Two-Period Consumption with CRRA Utility

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```
In[•]:= (* E1.1 Two-Period Consumption with CRRA Utility *)
        ClearAll;
```

The objective function

```
In[•]:= (* The objective function *)
        U = (c₀^(1 - η)) / (1 - η) + ρ * (((I₀ - c₀) * (1 + r))^(1 - η)) / (1 - η)
```

Out[•]=

$$\frac{\rho\ ((1+r)\ (i_0 - c_0))^{1-\eta}}{1-\eta} + \frac{c_0^{1-\eta}}{1-\eta}$$

Analytic solution for optimal consumption

```
(* dU(c₀)/dc₀ *)
```

```
In[•]:= Uprime = D[U, c₀]
```

Out[•]=

$$(-1-r)\ \rho\ ((1+r)\ (i_0 - c_0))^{-\eta} + c_0^{-\eta}$$

Proves the Analytical Solution

```
In[•]:= Uprime == 0
```

Out[•]=

$$(-1-r)\ \rho\ ((1+r)\ (i_0 - c_0))^{-\eta} + c_0^{-\eta} == 0$$

```
In[•]:= -((1 + r) ρ ((1 + r) (i₀ - c₀))^-η) + c₀^-η == 0
```

Out[•]=

$$-((1+r)\ \rho\ ((1+r)\ (i_0 - c_0))^{-\eta}) + c_0^{-\eta} == 0$$

```
In[•]:= c₀^-η == ((1 + r) (i₀ - c₀))^-η (1 + r) ρ
```

Out[•]=

$$c_0^{-\eta} == (1+r)\ \rho\ ((1+r)\ (i_0 - c_0))^{-\eta}$$

```
In[•]:= ((1 + r) ρ)^(-η/η) c₀^-η == ((1 + r) (i₀ - c₀))^-η
```

Out[•]=

$$\frac{c_0^{-\eta}}{(1+r)\ \rho} == ((1+r)\ (i_0 - c_0))^{-\eta}$$

In[•]:=
```
ρ = 1 / (1 + δ)
```
Out[•]=

$$\frac{1}{1+\delta}$$

In[•]:=
```
((1 + r) ρ)^(1/η) c₀ == (1 + r) (i₀ - c₀)
```
Out[•]=

$$\left(\frac{1+r}{1+\delta}\right)^{\frac{1}{\eta}} c_0 == (1+r)\,(i_0 - c_0)$$

In[•]:=
```
c₀ (((1 + r) / (1 + δ)) ^ (1 / η)) == I₀ * (1 + r) - c₀ * (1 + r)
```
Out[•]=

$$\left(\frac{1+r}{1+\delta}\right)^{\frac{1}{\eta}} c_0 == (1+r)\,i_0 - (1+r)\,c_0$$

In[•]:=
```
c₀ (((1 + r) / (1 + δ)) ^ (1 / η) + (1 + r)) == I₀ * (1 + r)
```
Out[•]=

$$\left(1 + r + \left(\frac{1+r}{1+\delta}\right)^{\frac{1}{\eta}}\right) c_0 == (1+r)\,i_0$$

In[•]:=
```
c₀ == I₀ * (1 + r) / (((1 + r) / (1 + δ)) ^ (1 / η) + (1 + r))
```
Out[•]=

$$c_0 == \frac{(1+r)\,i_0}{1 + r + \left(\frac{1+r}{1+\delta}\right)^{\frac{1}{\eta}}}$$

Analytic Solution + Solve + Maximize

```
(* c₀* computed from the analytic solution *)
```

In[•]:=
```
cstara := I₀ * (1 + r) / (((1 + r) / (1 + δ)) ^ (1 / η) + (1 + r));
```

```
(* c₀* computed using Mathematica to Solve dU/dc₀=0 *)
```

In[•]:=
```
cstarb := Solve[-(1 + r) * ρ * ((1 + r) * (I₀ - c₀)) ^ (-η) + c₀ ^ (-η) == 0, c₀];
```

In[•]:=
```
(* c₀* computed using Mathematica to maximize U(c₀) *)
cstarc := Maximize[(c₀ ^ (1 - η)) / (1 - η) + ρ * (((I₀ - c₀) * (1 + r)) ^ (1 - η)) / (1 - η), c₀];
(* Parameter Values *)
```

In[•]:=
```
I₀ = 1; r = 175 / 1000; δ = 2 / 100; ρ = 1 / (1 + δ); η = 1 / 4;
```

In[•]:=
```
Print["c₀*=", N[cstara]]
Print["c₀*=", N[cstarb]]
Print["c₀*=", N[cstarc]]
```

c₀*=0.400209

c₀*={{c₀→0.400209}}

c₀*={1.67636, {c₀→0.400209}}