Graphs and Equations

From: DLSU Physics Department. (n.d.). Skills Lab 3: Graphs and Equations. [URL] https://www.dlsu.edu.ph/wp-content/uploads/pdf/cos/physics/cos-graphs-and-equations.pdf

OBJECTIVES:

- ❖ To apply the rules in plotting the numerical results of an experiment.
- ❖ To linearize parabolic and hyperbolic graphs which will verify the actual relationship between two physical quantities.
- ❖ To interpret the graphs and determine the relationship between two physical quantities.
- ❖ Formulate an equation relating two or three quantities based on the data and the graphs.

THEORY:

A graphical presentation is often used as an effective tool to show explicitly how one variable varies with another. By plotting the numerical results of an experiment and observing the shape of the resulting graph, a relationship between two quantities can be established. The shape of the graph gives us a clue of the relationship of the variable involved. Some of the common ones are the following:

- ☐ A *straight-line* graph indicates a linear relationship between two quantities.
- ☐ A *hyperbolic* graph indicates an *inverse* relationship.
- ☐ A *parabolic* graph tells us of a specific kind of linear or direct relationship.

The specific equation relating the two variables of the graph can only be formulated when the graph is linearized. We will see how this can be done in the succeeding discussion.

A. Straight Line Graphs

A.1 ***Linear Relationship***

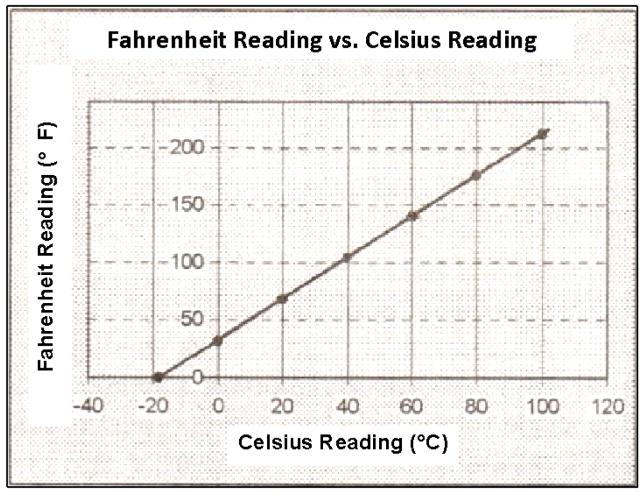
Figure 1 shows a straight-line graph that does not pass through the origin. This is a linear graph. it shows a *linear relationship* between the two variables. It means that there is a first-degree relationship between the Celsius readings and the Fahrenheit readings. The general equation for a linear graph is

$$y = mx + b \qquad (1)$$

where m and b are constants; m is the slope of the line and b is the y-intercept. The y-intercept of the line is the value of y when x is zero. If we take $y = 68°$, $x = 20°$, and $b = 32°$ in graph #1, the slope can be obtained using Eq. (1):

$$68° = m(20°) + 32° \Rightarrow m = 1.8 \; or \; 9/5 \qquad (2)$$

Celsius (°C)	Fahrenheit (°F)
0	32
20	68
40	104
60	140
80	176



Graph #1. Fahrenheit Reading vs. Celsius Reading

Substituting the value of the slope obtained in eq. (2) to Eq. (1) and considering that the y-axis is °F and the x-axis is °C, the equation relating Fahrenheit reading and Celsius reading is therefore:

$$°F = (9/5)°C + 32 \qquad (3)$$

We can also *extrapolate* values from the graph. If we extend the line downward until the temperature is $0°F$, we get the corresponding value in Celsius which is $17.8\,°C$. By *interpolation*, we get values within the line such as $50°C$ for the corresponding Fahrenheit reading of $122°F$.

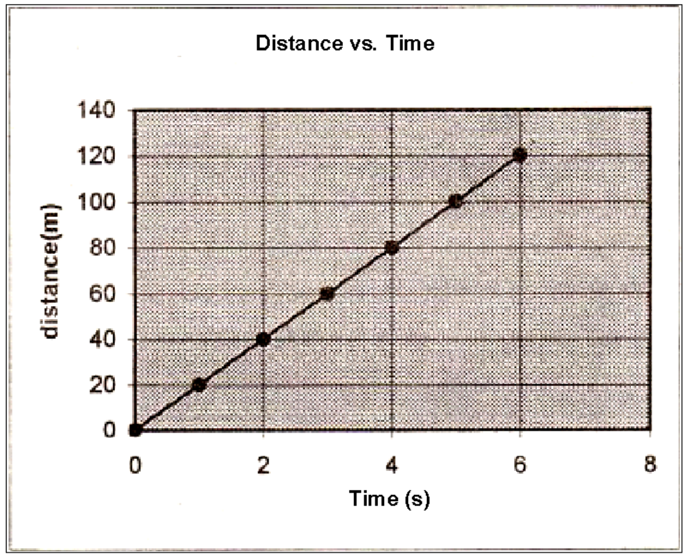
A.2 ***Direct Proportionality***

Figure #2 shows a straight line passing through the origin. The zero values for both variables simultaneously occur. When time is doubled the distance is also doubled. In this case, we say that the distance is directly proportional to time. In general, when two variables x and y are *directly proportional* to each other, the equation relating them is:

$$y \propto x \qquad \Rightarrow \qquad y = kx \quad \text{or} \quad k = \frac{y}{x} \tag{4}$$

where k is the constant of proportionality. This equation shows that the quotient of the two variables is always equal to a constant.

Time (s)	Distance (m)
0	0
1	20
2	40
3	60
4	80
5	100
6	120



Graph #2. Distance vs. Time Graph

In graph #2, the physical slope represents the constant *k*:

$$slope = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad \Rightarrow \quad \frac{\Delta d}{\Delta t} \tag{5}$$

The *physical slope* is always our concern in graphical analysis. The value is independent of the choice of scales and it expresses a significant fact about the relationship between the plotted variables. For example, the slope of the distance vs. time graph represents the average speed of the object.

On the other hand, the *geometrical slope* which is defined to be $tan\,,$ (where ϑ is the angle between the straight line connecting the points and the x-axis) depends on the inclination of the line and hence, on the choice of scales.

B. Parabolic Graphs

In general, a *parabolic graph* passing through the origin can be obtained for the quantities *x* and *y* obeying the following equations:

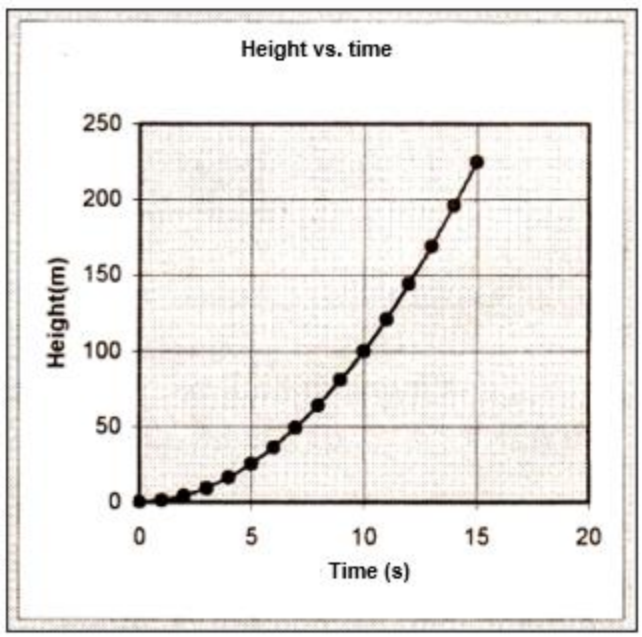
$$y = kx^2, y = kx^3, y = kx^4, ..., y = kx^n \quad (6)$$

The relationship between x and y can be expressed as $y \propto x^n$. Rewriting eq. (6),

$$\frac{y}{x^n} = k \ (constant)$$

the ratio of y and x^n is a constant. To verify the actual relationship, one has to *linearize* the graph, i.e., *plot* y *vs.* x^n, where *n* = 2,3,4…

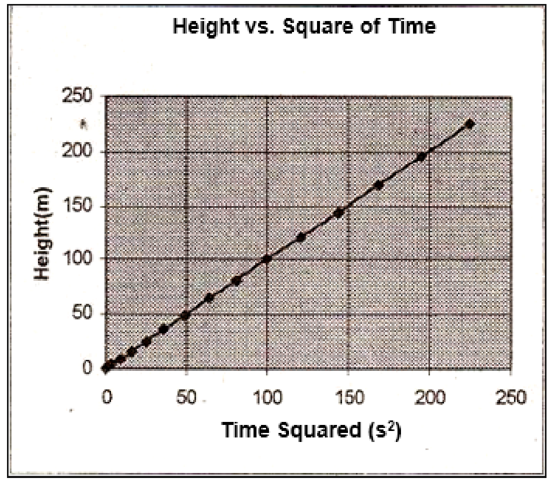
Time, *t* (s)	Height, *y* (m)
0	0
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225



Graph #3. A Parabolic Graph

Graph #3 shows a parabolic graph. From eq. (6), the value of n determines the specific equation relating x and y. By inspection, squaring the time in the data yields a *direct square relationship* between height and time. Thus, we say, "height is directly proportionally to the square of time." To verify this relationship, plot height vs. square of time. The result is shown in Graph #4.

Time, t (s)	Time squared, t^2 (s^2)	Height, y (m)
0	0	0
1	1	1
2	4	4
3	9	9
4	16	16
5	25	25
6	36	36
7	49	49
8	64	64
9	81	81
10	100	100
11	121	121
12	144	144
13	169	169
14	196	196
15	225	225



Graph #4. Linearized version of Graph #3

In general, if one quantity (y) varies directly with the square of another quantity (x^2) we write, $y \propto x^2$. In this case $n = 2$. Thus, the equation that correctly expresses the relationship of height (h) and time (t) in the data is:

$$\frac{h}{t^2} = k\ (constant) \quad \Rightarrow \quad h = kt^2$$

where the constant k represents the slope of *height* vs. *time squared* graph.

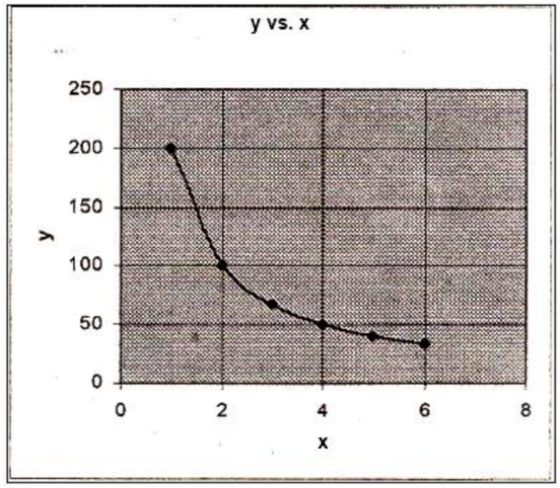
C. Hyperbolic Graphs

Hyperbolic graphs can be obtained for quantities obeying the following equations:

$$y = k/x, y = k/x^2, y = k/x^3, \ldots., y = \frac{k}{x^n} \tag{7}$$

A hyperbolic graph indicates an *inverse relationship* between two quantities i.e., $y\ \ 1/x^n$. The specific equation can be verified by determining the value of n. For $n = 1$, the equation is $y = k/x$.

x	y
1	200
2	100
3	67
4	50
5	40
6	33



Graph #5. A Hyperbolic Graph

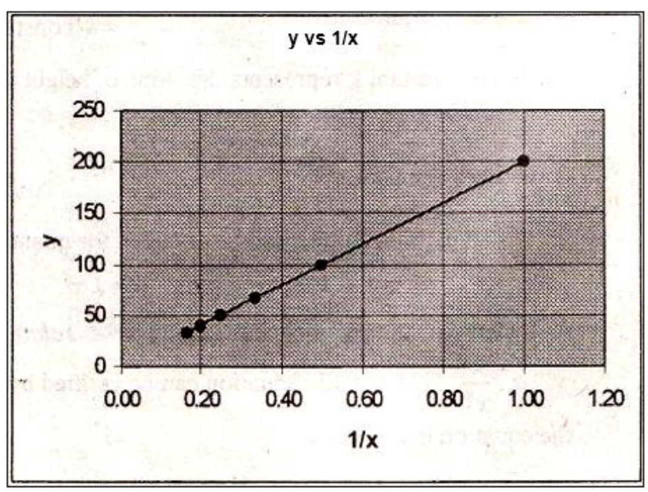
C.1 ***Inverse Proportionality***

Graph #5 shows a hyperbolic graph. To linearize it, try $n = 1$ such that $y = 1/x$. Plotting y vs. $1/x$ yields a straight-line graph as shown in Graph #6. Hence y is directly proportionally to $1/x$ or y is inversely proportional to x. In equation form

$$y \propto \frac{1}{x} \quad \Rightarrow \quad y = \frac{k}{x} \quad \Rightarrow \quad \text{or} \quad k = xy \tag{8}$$

where k is a constant which is equal to the slope of y vs. $1/x$ graph.

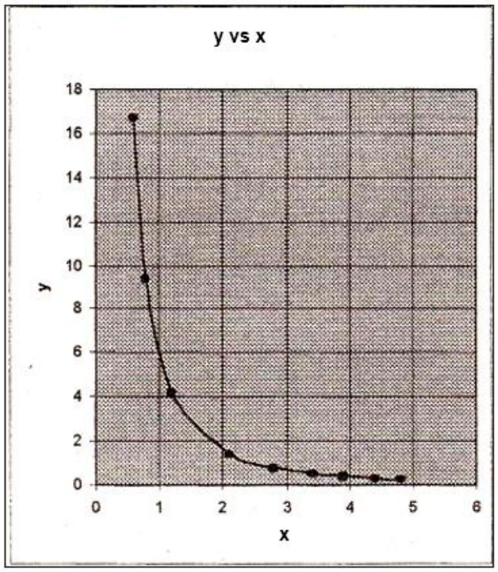
x	1/x	y
1	1	200
2	0.5	100
3	0.33	67
4	0.15	50
5	0.2	40
6	0.16	33



Graph #6. A linearized version of Graph #5

C.2 ***Inverse Square Proportionality***

Sometimes, plotting y vs. $1/x$ will not yield a straight line but plotting y vs. $1/x^2$ will yield one. This kind of relationship is called *inverse square proportionality.* The variable y is inversely proportional to the square of x. Graph #7 illustrates such a case.

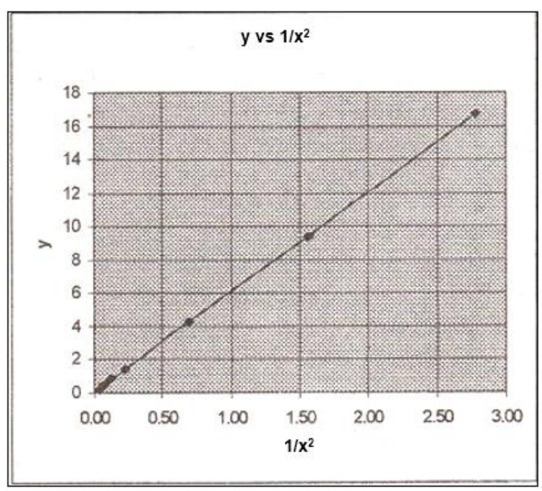


x	y
0.6	16.7
0.8	9.4
1.2	4.2
2.1	1.4
2.8	0.8
3.4	0.5
3.9	0.4
4.4	0.3

Graph #7. A Hyperbolic Graph

The linearized graph is shown in Graph #8. This can only be obtained if $n = 2$ such that $y = \frac{k}{x^2}$.

x	$1/x^2$	y
0.6	2.78	16.7
0.8	1.56	9.4
1.2	0.69	4.2
2.1	0.23	1.4
2.8	0.13	0.8
3.4	0.09	0.5
3.9	0.07	0.4
4.4	0.05	0.3
4.8	0.04	0.2



Graph #8. A linearized version of Graph #7

Rules for Drawing Graphs on Rectangular Coordinate Paper

1. Determination of Coordinates

Determine which of the quantities to be graphed the dependent variable is and which one is the independent variable. The *independent variable* is the quantity, which controls or causes a change in the other quantity (dependent variable) whenever it is increased or decreased. By convention, plot the independent variable along the *x-axis* and dependent variable on the *y-axis*.

2. Labeling the axes

Label each axis with the name of the quantity being plotted and its corresponding unit. Abbreviate all units in standard form.

3. Choosing the scale

Choose scales that are easy to plot and read. In general, choose scales for the coordinate axes so that the curve extends over most of the graph sheet. The same scale need not be used for both axes. In many cases it is not necessary that the intersection of the two axes represent the zero values of both variables. The number should increase from left to right and from bottom to top. In cases where the values to be plotted are exceptionally large or small, rewrite the numbers in scientific notation. Place the coefficients on the coordinate scale and the multiplying factor beside the unit used.

4. Location of Points

Encircle each point plotted on the graph to indicate that the value lies anywhere close to that point. Draw the curve up to the circle on one side. If several curves appear on the same sheet and the points might interfere, use squares and triangles to surround the dots of the second and third curves, respectively.

5. Drawing the curve

When the points are plotted, draw a smooth line connecting the points; ignore any points that are obviously erratic. “Smooth” suggests that the line does not have to pass exactly through each point but connects the general areas of significance. If there is a clue that the quantities are linear, then a straight line representing an average value should be used. There should be more or less equal number of points above and below the line. For nonlinear curves, points should be connected with a smooth curve so that the points average around the line. For Microsoft Excel users, this procedure is automatically done by a specific command.

6. Title of the Graph

At an open space near the top of the paper, state the *title* of the graph in the form of the *dependent variable* (y) *vs. the independent variable* (x).

EXERCISES:

1. The following data were obtained in an experiment relating time (t) (the independent variable) to the speed (v) of an accelerating object.

t(s)	0.5	1.0	1.5	2.0	2.5	3.0
v(m/s)	10	15	20	25	30	35

Plot these data on rectangular coordinate paper.
(a) Determine the slope of the graph
(b) What physical quantity does the slope represent?
(c) Determine the y-intercept of the graph. What does it represent?
(d) What is the equation of the curve?

2. The data below shows how the electric field (E) due to a point charge varies with distance (r).

Distance, r (meters)	1	2	3	4	5	6	7	8	9
Electric Field, *E* (N/C)	81	20.3	9.00	5.06	3.24	2.25	1.65	1.27	1.00

(a) Plot the given values (*E vs. r*). Select proper coordinate scales, label plot points, label the axes properly, and draw a smooth curve through points.
(b) Compare the slopes of the graph at r = 2 meters and r = 5 meters. Is the slope at r = 2 meters positive, negative, zero, or undefined? How about the slope at r = 5 meters? How do the slopes at the given points differ from each other?
(c) *Linearize* the graph. If necessary, compute different powers of variables and plot until you get a straight line.
(d) Determine the *equation of the line* obtained. Indicate the value of n, k, and other constants or intercepts present in the graph.

3. The following values represent the motion of a particle with a y-coordinate that varies in time.

Time, *t* (seconds)	0	1	2	3	4	5	6	7	8
Distance, *y* (meters)	0	15	20	15	0	-25	-60	-105	-160

(a) Plot the given values (*y vs. t*). Select proper coordinate scales, label plot points, label the axes properly, and draw a smooth curve through points.
(b) Compare the slopes of the graph at t = 1 second and t = 3 seconds. Is the slope at t = 1 second positive, negative, zero, or undefined? How about the slope at t = 3 seconds? How do the slopes at the given points differ from each other?
(c) *Linearize* the graph. If necessary, compute different powers of variables and plot until you get a straight line.
(d) Determine the *equation of the line* obtained. Indicate the value of n, k, and other constants or intercepts present in the graph.

4. Potential energy, U_s, as a function of *x*-coordinate for the mass-spring system.

x-coordinate (m)	-5	-4	-3	-2	-1	0	1	2	3	4	5
U_s (Joules)	375	240	135	60	15	0	15	60	135	240	375

(a) Plot the given values (U_s *vs. x*). Select proper coordinate scales, label plot points, label the axes properly, and draw a smooth curve through points.

(b) Compare the slopes of the graph at x = -2 meters, x = 0 meters, and x = 2 meters. Is the slope at x = -2 meters positive, negative, zero, or undefined? How about the slope at x = 0 meters? How about the slope at x = 2 meters? How do the slopes at the given points differ from each other?

(c) *Linearize* the graph. If necessary, compute different powers of variables and plot until you get a straight line.

(d) Determine the *equation of the line* obtained. Indicate the value of *n*, *k*, and other constants or intercepts present in the graph.