Three and Four Point Bending

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I. Introduction

Bones, a complex yet significant component of any living organism, are rigid organs that provide structural support and mobility. Despite all bones sharing the similarity in possessing a cortical external layer, they exist in a wide range of lengths and thicknesses to accomplish their assigned function. The foci of this experiment are bones which function as a lever mechanism. Normally performed by the long bones, the nature of their responsibility subjects them to various types of bending. Therefore, this experiment aims to determine how these bones, more specifically chicken wing bones, would react in the presence of physiological or traumatic bending stress. To test this thesis of how these rigid structures react, a three-point (3pt) and four-point (4pt) bending experiment will be performed on these specimens.

In this experiment, three different chicken bones, radius, ulna and humerus were utilized as specimens in the 3pt and 4pt bending testing on the CellScale machine. The purpose of the lab was to observe the properties, failure points and ultimate stress of these chicken proximal upper and lower chicken wing segments. Data using Sawbones samples were used to supplement analysis via comparison to chicken bones. Utilizing the obtained 3pt and 4pt bending test data, Young's modulus, moment of inertia (MOI), bending stress and strain and the toughness of the three bone specimens will be calculated and analyzed. These values will allow the understanding of each bone's properties and help determine the specimen that can withstand the most bending stress. Following a closer examination of the three different bone characteristics and functions, it is appropriate to hypothesize that the humerus will possess the highest Young's modulus and experience the most stress out of the three specimens.

II. Methods

A CellScale was used with UniVert testing software to conduct the experiments. Before testing, the length and diameter of the radius, ulna and humerus were measured. The distance between the supports in the CellScale was also noted. A bending profile was initialized, and the system was zeroed before each trial. The 3pt bending test was used for the radius and ulna and the humerus underwent a 4pt bending test. For 3pt bending, the sample was centred on the support and the point load was lowered until contact with the sample. The test was then conducted with an actuator speed of 4 and the results for each test were recorded on an excel sheet. After 3pt testing, the 4pt load was installed and the same procedure was repeated. After the bending, a picture of each bone's cross-section with a calliper measuring the data was taken.

Two sets of data were analyzed; the three different chicken bones and Sawbones. The Sawbones data had one sample in 3pt bending and one in 4pt bending. A linear force vs displacement graph was extrapolated for each test. The slope obtained using linear regression was used to find Young's Modulus and ultimate stress. Further plots were made with stress vs. load, stress vs. strain and radius of curvature vs. displacement. These plots provided insight into bone strength and how Sawbones compared to chicken bones. A force vs. displacement graph was also created for each chicken bone. MOI was obtained via SolidWorks and the ellipse approximation method. Using these values, Young's Modulus and ultimate stress were calculated for each chicken bone sample.

III. Results

A. *Sawbones Analysis*

The force-displacement graph is seen in Fig 1. The linear force-displacement plot with lines of best fit is seen in Fig 2. The MOI, Young's Modulus, ultimate stress and % difference are presented in Table 1. There was no difference between the MOI between 3pt and 4pt bending because the sample cross-section was the same. Sample calculations for MOI are in Appendix A. Young's Modulus and ultimate stress calculations for 3pt are in Appendix B and C respectively; for 4pt, they are in Appendix D and E respectively.

Further analysis of the 3pt data is seen in Fig 3 (Stress vs Load), Fig 4 (Stress vs Strain), and Fig 5 (Radius of Curvature vs Displacement). The Youngs Modulus was calculated using Fig 4 as seen in Appendix G. Using this graph, Youngs Modulus was 14.3 MPa. The ultimate strain and radius of curvature were calculated as seen in Appendix F and H respectively.

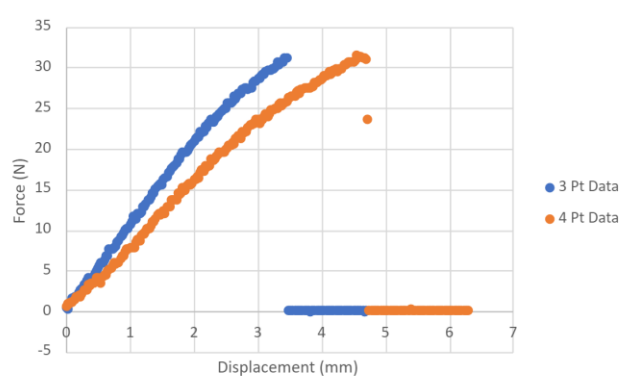

Fig. 1. Force vs. Displacement graph for 3pt and 4pt bending of Sawbones including the nonlinear region.

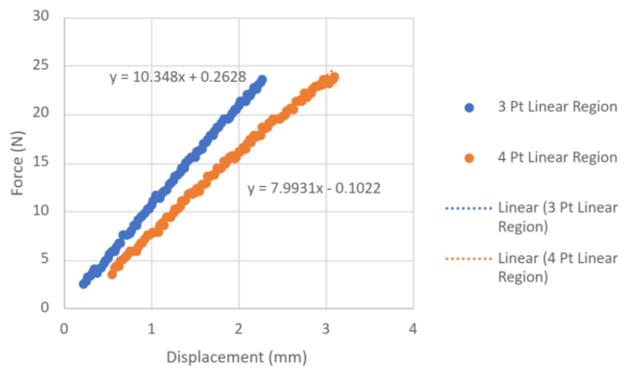

Fig. 2. Force vs. Displacement graph for 3pt bending and 4pt bending of Sawbones with lines of best fit.

TABLE I. MOMENT OF INERTIA, YOUNGS MODULUS AND ULTIMATE STRESS OF RECTANGULAR SAWBONES SAMPLES

	3pt Bending	4pt Bending	% Difference
Moment of Inertia(x) (m^4)	2.06×10^{-9}	2.06×10^{-9}	0
Moment of Inertia(y)(m^4)	6.1×10^{-10}	6.1×10^{-10}	0
Young's Modulus (E) (MPa)	13.9	17.4	
Ultimate Stress (MPa)	2.82	0.915	

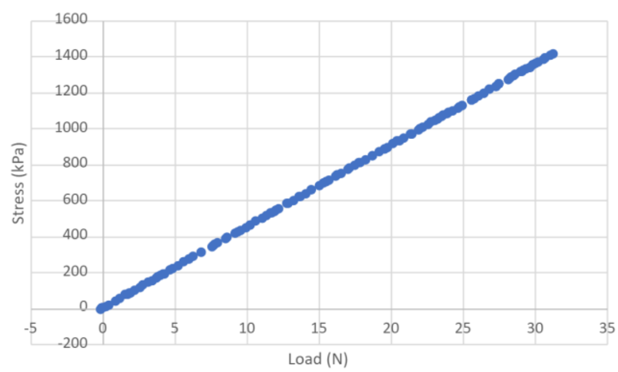

Fig. 3. Stress vs. Load graph of 3pt bending of Sawbones.

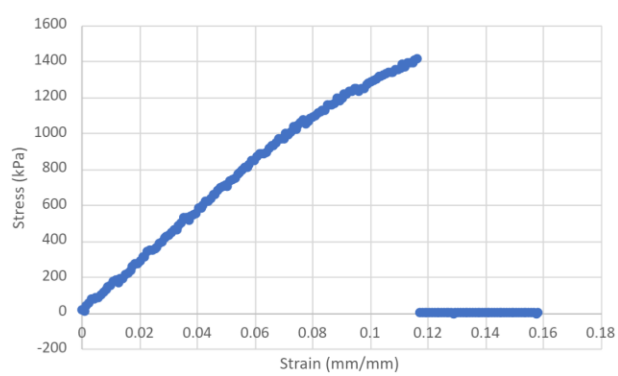

Fig. 4. Stress vs. Strain graph of 3pt bending of Sawbones.

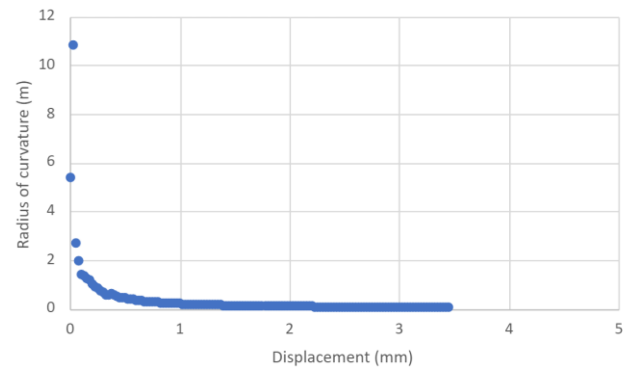

Fig. 5. Radius of Curvature vs. Displacement Graph of 3pt Bending of Sawbones.

B. *Chicken Bone Analysis*

The chicken bone force vs displacement graph is seen in Fig 6 with the linear portion in Fig 7. Appendix I contains the moment of inertia sample calculation using ellipse approximation. Table 2 contains the Young's Modulus and ultimate stress of the samples. Sample calculations are in Appendix J and K respectively. Table 3 tabulates the different moment of inertia methods used and corresponding resultant values.

The selection between the different moments of inertia was based on the axis. For this experiment, the axis of interest was the horizontal axis since the force was applied perpendicular to the horizontal surface. Moreover, the bones were set up in an orientation with the longer axis being horizontal. Therefore, when calculating the MOI using Solidworks, the longer portion was oriented horizontally and set as the L_x axis. Hence, L_x MOI was used when calculating the Young's Modulus of the different specimens.

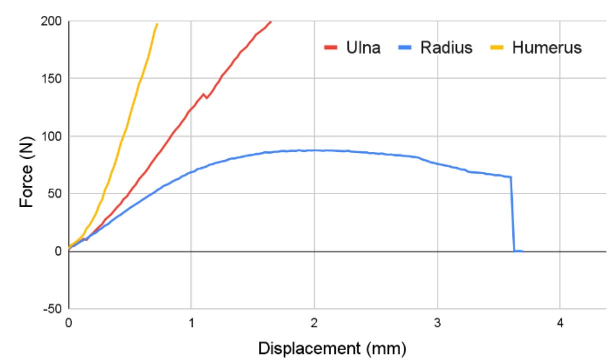

Fig. 6. Force vs. Displacement graph for 3pt bending of chicken radius and ulna, and 4pt bending on the humerus.

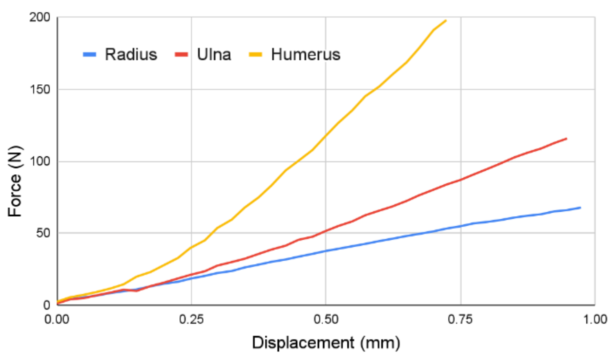

Fig. 7. Linear portion of Fig 6.

TABLE II. YOUNG'S MODULUS AND THE ULTIMATE STRESS OF THE 3 SPECIMENS

Specimen	**Young's Modulus (N/m^2)**	**Ultimate Stress (N/m^2)**
Radius	2.46×10^9	6.38×10^7
Ulna	9.28×10^8	NA
Humerus	4.17×10^8	NA

TABLE III. ASDF

	MOI: Ellipse (mm^4)	**MOI: Image (mm^4)**	**E: Ellipse (N/m^2)**	**E: Image (N/m^2)**	**σu: Ellipse (Pa)**	**σu: Image (Pa)**
Radius	Lx = 12.6 Ly = 15.66	Lx = 15.19 Ly = 19.14	2.96*10^9	2.46*10^9	4.81*10^7	6.38*10^7
Ulna	Lx = 65.53 Ly = 120.3	Lx = 72.21 Ly = 129.32	1.02*10^9	9.28*10^8	NA	NA
Humerus	Lx = 204.6 Ly = 230.3	Lx = 202.29 Ly = 275.58	4.12*10^8	4.17*10^8	NA	NA
% Diff.	**MOI**		**E**		**σu**	
Radius	Lx: 17% and Ly: 18%		20.60%		24%	
Ulna	Lx: 9% and Ly: 7%		10.20%		NA	
Humerus	Lx: 1% and Ly: 16%		1.13%		NA	

IV. Discussion

The hypothesis is not supported by the data; instead, the inverse is expressed. The radius has the highest Young's Modulus while the Humerus has the lowest. The ulna and humerus were both of the same order while the radius was one order higher. However, the humerus may still have the highest ultimate strength due to the limitations of the UniVert system.

The methods used minimized error by using image analysis for the MOI over ellipse approximation. Although useful when software for image analysis is unavailable, the ellipse approximation is not quite accurate compared to image analysis. Not only is the shape of the outer shell approximated, but the bone wall thickness measured can also vary the result as the approximation assumes the same thickness throughout the shape, while it is not the case in real life. The image analysis method allows for calculating MOI through a more precise shape approximation and accounting for different wall thicknesses throughout the shape.

The UniVert system was also flawed. The UniVert system can only exert a maximum force of 200 N. This forces the supports to be set specifically to 45 mm in 4pt bending to break specimens. By optimizing distance, ultimate stress can be reached without having to exert over 200N directly thanks to the 4pt bending relationships. Additionally, the UniVert cannot clamp the sample down when bending, allowing samples to shift as bending occurs skewing results. These factors may have contributed to the results seen in the bone samples.

The effect of 3pt versus 4pt bending also affects the results obtained [2]. 3pt bending concentrates the maximum stress under the point load. 4pt bending exerts stress over an entire area between the two point loads. 3pt loading creates stress concentration and is susceptible to breaking due to existing cracks in the portion directly under the load. In contrast, distributed loading avoids concentrating stress causing premature failure. When testing, a 4pt test is ideal for testing biological materials which may contain small microfractures or tears [3]. 4pt ensures better testing of general bone material over specific areas of the bone which is not representative of how bones are loaded [2]. Additionally, bending creates tension and compression. However, bones do not have the same compressive and tensile strength. Human femurs have 205 MPa ultimate compressive strength versus 135 MPa ultimate tensile strength. This is supported by the experiment. The radius cracked at the bottom side when being bent, indicating the bone failed under tension before compression. If 4pt bending was used on the radius, the load would have been distributed, changing the value of ultimate tensile strength by applying less tensile stress on a specific spot.

In comparison, the Sawbones samples showed similar values between 3pt and 4pt bending. However, the limitation due to bone orientation is shown by how MOI, ultimate stress, bending stiffness and Young's Modulus change if the Sawbone is rotated 90 degrees in 3pt bending [3]. The MOI would decrease if along the y-axis, as seen in Table 1. The ultimate stress would increase since values are divided by MOI. The bending stiffness would also change since there is less resistance to bending, allowing a lower force to create more displacement; this would lower the stiffness. The Young's Modulus would also change since stiffness is used to calculate it. A smaller stiffness results in a smaller young's modulus. Further 3pt vs 4pt bending is observed. Based on the flexure formula, the radius of curvature is inversely proportional to the bending moment [6]. Thus as the bending moment increases towards the center for 3pt testing, the radius of curvature decreases. At the center, the radius of curvature is at a minimum for 3pt testing. A similar relationship applies to 4 points.

Yield strength for chicken bones was not provided. An estimation of tensile yield strength from sawbones, considering bones yield in tension first, is 1200 kPa. This can be taken from the stress-strain curve since the stress values are absolute, indicating the bone undergoes this magnitude in both tension and compression on opposite surfaces of the bone. [4]

Despite the limitations, research on human bones showed the Young's Modulus of the humerus, ulna and radius as 2.35, 4.49 and 3.66 GPa respectively [5]. This mimics the experimental data with the lowest modulus as the chicken humerus and the highest modulus as the radius bone. The radius has a larger modulus than the humerus since it undergoes more bending and movement, thus requiring a larger range of elastic deformation. It also has the smallest cross-sectional area, resulting in higher stress and deformation. This larger deformation requires a larger region of elasticity for ideal biomechanics. In contrast, the humerus has the largest cross-sectional area and undergoes lower stress and deformation, requiring a smaller region of elasticity [5]. The paper cites the ulna with a larger modulus than the radial. However, the ulna used 4pt bending versus the radius' 3pt bending which may have skewed the results. The ultimate strength of a human humerus, ulna and radius was 128,135 and 80 MPa [5]. The radius was the only sample to break, implying it had the lowest ultimate strength, aligning with the research. The ulna and humerus did not have ultimate stress. The final value is stiffness measured using the slope of the force vs displacement graph. The humerus had the highest slope followed by the ulna and then the radius. This shows the humerus is the stiffest bone since it undergoes stress by muscle contractions from the bicep. The radius is the least stiff.

References

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Appendix A

Moment of Inertia Sample Calculation for 3pt and 4pt bending

$$I = \frac{1}{12}(7.91 \times 10^{-3})(14.62 \times 10^{-3})^3$$

Appendix B

Young's Modulus Sample Calculation for 3pt bending

$$\delta_{max} = \frac{PL^3}{48EI}$$

$$s = \frac{P}{\delta_{max}}$$

$$E = \frac{PL^3}{48\delta_{max}I}$$

$$E = \frac{sL^3}{48I}$$

$$E = \frac{(10{,}350)(51 \times 10^{-3})^3}{48(2 \times 10^{-9})}$$

Appendix C

Ultimate Stress Sample Calculation for 3pt bending

$$|\sigma| = \frac{My}{I}$$

$$M = \frac{PL}{4} = \frac{(31.215)(51 \times 10^{-3})}{4} = 0.39799125$$

$$|\sigma| = \frac{(0.39799125)(7.31 \times 10^{-3})}{(2.0 \times 10^{-9})}$$

Appendix D

Youngs Modulus Calculation for 4pt bending

$$\delta = \frac{P(L-a)}{6LEI}\left[\frac{L}{L-a}(x-a)^3 - x^3 + (L^2-(L-a)^2)x\right] + \frac{Pa}{6LEI}\left[\frac{L}{a}\left(x-(L-a)\right)^3 - x^3 + (L^2-a^2)x\right]$$

Taking $x = L/2$ and substituting existing values obtained, the expression reduces to

$$\delta = \frac{P}{E}(2172)$$

$$E = \frac{P}{\delta}(2172) = s(2172) = 17368909\text{Pa}$$

s = slope of the 4pt bending line. Thus the final value is obtained.

Appendix E

Ultimate Stress Calculation for 4pt bending

$$|\sigma| = \frac{M(7.31 \times 10^{-3})}{(2.0 \times 10^{-9})}$$

$$M = \frac{Pa}{2} = \frac{(31.534)(15.875 \times 10^{-3})}{2}$$

$$|\sigma| = \frac{(0.250301125)(7.31 \times 10^{-3})}{(2.0 \times 10^{-9})} = 914850.6119$$

Appendix F

Ultimate Strain Sample Calculation for 3pt Bending of Sawbones

$$|\epsilon| = \frac{12y\delta}{L^2}$$

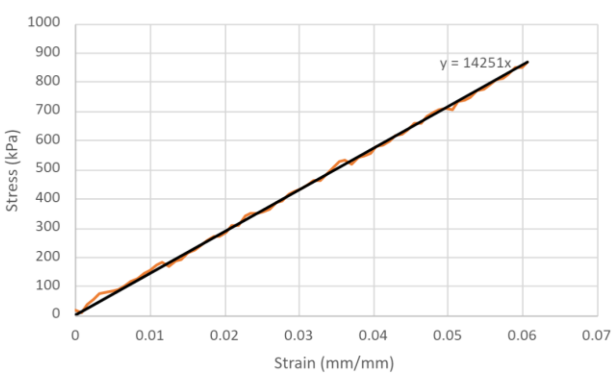
$$|\epsilon| = \frac{12(0.01462mm/2)(0.023mm)}{(51mm)^2}$$

$$|\epsilon| = 0.000775686mm/mm$$

Appendix G

Young's Modulus Calculation for 3pt Bending of Sawbones

Graph of linear portion of stress vs. strain graph with line of best fit is as follows.



Taking the slope of the line of best fit, E = 14251kPa = 14251000Pa.

Appendix H

Radius of Curvature Sample Calculation for 3pt Bending of Sawbones

$$\frac{E}{R} = \frac{\sigma}{y}$$

$$R = \frac{Ey}{\sigma}$$

$$R = \frac{(14251000Pa)(0.01462m/2)}{(53029.5349Pa)}$$

$$R = 1.964467729m$$