A simple model of the determination of the GDP with Mathematica

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In this post, I will show how to build a simple model of the determination of the GDP with government and trade. The idea is to find the equilibrium GDP, where desired aggregate expenditures are equal to the level of good and services produced. To this end, I will use Mathematica 12.3 and the chapter 16 of the following book: https://global.oup.com/ukhe/product/economics-9780198791034, that I use in my lecture of Principles of Macroeconomics at the University of Strasbourg. Let me explain, step by step, the notebook I reproduced below.

Presentation notebook of this work attached at the end of this post

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I. The simple model with a tax rate on GDP

First, I have to clear the symbols to be sure that the previous registered symbol will not interfere with your model . Use (*something*) to insert the comment "something" :

```
In[•]:= ClearAll["Global`*"] (*completely clear global symbols to start fresh*)
```

Then, I will use a first version of the model, with a tax rate. I name the equation 'eq' and I declare that my model can be reduced to one equation, where the GPD is in function of a consumption function, exogenous investment, exogenous government expenditures, exogenous exports, and an import function.

$$Y = C + F + G + (X - M) \tag{1}$$

$$C = c_0 + c_1 (1 - t) Y \tag{2}$$

$$M = m_1 Y \tag{3}$$

I use 'f' for the investment to not confuse with imaginary numbers and for the french "formation brute de capital fixe". You have to use "Ctrl+Enter" to execute the command. The above line gives this the following output:

```
In[•]:= eq = y == c0 + c1 * (1 - t) * y + f + g + x - m1 * y   (*f stands for the investment*)
Out[•]=
y == c0 + f + g + x - m1 y + c1 (1 - t) y
```

Then, I solve the model and I store the result in an object 'PIB' (the french acronym for GDP):

```
In[•]:= PIB = Solve[eq, y]
Out[•]=
```

$$\left\{\left\{y \to \frac{c0 + f + g + x}{1 - c1 + m1 + c1\, t}\right\}\right\}$$

Now, I want to compute the algebraic value of the multiplier effect of this model. By definition, the multiplier is the value of a change in the GDP after an increase in an exogenous expenditure. In this example, I choose to evaluate the value of change in the GDP after an increase in exogenous government spending. I store the value in an object called 'Multiplier'. The numerical value will depend on the value of the parameters of your models.

In[∘]:=
```
Multiplier = D[PIB, g]
```
Out[∘]=

$$\left\{\left\{0 \to \frac{1}{1 - c1 + m1 + c1\,t}\right\}\right\}$$

In the chapter 16 of the book, we can read that the multiplier is positively linked with the marginal propensity to consume and is negatively linked with the tax rate and the marginal propensity to import. I will derive the value of the multiplier relatively to these three last variables. The partial derivative of the multiplier effect relative to marginal propensity to import is indeed negative:

In[∘]:=
```
partialm1 = D[Multiplier, m1]
```
Out[∘]=

$$\left\{\left\{0 \to -\frac{1}{(1 - c1 + m1 + c1\,t)^2}\right\}\right\}$$

Besides, the partial derivative of the multiplier effect relative to the tax rate is also negative :

In[∘]:=
```
partialt = D[Multiplier, t]
```
Out[∘]=

$$\left\{\left\{0 \to -\frac{c1}{(1 - c1 + m1 + c1\,t)^2}\right\}\right\}$$

Finally, the partial derivative of the multiplier effect relative to the marginal propensity to consume is positive.

In[∘]:=
```
partialc1 = D[Multiplier, c1]
```
Out[∘]=

$$\left\{\left\{0 \to -\frac{-1 + t}{(1 - c1 + m1 + c1\,t)^2}\right\}\right\}$$

II. A little numerical exercise

Now, we run a numerical example to determine (a) the equilibrium GDP, (b) the size of the multiplier effect, and (c) the sign of the three partial derivatives by choosing vales for the exogenous variables and the parameters:

In[ ]:=
```
c0 = 100;
c1 = 0.8;
t = 0.2;
f = 1000;
g = 100;
x = 500;
m1 = 0.25;
PIB
Multiplier
partialm1
partialt
partialc1
```

Out[ ]=
```
{{y → 2786.89}}
```

Out[ ]=
```
{{0 → 1.63934}}
```

Out[ ]=
```
{{0 → -2.68745}}
```

Out[ ]=
```
{{0 → -2.14996}}
```

Out[ ]=
```
{{0 → 2.14996}}
```

I use '; ' to hide the output for the parameters . We found the equilibrium value for the GDP (2786.9), the size of the multiplier (1.63934), and the excepted signs for the partial derivative .

III. The simple model with lump-sum taxes

I can use a version of the model with lump-sum taxes to evaluate the balanced budget multiplier (a simultaneous and equivalent increase in public consumption and in taxes).

$$Y = C + F + G + (X - M) \tag{4}$$

$$C = c_0 + c_1 \, (Y - T) \tag{5}$$

$$M = m_1 \, Y \tag{6}$$

The above lines give the following output:

In[ ]:=
```
ClearAll["Global`*"] (*completely clear global symbols to start fresh*)
eq = y == c0 + c1 * (y - t) + f + g + x - m1 * y
(*f stands for the investment, t is now a lump-sum tax*)
PIB = Solve[eq, y]
Multiplier = D[PIB, g]
partialm1 = D[Multiplier, m1]
partialc1 = D[Multiplier, c1]
```

Out[ ]=

$$y == c0 + f + g + x - m1\, y + c1\,(-t + y)$$

Out[ ]=

$$\left\{\left\{y \to \frac{c0 + f + g - c1\, t + x}{1 - c1 + m1}\right\}\right\}$$

Out[ ]=

$$\left\{\left\{0 \to \frac{1}{1 - c1 + m1}\right\}\right\}$$

Out[ ]=

$$\left\{\left\{0 \to -\frac{1}{(1 - c1 + m1)^2}\right\}\right\}$$

Out[ ]=

$$\left\{\left\{0 \to \frac{1}{(1 - c1 + m1)^2}\right\}\right\}$$

A second little numerical exercise

To simplify, I use a version of this model with lump-sum taxes where the propensity to import is equal to zero and evaluate the balanced budget multiplier:

In[ ]:=
```
c0 = 100;
c1 = 0.8;
t = 100;
f = 1000;
g = 100;
x = 500;
m1 = 0.0;
PIB
Multiplier
partialm1
partialc1
```

Out[ ]=

$$\{\{y \to 8100.\}\}$$

Out[ ]=

$$\{\{0 \to 5.\}\}$$

Out[ ]=

$$\{\{0 \to -25.\}\}$$

Out[ ]=

$$\{\{0 \to 25.\}\}$$

After an increase in public expenditures and in taxes, we can observe that the balanced budget multiplier is equal to one since the equilibrium GDP moves from 8000 to 8001 (‘g’ moved from 100 to 101 and ‘t’ moved from 100 to 101), as in the balanced budget multiplier (Haavelmo, 1945):

```
In[•]:= c0 = 100;
c1 = 0.8;
t = 101;
f = 1000;
g = 101;
x = 500;
m1 = 0.0;
PIB
Multiplier
partialm1
partialc1

Out[•]=
{{y → 8101.}}

Out[•]=
{{0 → 5.}}

Out[•]=
{{0 → -25.}}

Out[•]=
{{0 → 25.}}
```

Bibliographic references

- Lipsey, R., & Chrystal, A. (2020). Economics. Oxford University Press. https://global.oup.com/ukhe/product/economics-9780198791034
- Haavelmo, T. (1945). Multiplier Effects of a Balanced Budget. Econometrica, 13(4), 311–318. https://doi.org/10.2307/1906924