Assignment 2

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Q.1

A2

$5 - \frac{4}{3} = \frac{15-4}{3} = \frac{11}{3} = 3.$

a)

No pivoting

$$\left[\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ 3 & 4 & 3 & 10 \\ 1 & 5 & -1 & 7 \\ 6 & 3 & 7 & 15 \end{array}\right]$$

$x \cdot (-3.6667) - 5 = 0$

$x \cdot (-3.6667) = 5$

$x = \frac{5}{-3.6667}$

$R2 - \frac{1}{3} R1 \longrightarrow R1$

$$\left[\begin{array}{ccc|c} 3 & 4 & 3 & 10 \\ 0 & +3.6667 & -2 & 3.6667 \\ 6 & 3 & 7 & 15 \end{array}\right]$$

$-5 = -\frac{5}{3.6667} \cdot 3.6667$

$R_3 - 2R_1 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 3 & 4 & 3 & 10 \\ 0 & +3.6667 & -2 & 3.6667 \\ 0 & -5 & 1 & 5 \end{array}\right]$$

$1 - \left(-2 \cdot \frac{-5}{3.6667}\right) = -1.7272$

$R_3 - \frac{-5}{3.6667} R_2 \longrightarrow R_3$

$$\left[\begin{array}{ccc|c} 3 & 4 & 3 & 10 \\ 0 & +3.6667 & -2 & 3.6667 \\ 0 & 0 & -1.7272 & 0 \end{array}\right]$$

$-1.7272 x_3 = 0$

$x_3 = 0$

$-3.6667 x_2 - 2x_3 = 3.6667$

$+3.6667 x_2 - 0 = 3.6667$

$x_2 = 1$

$3x_1 + 4x_2 + 3x_3 = 10$

$3x_1 + 4 + 0 = 10$

$3x_1 = 6$

$x_1 = 2$

$x_1 = 2$

$x_2 = 1$

$x_3 = 0$

$$\frac{5}{2} - \frac{7}{2}u = 0$$

$$\frac{5}{2} = \frac{7}{2}u$$

$$u = \frac{5}{7}$$

6)

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ 3 & 4 & 3 & 10 \\ 1 & 5 & -1 & 7 \\ 6 & 3 & 7 & 15 \end{array}$$

$R_1 \leftrightarrow R_3$

$$\begin{array}{ccc|c} 6 & 3 & 7 & 15 \\ 1 & 5 & -1 & 7 \\ 3 & 4 & 3 & 10 \end{array}$$

$R_2 - \frac{1}{6} R_1 \longrightarrow R_2$

$$\begin{array}{ccc|c} 6 & 3 & 7 & 15 \\ 0 & 4.5 & -2.1667 & 4.5 \\ 3 & 4 & 3 & 10 \end{array}$$

$R_3 - \frac{1}{2} R_1 \longrightarrow R_3$

$$\begin{array}{ccc|c} 6 & 3 & 7 & 15 \\ 0 & 4.5 & -2.1667 & 4.5 \\ 0 & 2.5 & -0.5 & 2.5 \end{array}$$

$R_3 - \frac{2.5}{4.5} R_2 \longrightarrow R_3$

$$\begin{array}{ccc|c} 6 & 3 & 7 & 15 \\ 0 & 4.5 & -2.1667 & 4.5 \\ 0 & 0 & 0.7037 & 0 \end{array}$$

$x_3 = 0$

$\cancel{x_2 =}\ 4.5x_2 - 2.1667x_3 = 4.5$

$4.5x_2 = 4.5$

$x_2 = 1$

$6x_1 + 3x_2 + 7x_3 = 15$

$6x_1 + 3 = 15$

$6x_1 = 12$

$x_1 = 2$

$x_1 = 2$
$x_2 = 1$
$x_3 = 0$

Q2

```
Epsilon:1.490200e-08, Relative Error X1:3.00e+00 , Relative Error X2:6.71e+07, Condition: 3.014213e+16
Epsilon:1.341180e-07, Relative Error X1:2.00e+00 , Relative Error X2:8.41e+02, Condition: 2.208770e+14
Epsilon:1.490200e-07, Relative Error X1:1.99e+00 , Relative Error X2:3.41e+04, Condition: 1.765839e+14
Epsilon:3.725500e-07, Relative Error X1:2.00e+00 , Relative Error X2:3.04e+02, Condition: 2.878851e+13
```

As we can see from the results, the farther you get away from the machine epsilon (1.4902e-08), the lower the condition number is. If is a condition number is high, that means it is ill-conditioned, and gives it more room for round off error.

Q3

```
Experiment a\b        No Pivoting        cond(A)
-------------------------------------------------------------
1    1.832783e-10     8.620528e-08       2.602101e+03
2    3.747209e-10     3.690514e-06       4.842738e+03
3    2.177635e-09     7.292065e-07       2.598909e+04
4    3.584733e-09     1.237084e-06       3.892096e+04
5    1.100695e-09     1.671659e-07       1.563455e+04
>>
```

The more ill-conditioned the matrix A is, the greater the errors are.

GE

```
function b = GE(A)
a = A;
b = gauss_eliml(a);

function A = gauss_eliml(A)
    [n, ~] = size(A);
    L = zeros(n);

    for k = 1:n - 1

        for i = k + 1:n
            m = A(i, k) / A(k, k);

            for j = k + 1:n
                A(i, j) = A(i, j) - m * A(k, j);
            end
            L(i, k) = m;
        end
        A(k + 1:n, k) = 0;
    end
    A = A + L;
end
```

```
end
```

backward

```
function x = backwards(B, b, ipivot)
b = b(ipivot, :);
z = [];
U = triu(B);
[n, n] = size(B);
L = tril(B, -1) + eye(n);


% Lower trianglular thing
z(1) = b(1);
for i = 2:n
    sum = 0;
    for j = 1:i-1
        sum = sum + L(i,j)*z(j);
    end
    z(i) = b(i) - sum;
end

%Upper Triangular ting
z = z';
x = zeros(1, n);
x(n) = z(n)/U(n, n);

for i = n - 1: -1: 1
    sum2 = 0;
    for j = i+1:n
        sum2 = sum2 + U(i,j)*x(j);
    end
    x(i) = (z(i) - sum2)/U(i,i);
end
end
```

main_ge

```
function main_ge
n = 2000;
experiment = 0;
fprintf("Experiment a\\b      No Pivoting      cond(A)\n")
fprintf("----------------------------------------------------------\n")
for i = 1:5
A = randn(n);
x = ones(n, 1);
b = A*x;
B = GE(A);
%Backwards Sub Stuff
ipivot = 1:size(A);
c = backwards(B, b, ipivot)';
better_c = A\b;
```

```
r = b - A*c;
error_c = cond(A)*((norm(r)/norm(b)));
r_better_c = b - A*better_c;
error_better_c = cond(A)*(norm(r_better_c)/norm(b));
experiment = experiment + 1;

fprintf("%d\t%d \t%d \t %d\n", experiment, error_better_c, error_c, cond(A))
end
end
```

Q4

4. a) $|f(x) - p_n(x)| \leq \frac{M}{4(n+1)} h^{n+1}$ $n=5$

$= \frac{M}{4(n+1)} h^{n+1}$ $M = \max_{a \leq t \leq b} |f^{(n+1)}(t)|$

$= \frac{e}{4(5+1)} \left(\frac{1}{5}\right)^{5+1}$ $= \max_{0 \leq t \leq 1} f(e^x) \rightarrow \max_{0 \leq t \leq 1} e^x$

$h = (b-a)/n = (1)/5$ $= e^1$ or e

$= \frac{e}{24} \left(\frac{1}{5^6}\right)$

$= \frac{e}{375000}$

b) $|10^{-8}| \leq \frac{\max f^{(n+1)}(e^x)}{4(n+1)} h^{n+1}$

$\leq \frac{e^1}{4n+4} (h)^{n+1}$

$|10^{-8}| \leq \frac{e^1}{4n+4} \cdot \left(\frac{1}{n}\right)^{n+1}$

Trying degrees

$n=5 \rightarrow e/375000$ or $7.24875 E-6$

$n=6 \rightarrow e/[illegible]$ or $3.46799 E-7$

$n=7 \rightarrow e/[illegible]$ or $1.47353 E-8$

$n=8 \rightarrow$ [illegible] or $5.62577 E-10$

degree $\underline{\underline{8}}$

5. a) $|error| \leq \frac{M}{4(3+1)} \frac{0.3}{3^{3+1}}$ $h = \frac{b-a}{n} \rightarrow \frac{0.3-0}{3}$

$= \frac{0.3}{3}$

$\leq$

c)

Q.5

```
f_x =

    1.0247    1.0724


M =

    0.9375


error =

   5.8594e-06


ans_c =

   1.0e-04 *

    0.0492    0.3053
```

Q.6

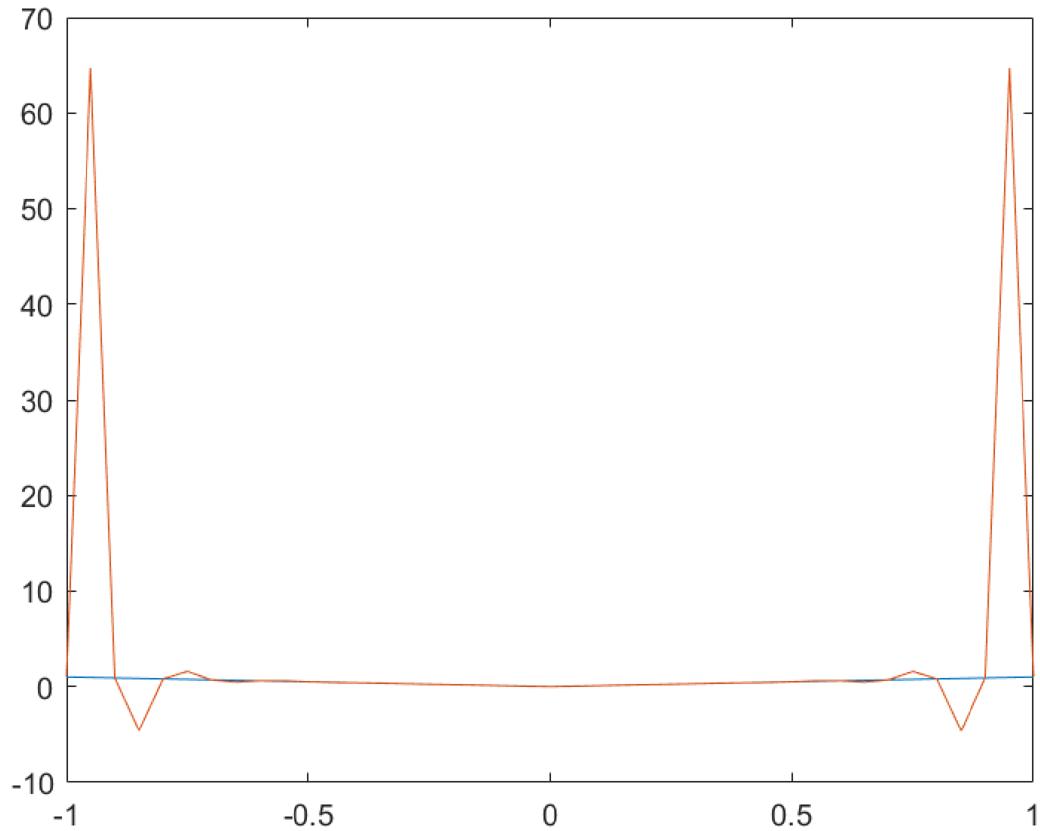

Figure 1: Question 6a

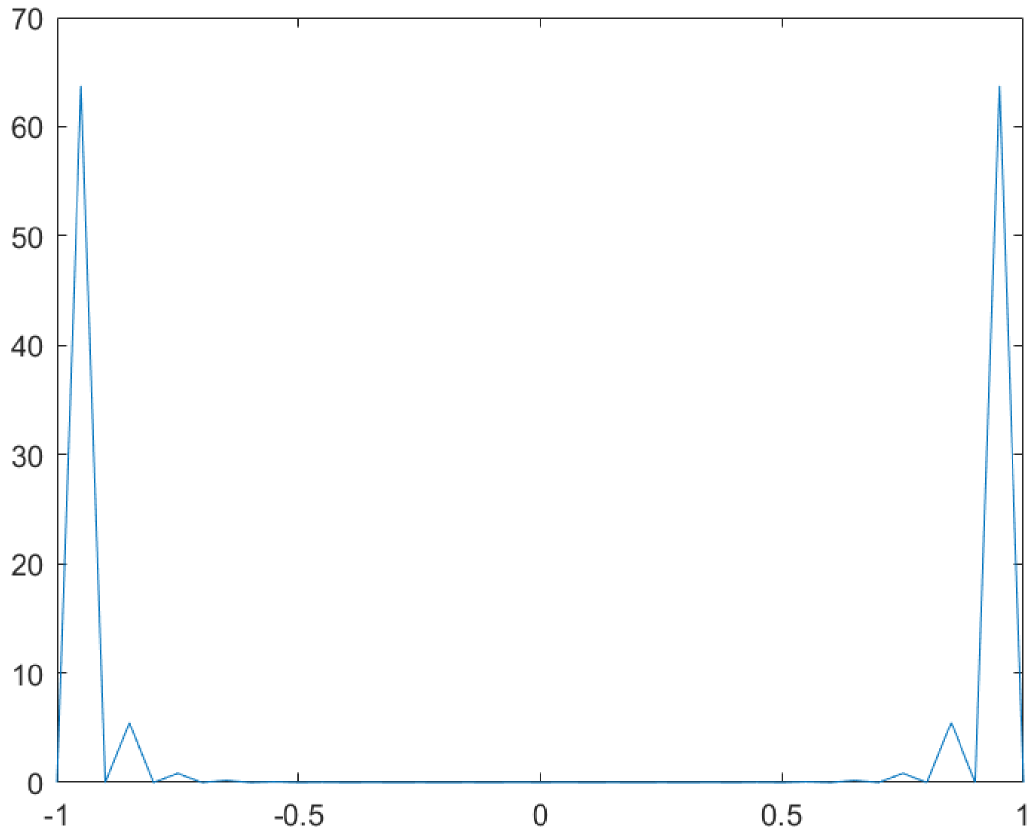

Figure 2: Question 6b

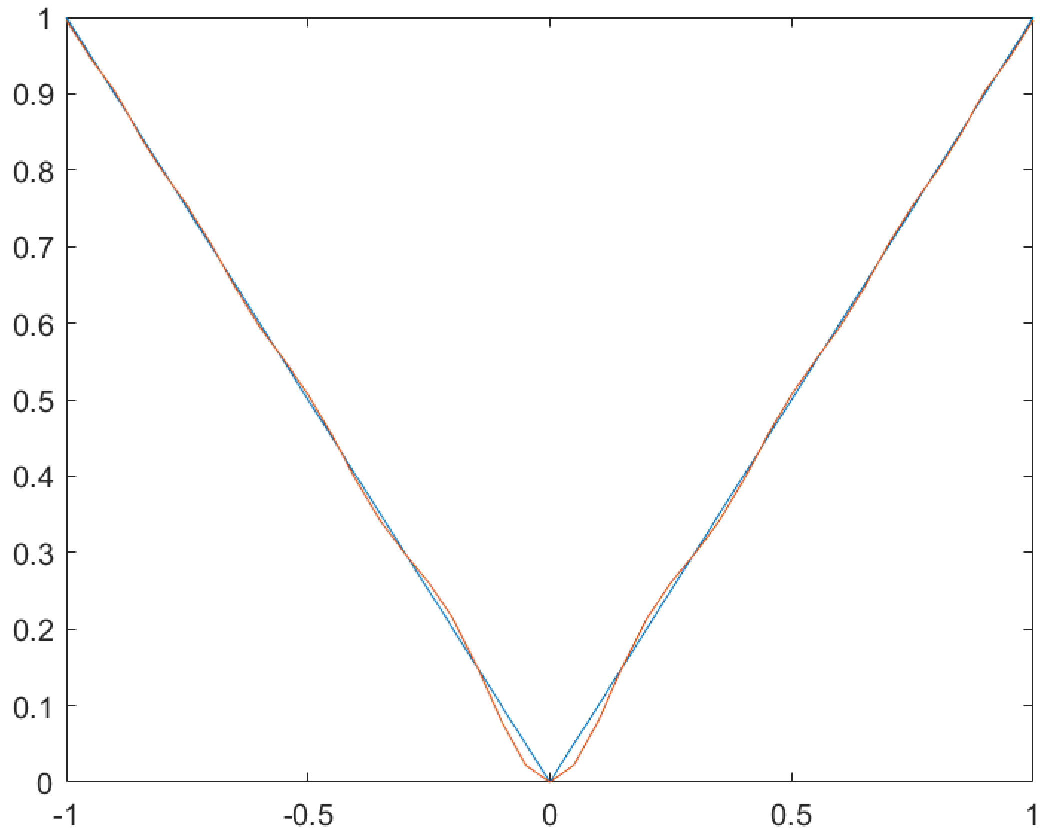

Figure 3: Question 6c

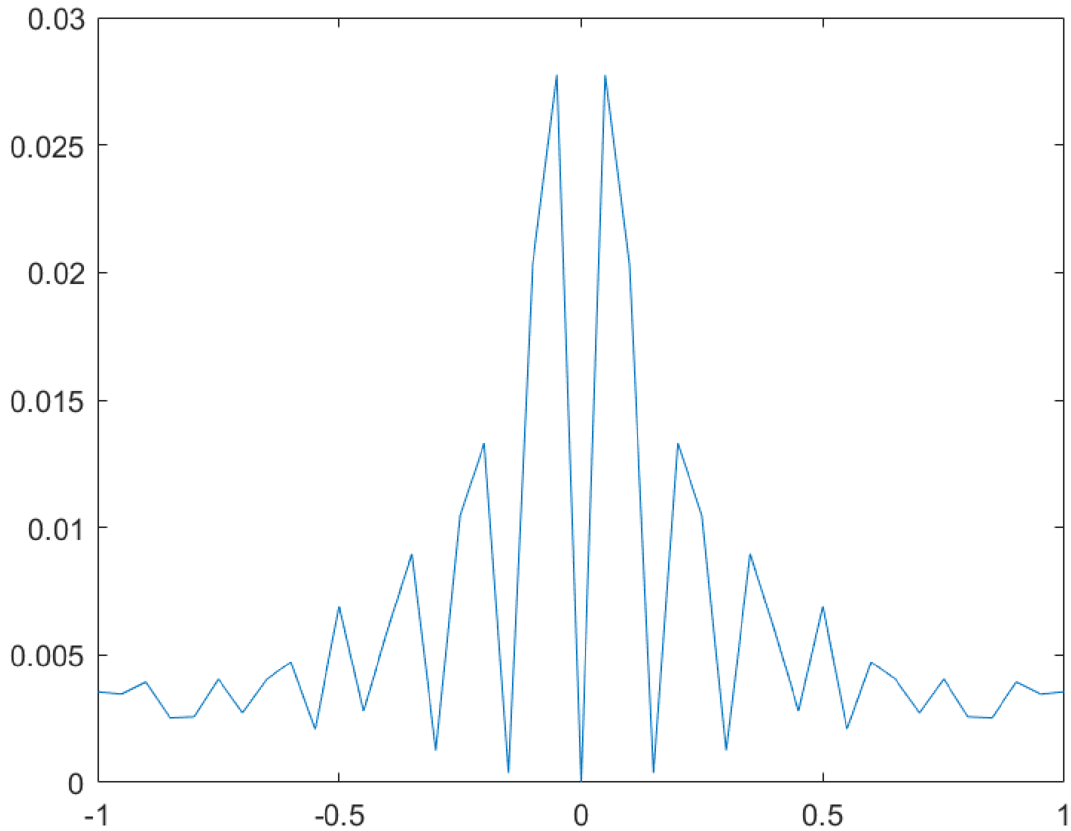

Figure 4: Chebyshev Error

Q.7

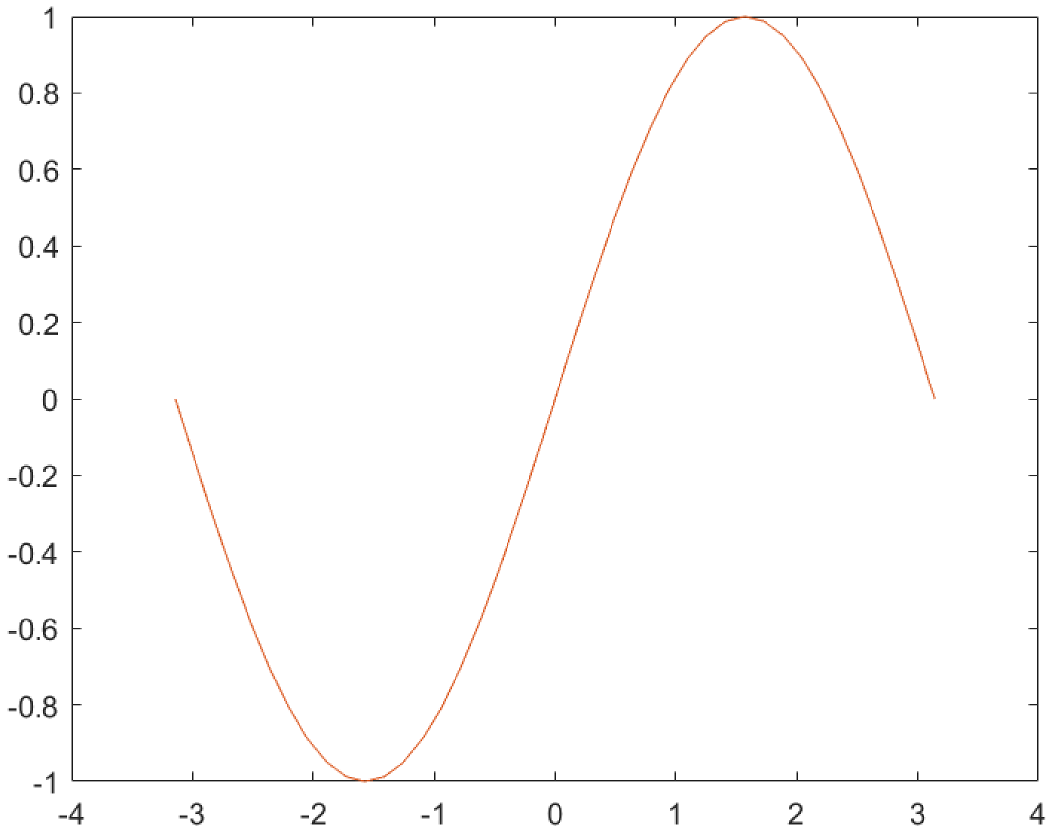

Figure 5:Question 7a

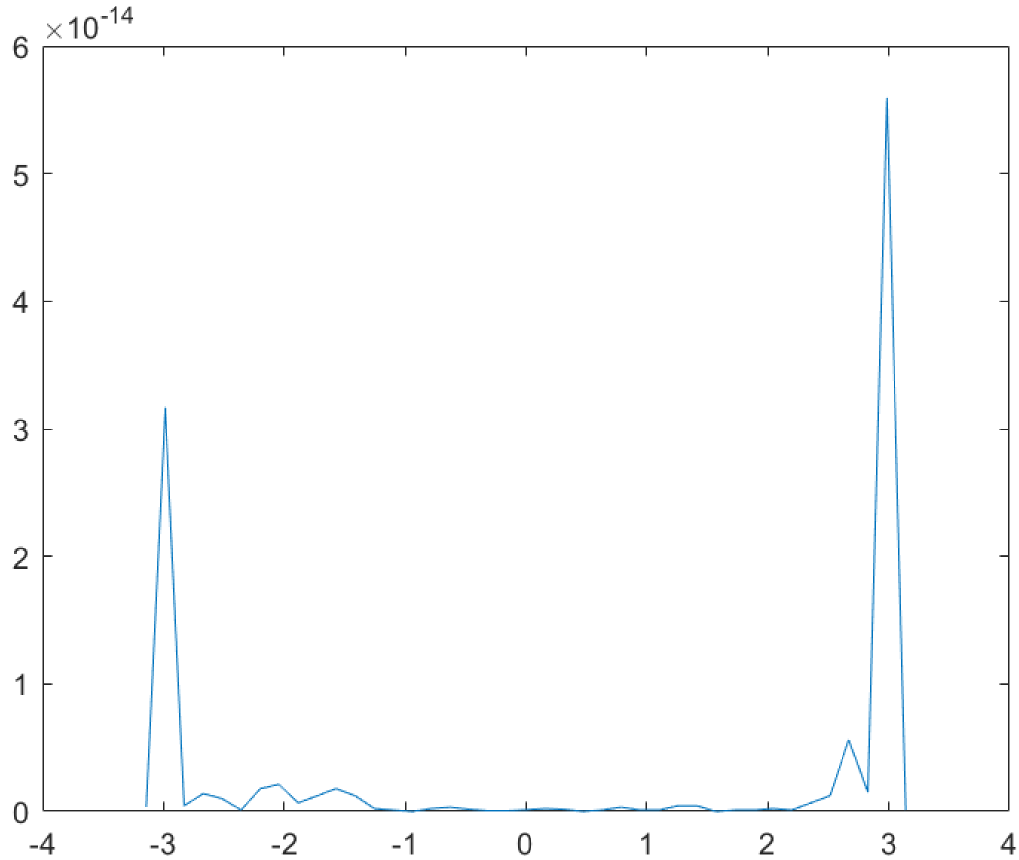

Figure 6:Question 7b

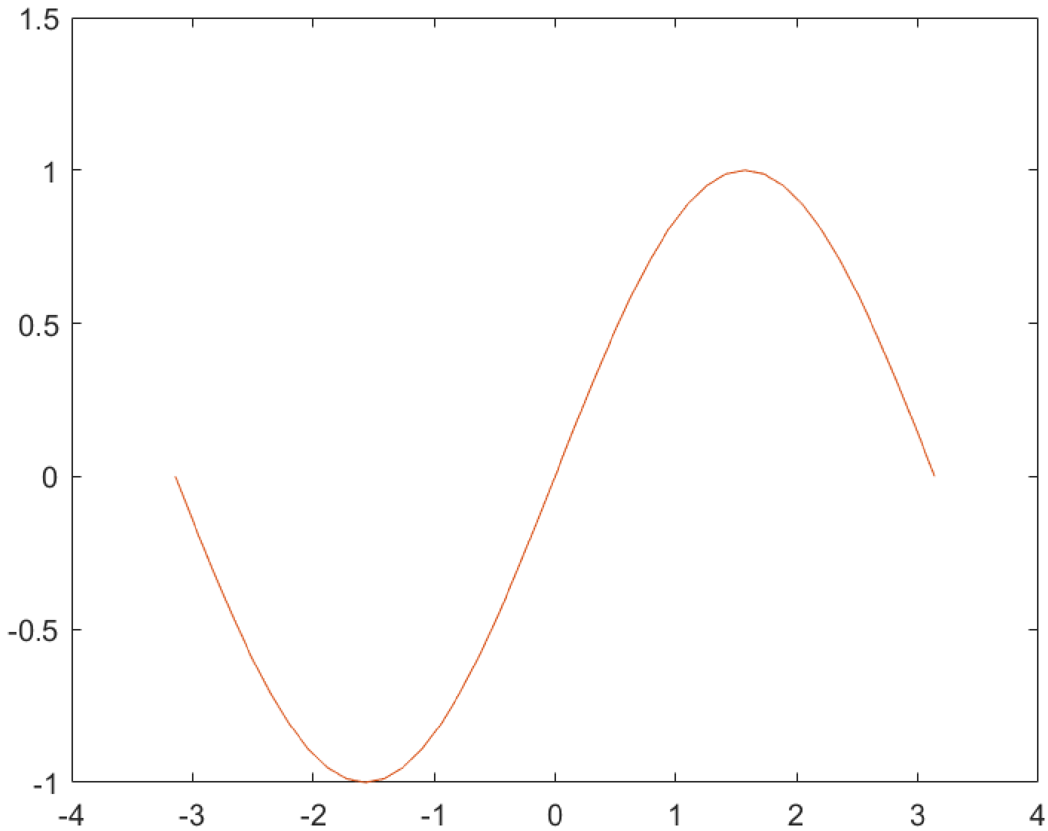

Figure 7: Question 7c Chebyshev

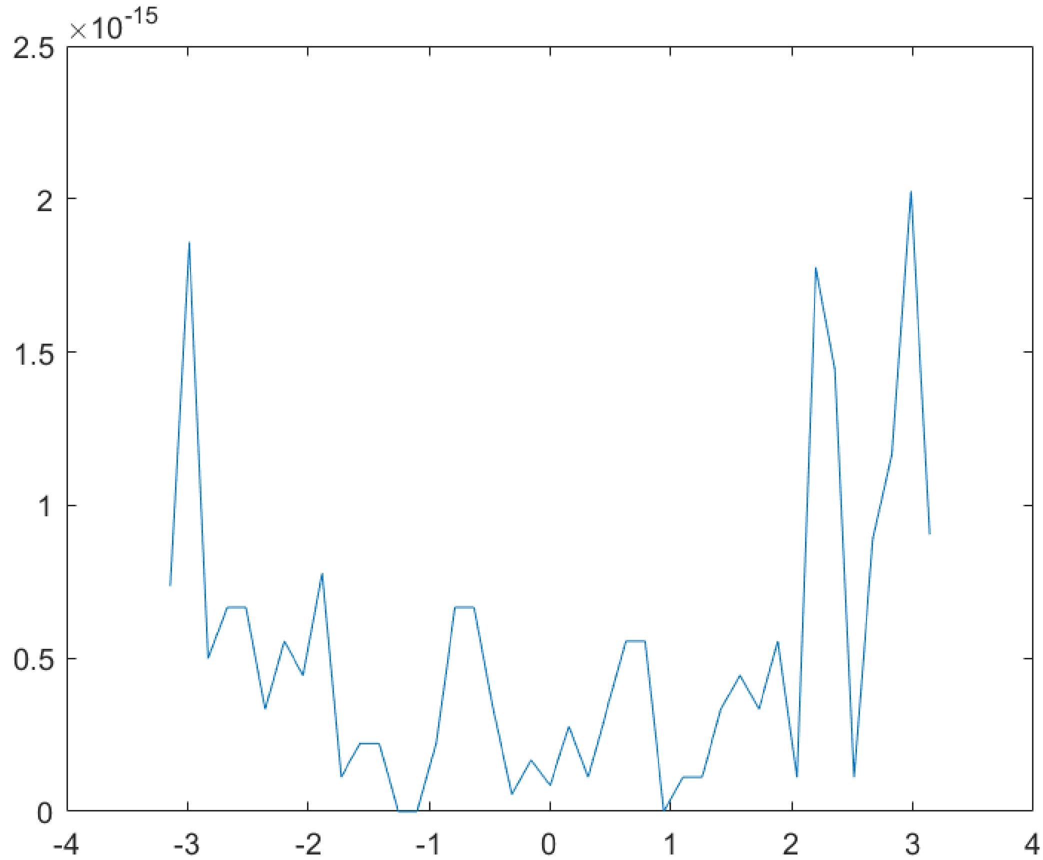

Figure 8: Question 7c Errors

Explanation

The errors are much higher for |x| compared to sin(x) for a variety of reasons. For our interpolation of |x| on equally spaced points between [-1, 1], we can see that there is a lot of oscillation near the end of the interval. This is the result of Runge's phenomenon, since we have equispaced interpolation points. For the Chebyshev nodes, these nodes are not evenly spaced, and the maximum error is guaranteed to reduce with increasing error, which typically increases Runge's phenomenon. For sin(x) over [-pi, pi], the interpolation is much nicer because there is a series expansion which turns it into a polynomial. We get very accurate results for both equally spaced points and Chebyshev points for sin(x), which allows us to conclude that the errors in |x| are primarily from Runge's phenomenon, as we can see Chebyshev nodes help to reduce the error. However, interpolating sin(x) will still have lower errors.

Question 8

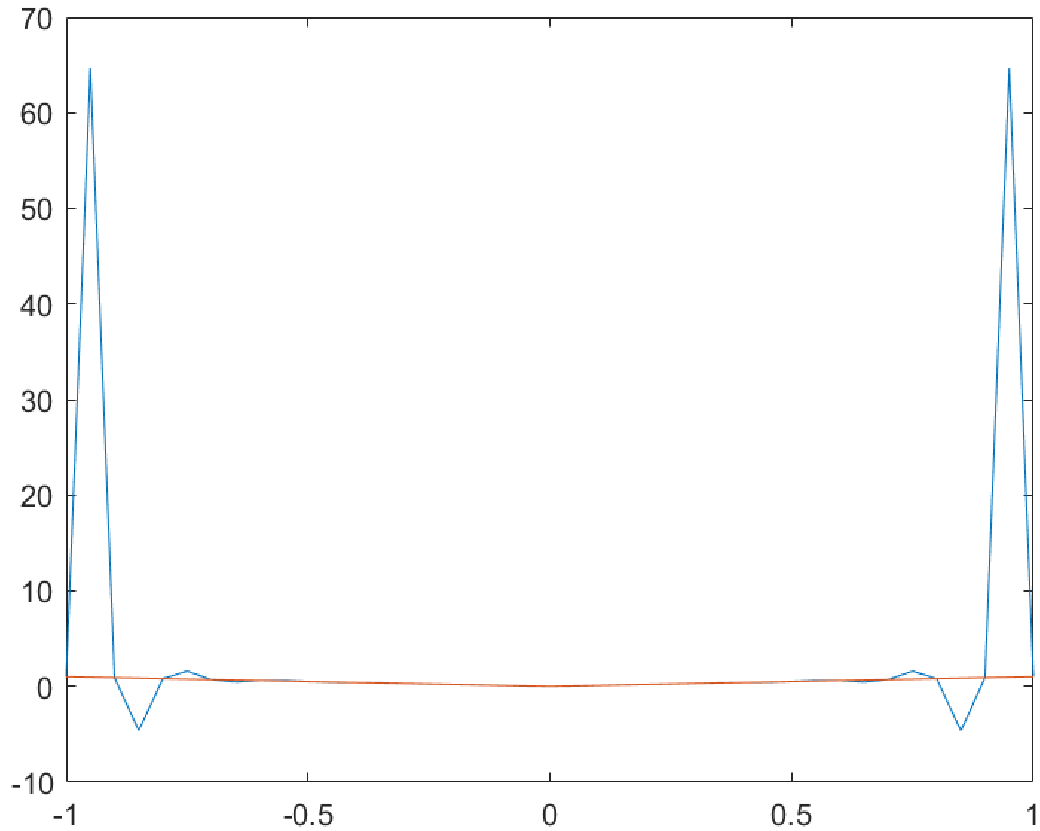

Figure 9: Question 8

Newton

```
function p = newton(x, y, degree)
t = sym('t');
a = divDif(x, y)
terms = 1;
poly_eq = a(1);
    for i = 2:degree+1;
        terms = terms * (t - x(i-1));
        poly_eq = poly_eq + terms*a(i);

    end
p = expand(poly_eq);
sym2poly(p);

function coeff = divDif(x, y)
n=length(x);

y=y';
y(n,n)=0;
    for column=2:n
        for row=column:n
```

```
        y(row,column)=(y(row,column-1)-y(row-1,column-1))/...
            (x(row)-x(row-column+1));
        end
    end
coeff(n)=0;
    for row=1:n
        coeff(row)=y(row,row);
    end
end
end
```

hornerN

```
function horny = hornerN(coefs, val, degree)
horny = coefs(1);
    for i = 2:degree + 1
        horny = coefs(i) + (val * horny);
    end
end
```