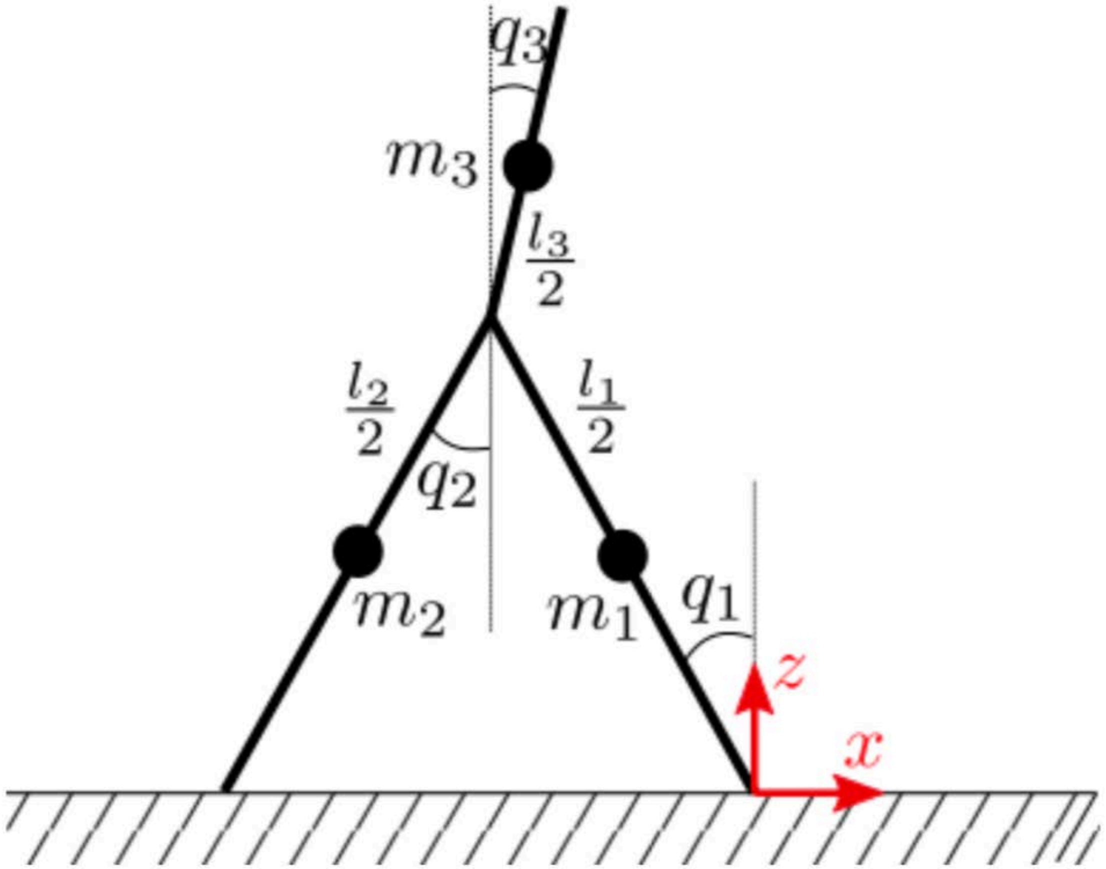
Modeling of a three-link 2D biped

Legged Robots

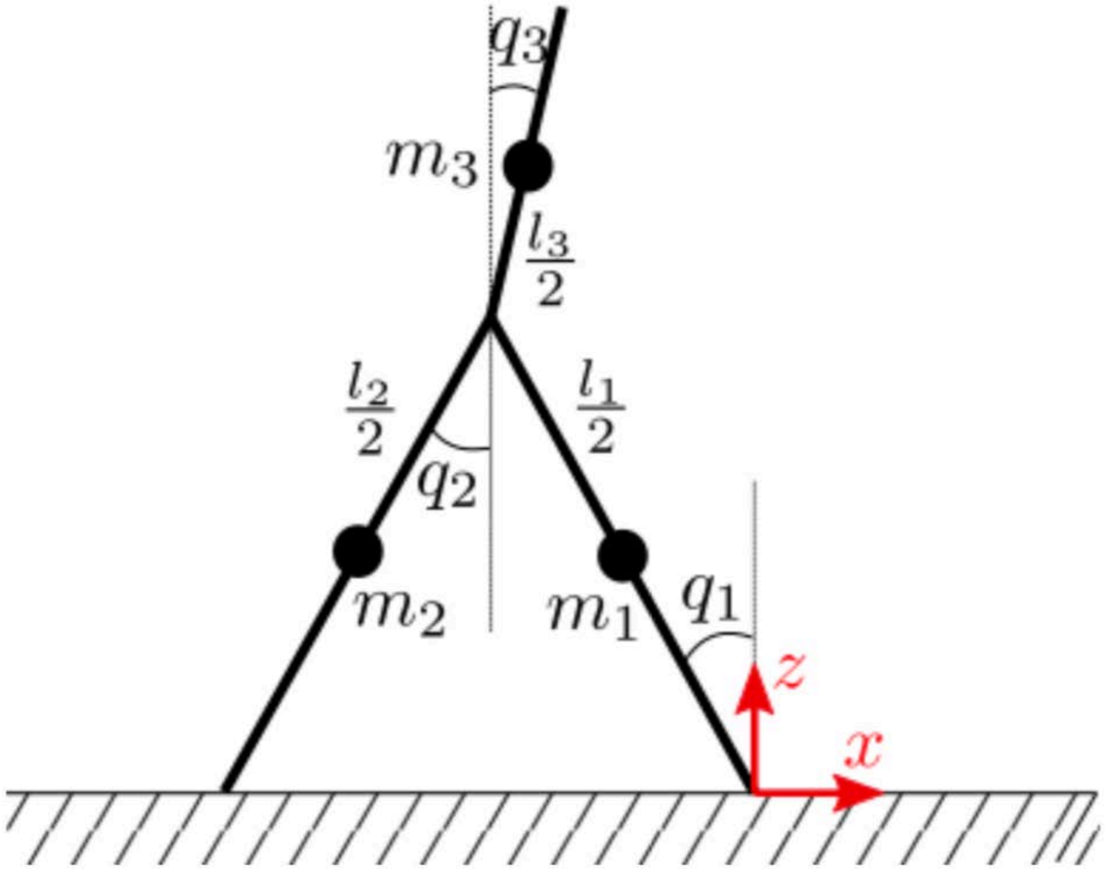
Overview

- Model and visualize the three-link biped
- Solve the equation of motion of the three-link biped
- Design walking controllers, evaluate the resulting gaits and compare them



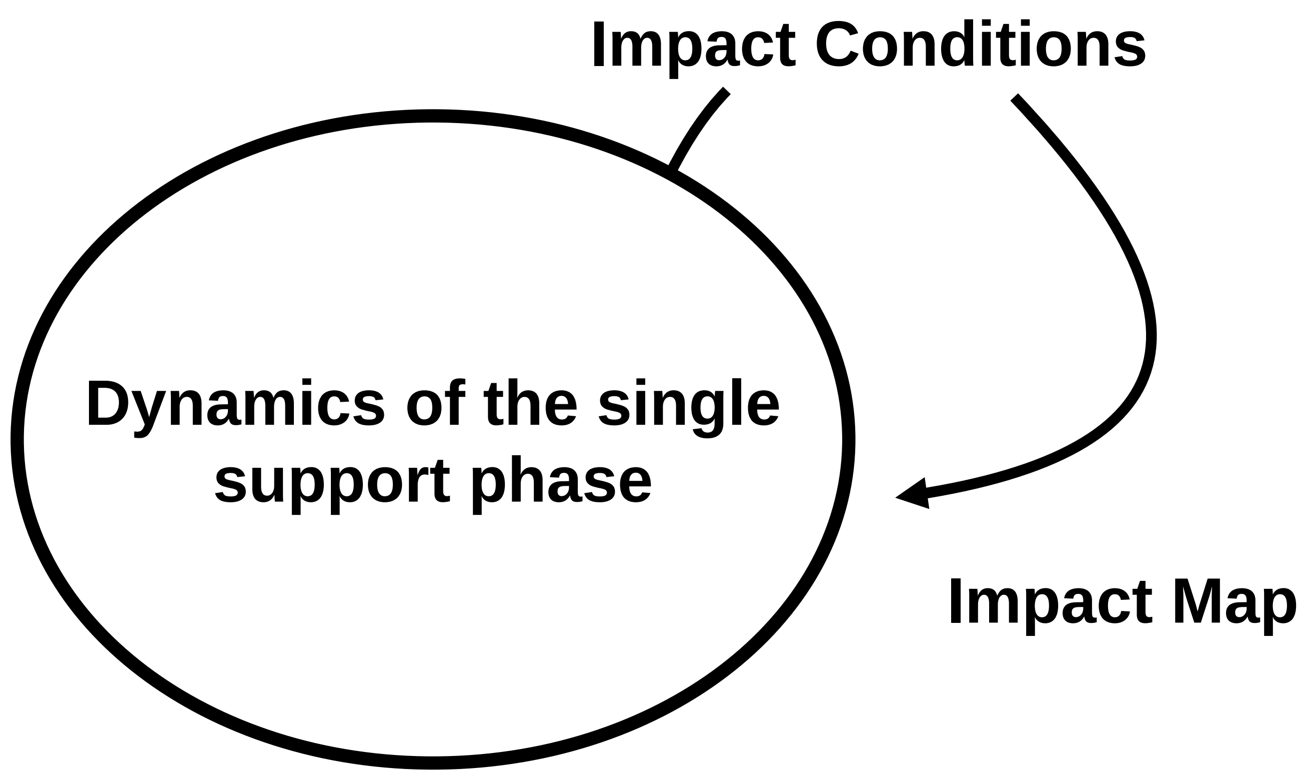
Model and visualize the three-link biped

- Kinematics
- Dynamics
- Impact



Hybrid model of walking

- Swing phase model
- Impact model



Feedback Control of Dynamic Bipedal Robot Locomotion, [Eric R. Westervelt, Jessy W. Grizzle, et al.]

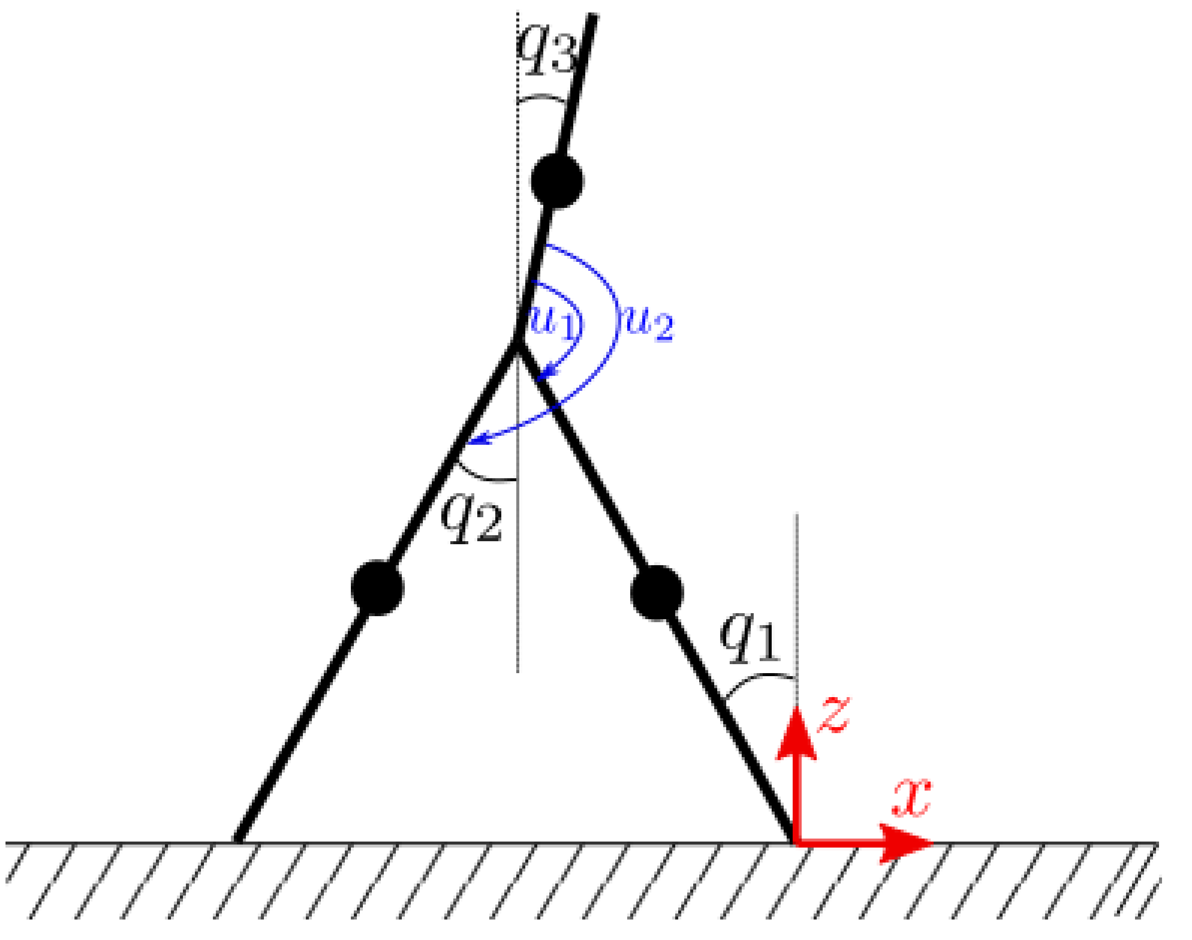
Swing phase model

- Pinned open kinematic chain
- Lagrangian method

$$L(q,\dot{q}) = T(q,\dot{q}) - V(q)$$

$$M\ddot{q} + C\dot{q} + G(q) = Bu$$

- Model is under actuated (why?)

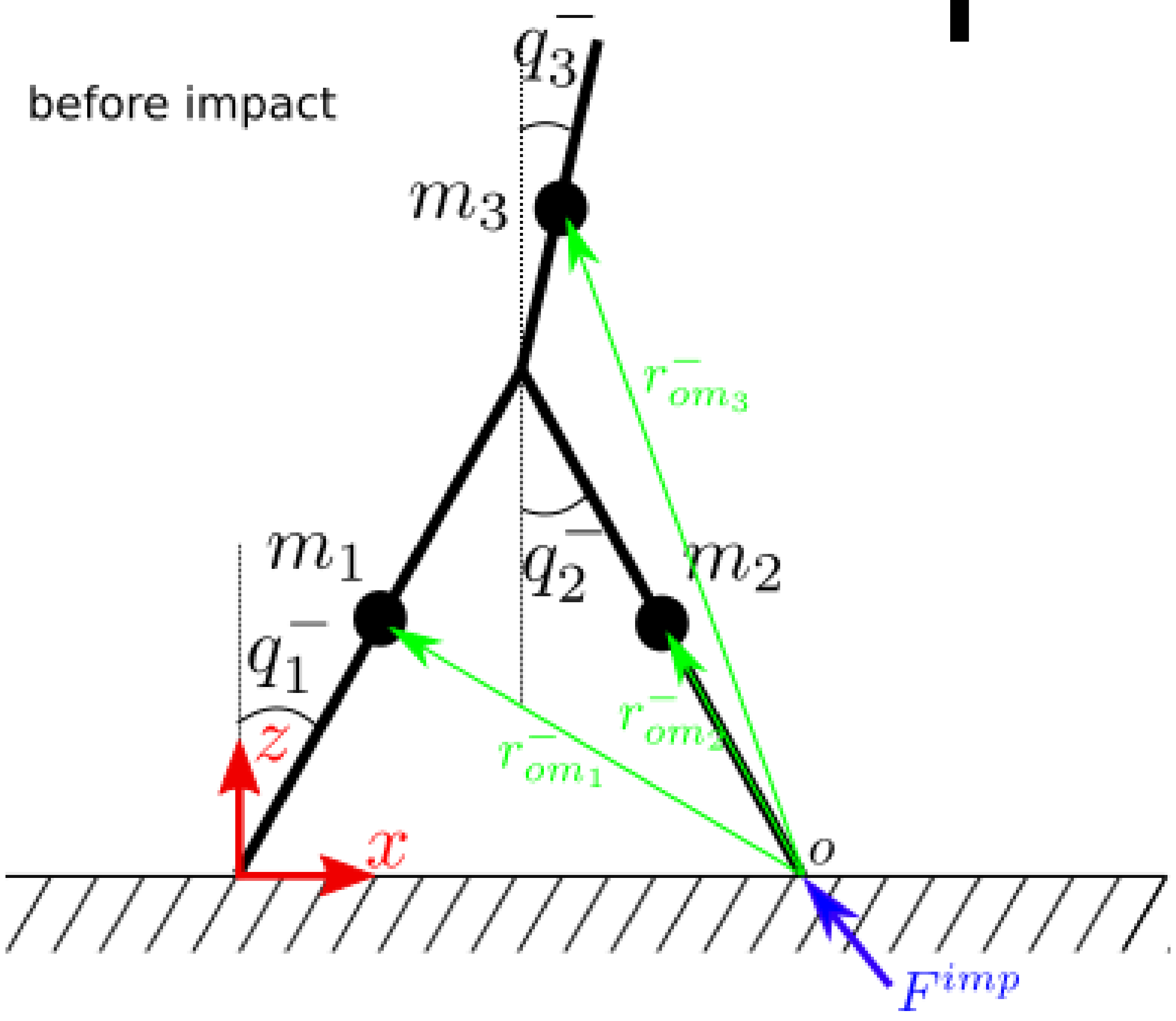


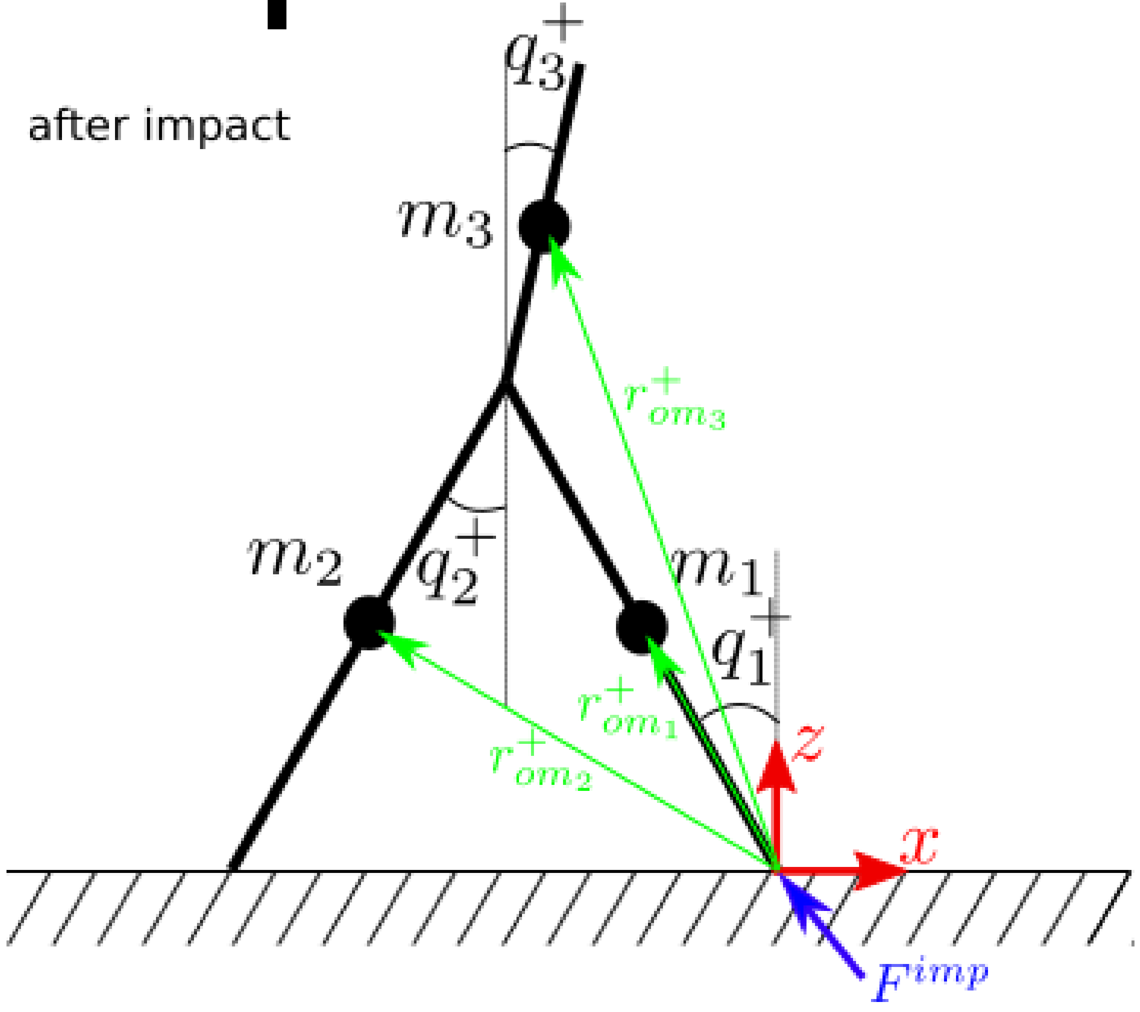
Clockwise rotation is considered positive!

Impact map

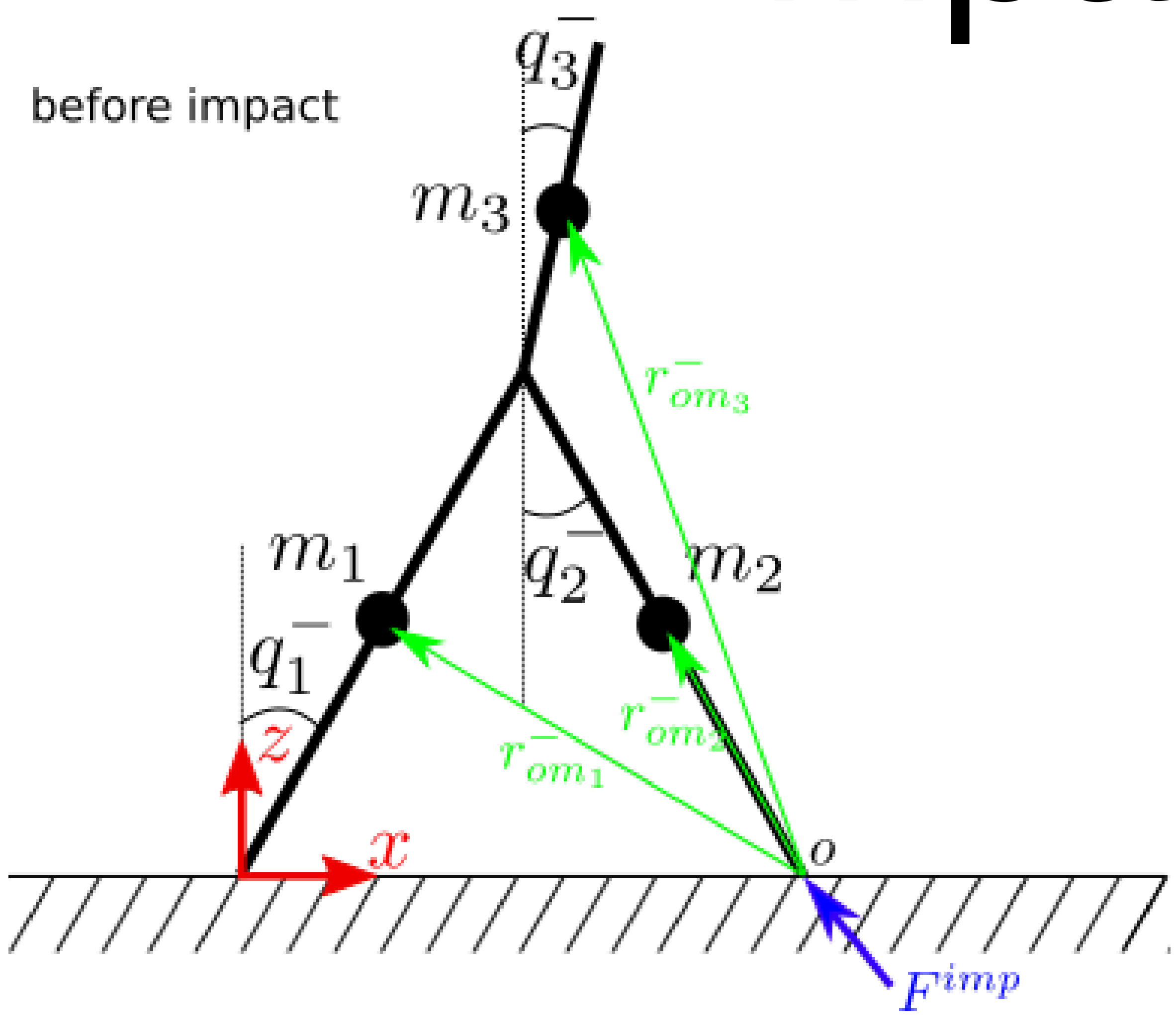
- The impact map, maps the state of the robot right before the impact to its state right after
- Assumption: the impact and switching of the leg roles are instantaneous and the stance leg does not slip

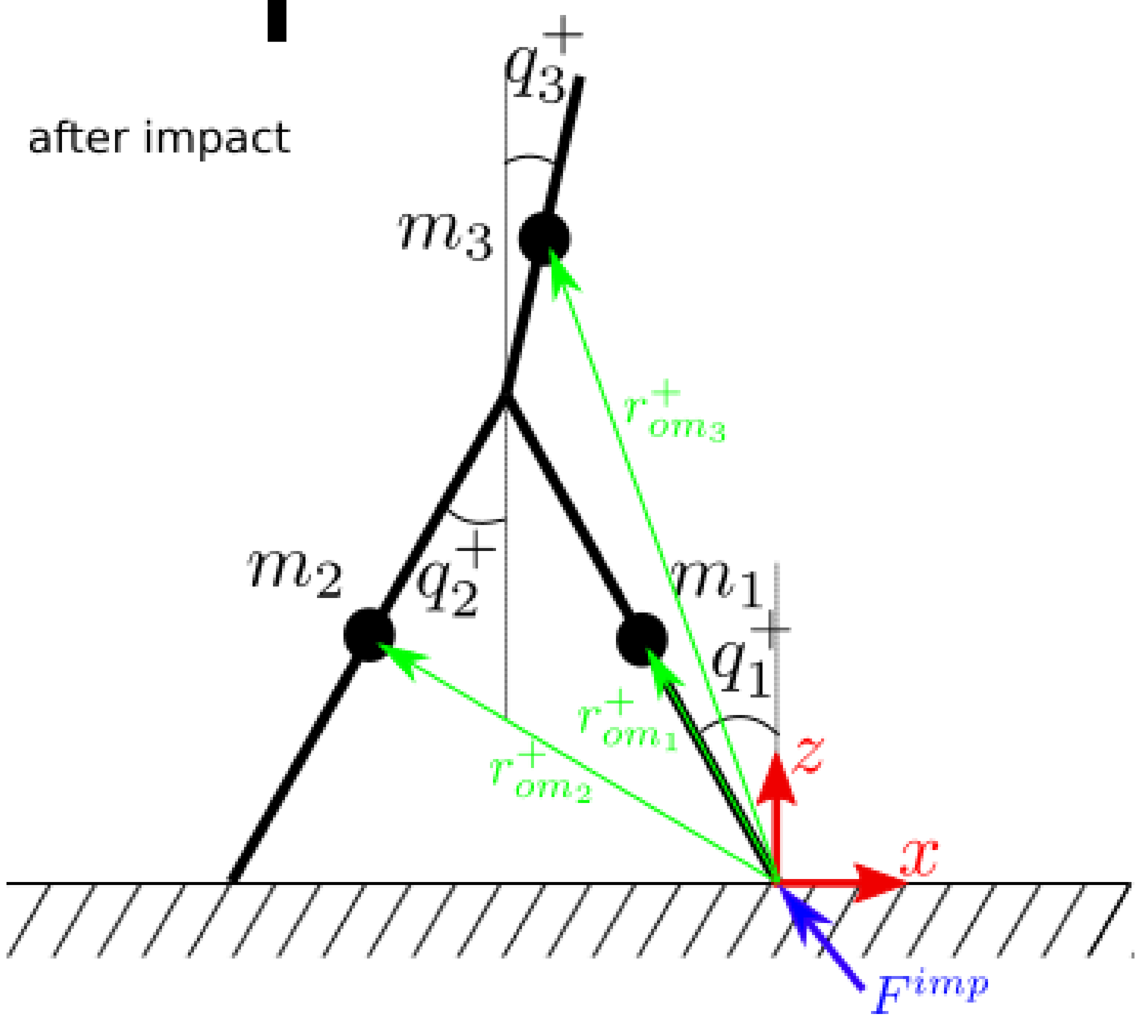
Impact map


before impact
q_3^-
m_3
$r^-_{om_3}$
m_1
q_2^-
m_2
q_1^-
$r^-_{om_2}$
z
$r^-_{om_1}$
x
o
F^{imp}


after impact
q_3^+
m_3
$r^+_{om_3}$
m_2
q_2^+
m_1
q_1^+
$r^+_{om_1}$
z
$r^+_{om_2}$
x
F^{imp}

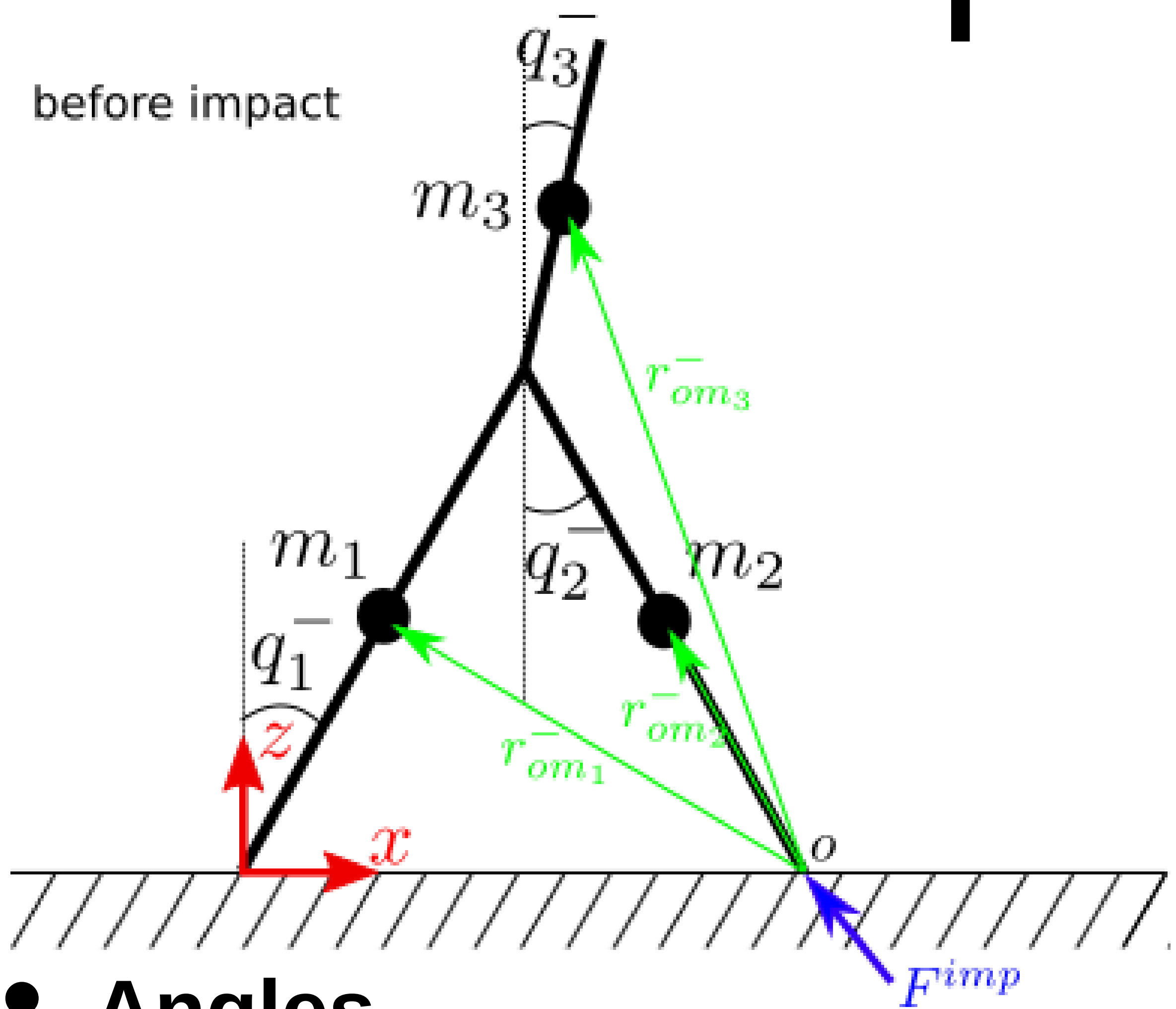
Impact map

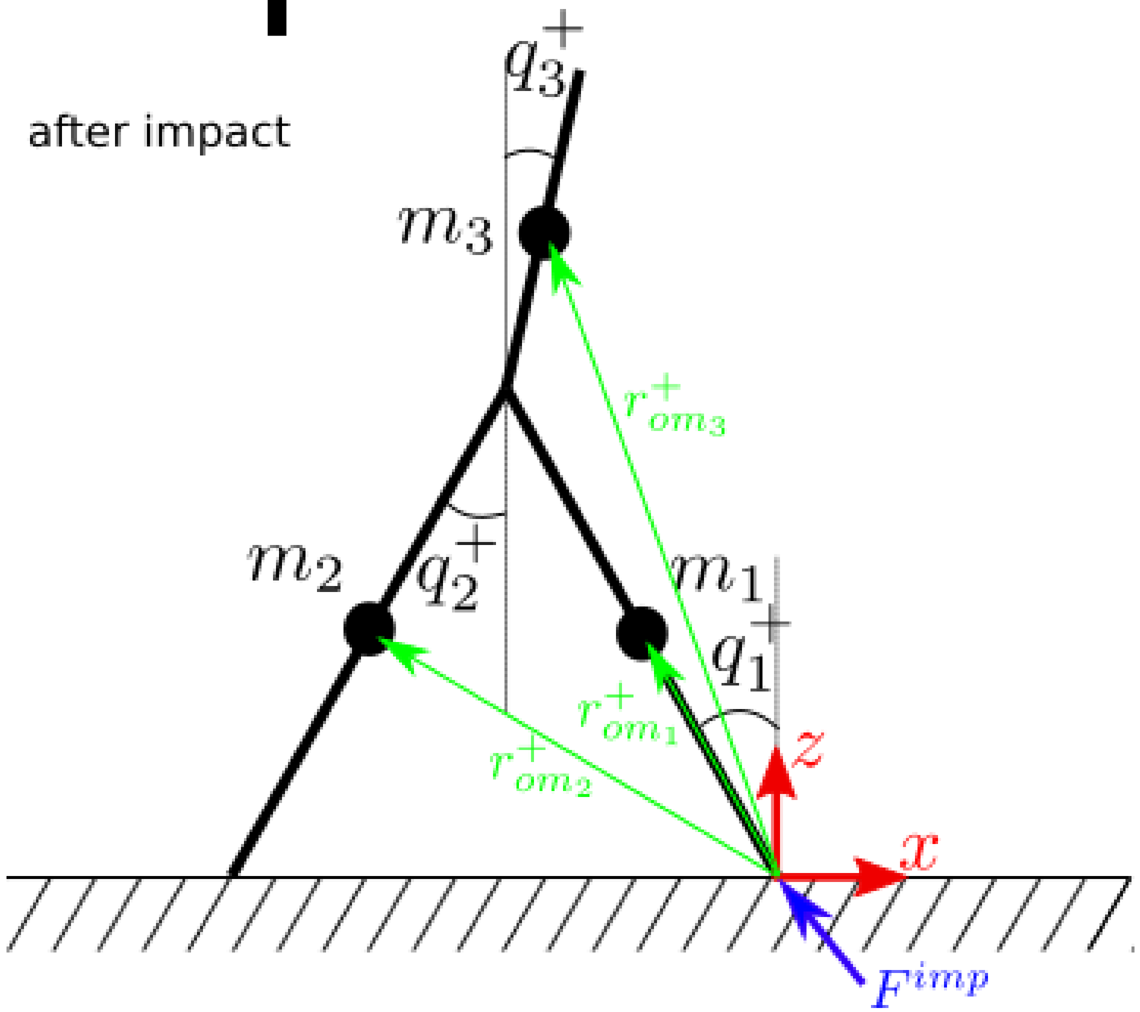




- **Differences**?
  - reference frame
  - masses (stance/swing)

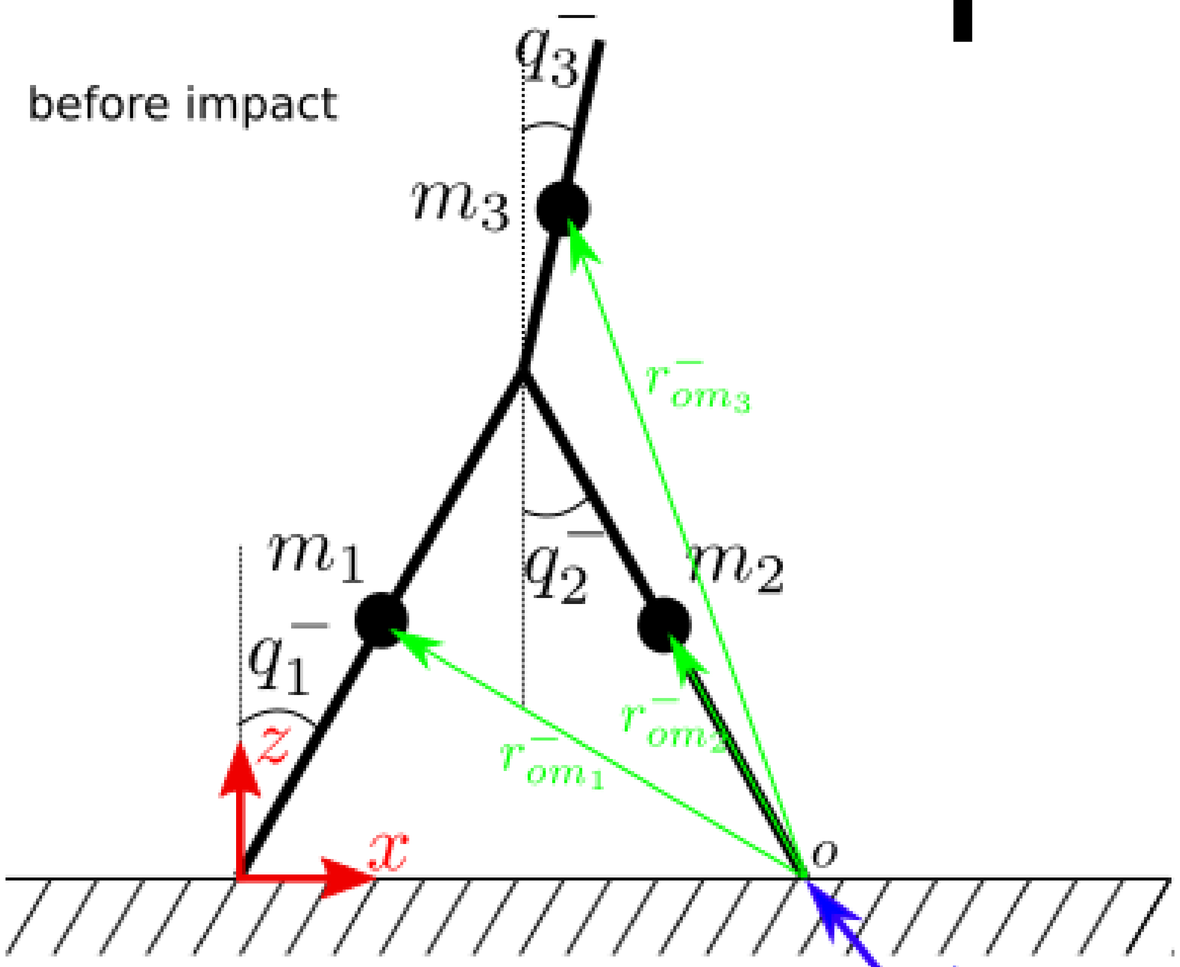
Impact map

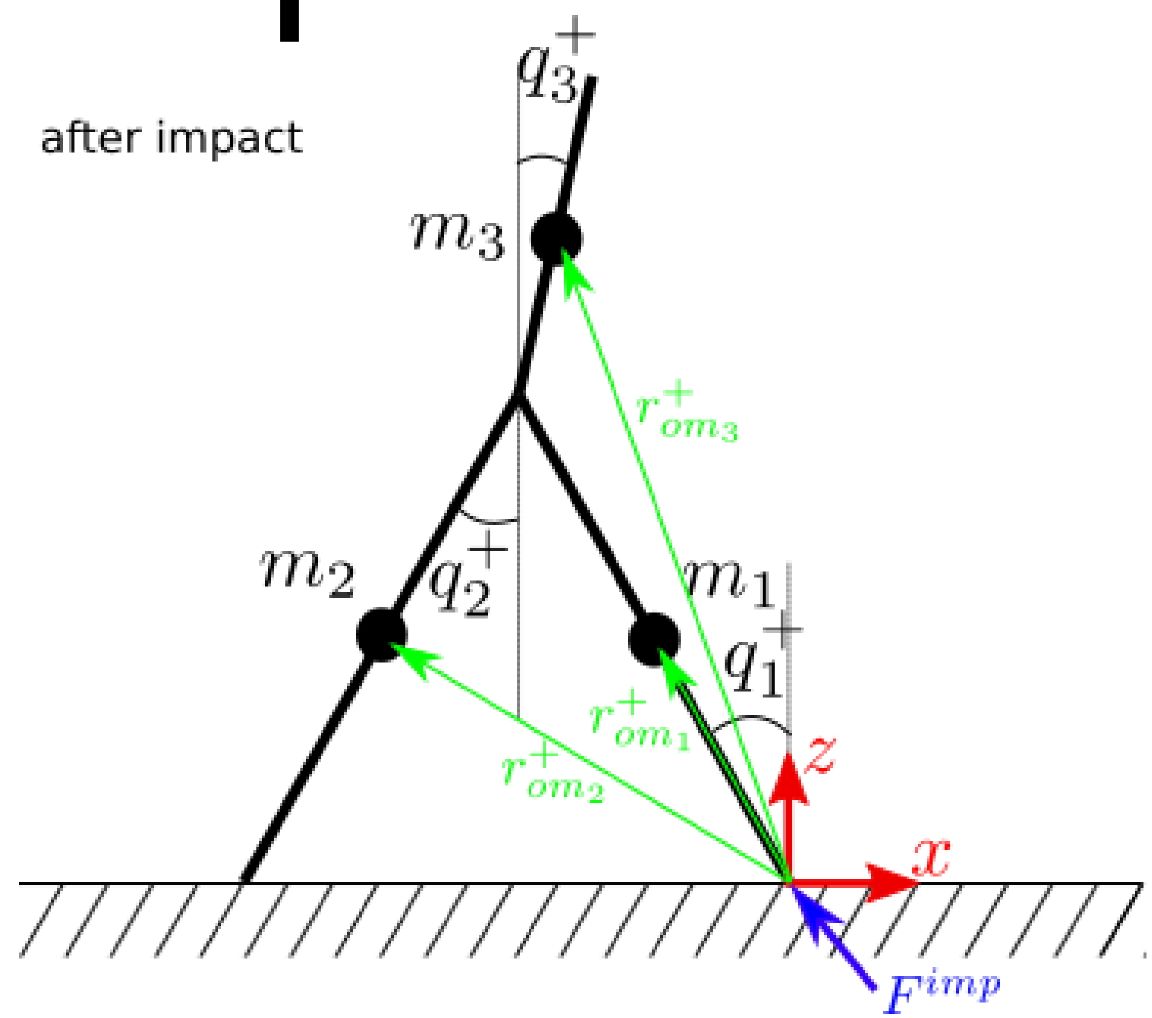




- **Angles**

Impact map

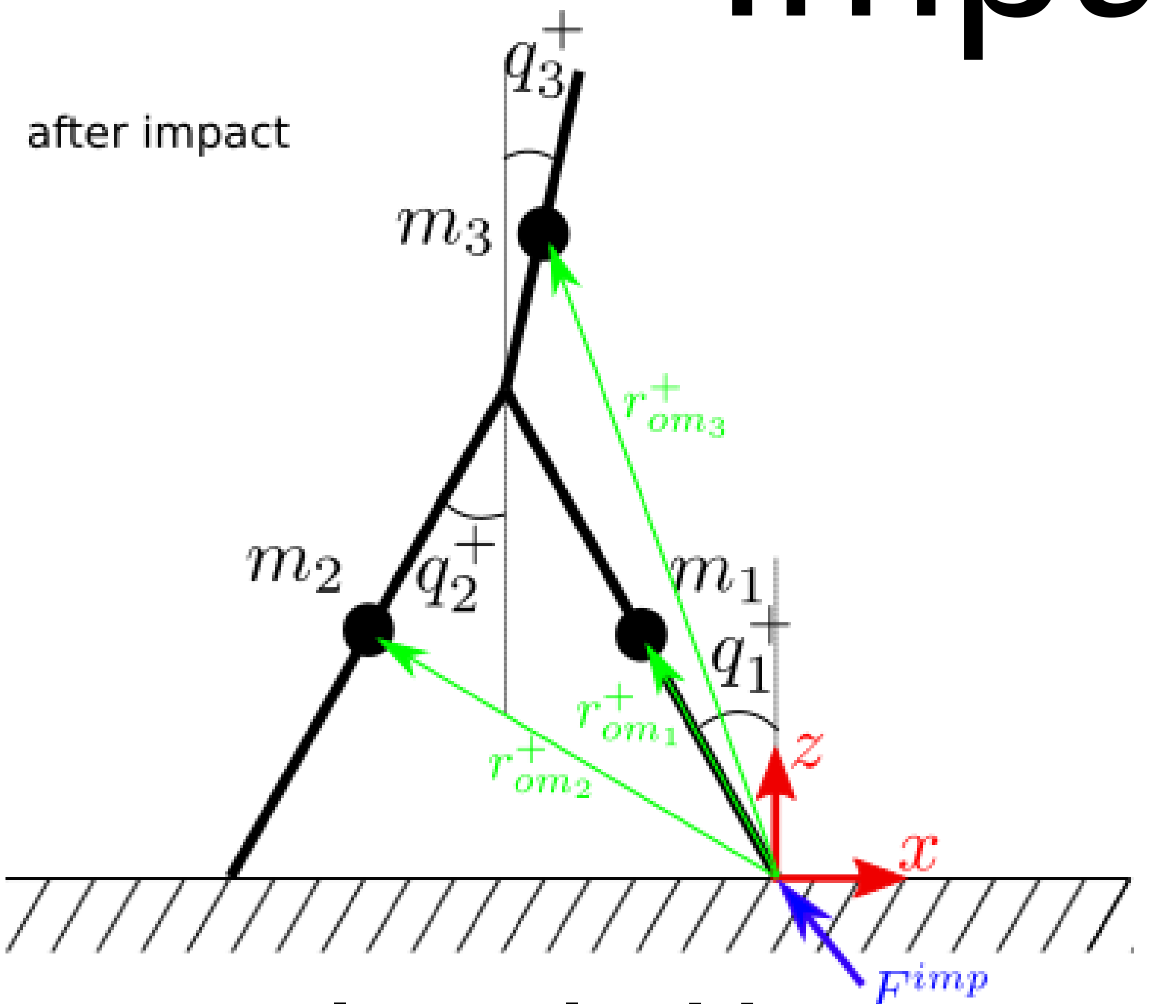


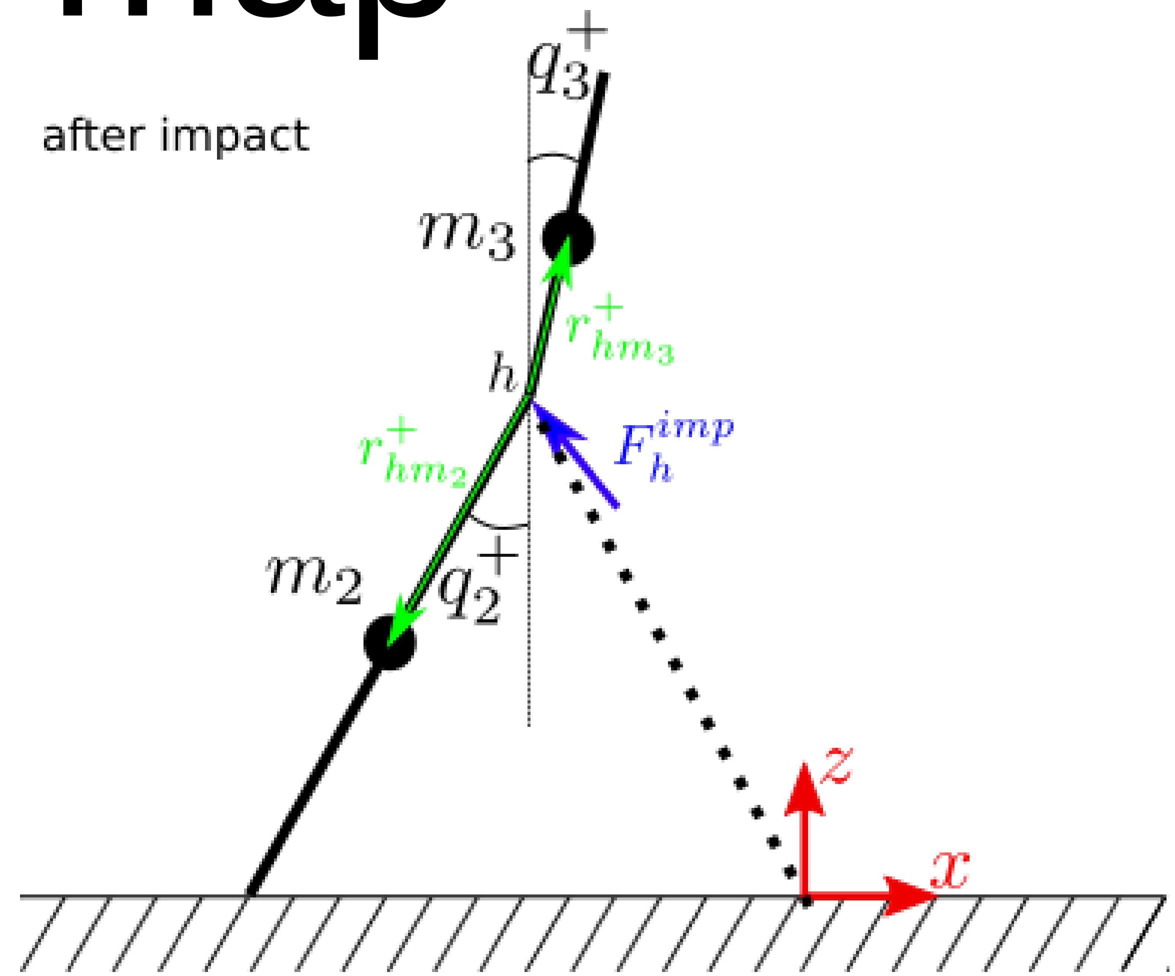


- **Angles**

$$\begin{bmatrix} q_1^+ \\ q_2^+ \\ q_3^+ \end{bmatrix} = \begin{bmatrix} q_2^- \\ q_1^- \\ q_3^- \end{bmatrix}$$

Impact map

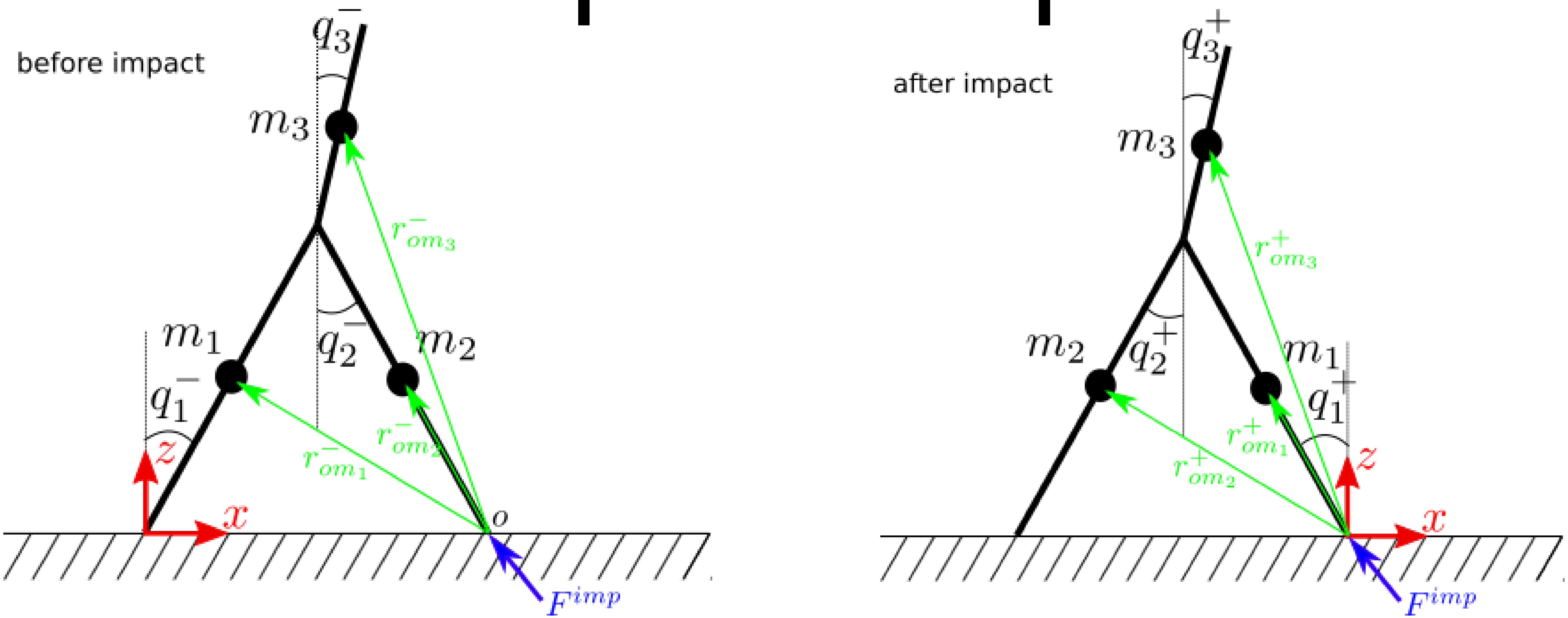




- **Angular Velocities**
  - Conservation of the angular momentum of **whole system about the post-transfer support point** and of the **trailing leg and torso about the hip**

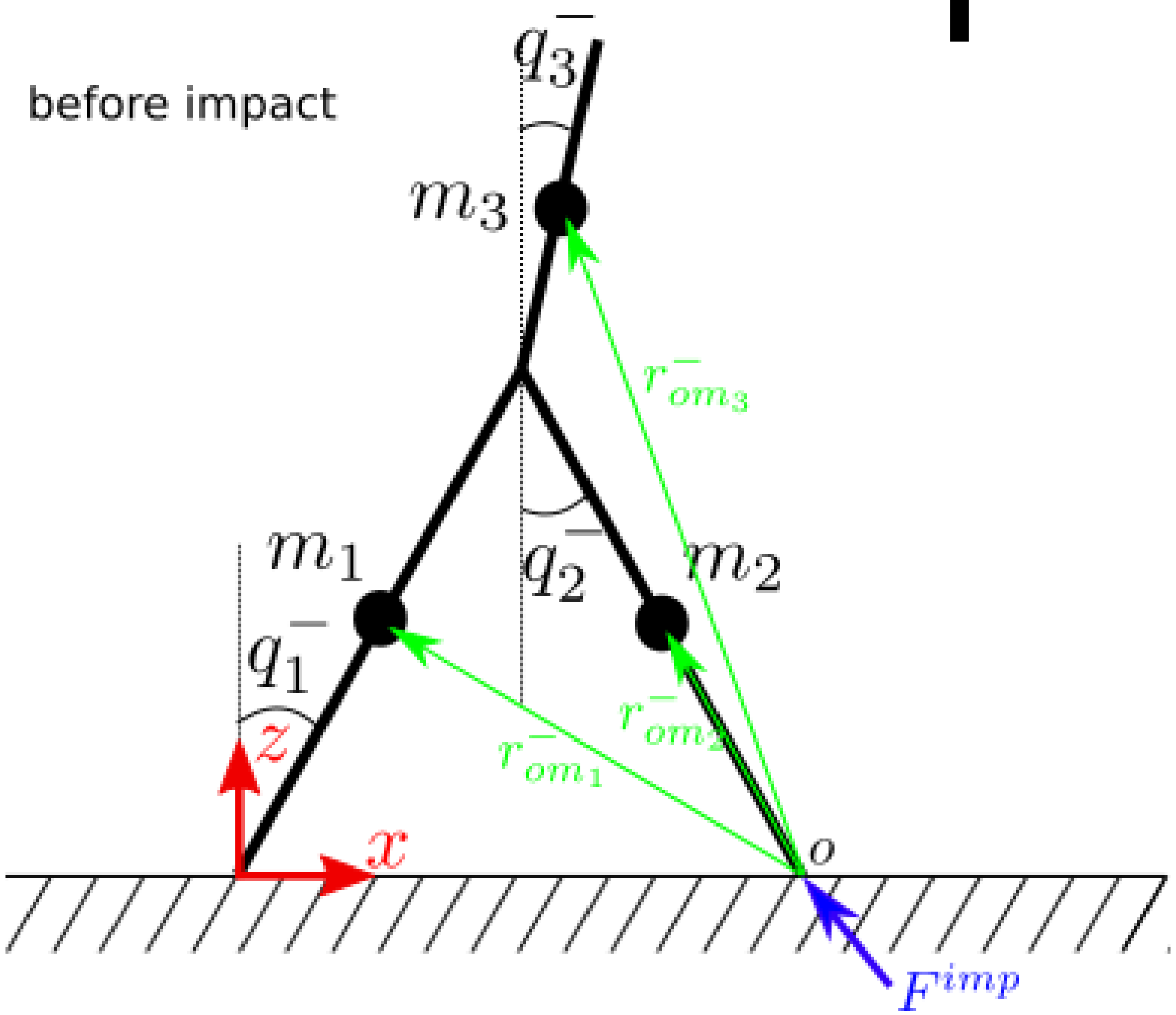
Stability and control of two dimensional bipedal walking, Tad McGeer, 1988

Impact map



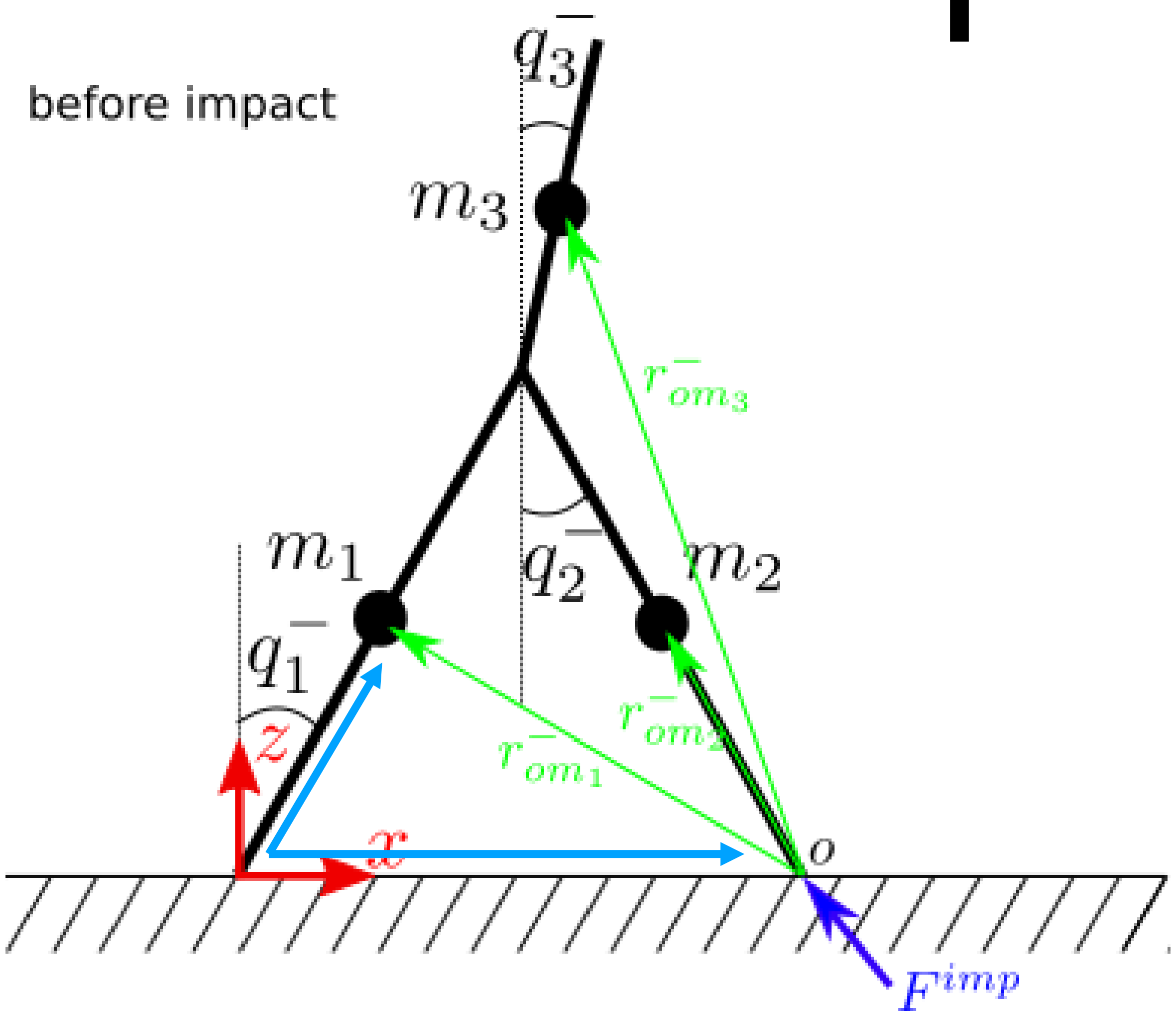
before impact
q_3^-
m_3
$r_{om_3}^-$
m_1
m_2
q_2^-
q_1^-
$r_{om_2}^-$
$r_{om_1}^-$
z
x
o
F^{imp}
after impact
q_3^+
m_3
$r_{om_3}^+$
m_2
q_2^+
m_1
q_1^+
$r_{om_1}^+$
$r_{om_2}^+$
z
x
F^{imp}

Impact map



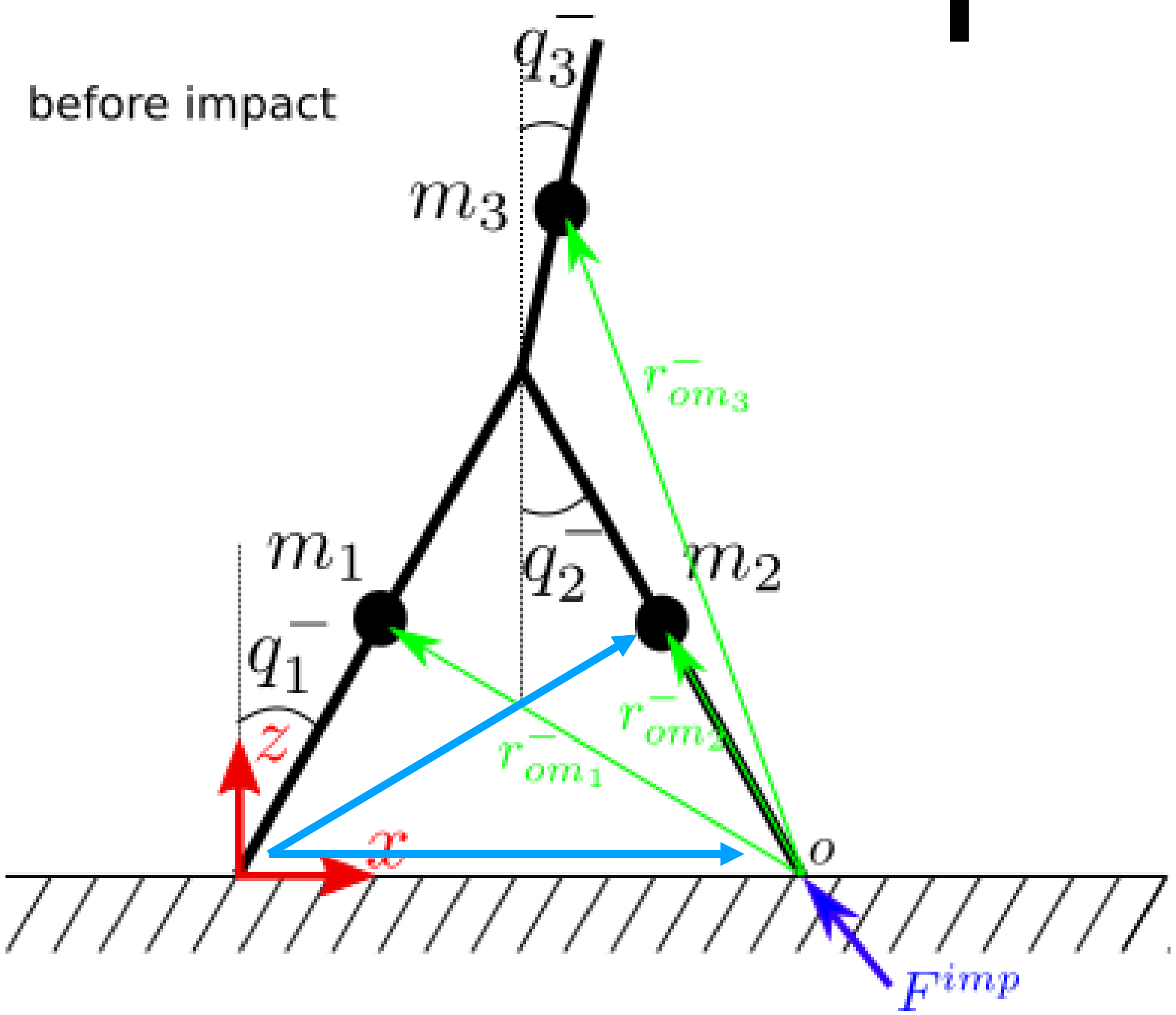
$$\boldsymbol{H_a^-} = \boldsymbol{m\, r_{om_1}^-} \times \boldsymbol{\dot{r}_1^-} + \boldsymbol{m\, r_{om_2}^-} \times \boldsymbol{\dot{r}_2^-} + \boldsymbol{m_3\, r_{om_3}^-} \times \boldsymbol{\dot{r}_3^-}$$

Impact map



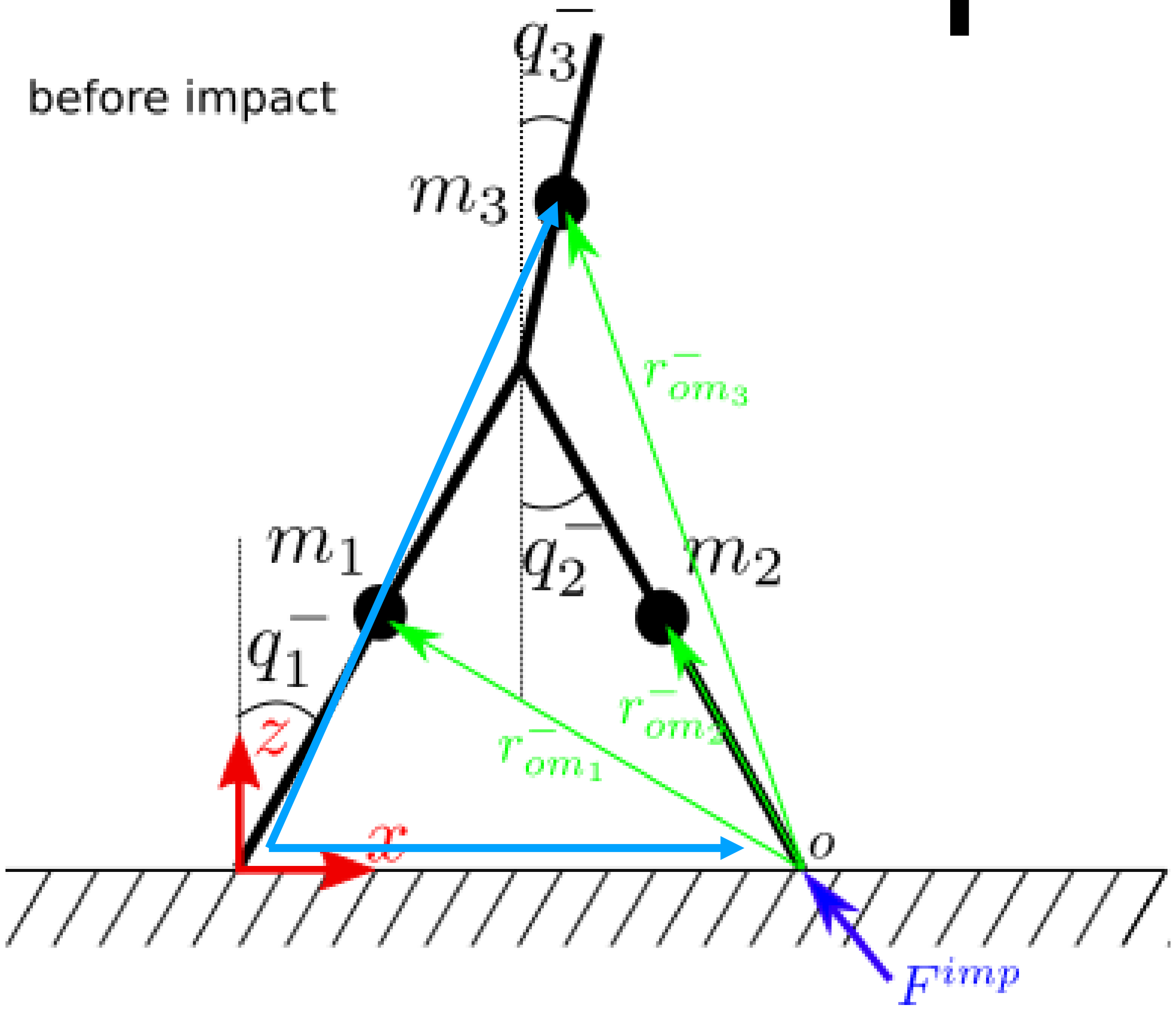
$$H_a^- = m\, r_{om_1}^- \times \dot{r}_1^- + m\, r_{om_2}^- \times \dot{r}_2^- + m_3\, r_{om_3}^- \times \dot{r}_3^-$$

Impact map



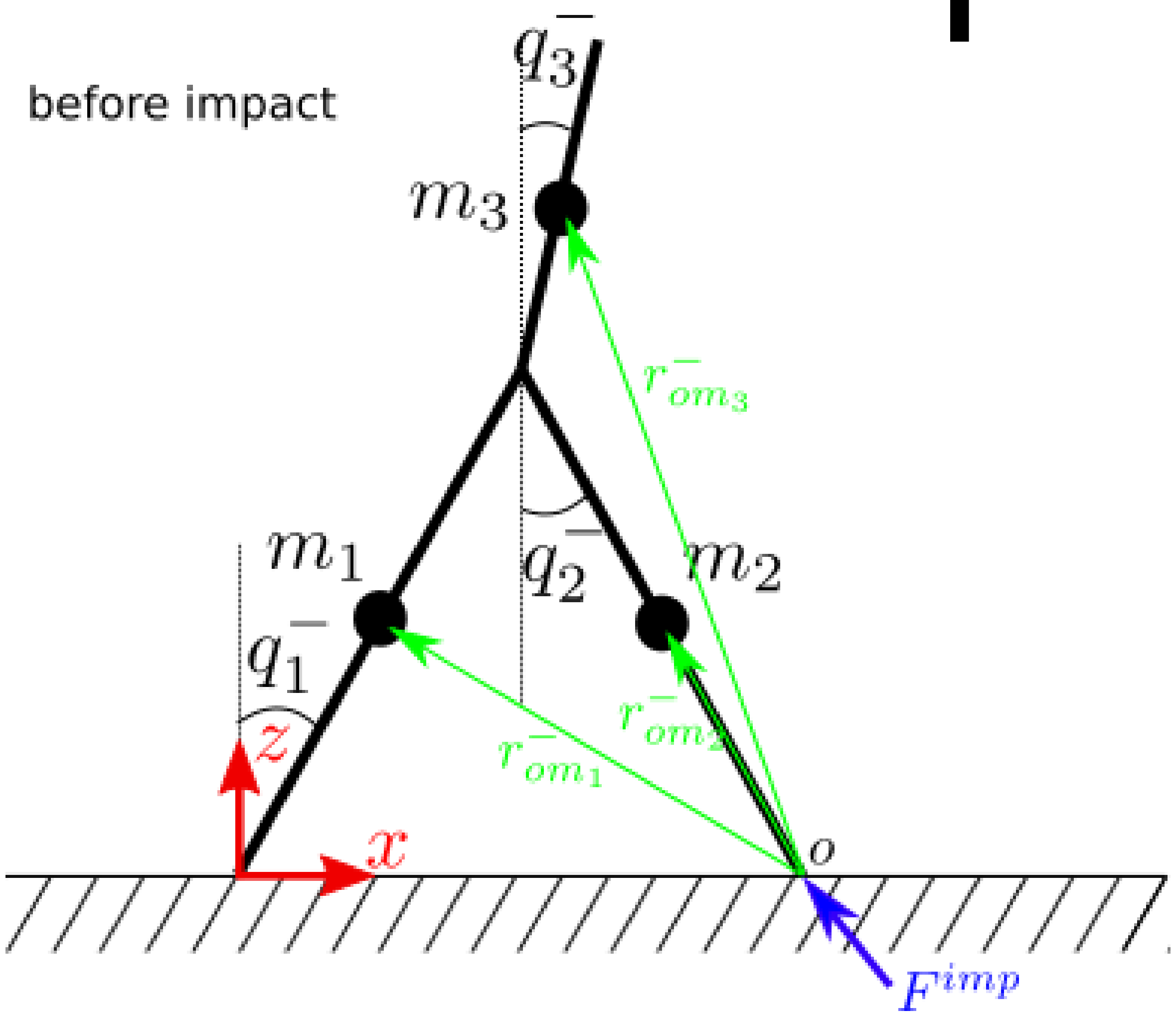
$$\boldsymbol{H_a^-} = \boldsymbol{m}\, \boldsymbol{r_{om_1}^-} \times \dot{\boldsymbol{r}}_{\boldsymbol{1}}^- + \boldsymbol{m}\, \boldsymbol{r_{om_2}^-} \times \dot{\boldsymbol{r}}_{\boldsymbol{2}}^- + \boldsymbol{m_3}\, \boldsymbol{r_{om_3}^-} \times \dot{\boldsymbol{r}}_{\boldsymbol{3}}^-$$

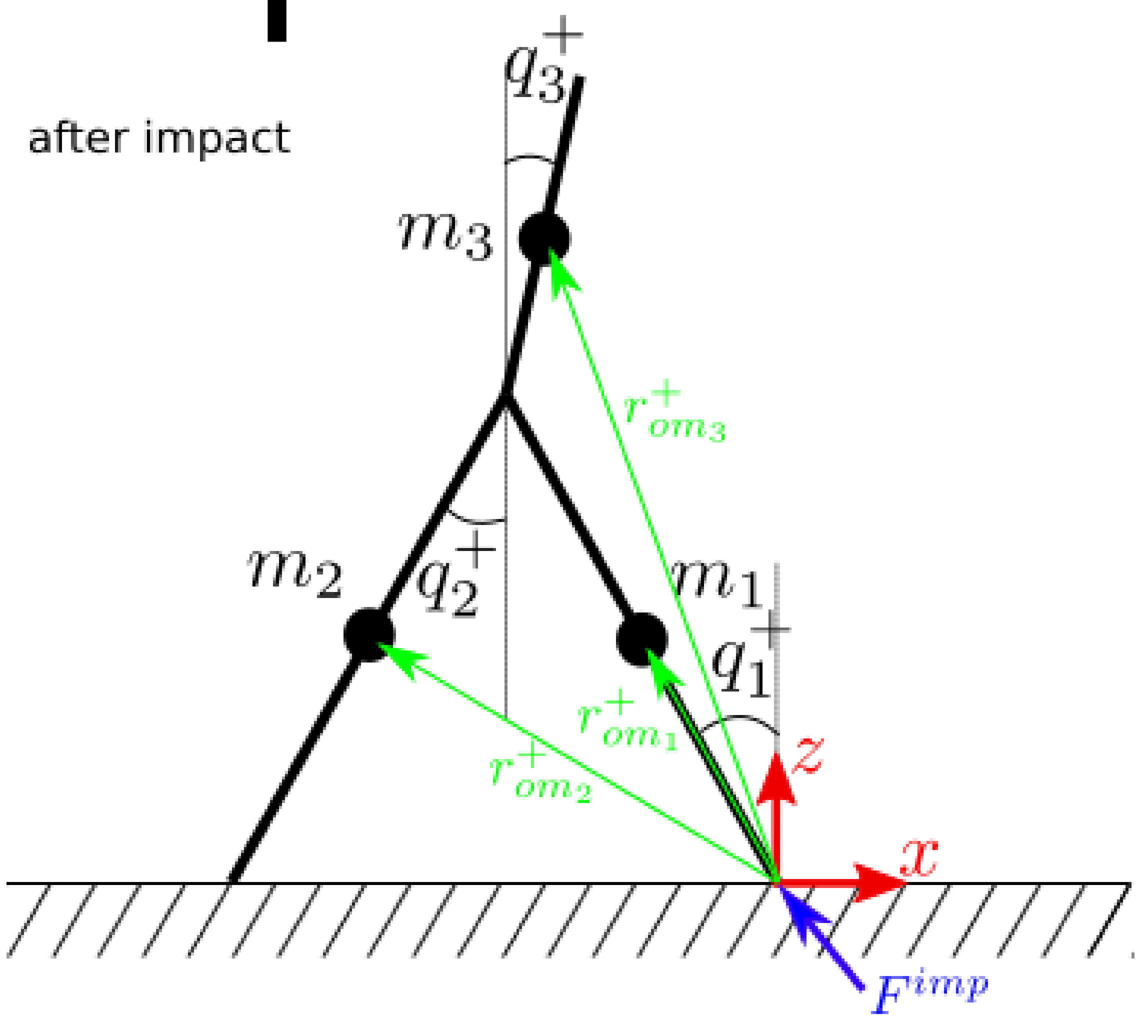
Impact map



$$H_a^- = m\, r_{om_1}^- \times \dot{r}_1^- + m\, r_{om_2}^- \times \dot{r}_2^- + m_3\, \boxed{r_{om_3}^-} \times \dot{r}_3^-$$

Impact map

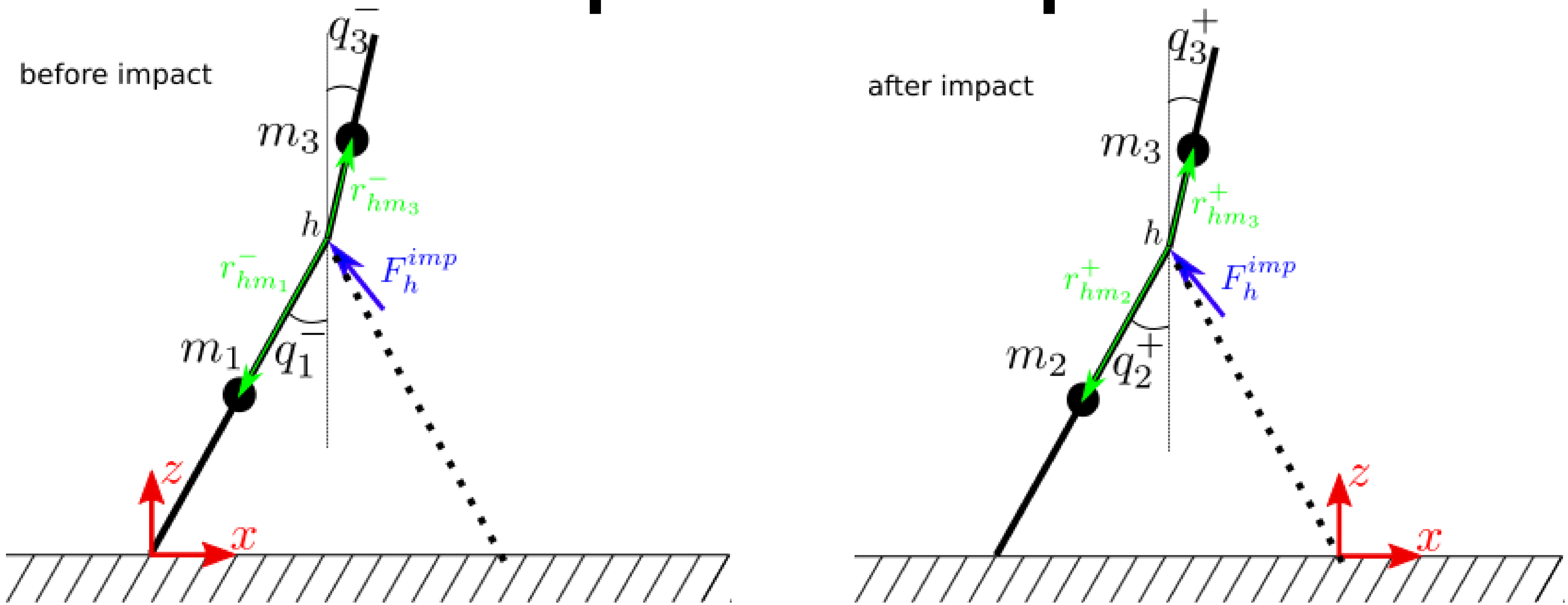




$$\boldsymbol{H_a^-} = \boldsymbol{m\, r_{om_1}^-} \times \boldsymbol{\dot{r}_1^-} + \boldsymbol{m\, r_{om_2}^-} \times \boldsymbol{\dot{r}_2^-} + \boldsymbol{m_3\, r_{om_3}^-} \times \boldsymbol{\dot{r}_3^-}$$

$$\boldsymbol{H_a^+} = \boldsymbol{m\, r_{om_1}^+} \times \boldsymbol{\dot{r}_1^+} + \boldsymbol{m\, r_{om_2}^+} \times \boldsymbol{\dot{r}_2^+} + \boldsymbol{m_3\, r_{om_3}^+} \times \boldsymbol{\dot{r}_3^+}$$

Impact map



- Conservation of the angular momentum of **trailing leg and torso about the hip**

$$\boldsymbol{H}_{\boldsymbol{b}}^{-} = \cdots \qquad \boldsymbol{H}_{\boldsymbol{c}}^{-} = \cdots$$

$$\boldsymbol{H}_{\boldsymbol{b}}^{+} = \cdots \qquad \boldsymbol{H}_{\boldsymbol{c}}^{+} = \cdots$$

Impact map

- Conservation of the angular momentum

$$H^- = [H_a^-; H_b^-; H_c^-] \qquad H^+ = [H_a^+; H_b^+; H_c^+]$$

$$H^+ = H^-$$

$$\dot{q}^+ = (A^+)^{-1} A^- \dot{q}^-$$