Investigation of Cantilever Bending with Strain Gauges

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I. Introduction

When performing material selection for products of any application, it is essential that the chosen materials can withstand relevant external forces during usage. Strain gauges can assess the deformation resistance of materials to these forces by measuring the strain experienced by materials under some applied load. By applying loads that parallel real-world external forces, users can simulate a product's use cases and realistically analyze material deformation. Strain gauges convert applied forces into measurable electrical signals [1], and can be assembled into quarter-, half-, or full-bridge configurations [2]. This lab aims to identify three unknown samples using their densities and strain resistances under three distinct loads. Different materials have different moduli of elasticity, and thus, demonstrate varying levels of deformation when a load is applied. It is hypothesized that all three samples will possess unique moduli of elasticity, ensuring they will be differentiated and subsequently identified. .

II. Materials and Methods

The three samples were individually fastened to three separate strain gauges in quarter-bridge configurations. 50-g, 200-g, and 1-kg masses were then singly hooked onto the samples. Five measurements for the longitudinal and transverse strains were taken for each mass-sample pair. Afterwards, a digital scale and vernier caliper were used to respectively determine the mass and volume of each sample.

III. Results

See Appendix A for the microstrain (με) values measured from the strain gauge and Appendix B for the mass values and dimensions. Density, stress, and the modulus of elasticity were computed for each of the three materials (see Table 1), with sample calculations included in Appendix C, D, and E respectively.

TABLE 1. Density, Stress, and Modulus of Elasticity of Each Sample Calculated Using Strain Gauge

Sample	**Load (g)**	**Density (kg/m^3)**	**Stress (GPa)**	**Modulus of Elasticity (GPa)**
A	50	7210.17761	0.0019805	514.1764222
	200		0.0079222	569.1498659
	1000		0.03961112	222.409451
B	50	7275.81758	0.0017823	48.64162275
	200		0.0071295	124.5653871
	1000		0.0356475	156.0960535
C	50	2550.34712	0.0024189	73.06879668
	200		0.0096758	73.52497972
	1000		0.0483791	71.00783383

IV. Discussion

As hypothesized, the three material samples showed different moduli of elasticity, as displayed in Table 1. In the cantilever bending tests for samples A and B, loads with a smaller mass were found to display different moduli of elasticity compared to larger weights. This can be attributed to the general sensitivity of the quarter-bridge configuration, which possesses half the sensitivity of a half-bridge configuration [3]. In other words, smaller loads resulting in smaller strains were not as easily detected by the load cell. As a result, for the smaller loads, changes in moduli of elasticity greater than 20 percent were neglected; the average of the other moduli were taken instead. The moduli of elasticity of samples A, B, and C were found to be 222.4 GPa, 140.3 GPa, and 72.53 GPa respectively. Comparing the modulus of elasticity of sample A to a table found in [4], A had a modulus of elasticity similar to a carbon-infused steel with a modulus of elasticity of 200 GPa. Sample B had a modulus of elasticity akin to grey cast iron with a modulus of elasticity of 130-157 GPa, as detailed in [5].

Additionally, Sample B exhibited noticeable brown discolourations on its surface (see Fig 1. in Appendix F). These discolourations were hypothesized to be rust forming from the oxidized iron contained within the sample. Sample C had a modulus of elasticity similar to that of aluminum 2014-T6, which was described as 73.1 GPa in [6].

The calculated moduli of elasticity were not equivalent to the moduli of elasticity measured by other sources. This could be on account of the sensitivity of the load cell in the quarter-bridge configuration as mentioned earlier. This could also be due in part to the wobble caused by loads being placed near the ends of the samples. Vibrations caused by the wobble could affect the readings listed by the gauge. The positioning of the sample was also a major factor in affecting the readings of the gauge. As discussed in [2], it is possible that the samples carry anisotropic properties, and thus, varying cuts in the metal could result in different strain values.

In the design space, there were several constraints that had to be met. To ensure the material and wall-thickness combinations were suitable for use in the walker, the maximum strain when a 105-kg patient placed their full weight onto one of the grips was calculated for every combination. Then, using the given safety factor of 2, failure strain was computed. Ultimately, none of the combinations surpassed the failure strain, and thus, proved suitable in the walker. Sample calculations for maximum and failure strain are included in Appendix G and H respectively.

Cost, weight, and stiffness were taken into consideration for determining the optimal combination between material and wall thickness.These values were tabulated in Appendix I.

If an instrumented walker were to be created with one of the materials tested in this lab, sample A with a wall thickness of 0.049 inches, an outer diameter of 1.5 in, and a full bridge gauge configuration would be most useful. Using carbon-infused steel with a wall thickness of 0.049 in allows the handle to resist the force required to achieve the factor safety when a 105-kg load is applied on the handle. It also allows the handles to be as light as possible with a mass of 0.317 kg. Lighter handles decrease the force required for the user to move the walker as they walk. This makes using the walker less tiring. Using larger inner diameters with sample A could also be a viable option in achieving the targeted factor of safety, but it will add additional weight to the walker. Carbon infused steel also has the greatest modulus of elasticity out of the three materials. Therefore, it is less likely to deform after loads are repeatedly applied. The biggest downside to using carbon infused steel is its high melting point of over 1400 ℃ [7]. This makes it more difficult to shape the steel into the needed handle shape.

Other possible configurations include using samples B or C with a wall thickness of 0.049 in or greater. Aluminum is lighter than carbon-infused steel. Using aluminum would make the handles lighter. It also has a lower melting point of 660.5 ℃ [8], allowing for the handles to be easily shaped. However, the cost of aluminum is much higher than the cost of carbon infused steel. Carbon-infused steel costs 0.36 USD per pound [9] while aluminum 2014-T6 costs 1.21 USD per pound [10]. Aluminum also has the lowest modulus elasticity as shown in Appendix I. Therefore, it is the most likely to deform under extreme loads. Grey cast iron could also be used but it is heavier than carbon infused steel. A handle with a wall thickness of 0.049 inches and an outer diameter of 1.5 inches would have a mass of 0.320 kilograms. Grey cast iron also has a higher cost than carbon infused steel. Grey cast iron costs about 0.57 USD per pound [11].

For gauge configuration, a full-bridge configuration would be the most useful. A full-bridge configuration allows for the highest sensitivity [12]. Since this walker is used to monitor how much body weight the user is applying to the device, the device should be able to measure small changes in the weight applied to the handles. A full-bridge or a half-bridge configuration both have higher sensitivity than a quarter-bridge configuration [3]. This enables the device to more accurately measure the changes in the strain experienced by the handles and will therefore give more accurate data.

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Appendix A

Table Containing All Microstrain (me) Values Measured From the Strain Gauges for Each Material

	Strain (με)											
Load	MATERIAL A				MATERIAL B				MATERIAL C			
	Longitudinal		Transverse		Longitudinal		Transverse		Longitudinal		Transverse	
	Raw	Mean	Raw	Mean	Raw	Mean	Raw	Mean	Raw	Mean	Raw	Mean
0g	35.89	34.738	40.8001	40.3881	-77.065	-76.073	NR	349.19	NR	88.5028	-77.058	-76.258
	36.37		40.2701		-76.935		NR		NR		-78.728	
	33.79		40.3301		-78.975		NR		NR		-76.848	
	34.52		39.8901		-74.395		NR		NR		-74.328	
	33.12		40.6501		-72.995		NR		NR		-74.328	
50g	31.5101	30.8861	42.3641	41.8081	-118.71	-112.716	354.01	356.28	113.878	121.608	-64.671	-65.169
	29.0401		41.9841		-117.95		347.28		119.288		-60.591	
	33.1701		42.7741		-107.57		357.53		123.828		-71.291	
	29.9101		41.2041		-105.32		376.08		125.608		-69.471	
	30.8001		40.7141		-114.03		346.5		125.438		-59.821	
200g	23.5006	20.8186	42.7541	43.0321	-130.62	-133.308	343.74	349.91	213.598	220.102	-34.081	-31.877
	19.7806		42.3941		-126.91		368.45		219.668		-26.861	
	21.0706		43.9641		-130.49		346.52		221.498		-31.931	
	20.5406		42.9541		-144.57		334.33		225.178		-33.101	
	19.2006		43.0941		-133.95		356.51		220.568		-33.411	
1000g	-148.9	-143.362	33.1701	32.5617	-300.09	-304.442	NR	303.732	NR	769.824	133.268	139.1972
	-147.03		29.0401		-304.9		NR		NR		144.238	
	-146.3		31.5101		-305.92		NR		NR		139.548	
	-139.35		34.6541		-309.76		NR		NR		135.001	
	-135.23		34.4341		-301.54		NR		NR		143.931	
50g-0g	-3.8519		1.42		-36.643		7.09		33.1052		11.089	
200g-0g	-13.9194		2.644		-57.235		0.72		131.5992		44.381	
1000g-0g	-178.1		-7.8264		-228.369		-45.458		681.3212		215.4552	

Appendix B

The Dimensions and Mass of the Samples Given

Sample	Mass (kg)	Length (m)	Width (m)	Height (m)
A	0.112365	0.2032	0.02474	0.0031
B	0.1213	0.2032	0.02548	0.00322
C	0.031525	0.2032	0.01901	0.0032

Appendix C

Sample Density Calculation (Material A)

Finding Cross-Sectional Area:

$$A = dh$$

$$= (0.02474m)(0.0031m)$$

$$= 0.000076694m^2$$

Finding Volume:

$$V = Al$$

$$= (0.000076694m^2)(0.2032m)$$

$$= 1.55842 \times 10^{-15}m^3$$

Finding Density:

$$\rho = \frac{M}{V}$$

$$= \frac{0.112365kg}{1.55842 \times 10^{-15}m^3}$$

$$= \frac{7210.177618kg}{m^3}$$

Appendix D

Sample Stress Calculation (Material C, 1kg Load)

Finding Moment of Inertia (I_x):

$$I_x = \frac{1}{12}dh^3$$

$$= \frac{1}{12}(0.01901m)(0.0032m)^3$$

$$= 5.191 \times 10^{-11}m^4$$

Finding Bending Moment:

$$M = FL$$

$$= (9.81N)(0.16m)$$

$$= 1.5696Nm$$

Finding Stress:

$$\sigma = \frac{My}{I_x}$$

$$= \frac{(1.5696Nm)(0.0016m)}{5.191 \times 10^{-11}m^4}$$

$$= 0.048379143GPa$$

Appendix E

Sample Modulus of Elasticity Calculation (Material B, 1kg Load)

Absolute value of strain taken as symmetry in the geometry of samples make compression = tension.

$$E = \frac{\sigma}{\varepsilon}$$

$$= \frac{0.0356475GPa}{0.000228369\varepsilon}$$

$$= 156.0960535GPa$$

Appendix F

Fig. 1. Sample A, B, and C used in the strain gauge tests.

Appendix G

Sample Max Strain Calculation (Carbon Infused Steel, Wall Thickness 0.049")

Finding inner diameter (I.D.):

$$I.D. = O.D. - \frac{Wall\ Thickness}{2}$$

$$= 0.0381m - \frac{0.0012446m}{2}$$

$$= 0.0356108m$$

Finding cross-sectional area:

$$A = \frac{\pi}{4}(O.D.^2 - I.D.^2)$$

$$= \frac{\pi}{4}((0.0381m)^2 - (0.0356108m)^2)$$

$$= 0.000144106m^2$$

Finding distance from neutral axis to centroid of upper half of cross-section (for maximum value of Q) [13]:

$$\bar{y}' = \frac{4(O.D.^3 - I.D.^3)}{3\pi(O.D.^2 - I.D.^2)}$$

$$= \frac{4((0.0381m)^3 - (0.0356108m)^3)}{3\pi((0.0381m)^2 - (0.0356108m)^2)}$$

$$= 0.023471795m$$

Finding Q:

$$Q = A'\bar{y}'$$

$$= \frac{0.000144106m^2}{2}(0.023471795m)$$

$$= 1.69121 \times 10^{-6}m^3$$

Finding Moment of Inertia (I_x) [14]:

$$I_x = \frac{\pi}{64}(O.D.^4 - I.D.^4)$$

$$= \frac{\pi}{64}((0.0381m)^4 - (0.0356108m)^4)$$

$$= 2.4495 \times 10^{-8}m^4$$

Finding max stress applied:

$$\tau = \frac{VQ}{I_x t}$$

$$= \frac{(1030.05N)(1.69121 \times 10^{-6}m^3)}{(2.4495 \times 10^{-8}m^4)(0.0024892m)}$$

$$= 0.028569824GPa$$

Finding maximum strain:

$$\varepsilon = \frac{\tau}{E}$$

$$\frac{(0.028569824GPa)}{(200GPa)}$$

$$= 142.8491218\mu\varepsilon$$

Appendix H

Sample Failure Strain Calculation (Carbon Infused Steel, Wall Thickness 0.049")

$$\varepsilon_{failure} = FS\varepsilon_{maximum}$$

$$= 2(142.8491218\mu\varepsilon)$$

$$= 285.6982436\mu\varepsilon$$

Appendix I

Table Containing Stiffness, Mass, and Cost of Each Combination of Material/Wall Thickness Along with Max Strain and Failure Strain Values

	E (GPa)	Wall Thickness (")	Max Strain (με)	Failure Strain (με)	Mass (kg)	Cost ($USD)
Carbon Infused Steel (A)	200	0.049	142.8491218	285.6982436	0.3166954	0.2537931
		0.058	121.3975285	242.795057	0.3725388	0.2985449
		0.065	108.8209491	217.6418982	0.4154737	0.332952
		0.083	86.22475192	172.4495038	0.5238732	0.4198212
		0.125	58.81371101	117.627422	0.7655806	0.6135205
		0.25	31.69129339	63.38258679	1.3919648	1.1154919
Grey Cast Iron (B)	143.5	0.049	199.0928527	398.1857054	0.3195785	0.401593
		0.058	169.1951616	338.3903233	0.3759303	0.4724066
		0.065	151.666828	303.3336561	0.419256	0.5268511
		0.083	120.1738703	240.3477405	0.5286424	0.6643097
		0.125	81.97032894	163.9406579	0.7725503	0.9708125
		0.25	44.16905003	88.33810005	1.404637	1.7651137
Aluminum 2014-T6 (C)	73.1	0.049	390.8320706	781.6641412	0.1120199	0.2988231
		0.058	332.1409808	664.2819616	0.1317725	0.3515151
		0.065	297.7317349	595.4634699	0.1469592	0.392027
		0.083	235.909034	471.818068	0.1853017	0.4943091
		0.125	160.913026	321.826052	0.2707973	0.7223761
		0.25	86.70668507	173.4133701	0.4923587	1.3134112