Research Software Engineer Apprentice – Scientific Computing Department

Assessment Task 2

Finding the value of an integral is a typical example of tasks in scientific computing. In this problem, we wish to determine the value of π. This can be done by integration using a range of methods. We would like you to determine an approximate value for π by integrating the function:

$$I = \int_a^b f(x)dx$$

where a and b are the lower and upper limits, respectively. The Trapezoidal rule is one method that can be used to perform the integration and is given by:

$$I = \int_a^b f(x)dx \approx \Delta x \left\{ \sum_{i=1}^{n-1} f(x_i) + \frac{f(x_n) + f(x_0)}{2} \right\}$$

Here, $\Delta x = (b - a)/n$ is the width of the interval and n is the number of subintervals.

Task 1: **Write a program, for any value of n, to determine π using the Trapezoidal rule.**

The function, $f(x)$, is given by:

$$f(x) = \frac{4}{1 + x^2}$$

The integral to solve for π is given by:

$$I = \int_0^1 \frac{4}{1 + x^2} dx$$

Use $n = 4$ subintervals to approximate the value of π.

The percentage error in your answer can be found from:

$$\varepsilon = \left| \frac{True\ value - Approximate\ value}{True\ value} \right| \times 100\%$$

For this analysis, assume the true value for π is 3.1415667. What is the percentage error for $n = 4$?

Task 2: **An alternative approach is to use Simpson's rule. The integration is given by:**

$$I = \int_a^b f(x)dx \approx \frac{\Delta x}{3}\left\{f(x_0) + 4 \times \sum_{i=1,3,5}^{n-1} f(x_i) + 2 \times \sum_{i=2,4,6}^{n-2} f(x_i) + f(x_n)\right\}$$

As an example, for the case of $n = 4$, the integration is given by:

$$I \approx \frac{\Delta x}{3}\{f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\}$$

Write a program, for any value of n, to determine π using Simpson's rule. If $n = 4$, determine the error assuming the true value for π is 3.1415667.

Task 3: **For the Trapezoidal rule, what is the value of π when $n = 12$ and determine the error.**

Using your results, please complete the following table:

Method	N	Value of π	Error (%)
Trapezoidal	4		
Simpson	4		
Trapezoidal	12		

When complete, please your answers to the three tasks above to stfcapprentice@ukri.org with the subject line: RSE Assessment Task 2