CITE THIS NOTEBOOK: The fragility of an incomplete monetary union by *Jamel Saadaoui*. Wolfram Community MAR 4 2023.

In this blog, I will show how to build a model of the fragility of an incomplete monetary union. The idea is to find the optimal inflation rule and to examine different configurations for fixed exchange rate and expectation of a devaluation. To this end, I will use Mathematica and the chapter 5 of this book (https://global.oup.com/ushe/product/economics-of-the-monetary-union-9780198849544), that I use in my lecture of Economics of Monetary Union. Allow me to explain the different steps in the notebook below. I would like to publicly thank Paul De Grauwe for providing his very clear lecture notes.

The model

In[∘]:= **ClearAll["Global`*"] (*completely clear global symbols to start fresh*)**

- The Phillips curve:

$$U = U_N + a(\pi^e - \pi) + \varepsilon \qquad (1)$$

In[∘]:= **U = UN + a (pe − p) + ϵ**

Out[∘]=

a (−p + pe) + UN + ϵ

- Alternatively, you can specify the supply curve

$$Y = Y_n + \theta(\pi - \pi^e) + \varepsilon \qquad (2)$$

- Loss function of authorities

$$L = \pi^2 + \beta\left(U - \bar{U}\right)^2 \qquad (3)$$

L = p² + β (U − UB)²

Out[∘]=

$p^2 + \beta(a(-p + pe) - UB + UN + \epsilon)^2$

with UB = target unemployment

$$\bar{U} = \lambda U_N \qquad (4)$$

In[∘]:= **UB = λUN**

Out[∘]=

λUN

where λ < 1

Out[∘]=

$p^2 + \beta(a(-p + pe) + UN + \epsilon - \lambda UN)^2$

In[∘]:= $\mathbf{p^2 + \beta(a(-p + pe) + UN + \epsilon - \lambda UN)^2}$

Out[∘]=

$p^2 + \beta(a(-p + pe) + UN + \epsilon - \lambda UN)^2$

$$L = \pi^2 + \beta\left[(1 - \lambda)U_N + a(\pi^e - \pi) + \varepsilon\right]^2 \qquad (5)$$

- Workers set wages based on pe
- Authorities minimize loss given these expectations
- Note : authorities directly control inflation

■ Authorities minimize loss given these expectations

$$\frac{dL}{d\pi} = 2\pi + 2\beta(-a)\left[(1-\lambda)U_N + a\left(\pi^e - \pi\right) + \varepsilon\right] = 0 \tag{6}$$

In[∘]:= **CBLoss = D[L, p]**

Out[∘]=

2 p – 2 a β (a (–p + pe) + UN + ϵ – λUN)

In[∘]:= **CBLoss1 = Simplify[Collect[CBLoss, βa]/2]**

Out[∘]=

p – a β (a (–p + pe) + UN + ϵ – λUN)

In[∘]:= **Collect[CBLoss1, p]**

Out[∘]=

$-a^2\,pe\,\beta - a\,UN\,\beta + p\left(1 + a^2\beta\right) - a\,\beta\,\epsilon + a\,\beta\,\lambda UN$

$$\pi\left(1 + \beta a^2\right) - \beta a(1-\lambda)U_N - \beta a^2\pi^e - \beta a\varepsilon = 0 \tag{7}$$

$$\pi\left(1 + \beta a^2\right) - \beta a(1-\lambda)U_N - \beta a^2\pi^e - \beta a\varepsilon = 0$$

In[∘]:= **p (1 + βa ^ 2) – βa (1 – λ) UN – βa ^ 2 pe – βaϵ == 0**

Out[∘]=

$-pe\,\beta a^2 + p\left(1 + \beta a^2\right) - \beta a\epsilon - UN\,\beta a\,(1-\lambda) == 0$

In[∘]:= **Solve[$-pe\,\beta a^2 + p\left(1 + \beta a^2\right) - UN\,\beta a\,(1-\lambda) - \beta a\epsilon == 0$, p]**

Out[∘]=

$$\left\{\left\{p \to \frac{UN\,\beta a + pe\,\beta a^2 + \beta a\epsilon - UN\,\beta a\,\lambda}{1 + \beta a^2}\right\}\right\}$$

$$\pi = \frac{\beta a(1-\lambda)U_N}{1+\beta a^2} + \frac{\beta a^2\pi^e}{1+\beta a^2} + \frac{\beta a\varepsilon}{1+\beta a^2} \tag{8}$$

This is optimal policy rule of authorities given expectations
Agents know this rule
Thus when forming expectations pe they use this rule

$$\pi = \frac{\beta a(1-\lambda)U_N}{1+\beta a^2} + \frac{\beta a^2\pi^e}{1+\beta a^2} + \frac{\beta a\varepsilon}{1+\beta a^2} \tag{9}$$

In[∘]:= **$p = \frac{\beta a\,(1-\lambda)\,UN + pe\,\beta a^2 + \beta a\epsilon}{1 + \beta a^2}$**

Out[∘]=

$$\frac{pe\,\beta a^2 + \beta a\epsilon + UN\,\beta a\,(1-\lambda)}{1 + \beta a^2}$$

1. Set p = pe and assume first no shocks ($\epsilon = 0$)

$$\pi = \frac{\beta a(1-\lambda)U_N}{1+\beta a^2} + \frac{\beta a^2\pi^e}{1+\beta a^2} + \frac{\beta a\varepsilon}{1+\beta a^2} \tag{10}$$

$$\pi^e = \frac{\beta a(1-\lambda)U_N}{1+\beta a^2} + \frac{\beta a^2\pi^e}{1+\beta a^2} \tag{11}$$

In[∘]:= Solve$\left[\frac{\beta a\,(1-\lambda)\,UN + pe\,\beta a^2}{1+\beta a^2} - pe == 0, pe\right]$

Out[∘]=

$\{\{pe \to -UN\,\beta a\,(-1+\lambda)\}\}$

$$\pi = \pi^e = \beta\, a\,(1-\lambda)\; U_N \tag{12}$$

(*there is an inflation bias (non–zero inflation), which increases with β,a.*)

When shocks (e . g ., deterioration of trade balance) occur, solution is

$$\pi = \pi^e + \frac{\beta\, a}{1+\beta\, a^2}\;\varepsilon = \beta\, a\,(1-\lambda)\; U_N + \frac{\beta\, a}{1+\beta\, a^2}\;\varepsilon \tag{13}$$

Authorities react to shocks in a stabilizing manner depending on β

- if $\beta = 0$: no stabilization (strict inflation targeting)
- if $\beta > 0$: some stabilization

Substitute optimal inflation $\pi^e - \pi$ in Phillips curve

$UN + a\left(-\frac{\beta a}{1+\beta a^2}\,\epsilon\right) + \epsilon$

Out[∘]=

$UN + \epsilon - \frac{a\,\beta a\,\epsilon}{1+\beta a^2}$

In[∘]:= Together$\left[UN + \epsilon - \frac{a\,\beta a\,\epsilon}{1+\beta a^2}\right]$

Out[∘]=

$\frac{UN + UN\,\beta a^2 + \epsilon - a\,\beta a\,\epsilon + \beta a^2\,\epsilon}{1+\beta a^2}$

In[∘]:= FullSimplify$\left[\frac{UN + UN\,\beta a^2 + \epsilon - a\,\beta a\,\epsilon + \beta a^2\,\epsilon}{1+\beta a^2}\right]$

Out[∘]=

$UN + \frac{(1 - a\,\beta a + \beta a^2)\,\epsilon}{1+\beta a^2}$

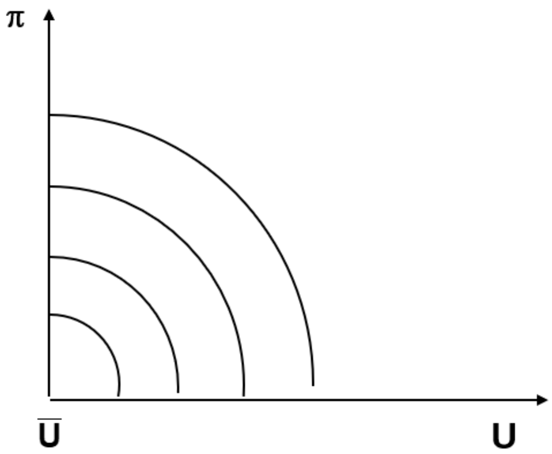
$$U = U_N + \frac{1}{1+\beta\, a^2}\;\varepsilon \tag{14}$$

If $\beta = 0$,

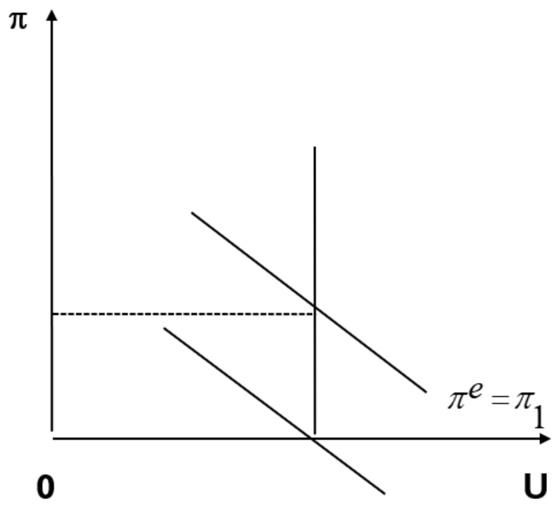
var U = var ϵ : no stabilization.

More generally, Var U = $\left(\frac{1}{1+\beta\, a^2}\right)^2$ var $\epsilon < 1$ depending on β.

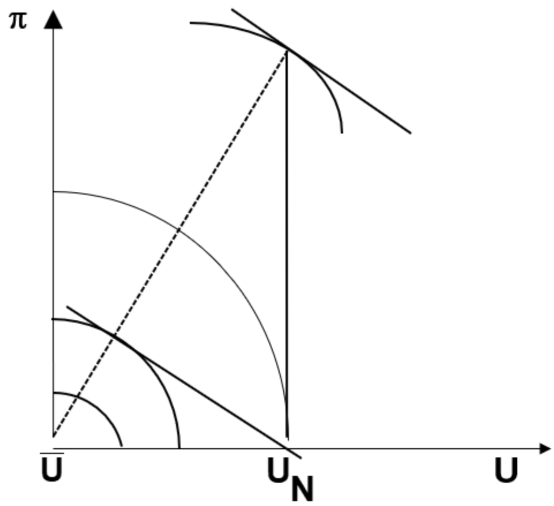
Indifference curves between inflation and excess unemployment:



Phillips curve with forward looking expectations:



Credibility of the monetary authority

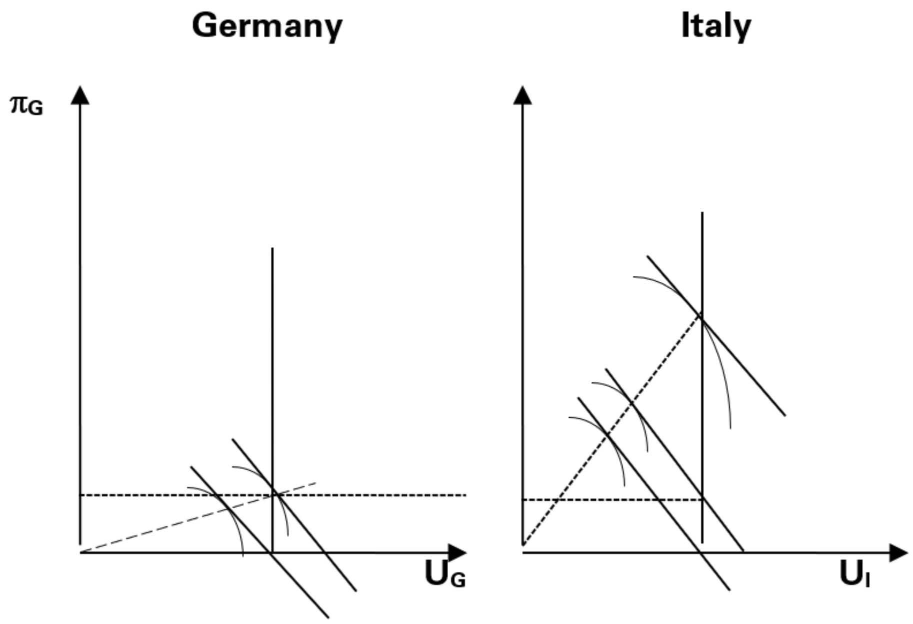


We use this model to analyze crisis

Assume PPP : $\dot{S} = \pi - \pi^*$ set $\pi^* = 0$

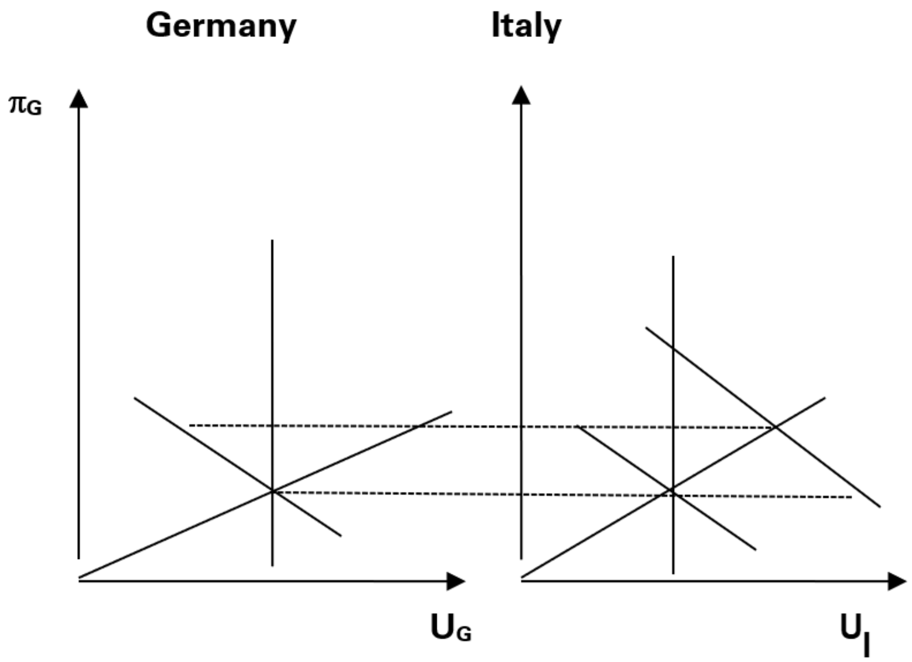
Fixing exchange rate is equivalent to announcing zero inflation
If there is no cost associated to devaluation fixing exchange rate is never credible

Italy will not follow its commitment to the announced FX



- If no cost of devaluation, the peg by Italy is not credible and will be attacked
- Two countries must have same preferences
- Even if they have same preferences, asymmetric shocks may reduce credibility

Italy must now accept too high unemployment to maintain the FX



- Italy must now accept too high unemployment
- Italian authorities would like Germany to accommodate

- If Germany refuses
  - conflict
  - loss of credibility
- Attack on the system
- Case study : EMS during 1992 - 1993
- If there is a cost of devaluation, the fixed exchange rate can be made credible

The model with multiple equilibria

We will compute the loss of the authorities under different expectations of private agents

1. Loss under discretion

This is the loss when CB follows discretionary policy which is fully expected
This is solution derived earlier

$$\pi = \pi^e = \beta a (1 - \lambda) U_N \quad (15)$$

$$U = U_N \quad (16)$$

$$\bar{U} = \lambda U_N \quad (17)$$

Substitute into loss function : $L = \pi^2 + \beta \left(U - \bar{U}\right)^2$

$$L_{DIS} = \beta^2 a^2 [(1 - \lambda) U_N]^2 + \beta [U_N (1 - \lambda)]^2 \quad (18)$$

$$L_{DIS} = \beta [(1 - \lambda) U_N]^2 (\beta a^2 + 1) = \beta k^2 (1 + \beta a^2) \quad (19)$$

LDIS = βk² (1 + βa²)

Out[∘]=

$(1 + \beta a^2)\ \beta k^2$

2. Loss of fixed exchange rate (zero inflation) when agents expect fixed exchange rate

$$\pi = \pi^e = 0 \quad (20)$$

$$U = U_N \quad (21)$$

$$L_{0,\pi^e=0} = \beta \left[U_N - \bar{U}\right]^2 = \beta [(1 - \lambda) U_N]^2 = \beta k^2 \quad (22)$$

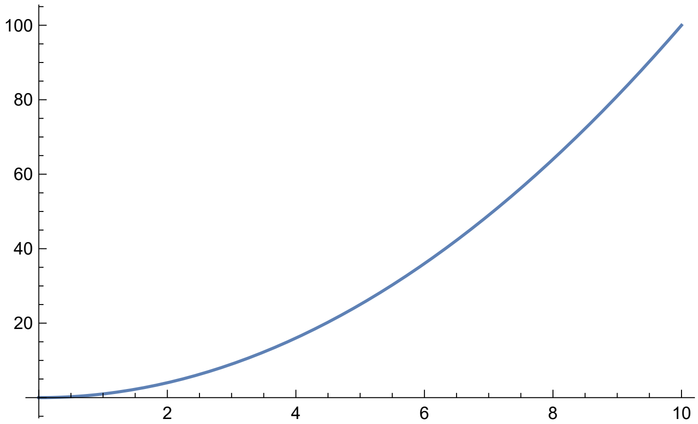
In[∘]:= **LFXZERO = βk²**

Out[∘]=

βk^2

In[•]:=
```
Plot[LFXZERO, {βk, 0, 10}]
```

Out[•]=



3. Loss when the CB devalues (cheats) while agents expect fixed exchange rate

Take optimal inflation from first order condition and set pe = 0 (also $\epsilon = 0$)

$$\pi = \frac{\beta a (1-\lambda) U_N}{1+\beta a^2} + \frac{\beta a^2 \pi^e}{1+\beta a^2} + \frac{\beta a \varepsilon}{1+\beta a^2} \tag{23}$$

$$\pi = \frac{\beta a (1-\lambda) U_N}{1+\beta a^2} \tag{24}$$

$$U = U_N + a\left[-\frac{\beta a (1-\lambda) U_N}{1+\beta a^2}\right] \tag{25}$$

Substitute into loss function

$$\frac{\beta a\ (1-\lambda)\ \mathrm{UN}}{1+\beta a^2}$$

Out[•]=

$$\frac{\mathrm{UN}\ \beta a\ (1-\lambda)}{1+\beta a^2}$$

In[•]:=

$$\mathrm{UN} + a\left[-\frac{\mathrm{UN}\ \beta a\ (1-\lambda)}{1+\beta a^2}\right]$$

Out[•]=

$$\mathrm{UN} + a\left[-\frac{\mathrm{UN}\ \beta a\ (1-\lambda)}{1+\beta a^2}\right]$$

$$\begin{aligned} L_{cheat} &= \left[\tfrac{\beta a (1-\lambda) U_N}{1+\beta a^2}\right]^2 + \beta\left[(1-\lambda) U_N - \tfrac{\beta a^2 (1-\lambda) U_N}{1+\beta a^2}\right]^2 = \left[\tfrac{\beta a (1-\lambda) U_N}{1+\beta a^2}\right]^2 + \beta\left[(1-\lambda) U_N\left(1 - \tfrac{\beta a^2}{1+\beta a^2}\right)\right]^2 \\ &= \left[\tfrac{\beta a (1-\lambda) U_N}{1+\beta a^2}\right]^2 + \beta\left[\tfrac{(1-\lambda) U_N}{1+\beta a^2}\right]^2 = \left[\tfrac{(1-\lambda) U_N}{1+\beta a^2}\right]^2 \beta(\beta a^2 + 1) \\ &= \tfrac{\beta [(1-\lambda) U_N]^2}{1+\beta a^2} = \tfrac{\beta k^2}{1+\beta a^2} \end{aligned} \tag{26}$$

In[•]:=

$$(\beta a)^2 \left(\frac{(1-\lambda)\ \mathrm{UN}}{1+\beta a^2}\right)^2 + \beta\left(\frac{(1-\lambda)\ \mathrm{UN}}{1+\beta a^2}\right)^2$$

Out[•]=

$$\frac{\mathrm{UN}^2\ \beta\ (1-\lambda)^2}{\left(1+\beta a^2\right)^2} + \frac{\mathrm{UN}^2\ \beta a^2\ (1-\lambda)^2}{\left(1+\beta a^2\right)^2}$$

In[∘]:= $\beta(\beta a^2 + 1)$

Out[∘]=

$\beta\left[1 + a^2\,\beta\right]$

In[∘]:=
```
% == (βa)^2 + β
```

Out[∘]=

$\beta\left[1 + a^2\,\beta\right] == \beta + \beta a^2$

In[∘]:=
```
Equal[%]
```

Out[∘]=

```
True
```

In[∘]:= LCHEAT $= \dfrac{\beta\,k^2}{1 + \beta\,a^2}$

Out[∘]=

$$\frac{k^2\,\beta}{1 + a^2\,\beta}$$

The difference between LFXZERO and LCHEAT and measures the temptation to devalue

In[∘]:=
```
LFXZERO - LCHEAT
```

Out[∘]=

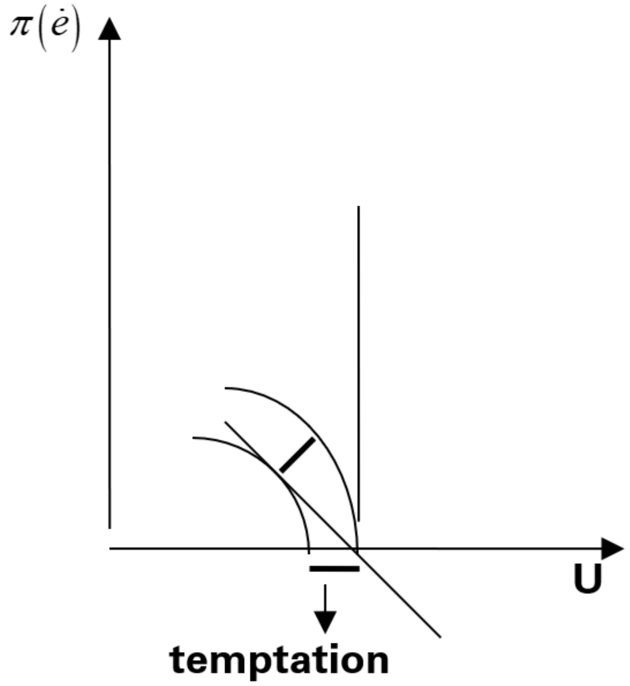
$$-\frac{k^2\,\beta}{1 + a^2\,\beta} + \beta k^2$$

In[∘]:= Together$\left[-\dfrac{k^2\,\beta}{1 + a^2\,\beta} + \beta k^2\right]$

Out[∘]=

$$\frac{-k^2\,\beta + \beta k^2 + a^2\,\beta\,\beta k^2}{1 + a^2\,\beta}$$

$$\begin{aligned} L_{0,\pi^e=0} - L_{cheat} &= \beta\,k^2 - \tfrac{\beta\,k^2}{1+\beta\,a^2} \\ &= \beta\,k^2\left(1 - \tfrac{1}{1+\beta\,a^2}\right) \\ &= \tfrac{\beta^2\,k^2\,a^2}{1+\beta\,a^2} > 0 \end{aligned} \tag{27}$$



Suppose there is a fixed cost of devaluation (e.g. it is costly politically; cf. R. Cooper)
Call this cost C

Then if C > temptation = $\frac{\beta^2 k^2 a^2}{1+\beta a^2}$ fixed exchange rate is credible

This does not yet mean, however, that it will last

4. Loss when authorities maintain fixed exchange rate while agents expect devaluation (discretion)

Call this loss under stabilisation

$$\begin{aligned} &\pi = 0 \\ &\pi^e = \beta a(1-\lambda) U_N = \beta a k \\ &U = U_N + a(\beta a(1-\lambda) U_N) \end{aligned} \tag{28}$$

$$\begin{aligned} L_{stab} &= 0 + \beta\left[U_N + a(\beta a(1-\lambda) U_N) - \bar{U}\right]^2 = \beta\left[(1-\lambda) U_N + a^2 \beta(1-\lambda) U_N\right]^2 \\ &= \beta\left[(1-\lambda) U_N(1 + a^2 \beta)\right]^2 = \beta k^2 \left(1 + a^2 \beta\right)^2 \end{aligned} \tag{29}$$

In[∘]:=
```
β ((1 - λ) UN + a^2 β (1 - λ) UN)^2
```
Out[∘]=

$\beta \left(\text{UN}\ (1-\lambda) + a^2\, \text{UN}\, \beta\ (1-\lambda)\right)^2$

In[∘]:=
```
Collect[β (UN (1 - λ) + a^2 UN β (1 - λ))^2, (1 + 2 aβ)]
```
Out[∘]=

$\beta \left(\text{UN}\ (1-\lambda) + a^2\, \text{UN}\, \beta\ (1-\lambda)\right)^2$

In[∘]:=
```
Equal[UN (1 - λ) + a^2 UN β (1 - λ) == UN (1 - λ) (1 + a^2 β)]
```
Out[∘]=
```
True
```

In[∘]:= **LSTAB = $\beta k^2 \left(1 + a^2 \beta\right)^2$**

Out[∘]=

$\left(1 + a^2 \beta\right)^2 \beta k^2$

Comparing LDIS with LSTAB allows us to find the cost of defending the peg

$$\begin{aligned} L_{stab} - L_{dis} &= \beta k^2 \left(1 + a^2 \beta\right)^2 - \beta k^2 \left(1 + a^2 \beta\right) = \beta k^2 \left(1 + a^2 \beta\right) a^2 \beta \\ &= \beta^2 a^2 k^2 \left(1 + a^2 \beta\right) > 0 \end{aligned} \tag{30}$$

In[∘]:= **LSTAB - LDIS**

Out[∘]=

$\left(1 + a^2 \beta\right)^2 \beta k^2 - \left(1 + \beta a^2\right) \beta k^2$

In[∘]:= **Simplify$\left[\left(1 + a^2 \beta\right)^2 \beta k^2 - \left(1 + \beta a^2\right) \beta k^2\right]$**

Out[∘]=

$\left(-1 + \left(1 + a^2 \beta\right)^2 - \beta a^2\right) \beta k^2$

In[∘]:= **Equal$\left[\left(-1 + \left(1 + a^2 \beta\right)^2 - \beta a^2\right) \beta k^2 == \beta\, k^2 \left(1 + \beta\, a^2\right) a^2 \beta\right]$**

Out[∘]=

True

In[∘]:= **$\left(1 + a^2 \beta\right)^2$**

Out[∘]=

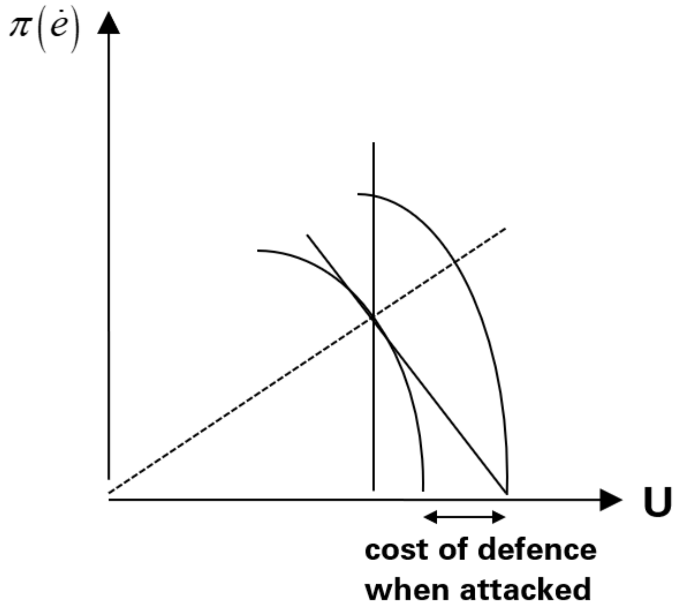
$\left(1 + a^2 \beta\right)^2$

In[∘]:= **Expand$\left[\left(1 + a^2 \beta\right)^2\right] == 1 + 2\, a^2 \beta + \left(a^2 \beta * a^2 \beta\right)$**

Out[∘]=

True

Cost of defending when attacked > Temptation when no attack



If C > cost of defense = $\beta^2\, a^2\, k^2\, (1 + a^2\, \beta)$ authorities will not yield

Speculators know this

Therefore they will not attack

Three possible cases

1. $\frac{\beta^2\, k^2\, a^2}{1+a^2\, \beta} < \beta^2\, k^2\, a^2\, (1 + a^2\, \beta) < C$

Temptation < cost of defense when attacked < C

Fixed exchange rate is credible and will not be attacked

2. $C < \frac{\beta^2\, k^2\, a^2}{1+a^2\, \beta} < \beta^2\, k^2\, a^2\, (1 + a^2\, \beta)$

Fixed exchange rate has no credibility and will be attacked with certainty

3. $\frac{\beta^2\, k^2\, a^2}{1+a^2\, \beta} < C < \beta^2\, k^2\, a^2\, (1 + a^2\, \beta)$

Two expectations are model consistent:

- Agents do not expect a devaluation;
In that case authorities temptation to devalue is smaller than the cost of devaluation
So, authorities do not devalue

- Agents expect a devaluation;
In this case the authorities cost of defending the peg exceeds the cost of devaluation
So, authorities devalue

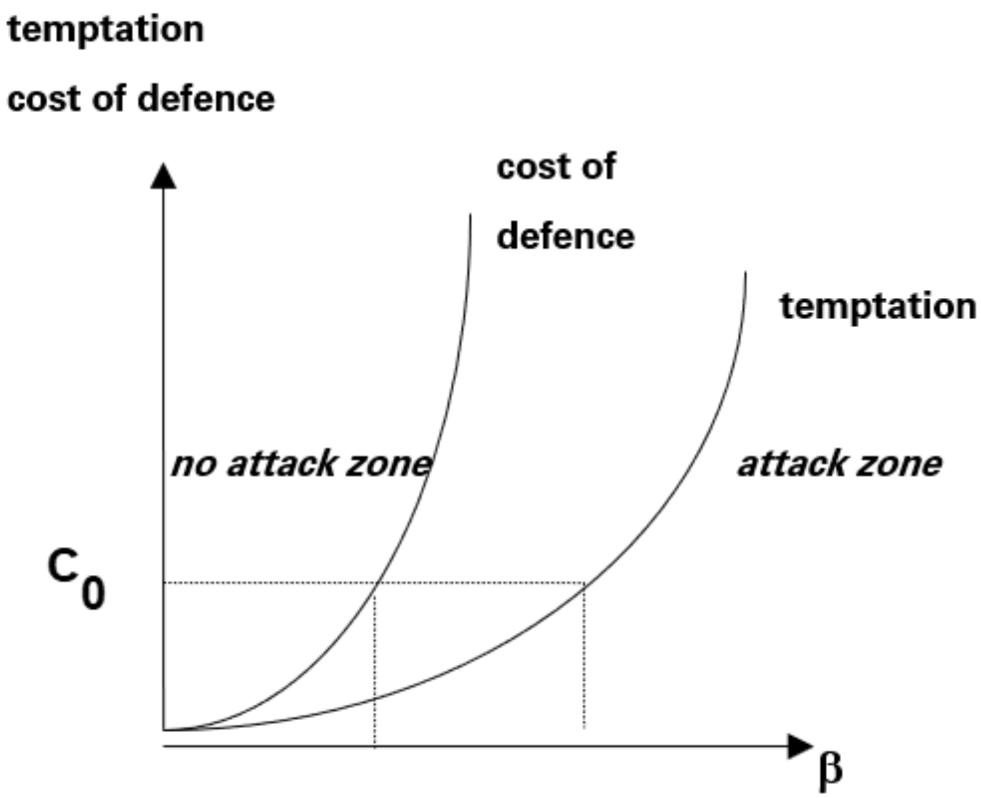
There are two possible equilibria

Which on will prevail solely depends on expectations which become self - fulfilling

Importance of weight attached to stabilization (β)

Recall that $\frac{a^2\, \beta^2\, k^2}{1+a^2\, \beta}$: Temptation to devalue

Recall that $a^2\, \beta^2\, k^2\, (1 + a^2\, \beta)$: Cost of defense when attacked



- If $\beta = 0$ (no desire to stabilize) then C > 0 ensures there is never an attack
- With cost of devaluation C_0
  - ⇒ range of β between β_1 and β_2 with multiple equilibria
- With β sufficiently ⇒ high always attack

Two conclusions

1) Too strong ambitions to stabilize make fixed exchange rate regime fragile

2) Making devaluation more costly, improves conditions for survival of fixed exchange rate

- Case study : ERM during 1996 - 1998

- Cost of devaluation was drastically increased (cf. convergence criteria)

Reference

Economics of the Monetary Union, Thirteenth Edition, 2020, ISBN : 9780198849544, 304 pages
https://global.oup.com/ushe/product/economics-of-the-monetary-union-9780198849544