Zombie Epidemic

Cali Ho, Pranav Kode, Ruochao Pei, James Zhang

Outline

1. Train Model (SIZ)
2. City Model (SIZR)
3. Transport Models (SIZ and SIZR)

Train Model: SIZ

$$S_T \xrightarrow{\beta S_T Z_T} I_T \xrightarrow{\rho I_T} Z_T$$

become infected

become zombie

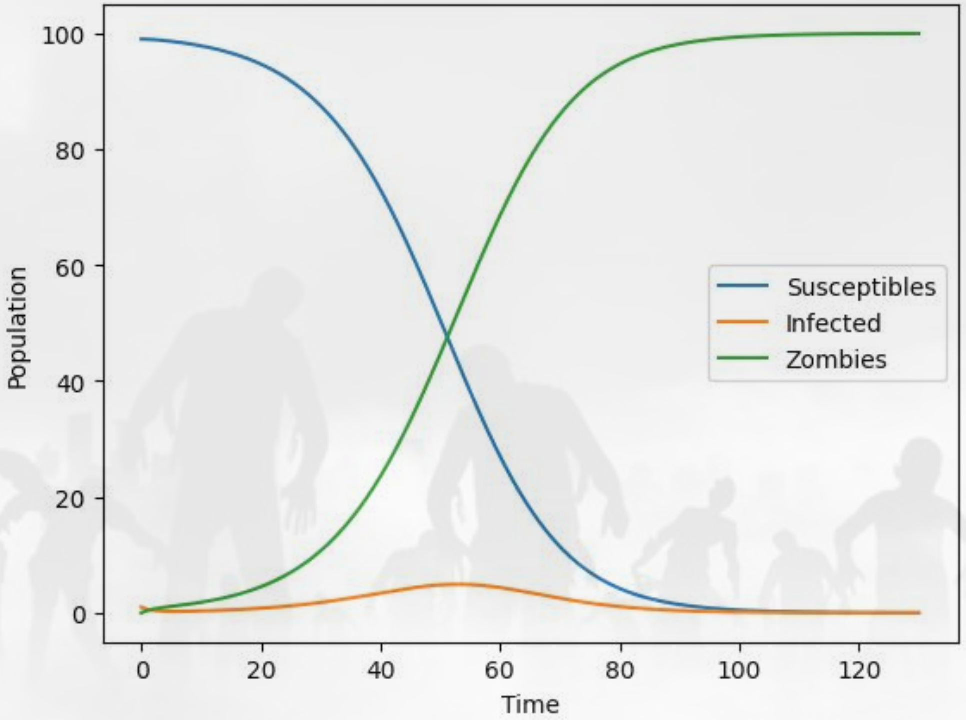
$$\begin{cases} S'_T = -\tilde{\beta} S_T Z_T \\ I'_T = -\rho I_T + \tilde{\beta} S_T Z_T \\ Z'_T = \rho I_T \end{cases}$$

Train to Busan Parameters

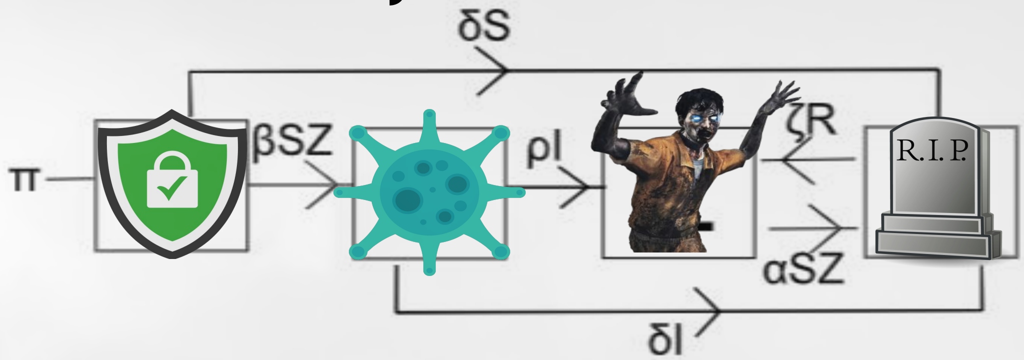
$\beta = 0.0011$
$\rho = 0.5$

$S_T(0) = 99$
$I_T(0) = 1$
$Z_T(0) = 0$

$S_T(50) = 50$



City Model: SIZR



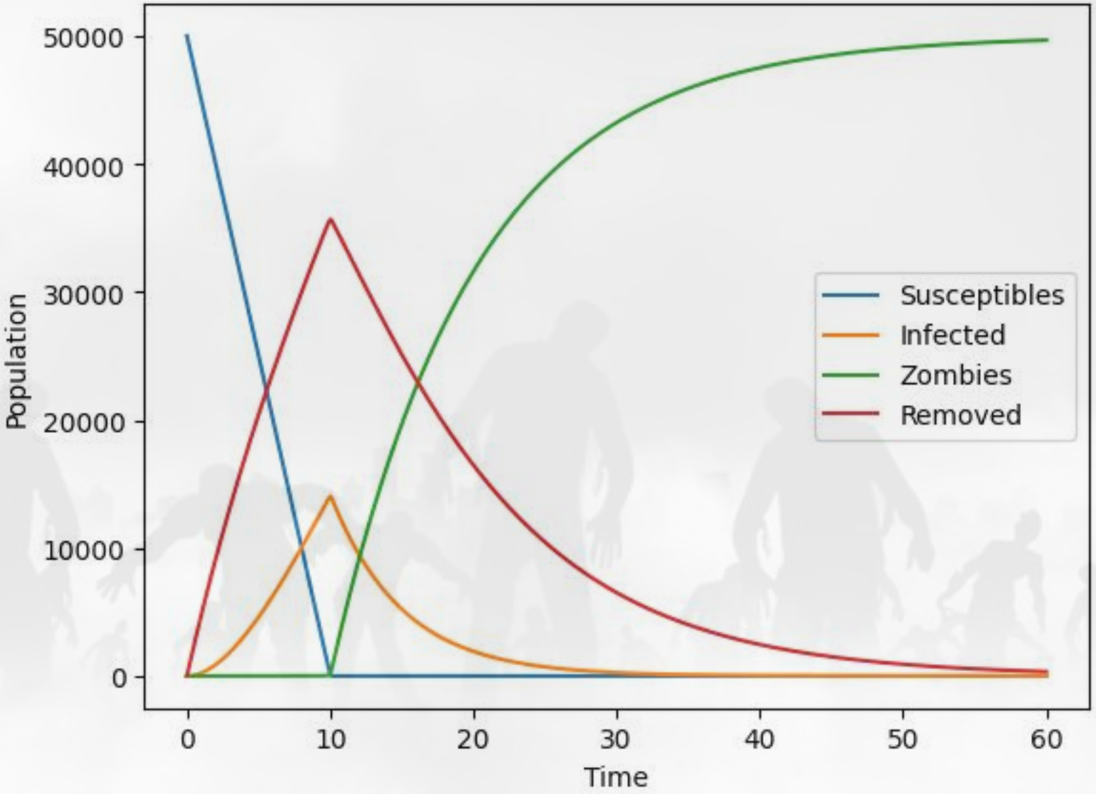
birth | become infected | death of susceptible

become zombie | death of infected

resurrection | zombie killed

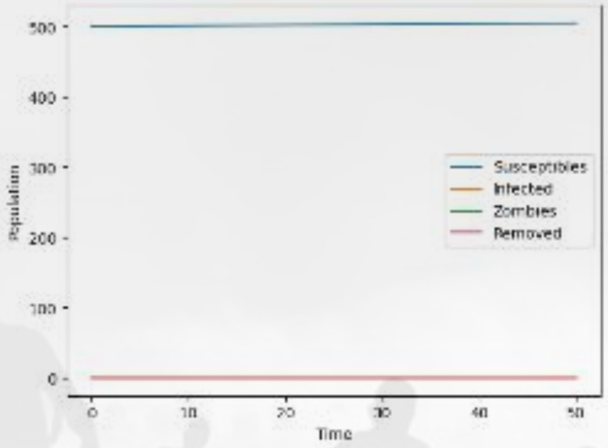
$$\begin{cases} S' = \Pi - \beta S Z - \delta S \\ I' = \beta S Z - \rho I - \delta I \\ Z' = \rho I + \zeta R - \alpha S Z \\ R' = \delta S + \delta I + \alpha S Z - \zeta R \end{cases}$$

SIZR Simulation



Linear Stability Analysis (SIZR)

- Disease Free Equilibrium (DFE)
  - At the DFE, (S, I, Z, R) = (N, 0, 0, 0), where N is the total population.
  - The eigenvalues are all positive



$$J(DFE) = \begin{bmatrix} -\delta & 0 & -\beta N & 0 \\ 0 & -\rho - \delta & \beta N & 0 \\ 0 & \rho & -\alpha N & \zeta \\ \delta & \delta & \alpha N & -\zeta \end{bmatrix}$$

Linear Stability Analysis (SIZR)

- Final Equilibrium (FE)
  - At the FE, (S, I, Z, R) = (0, 0, Z, 0), where Z is the number of zombies.

$$J(FE) = \begin{bmatrix} -\beta Z^* - \delta & 0 & -\beta S^* & 0 \\ \beta Z^* & -\rho - \delta & \beta S^* & 0 \\ -\alpha Z^* & \rho & -\alpha S^* & \zeta \\ \alpha Z^* + \delta & \delta & \alpha S^* & -\zeta \end{bmatrix}$$

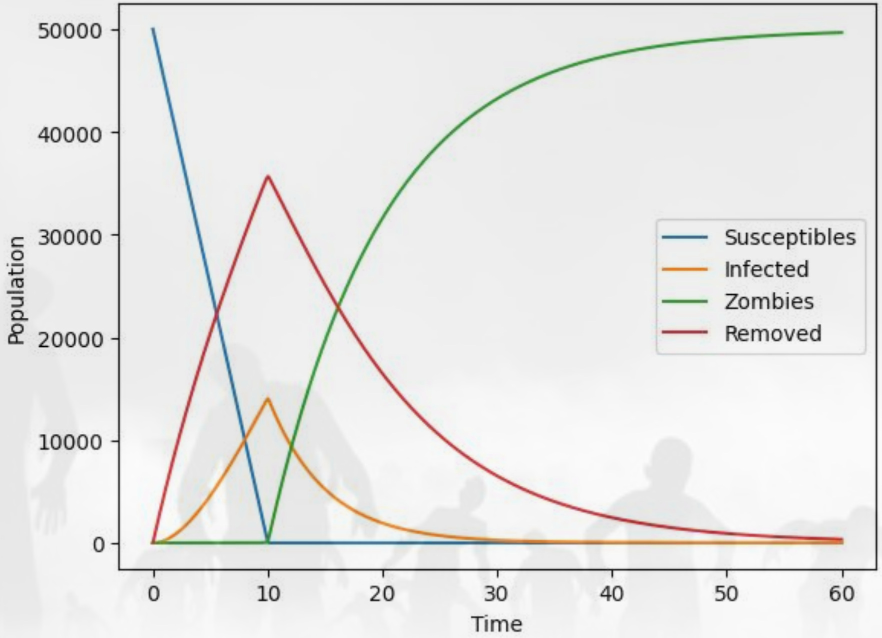
- The eigenvalues are

$$\lambda = 0, -\beta \bar{Z}, -\rho, -\zeta$$

- This indicates that this state is stable

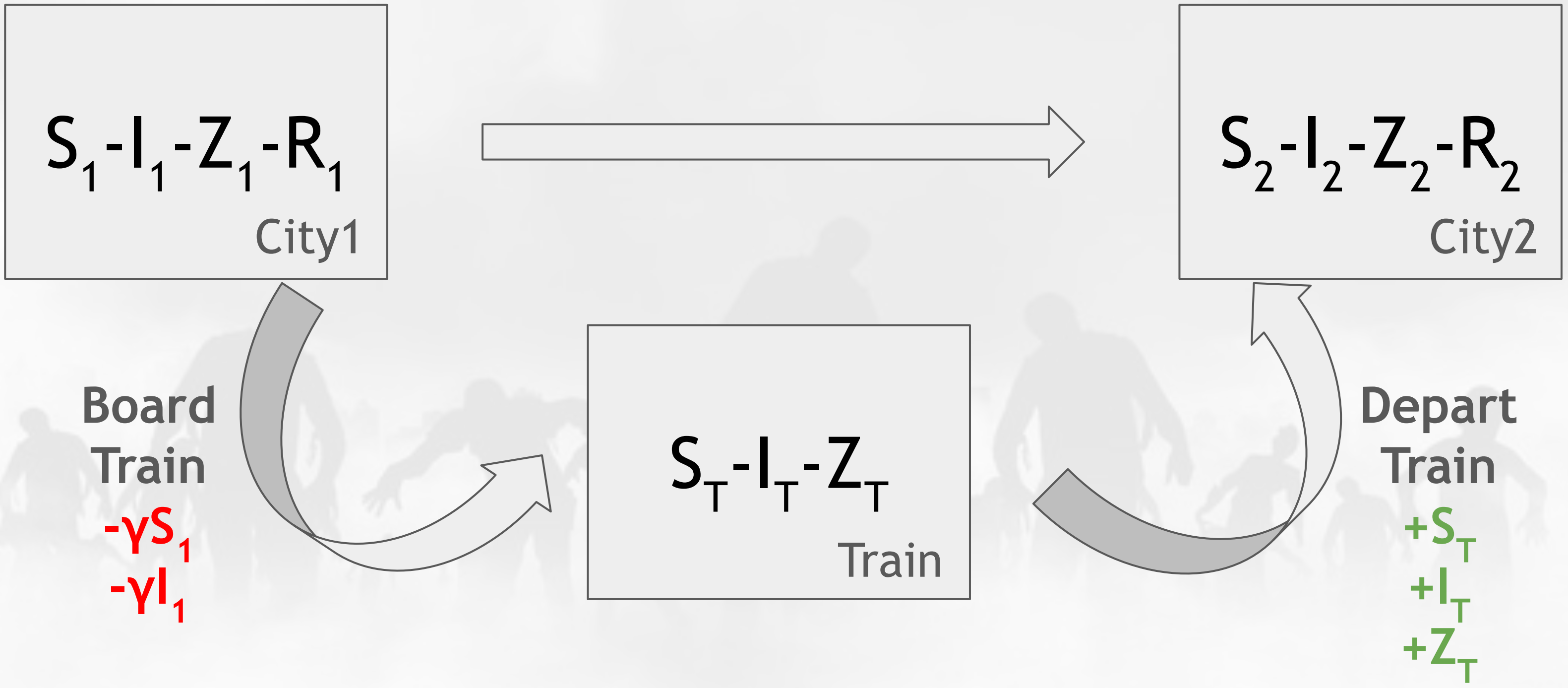
Linear Stability Analysis Conclusion

- The disease free equilibrium is unstable
- The final equilibrium is stable



- ZOMBIES!!!

One-way Train Diagram



One-Way Train Equations

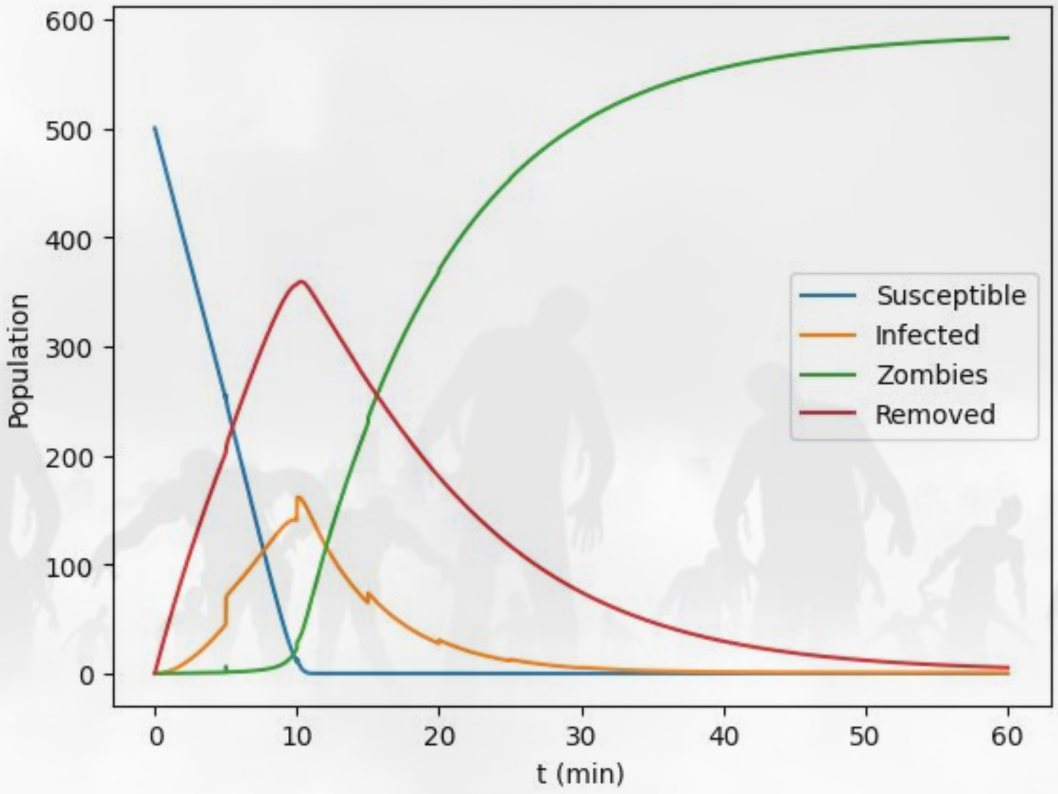
$$\begin{cases} S_1' = \Pi - \beta S_1 Z_1 - \delta S_1 - \gamma S_1 \\ I_1' = \beta S_1 Z_1 - \rho I_1 - \delta I_1 - \gamma I_1 \\ Z_1' = \rho I_1 + \zeta R_1 - \alpha S_1 Z_1 \\ R_1' = \delta S_1 + \delta I_1 + \alpha S_1 Z_1 - \zeta R_1 \end{cases}$$

$$\begin{cases} S_T' = -\widetilde{\beta} S_T Z_T \\ I_T' = -\rho I_T + \widetilde{\beta} S_T Z_T \\ Z_T' = \rho I_T \\ S_T(0) = \gamma S_1, I_T(0) = \gamma I_1 \end{cases}$$

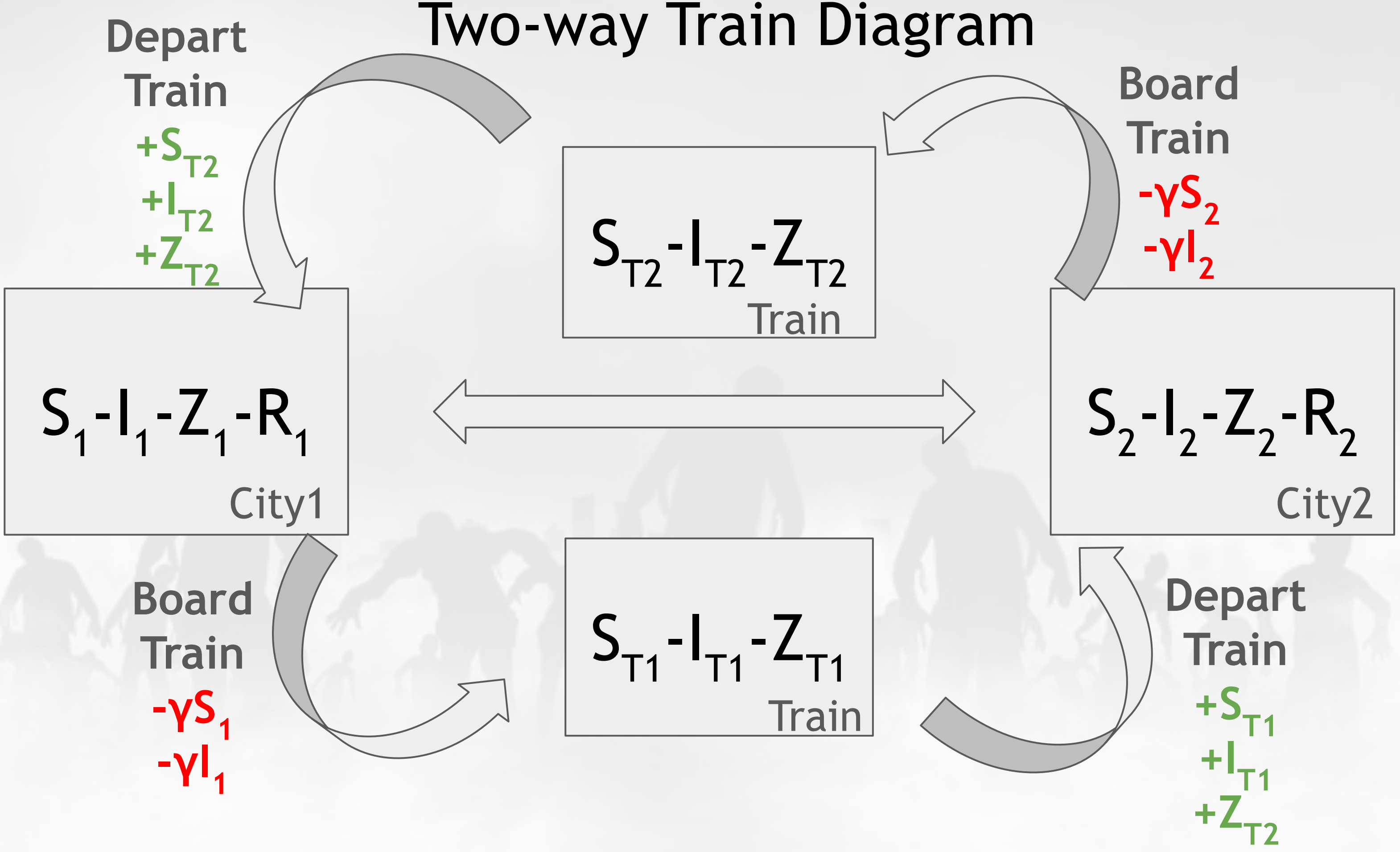
$$\begin{cases} S_2' = \Pi - \beta S_2 Z_2 - \delta S_2 + S_T \\ I_2' = \beta S_2 Z_2 - \rho I_2 - \delta I_2 + I_T \\ Z_2' = \rho I_2 + \zeta R_2 - \alpha S_2 Z_2 + Z_T \\ R_2' = \delta S_2 + \delta I_2 + \alpha S_2 Z_2 - \zeta R_2 \end{cases}$$

One-Way Train Timetable

Dynamic Population of City 2 with One-Way Train Interaction



Two-way Train Diagram



Two-way Train Equations

$$\begin{cases} S_1' = \Pi_1 - \beta S_1 Z_1 - \delta S_1 - \gamma S_1 + S_{T_2} \\ I_1' = \beta S_1 Z_1 - \rho I_1 - \delta I_1 - \gamma I_1 + I_{T_2} \\ Z_1' = \rho I_1 + \zeta R_1 - \alpha S_1 Z_1 + Z_{T_2} \\ R_1' = \delta S_1 + \delta I_1 + \alpha S_1 Z_1 - \zeta R_1 \end{cases}$$

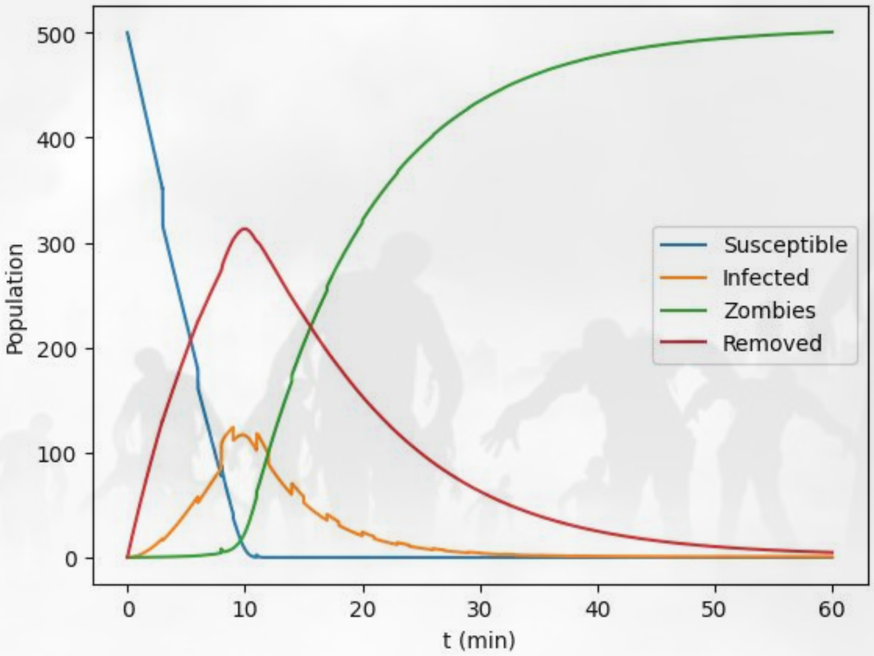
$$\begin{cases} S_{T_1}' = -\widetilde{\beta} S_{T_1} Z_{T_1} \\ I_{T_1}' = -\rho I_{T_1} + \widetilde{\beta} S_{T_1} Z_{T_1} \\ Z_{T_1}' = \rho I_{T_1} \\ S_{T_1}(0) = \gamma S_1, I_{T_1}(0) = \gamma I_1 \end{cases}$$

$$\begin{cases} S_2' = \Pi_2 - \beta S_2 Z_2 - \delta S_2 + S_{T_1} - \gamma S_2 \\ I_2' = \beta S_2 Z_2 - \rho I_2 - \delta I_2 + I_{T_1} - \gamma I_2 \\ Z_2' = \rho I_2 + \zeta R_2 - \alpha S_2 Z_2 + Z_{T_1} \\ R_2' = \delta S_2 + \delta I_2 + \alpha S_2 Z_2 - \zeta R_2 \end{cases}$$

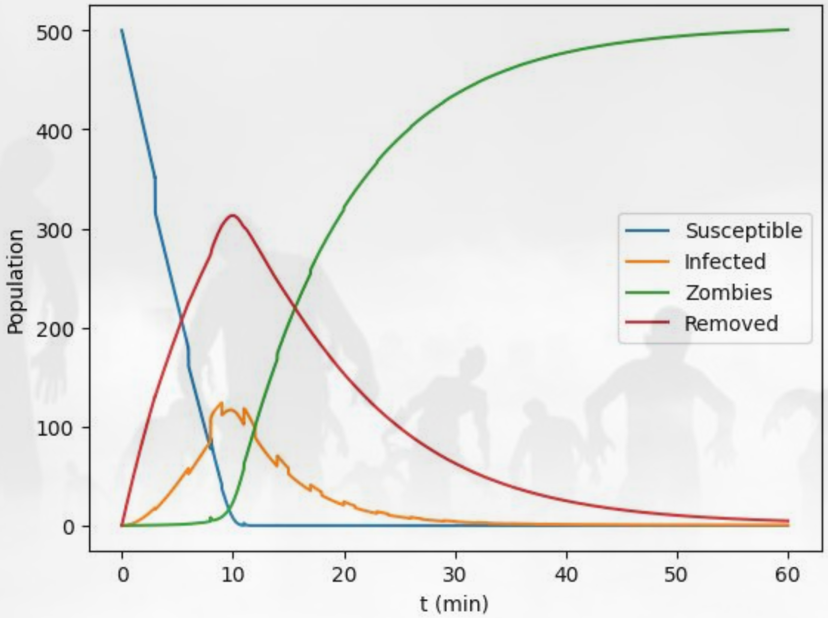
$$\begin{cases} S_{T_2}' = -\widetilde{\beta} S_{T_2} Z_{T_2} \\ I_{T_2}' = -\rho I_{T_2} + \widetilde{\beta} S_{T_2} Z_{T_2} \\ Z_{T_2}' = \rho I_{T_2} \\ S_{T_2}(0) = \gamma S_2, I_{T_2}(0) = \gamma I_2 \end{cases}$$

2-Way Train interaction for Two Big Cities

City 1 Population



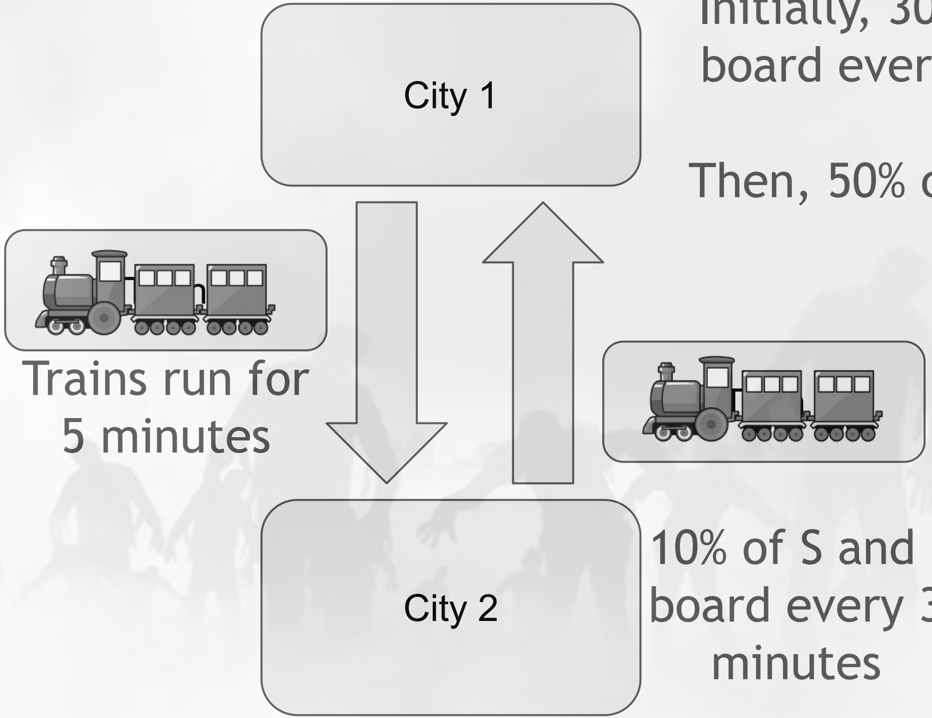
City 2 Population



Two-Way Train Timetable

Initially, 30% of S and I board every 3 minutes

Then, 50% of S and I board

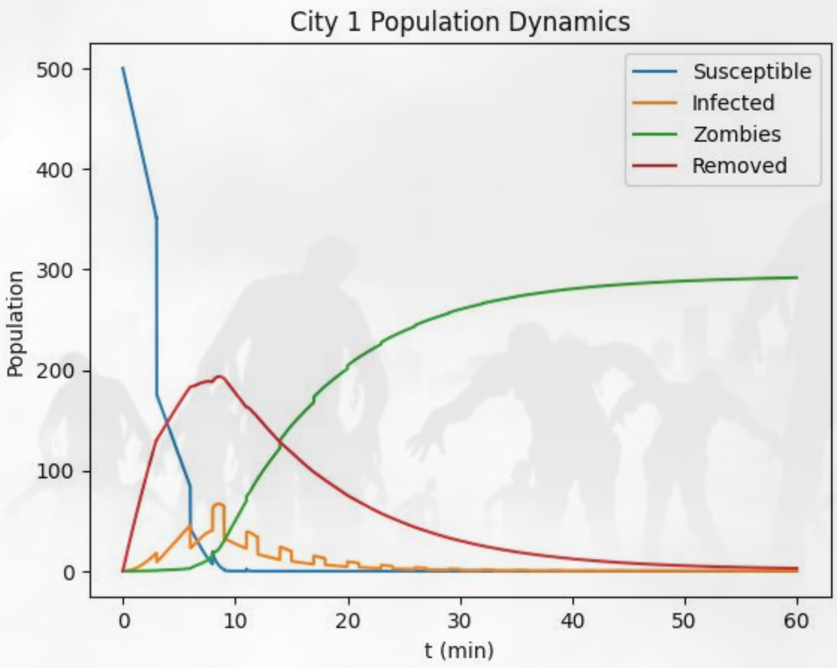


10% of S and I board every 3 minutes

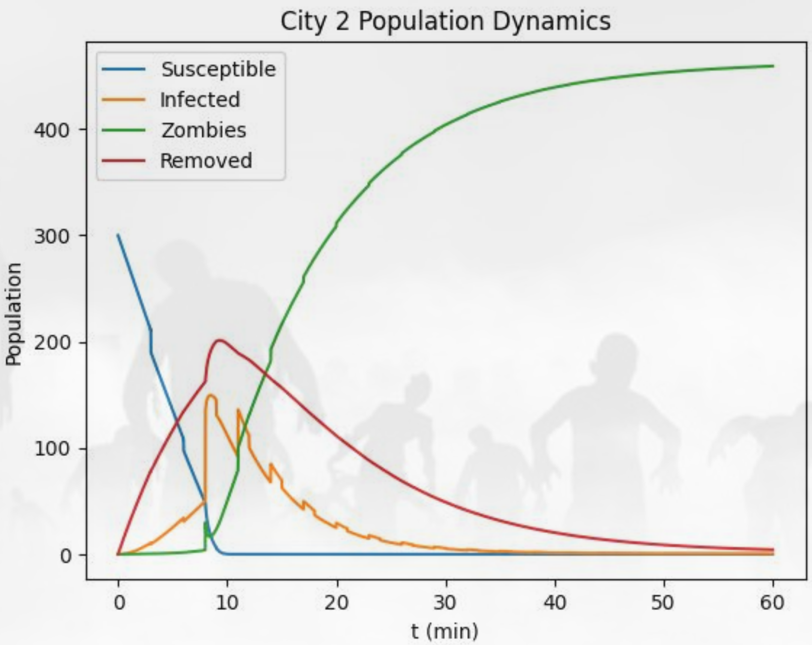
Time	Train Schedule
0	
1	
2	
3	Train A departs
4	
5	
6	Train B departs
7	
8	Train A arrives
9	Train C departs
10	
11	Train B arrives
12	Train D departs
13	
14	Train C arrives
15	Train E departs

2-Way Train interaction for Major Hub and Small City

Major Hub Population



Small City Population



Multi-city Network