \exists y \, P(x,y)$ $\qquad$ $\forall x \, \forall y \, \exists z \underbrace{(x+y=z)}_{P(x,y,z)}$

Exp $\qquad P(x,y) : \; x+y=y+x$

$\underbrace{\forall x \, \forall y \, P(x,y)}_{\text{true}}$ $\qquad$ $\underbrace{\forall y \, \forall x \, P(x,y)}_{\text{true}}$

D : set of real number

Exp $\quad Q(x,y): \quad x+y=0$

$\exists y \forall x \, Q(x,y)$ $\qquad \underbrace{\forall x \exists y \, Q(x,y)}_{\text{true}}$

$D = \mathbb{R}$

$\exists x \forall y \; x+y=0$

False

Exp $\quad Q(x,y,z): \underbrace{x+y=z}_{}$

$\forall x \forall y \exists z \, Q(x,y,z)$ true

$\exists z \forall x \forall y \, Q(x,y,z)$ false. $\quad D=\mathbb{R}$

Rule of Inference

Propositional logic

Defn An **argument** is a set of propositions.

$$\left.\begin{array}{l} P_1 \\ P_2 \\ \vdots \\ P_n \end{array}\right\} \text{ Premises}$$

p_1, q_2, p_n

$p \to q$

v v s

$$\frac{\quad}{C} \quad \text{conclusion}$$

Argument form

Argument form

$$\begin{array}{ll} C_1 & r \vee s \\ C_2 & p \vee q \\ \vdots & \vdots \\ C_n & p \to r \\ \hline C & \end{array}$$

$$\begin{array}{c} 5 \\ 2^5 \\ \hline \end{array}$$

$\{C_1, C_2 \dots C_n\} \Rightarrow C$

C logically follows from $C_1, C_2 \dots C_n$

Rule of Inference

p

$p \to q$

$\therefore q$

Modus Pons

$[p \wedge (p \to q)] \to q$ Modus Pons

$\{p, p \to q\} \Rightarrow q$

	p	q	$p \to q$	q
1	T	T	T	T
2	T	F	F	F
3	F	T	T	T
4	F	F	T	F
	T	F	T	F

$\{p, p \to q\} \Rightarrow q$: T, T, T

2^2

n

2^n

$$\begin{array}{l} \neg q \\ p \to q \\ \hline \neg p \end{array}$$ ✓

Modus Tollens

$$\begin{array}{l} p \to q \\ q \to r \\ \hline p \to r \end{array}$$ ✓

Hypothetical syllogism

$$\begin{array}{l} p \lor q \\ \neg p \\ \hline q \end{array}$$ ✓

Disjunctive syllogism

$$\frac{p}{p \vee q} \qquad p \Rightarrow p \vee q$$

$$\frac{p \wedge q}{p} \qquad p \wedge q \Rightarrow p$$

$$\frac{\begin{array}{c} p \\ q \end{array}}{p \wedge q}$$

$$\frac{\begin{array}{c} p \vee q \\ \neg p \vee v \end{array}}{q \vee v}$$

Exp

It is not sunny this after noon and it is colder than yesterdy
We will go swimming only if it is sunny. If we dont go swimmng then we will take a canoe trip. If we take a canoe trip the we will be home .by sunset. Lead to the conclusion 'we will be home by sunset

$\neg p \wedge q$

p: It is sunny this afternn

q: It is colder than yester

$r \to p$

r: We will go swimng.

s:

It is not sunny this afternoon and it is colder than yesterday

Exp

We will go swimming only if it is sunny. If we dont go swimming then we will take a canoe trip. If we take a canoe trip then we will be home by sunset. lead to the conclusion 'we will be home by sunset

$\neg p \wedge q$ p : It is sunny this afternoon

q : It is colder than yesterday

$r \to p$ r : We will go swimming.

s :

t :

$$\{\neg p \wedge q,\ r \to p,\ \neg r \to s,\ s \to t\} \Rightarrow \underline{t}$$

1. $\neg p \wedge q$ $\quad p \quad q \quad r \quad s \quad t \quad \neg p$
2. $\neg p$
3. $r \to p$
4. $\neg r$
5. $\neg r \to s$
6. s
7. $s \to t$
8. $\underline{t}$

$\{\neg p \wedge q,\ r \to p,\ \neg r \to s,\ s \to t\} \models t$

T T T T

	p	q	r	s	t	$\neg p \wedge q$	$r \to p$	$\neg r \to s$	$s \to t$	t
1.	T	T	T	T	T					
2.	T	T	T	T	F	T	T	T	T	T
⋮										
5						T	T	T	T	T
6						T	T	T	T	F
						T	T	T	T	T

2^5

Resolution method

$$\frac{\begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{matrix}}{C}$$

literal p, $\neg p$,

conjunction of literal $p \wedge q \wedge \neg r$
Product

disjunction of literal $p \vee q \vee \vee$
Sum, clause

$\underline{Res(C_1, C_2)} = q \vee r \vee q$
$= \underline{q \vee r}$

C_1^{L}, $C_2^{\neg L}$
clause, clause

$C_1 = \cancel{p} \vee q$

$C_2 = \cancel{\neg p} \vee r \vee q$

Given two clauses C_1 and C_2

a resolution C is a

logical consequence of C_1 and C_2

$$\{C_1, C_2\} \Rightarrow \underbrace{Res(C_1, C_2)}_{C = Res(C_1, C_2)} = \underline{C}$$

Resolution Derivation

$$\frac{\begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{matrix}}{\begin{matrix} C_{11} \\ C_{12} \\ \vdots \\ C_{1n} \end{matrix}} \qquad \frac{\begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_n \\ \neg C \end{matrix}}{\Box}$$

$$\overline{C}$$

$\cancel{\{C_1, C_2, \ldots C_n\} \overset{\Rightarrow}{\vdash} C}$

$C_{11} = Res(C_i, C_j)$

$C_1 = p$

$C_2 = \neg p$

$\Box$

$Res(C_1, C_2) = \Box$

$$\{p,\ p \to q\} \Rightarrow \underline{q}$$

1. $p \equiv p$
2. $\underline{p \to q} \equiv \neg p \lor q$
3. $\neg q \equiv \neg q$

$$\overline{\qquad\qquad\qquad\qquad}$$

q

$\cancel{p},\ \neg\cancel{p} \lor q$

$\{p,\ p \to q\} \Rightarrow q$

1. $p \equiv p$ —
2. $p \to q \equiv \neg p \vee q$ —
3. $\neg q \equiv \neg q$
4. q fm (1), (2)

$[\,]$ fm (3), 4.

$\cancel{p}$, $\neg\cancel{p} \vee q$