1. For the following matrices $A \in \mathbb{R}^{m\times n}$ and vectors $b \in \mathbb{R}^m$, find whether there will be unique, multiple or no solutions.

   (a) $A = \begin{bmatrix} 2 & 3 \\ -10 & -15 \\ 1 & -2 \end{bmatrix}, b = \begin{bmatrix} 3 \\ -15 \\ -2 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 2 & -10 & 1 \\ 3 & -15 & -2 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, b = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$

   (d) $A = \begin{bmatrix} 2 & 3 \\ -1 & -1 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

2. Let $A \in \mathbb{R}^{m\times n}$ be a matrix, and let $b \in \mathbb{R}^m$ be a given vector. Let $y, z \in \mathbb{R}^n$ be two distinct solutions to the equations $Ax = b$. Is it possible that the system of equations $Ax = b_1$, where $b_1 \in \mathbb{R}^m, b_1 \neq b$ has a solution? When it does, is the solution unique?

3. Let $v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}, v_i \in \mathbb{R}, i = 1, \ldots, n$ be any arbitrary non-zero column vector. Let $A = vv^T$, where $v^T = \begin{bmatrix} v_1 & v_2 & \ldots & v_n \end{bmatrix}$ is called the transpose of v. What can you conclude about the solutions to the system of equations $Ax = b$ for any $b \in \mathbb{R}^n$.

4. Let $w = (w_x, w_y, w_z)$ be a fixed but non-zero given vector in $\mathbb{R}^3$.

   (a) For any $v \in \mathbb{R}^3$, let $u = w \times v$ be the *vector cross product*[1] of the vectors w and v. Find the matrix A such that $u = Av$.

   (b) What can you conclude about the solutions to the system of equations $Ax = b$ for any $b \in \mathbb{R}^3$.

[1]Hope you remember this from the Electromagnetic Theory course.