Home Assignment 1 – Burst Image Deblurring

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1 Theoretical questions

Q1 Solution:

While taking the image we do two processes: <u>Translation and Digitalization(Sampling)</u>, thus we get the term:

$$\boxed{h_k = \int_{t}^{t+1msec} \underbrace{\varphi_0(s)}_{\text{Translation}}\underbrace{\delta}_{\text{Sampling}}\, ds}$$

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Q2 Solution: **Suppose that** $\mathcal{F}\{\varphi_0(t)\}_{(\omega)} = \Phi_0(\omega)$ **, that's why we get that :**

$$\mathcal{F}(h_k)[\omega] = \mathcal{H}_k[\omega] = = \mathcal{F}\left(\int_k^{k+1} \varphi_0(s)\delta\, ds\right) \underset{\text{Linearity}}{=} \boxed{\int_k^{k+1} \phi_0(s)\, ds}$$

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Q3 Solution: **Let's denote the camera shutter exposure time (in the digital domain) as Rect[n]**

(Rect[n] is the Box function, the Fourier Transform of the Box function is a sinc, as seen in the lectures and tutorials).

Cont.

And thus, we get that:

$G_k[\omega] = \mathcal{F}(g_k[n] * \text{Rect}(n)) = \mathcal{F}(f(n) * h_k(n) * \text{Rect}[n]) =$

$= \mathcal{F}[\omega] \cdot H_k[\omega] \cdot \text{sinc}(\omega)$

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And here we also got that: $P_k = H_k$

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Q4 Solution:

$$|P_k| = |H_k| = \left|\int_k^{k+1} \phi_0(s)\, ds\right| \leq \int_k^{k+1} |\phi_0(s)|\, ds =$$

$$= \int_k^{k+1} |\exp(-2\pi j w^T o(s)\quad)|\, ds \leq \int_k^{k+1} |1|\, ds = 1$$

Euler

Thus, the upper value of $|P_k[w]|$ is 1

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Q5 Solution:

From Q4 we got that $P_k[w] = \int_k^{k+1} \exp(-2\pi j w^T o(s)\quad)\, ds$,

and in our case, we get that:

$$P_k[w] = \int_k^{k+1} \exp(-2\pi j w^T (s - k - 0.5) v e_1)\, ds$$

$$= exp\left(2\pi j w^T v e_1 (k + 0.5)\right) \int_k^{k+1} \exp(-2\pi j w^T s v e_1)\, ds =$$

$$= \frac{exp\left(2\pi j w^T v e_1 (k+0.5)\right)\left[exp\left(-2\pi j w^T v e_1 (k+1)\right) - exp\left(-2\pi j w^T k v e_1\right)\right]}{-2\pi j w^T v e_1}$$

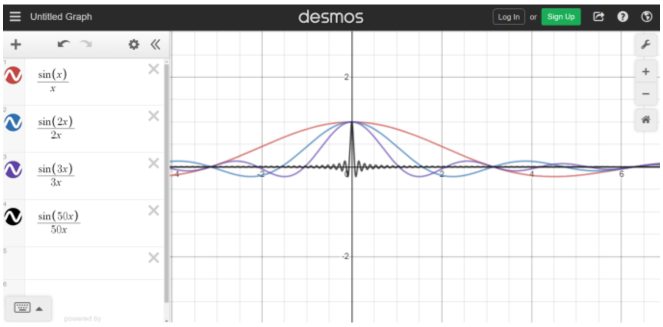
Cont.

$$=\frac{sin(\pi w^T v e_1)}{\pi w^T v e_1} = \boxed{sinc(w^T v e_1)}$$

<u>How does |Pk[w]| depend on v?</u>

To answer this question, we should consider the properties of the sinc function, whenever v is greater then the center of the censor would move faster through space and thus we gather more spatial information and <u>thus the sinc function (which is in the frequency domain) would become narrower, and so the space of available frequencies is smaller and we begin to see blurry images.</u>

<u>A demonstration to make matters clearer:</u>



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<u>Q6 Solution:</u>

Using the formulas from Q4 and Q5 we get that:

Pk[w]= $\int_k^{k+1} \exp\left(-2\pi j w^T(q+sv)\right) ds =$

$$= \int_k^{k+1} \exp(-2\pi j w^T[(s-k-0.5)v + (k+0.5) +])\, ds$$

$$= \boxed{\text{sinc}(w^T v).\ exp(-2\pi j w^T[(k+0.5)v+q)}$$

We get a very similar result to the previous question, now we have an additional factor that we got because the movement is not constrained in one direction.

How does $|P_{k[w]}|$ depend on v?

$|P_k[w]| = |\text{sinc}(w^T v).\exp(-2\pi j w^T[(k+0.5)v+q)| =$

$$=\text{sinc}(w^T v)$$

So we get the same results as in Q5 but the blur now can occur along three axis.

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Q7 Solution:

In order to recover the unblurred image we used the paper:

Removing Camera Shake via Weighted Fourier Burst Accumulation " by Delbracio and Sapiro, we will explain the algorithm in the following lines:

From previous questions we concluded that in the given case of Q7 we get $|P_k| = |H_k| = 1$, and this is a very crucial conclusion because it means that each blurring kernel in every frame does not amplify the spectrum, so we get that the Power Spectral Density before the blur has the same energy as of that after the blur.

We know that the movement of the hand during the capture of the image is random, and thus would be different from one frame to another, that is why in different frames each frequency will be affected and manipulated differently, in other words whenever a blur happens some of the energy in a specific frequency will be "spread" onto neighboring frequencies and the energy of this certain frequency will become smaller, the idea and the intuition of our algorithm is

in a given frame, the frequency with a very high energy (relative to the energy of the same frequency in other frames) will be most likely affected less by the blur in this frame and would indicate a value close to the energy of the frequency in the original pre-blurred image. Thus, we want through our algorithm to combine the "strongest" frequencies from every frame into one frame which in theory would be the frame which is very similar to the original pre-blurred image.

The algorithm:

If we denote the original picture to be f, the blur kernel of frame k to be h_k and the blurred picture of frame k to be v_k, then we get that v_k=h_k*f, for k=1,2……,N.

Let p be a positive integer.

For every frame k we give a weight : $w_k = \frac{|V_k|^p}{\sum_{i=1}^{N}|V_k|^p}$,as V_k is the Fourier transform of v_k.

We denote the recovered image by $\boxed{\hat{f} = F^{-1}(\sum_{k=1}^{N} w_k \cdot v_k)(x)}$

And so, we get the recovered image. The algorithm is very simple after all. Something very important about raising to the power of p: Raising the values to a power of positive integer p makes frequencies with high energy values very prominent and "indicates" to the algorithm that this might be the frame where this frequency has the energy which is the closest to that in the original image.

<u>Note:</u> Given high weights to frames with high energy in a certain frequency is sufficient to give a very close prediction about this certain frequency and assume that it has almost the same properties of the same frequency in the original picture because we are talking about relatively small motion

changes and that's why the kernel normally don't affect the phase that much…

And so, we reached the end of the homework…

We hope that you have enjoyed reading it 😊