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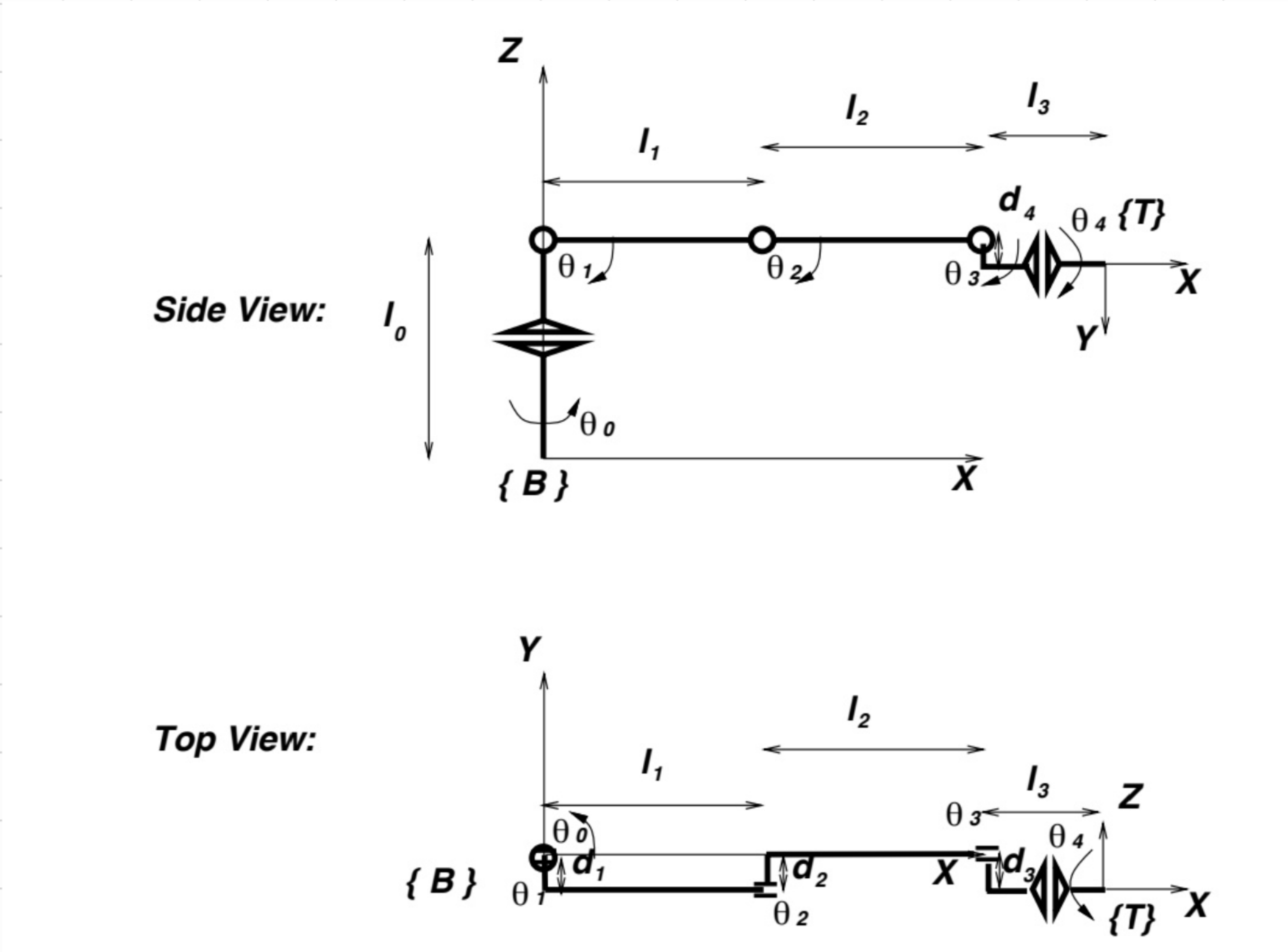
Robotics HW1

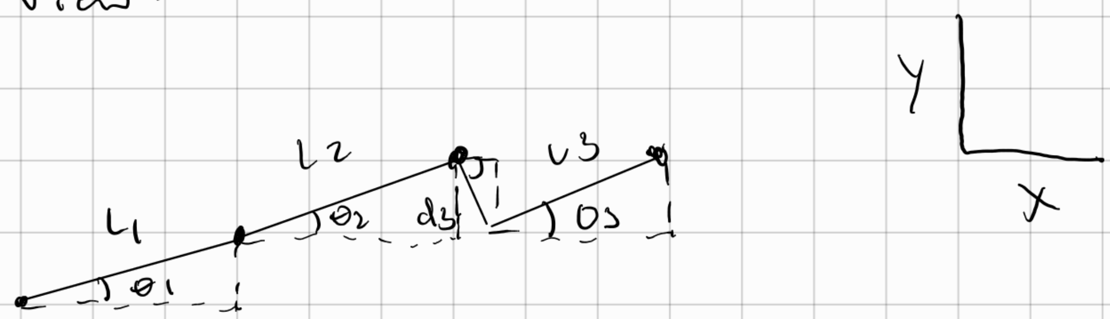


Forward Kinematics:

- To find the forward kinematics I used the Trigonometric solution.

- Because everything is effected by θ_0 I can make it so that for x and y to be seperated into a 2D space for easier equation building

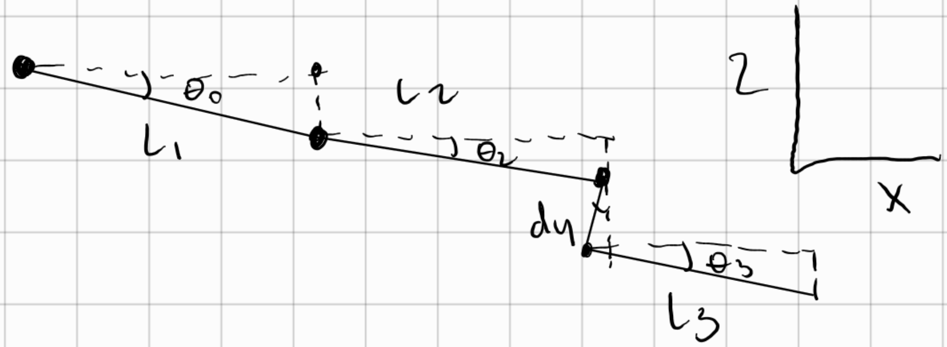
Top View:



$$x = \cos(\theta_0)\,(L_1\cos(\theta_1) + L_2(\cos(\theta_1+\theta_2))$$
$$+ D_4\sin(\theta_1+\theta_2+\theta_3) + L_3\cos(\theta_1+\theta_2+\theta_3)) - (D_3\sin(\theta_0))$$

$$y = \sin(\theta_0)\,(L_1\cos(\theta_1) + L_2(\cos(\theta_1+\theta_2))$$
$$+ D_4\sin(\theta_1+\theta_2+\theta_3) + L_3\cos(\theta_1+\theta_2+\theta_3)) - (D_3\cos(\theta_0))$$

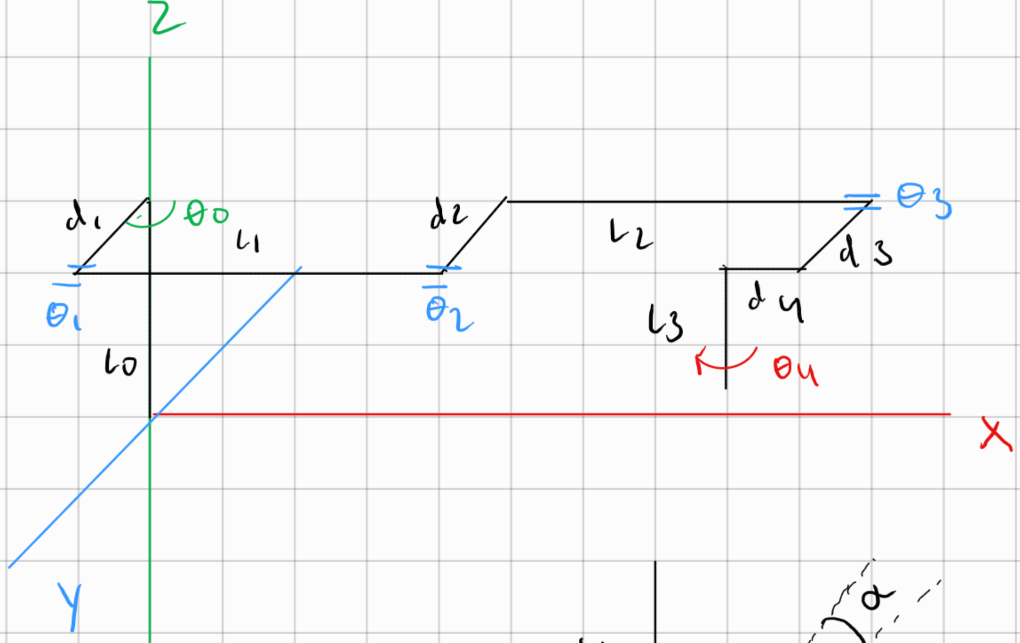
Side View:

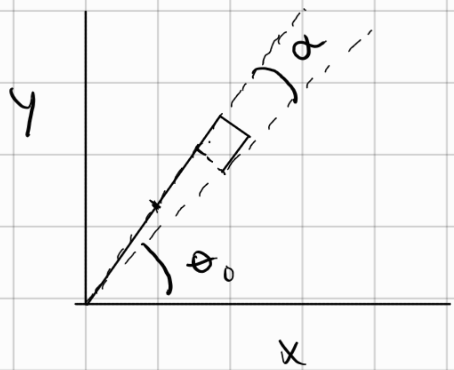


$$z = L_0 - L_1(\sin(\theta_1)) - L_2(\sin(\theta_1+\theta_2))$$
$$+ D_4(\sin(\theta_1+\theta_2+\theta_3)) - L_3(\sin(\theta_1+\theta_2+\theta_3));$$

Inverse kinematics:

- First thing we need to do is find the offset of α as there is an offset of d3 in this robotic arm (From Top View).





$$\text{offset} = \alpha$$

$$\theta_0 + \alpha = \arctan\left(\frac{y}{x}\right)$$

$$\sin(\alpha) = \frac{d3}{\sqrt{x^2+y^2}} \rightarrow \alpha = \arcsin\left(\frac{d3}{\sqrt{x^2+y^2}}\right)$$

$$\theta_0 = \arctan\left(\frac{y}{x}\right) + \alpha$$

- With this we have found the offset and θ_0
- if we did not calculate the offset our θ_0 will be off by that constant offset
- Next I will move the offset by subtracting L0 as well as d_1 and d_2

so from θ_0 to θ_3

$$\begin{cases} x_1 = x \\ y_1 = y \\ z_1 = z - L0 \end{cases}$$

} Sence d_1 and d_2 cancels out you dont have to do any thing for x and y

was not needed in code so skip to

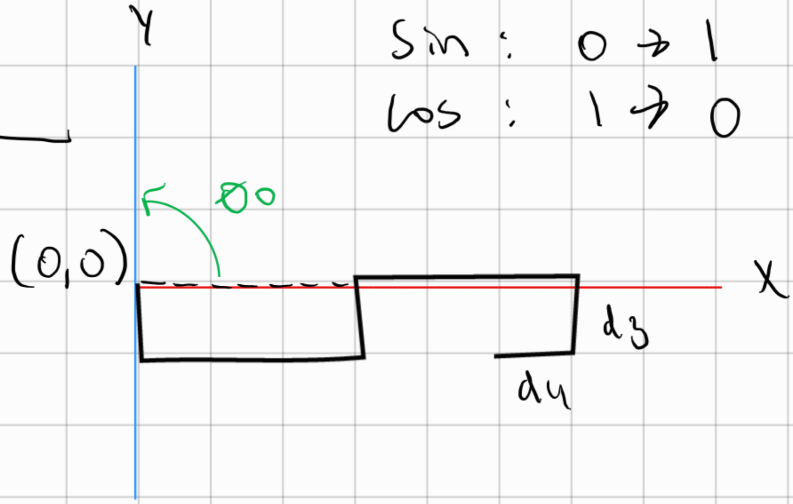
θ_3 to wrist

$$\begin{cases} x_2 = x - d4\cos(\theta_0) + d3(\sin(\theta)) \\ y_2 = y - d4\sin(\theta_0) - d3(\cos(\theta)) \\ z_2 = z - L0 + L3 \end{cases}$$

we are trying to move every thing to the wrist

Sin : 0 to 1

Cos : 1 to 0



- Now that we got that we can get the 2D plane for the 3DOF robot.

- I used the equations in the slides
- x needs to be aligned with θ_1
- y needs to align with -z so that

the equation in class works

$x_3 = \sqrt{x_2^2 + y_2^2}$
$y_3 = -z_2$

$S = x_3^2 + y_3^2$
$B = \arctan(y_3 / x_3)$
$A = \arccos\left(\frac{S^2 + L_1^2 + L_2^2}{2 S L_1}\right)$

$\theta_2 = \arccos \frac{S^2 - L_1^2 - L_2^2}{2 L_1 L_2}$

* There are only 2 valid solutions

$\theta_2 > 0$ then $= B - A = \theta_1$
$\theta_2 < 0$ then $= B + A = \theta_1$

- in the code there were while loops to check A and θ_2 before they use arccos, as arccos only accepts values between $(1, -1)$

- for θ_3 because we rotated on the z axis to move it back we have to to move it 90° so $\pi/2$

$\theta_3 = \frac{\pi}{2} - \theta_1 - \theta_2$

Jacobian Matrix

$$J(\theta) = \begin{Bmatrix} \frac{dx}{d\theta} & \frac{dx}{d\theta_1} & \frac{dx}{d\theta_2} & \frac{dx}{d\theta_3} \\ \frac{dy}{d\theta} & \frac{dy}{d\theta_1} & \frac{dy}{\theta_2} & \frac{dy}{\theta_3} \\ \frac{dz}{d\theta} & \frac{dz}{d\theta} & \frac{dz}{\theta_2} & \frac{dz}{\theta_3} \end{Bmatrix}$$

I used my forward Kinematics for my Jacobian b/c I used Trig

$$x = \cos(\theta_0)(L_1 \cos(\theta_1) + L_2(\cos(\theta_1+\theta_2)) + D_4 \sin(\theta_1+\theta_2+\theta_3) + L_3 \cos(\theta_1+\theta_2+\theta_3) - (D_3 \sin(\theta_0)$$

$$y = \sin(\theta_0)(L_1 \cos(\theta_1) + L_2(\cos(\theta_1+\theta_2)) + D_4 \sin(\theta_1+\theta_2+\theta_3) + L_3 \cos(\theta_1+\theta_2+\theta_3) - (D_3 \cos(\theta_0)$$

$$z = L_0 - L_1 \sin(\theta_1) - L_2 \sin(\theta_1+\theta_2) + D_4 \cos(\theta_1+\theta_2+\theta_3) - L_3 \sin(\theta_1+\theta_2+\theta_3)$$

$$dx/d\theta = -\sin(\theta)(L_1 \cos(\theta_1) + L_2 \cos(\theta_1+\theta_2) + D_4 \sin(\theta_1+\theta_2+\theta_3) + L_3 \cos(\theta_1+\theta_2+\theta_3)) - (D_3 \cos(\theta_0)$$

$$dx/d\theta_1 = \cos(\theta)(-L_1 \sin(\theta_1) - L_2 \sin(\theta_1+\theta_2) + D_4 \cos(\theta_1+\theta_2+\theta_3) - L_3 \sin(\theta_1+\theta_2+\theta_3))$$

$$dx/d\theta_2 = \cos(\theta)(-L_2 \sin(\theta_1+\theta_2) + D_4 \cos(\theta_1+\theta_2+\theta_3) - L_3 \sin(\theta_1+\theta_2+\theta_3))$$

$$dx/d\theta_3 = \cos(\theta)(D_4 \cos(\theta_1+\theta_2+\theta_3) - L_3 \sin(\theta_1+\theta_2+\theta_3))$$

$$dy/d\theta_0 = \cos(\theta)(L_1 \cos(\theta_1) + L_2 \cos(\theta_1+\theta_2) + D_4 \sin(\theta_1+\theta_2+\theta_3) + L_3 \cos(\theta_1+\theta_2+\theta_3)) - (D_3 \sin(\theta)$$

$$dy/d\theta_1 = \sin(\theta)(-L_1 \sin(\theta_1) - L_2 \sin(\theta_1+\theta_2)$$
$$+ D_4 \cos(\theta_1+\theta_2+\theta_3) - L_3 \sin(\theta_1+\theta_2+\theta_3))$$

$$dy/d\theta_2 = \sin(\theta)(-L_2 \sin(\theta_1+\theta_2)$$
$$+ D_4 \cos(\theta_1+\theta_2+\theta_3) - L_3 \sin(\theta_1+\theta_2+\theta_3))$$

$$dy/d\theta_3 = \sin(\theta)(D_4 \cos(\theta_1+\theta_2+\theta_3) - L_3 \sin(\theta_1+\theta_2+\theta_3))$$

$$dz/d\theta_0 = \theta$$

$$dz/d\theta_1 = -L_1 \cos(\theta_1) - L_2 \cos(\theta_1+\theta_2)$$
$$- D_4 \sin(\theta_1+\theta_2+\theta_3) - L_3 \cos(\theta_1+\theta_2+\theta_3)$$

$$dz/d\theta_2 = -L_2 \cos(\theta_1+\theta_2) - D_4 \sin(\theta_1+\theta_2+\theta_3)$$
$$- L_3 \cos(\theta_1+\theta_2+\theta_3)$$

$$dz/d\theta_3 = - D_4 \sin(\theta_1+\theta_2+\theta_3) - L_3 \cos(\theta_1+\theta_2+\theta_3)$$