11.1 Sequences

$\{a_1, a_2, a_3, \dots\} = a_n{}_{1}^{\infty}$

n usually starts @1

Ex: Write the sequence in a formula

$\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots\} \quad a_n = \boxed{\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}}$

Ex 2: $\{-\frac{2}{3}, \frac{3}{9}, -\frac{4}{27}, \frac{5}{81}, \dots\}$

$$= \frac{n+1}{3^n} \underbrace{(-1)^n}_{\text{do this for alternating}}$$

Ex 3: $\{\frac{3}{5}, \frac{-4}{25}, \frac{5}{125}, \frac{-6}{625}, \dots\}$

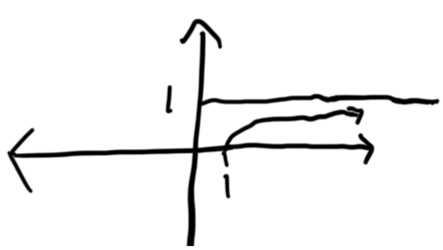
$$\frac{2+n}{5^n} (-1)^{n+1} \quad \text{or} \quad (-1)^{n-1}$$

☆ Doing this so we can offset − sign, in ex 2 it started with −, but here it starts with + so we want to offset it

Ex 4: $\left\{\frac{n}{n+1}\right\}_1^{\infty} = \boxed{a_n = \frac{n}{n+1}}$

If limit exists, it is $\boxed{\text{Convergent}}$

n=1 0.5
n=2 0.66
n=3 0.75
n=4 0.8
n=5 0.83



$n=5$ 0.83
$n=10$ 0.90

↓

Def: A seq $\{a_n\}$ has a limit L $\lim_{n\to\infty} a_n = L$

If $\lim_{n\to\infty}$ exist, seq is convergent, otherwise, it is divergent

Ex 5:

$a_n = (-1)^n$ This is divergent, oscillating between -1 and 1.

Limit Laws

$\{a_n\}$ and $\{b_n\}$ are convergent, a sequence, and C is a constant

1) $\lim_{n\to\infty} (a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$

2) $\lim_{n\to\infty} (a_n - b_n) = \lim_{n\to\infty} a_n - \lim_{n\to\infty} b_n$

3) $\lim_{n\to\infty} c\, a_n = c \lim_{n\to\infty} a_n$

4) $\lim_{n\to\infty} (a_n \cdot b_n) = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n$

5) $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n}$

6) $\lim_{n\to\infty} [a_n]^p = [\lim_{n\to\infty} a_n]^p$

$n \to \infty$ $\quad$ $n \to \infty$

Ex: Find the limit $\lim\limits_{n\to\infty} \dfrac{n/n}{\frac{n}{n}+\frac{1}{n}}$

Divide by the highest Power

$$= \lim_{n\to\infty} \frac{1}{1+\underbrace{\frac{1}{n}}_{=0}} = 1$$

Ex: $\{4, -1, 1/4, -1/16, 1/64, \ldots\}$

$$\boxed{(-1)^{n+1}\, 4^{2-n}}$$

Ex: $\{-3, 2, -\frac{4}{3}, \frac{8}{9}, \frac{-16}{27}, \ldots\}$

$$(-3)\left(-\frac{2}{3}\right)^{n-1} \quad \text{or} \quad \frac{2^{n-1}}{(-3)^{n-2}}$$

☆ Study

Ex: $\{5, 8, 11, 14, 17\}$

$$3n+2$$

Ex: $\{1, 0, -1, 0, 1, 0 \ldots\}$

☆ Trigonometric function ☆

$$\sin \frac{n\pi}{2} \quad \text{or} \quad \frac{\cos (n-1)\pi}{2}$$

Find the limits

$$\lim_{n\to\infty} a_n = \frac{3/n^2 + 5n^2/n^2}{n/n^2 + n^2/n^2} = \frac{5 + \frac{3}{n^2}}{1 + \frac{1}{n}} = \frac{5}{1} = \boxed{5}$$

0

0

Theorem: The seq r^n is convergent if $-1 < r \leq 1$, else it is divergent for all others

Ex: $\lim_{n\to\infty} a_n = \frac{n^4/n^4}{n^3/n^4 - 2n/n^4} = \frac{1}{\frac{1}{n} - \frac{2}{n^3}} =$

0

its actually divergent wtf? research

This is convergent b/c

Ex: $a_n = \frac{4^n/9^n}{1/9^n + 9^n/9^n} = \frac{\frac{4^n}{9^n}}{1 + \frac{1}{9^n}} = \frac{0}{1} = 0$

On exam before:

11-1 #52 Determine if sequence converges or diverges, if converges, find the limit

$a_n = n - \sqrt{n+1}\ \sqrt{n+3}$

Try this one

Theorem: A seq $\{a_n\}$ is called increasing $a_n < a_{n+1}$ for all $n \geq 1$

Called decreasing if $a_n > a_{n+1}$ for all $n \geq 1$

A sequence is called monotonic if its either completely increasing or completey decreasing (Possibly constant in some cases)

Ex) Show that the sequence

$\frac{3}{an+5}$ is dec.

★ Finish UP ★

$$\frac{3}{an+5} \overset{?}{>} \frac{3}{a(n+1)5}$$