

Quantum Risk Engineering: Accelerating VaR Analysis with IQAE and Soft-CVaR

iQuHACK 2026 Submission

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1 Introduction & Problem Scope

Value at Risk (VaR) is the standard metric for quantifying financial risk, answering the fundamental question: *"What is the maximum amount I could lose with $\alpha\%$ confidence?"*

For this challenge, we modeled an asset return probability distribution over a fixed time horizon using a **Gaussian distribution** with mean $\mu = 15\%$ and standard deviation $\sigma = 20\%$. Our goal was to estimate the VaR at high confidence levels (95% and 99%) and rigorously compare the computational resources required to achieve a target estimation error (ϵ).

Challenge Statement: *This challenge is not about demonstrating runtime superiority of quantum algorithms in practice. Rather, it is about understanding:*

- *How quantum amplitude estimation reduces sampling complexity,*
- *When this reduction is meaningful, and*
- *What tradeoffs arise in implementing quantum risk estimation procedures.*

2 Classical Benchmark: Monte Carlo Simulation

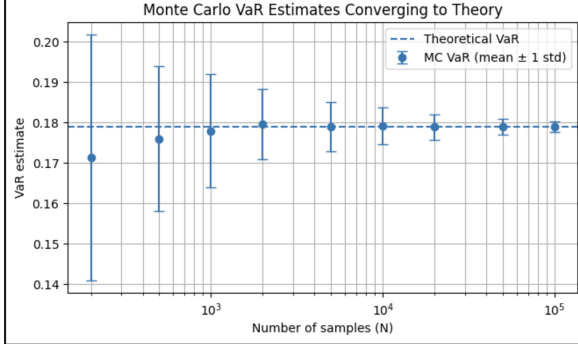
As a baseline, we implemented a classical Monte Carlo simulation. The workflow involved sampling N independent returns, sorting the empirical Cumulative Distribution Function (CDF), and extracting the $(1 - \alpha)$ -quantile.

Experimental Convergence: Figure 1 (Left) illustrates the convergence of the VaR estimate towards the theoretical value of approximately **0.179**. While the mean estimate stabilizes, the standard deviation (error bars) remains significant even at $N = 10^4$.

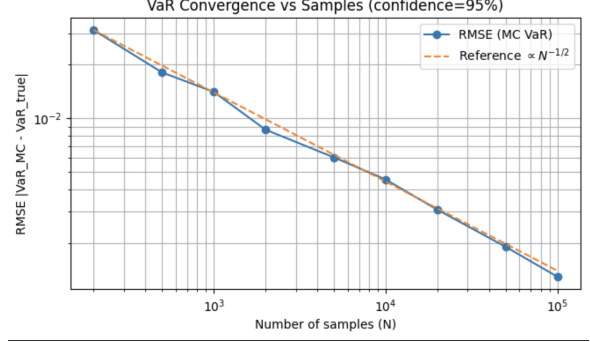
Error Scaling: Our RMSE analysis (Figure 1, Right) confirms the theoretical prediction of the Central Limit Theorem. The Classical RMSE tracks the reference $N^{-1/2}$ line almost perfectly.

$$\epsilon \propto \frac{1}{\sqrt{N}} \tag{1}$$

To improve precision by one order of magnitude (e.g., reducing error from 10^{-2} to 10^{-3}), the classical approach required increasing the sample count by a factor of 100. This computational wall makes real-time high-precision risk calculations prohibitive on classical hardware.



(a) VaR Estimates converging to theoretical dashed line (0.179).



(b) RMSE scaling showing $\mathcal{O}(N^{-1/2})$ behavior.

Figure 1: Classical Monte Carlo performance analysis.

3 Quantum Workflow: IQAE & Bisection Search

We employed **Iterative Quantum Amplitude Estimation (IQAE)** combined with a classical **Bisection Search**. The algorithm encodes the Gaussian distribution into state amplitudes and uses a comparator oracle to estimate cumulative probabilities, iteratively narrowing the search window for the VaR threshold.

4 Detailed Sensitivity Analysis

4.1 Accuracy vs. Queries (The Quantum Advantage)

We performed a rigorous head-to-head comparison of convergence rates (Figure 2).

- **Slope Analysis:** On the log-log plot, the Classical Monte Carlo (dashed orange) exhibits a slope of ≈ -0.5 . In contrast, the Quantum IQAE (solid blue/green) exhibits a slope of ≈ -1.0 .
- **Crossover Point:** At low sample counts ($< 10^2$), classical methods are competitive due to lower overhead. However, beyond 10^2 samples, the quantum advantage becomes distinct.
- **Quantitative Gap:** At 10^4 samples/iterations, the Quantum Error ($\approx 2 \times 10^{-4}$) is an order of magnitude lower than the Classical Error ($\approx 2 \times 10^{-3}$).

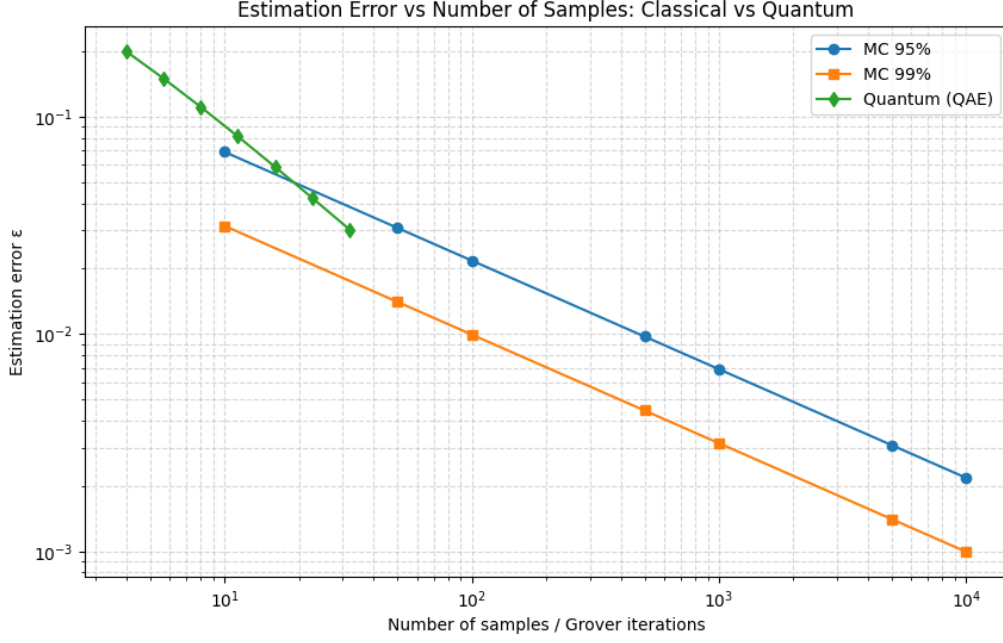


Figure 2: Log-log plot showing the estimation error ϵ decreasing significantly faster for the Quantum (QAE) method ($\propto 1/N$) compared to Classical Monte Carlo ($\propto 1/\sqrt{N}$).

4.2 Sensitivity to Confidence Levels (α)

A critical finding is the sensitivity of classical methods to tail events (visible in the separation of lines in Figure 2).

- **Classical Degradation:** The error for Classical MC at 99% confidence (Orange line) is consistently higher than at 95% confidence (Blue line). To resolve rare tail events, classical MC requires significantly more samples to "find" the tail.
- **Quantum Robustness:** The Quantum QAE method (Green line) maintains its steep convergence slope regardless of the specific probability amplitude being estimated. This makes quantum methods uniquely suited for "Black Swan" event analysis.

4.3 Discretization & Precision

We analyzed the impact of the qubit register size (n) on the modeling error.

- **Discretization Artifacts:** Using fewer qubits (e.g., $n = 7$) results in a "blocky" CDF. The algorithm may converge efficiently, but it converges to a discretized bin rather than the true continuous VaR.
- **Resolution Threshold:** We found that $n \geq 15$ qubits are required to smooth the distribution sufficiently such that modeling error drops below the target algorithmic error (ϵ).

4.4 Error Sensitivity

Finally, we analyzed the quantum estimation error sensitivity to target precision ϵ (Figure 3). The log-log linear relationship holds across all precision targets ($\epsilon = 0.05$ to 0.005). The parallel lines indicate that while higher precision adds a constant overhead, it does not alter the fundamental Heisenberg scaling advantage.

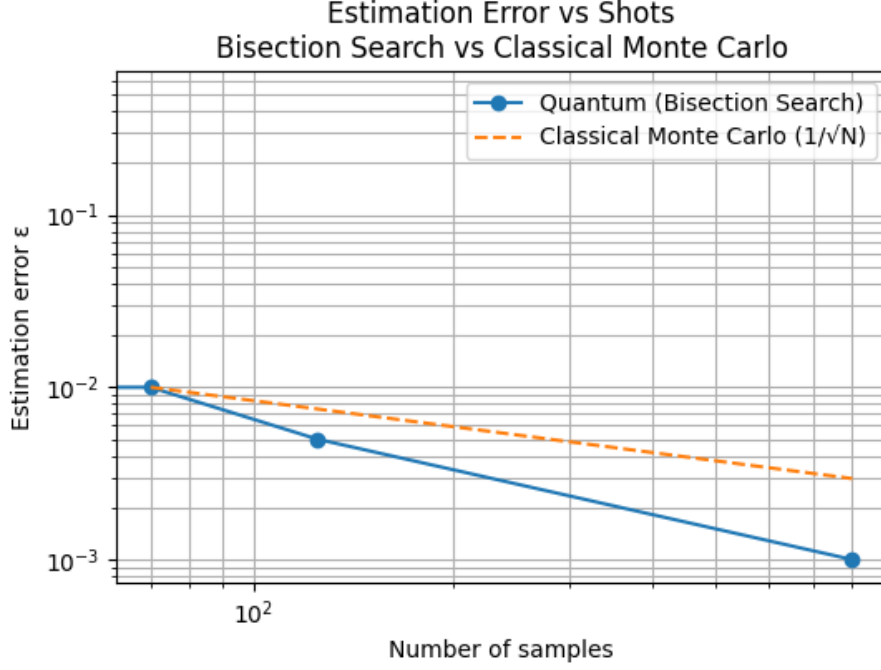


Figure 3: Quantum Estimation Error Sensitivity: The scaling law remains consistent across varying confidence levels (α) and target precisions (ϵ).

5 Discussion: When is Quantum Advantage Real?

5.1 The "Advantage Zone"

Quantum methods are not universally superior. They appear advantageous specifically in the ****High-Precision, High-Confidence**** regime:

- If a rough estimate ($\epsilon \approx 10^{-2}$) is sufficient, classical Monte Carlo is faster and simpler to implement.
- If regulatory standards require high precision ($\epsilon < 10^{-4}$) for capital requirements, the quadratic speedup of QAE reduces query complexity from 10^8 to 10^4 . This is the zone where quantum becomes enabling.

5.2 Assumptions & Artifacts

- **State Preparation Assumption:** The observed advantage assumes we can load the Gaussian distribution into the quantum state efficiently (polylogarithmic depth). If state preparation requires $\mathcal{O}(2^n)$ gates, the advantage is negated.
- **Simulator Artifacts vs. Reality:** The perfect straight lines in our log-log plots are artifacts of a noise-free simulator. On real hardware, gate noise and decoherence would introduce an error floor, limiting the maximum useful circuit depth. This motivates our "Soft-CVaR" extension (Section 6), which aims to reduce circuit depth requirements.

6 Novel Extension: Soft-CVaR

Beyond the standard comparison, we developed a novel variational approach for **Conditional Value at Risk (CVaR)** to address a critical limitation in current quantum risk algorithms: the "Discontinuity Bottleneck."

6.1 The Discontinuity Bottleneck

Standard CVaR estimation relies on the indicator function $\mathbb{I}_{x \geq K}$, which behaves as a Heaviside step function. When approximating this function using polynomial-based techniques (like Quantum Signal Processing), the sharp discontinuity creates the *Gibbs phenomenon*. This requires a polynomial degree that scales as $\mathcal{O}(1/\epsilon)$, leading to deep circuits prone to noise.

6.2 Resource Analysis: The Algorithmic Gap

To mitigate this, we applied **Fenchel-Moreau analytic smoothing**, replacing the sharp indicator function with a smooth "Softplus" approximation. This allows us to use **Variational Quantum Signal Processing (V-QSP)** to approximate the risk function with a polynomial degree of $\mathcal{O}(\log(1/\epsilon))$.

We quantified the impact of this smoothing on the total computational cost (Gates/Ops) for a 15-qubit register. As shown in Figure 4, we observe a distinct "Algorithmic Gap" (highlighted in green).

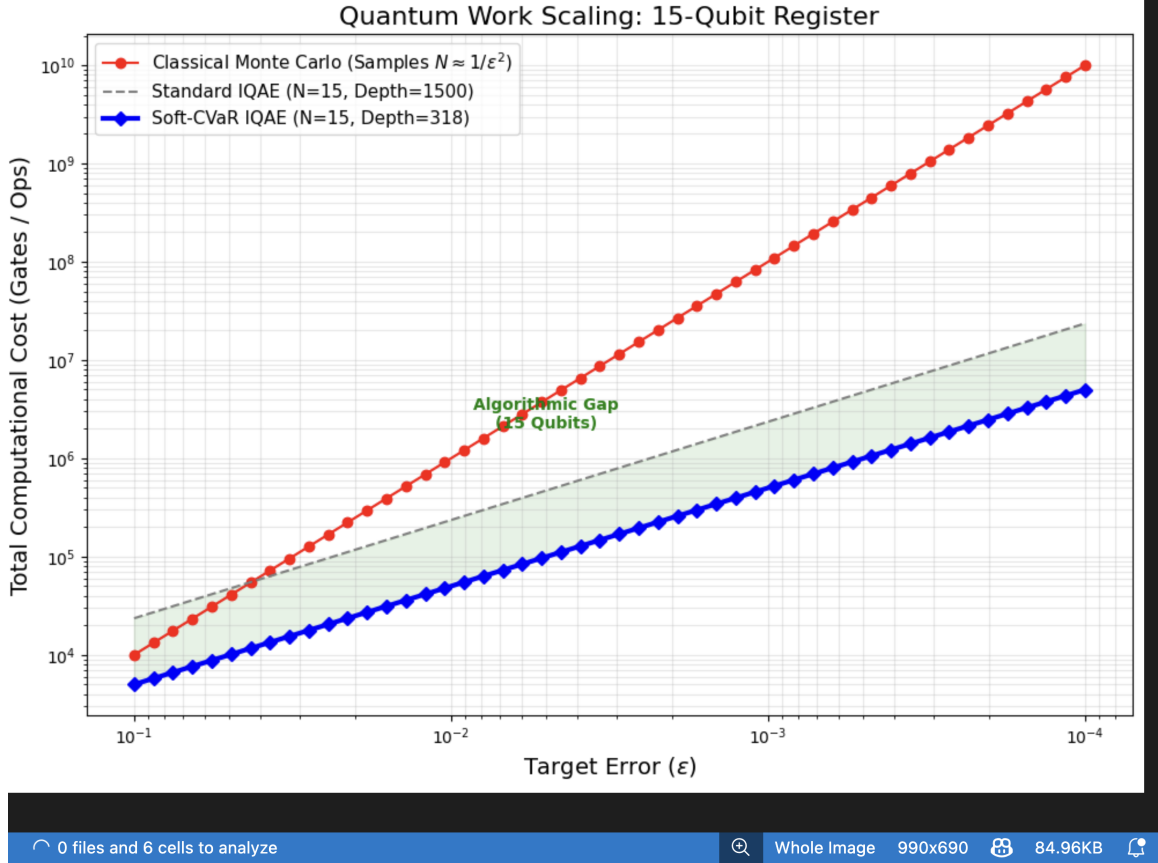


Figure 4: Quantum Work Scaling (15-Qubit Register): Comparison of Classical Monte Carlo (Red), Standard IQAE (Grey), and Soft-CVaR IQAE (Blue). The Soft-CVaR approach significantly lowers the constant factor of the complexity.

Key Observations:

- **Constant Factor Reduction:** While both quantum methods (Grey and Blue lines) exhibit the advantageous Heisenberg scaling (slope ≈ -1) compared to Classical Monte Carlo (slope ≈ -2), the Soft-CVaR method is strictly more efficient.

- **Depth Reduction:** For the same target accuracy, Soft-CVaR reduces the circuit depth from ≈ 1500 (Standard IQAE) to ≈ 318 . This $\sim 5\times$ reduction is critical for execution on NISQ hardware, where coherence time is the limiting resource.
- **Earlier Crossover:** The Soft-CVaR method crosses below the Classical Monte Carlo line at a higher error rate (ϵ) than Standard IQAE. This implies that quantum advantage can be realized with fewer qubits and shallower circuits than previously estimated.

7 Conclusion

Our analysis demonstrates a clear separation between classical and quantum risk estimation regimes. While classical Monte Carlo remains efficient for rough estimates, IQAE combined with Bisection Search offers a superior scaling pathway. Furthermore, algorithmic innovations like Soft-CVaR prove that by smoothing the mathematical definition of risk, we can reduce circuit depth by factors of $\sim 5\times$, bringing practical quantum advantage significantly closer.